Tetraquarks and quark three-body forces

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- New three-quark potentials
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Introduction

Three-quark potentials in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ \left.\begin{array}{l} d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{array}\right\} \text{Intrinsic 3-body forces} \end{cases}$$

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc} \mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc} \bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{6}\left[C_{i+j+k}^{(3)} - \frac{5}{2} C_{i+j+k}^{(2)} + \frac{20}{3} \right]$$

$$\langle C^{(2)} \rangle = \frac{1}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$$

$$\langle C^{(3)} \rangle = \frac{1}{18}(\lambda - \mu)(2\lambda + \mu + 3)(\lambda + 2\mu + 3)$$

V. Dmitrašinovic, PLB 499, 135(2001)
S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Three-quark potentials in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$d^{abc}F_i^a F_j^b F_k^c$$
$$= \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9}I - \frac{1}{6}[(ij) + (ik) + (jk)],$$
$$f^{abc}F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc}\mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc}\bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

V. Dmitrašinovic, PLB 499, 135(2001)
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Three-quark potentials in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc}\mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc}\bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

$$d^{abc}F_i^a F_j^b F_k^c$$
$$= \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9}I - \frac{1}{6}[(ij) + (ik) + (jk)],$$
$$f^{abc}F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

For a baryon,

$$d^{abc}F_i^a F_j^b F_k^c = \frac{10}{9},$$

$$f^{abc}F_i^a F_j^b F_k^c = 0.$$

V. Dmitrašinovic, PLB 499, 135(2001)
S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Three-quark potentials in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9} I - \frac{1}{6}[(ij) + (ik) + (jk)],$$

$$f^{abc} F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc} \mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc} \bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

For a baryon,

$$d^{abc} F_i^a F_j^b F_k^c = \frac{10}{9},$$

$$f^{abc} F_i^a F_j^b F_k^c = 0.$$

For a tetraquark,

$$V_s = c \frac{5}{18} \omega^2 (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2),$$

$$V_a = -c \frac{5}{9} \omega^2 (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2).$$

$$[s \equiv 8, a \equiv 1]$$

$$-\frac{3}{2} < c < \frac{78}{5}.$$

V. Dmitrašinovic, PLB 499, 135(2001)

S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Casimir Invariants

There are two Casimir operators in SU(3).

C_2: quadratic Casimir operator

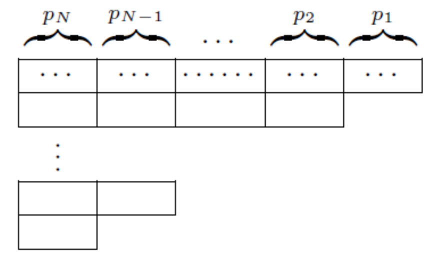
C_3: cubic Casimir operator

$$C_2 = \sum_{i,j} F_i^c F_j^c = \sum_i F_i^c F_i^c + 2 \sum_{i<j} F_i^c F_j^c$$

$$\begin{aligned} C_3 &= \sum_{i,j,k} d^{abc} F_i^a F_j^b F_k^c \\ &= \sum_i d^{abc} F_i^a F_i^b F_i^c + 6 \sum_{i<j} d^{abc} F_i^a F_i^b F_j^c + 6 \sum_{i<j<k} d^{abc} F_i^a F_j^b F_k^c \end{aligned}$$

Casimir Invariants

For SU(N),



total number of box : $n = p_1 + 2p_2 + \cdots + Np_N = \sum_{k=1}^{N} kp_k$

$$C_2^{SU(N)} = \frac{n}{2N}(N^2-1) + \frac{1}{N}(N-1)\left[\sum_{i=1}^{N}\binom{\sum_{j=i}^{N} p_j}{2}\right] - \sum_{k=1}^{N-1}\left(\sum_{j=k+1}^{N}(j-k)p_j\right)\left(1+\frac{1}{N}\sum_{i=k}^{N} p_i\right)$$

$$C_2^{\mathrm{SU}(3)} = p_1 + \frac{p_1^2}{3} + p_2 + \frac{1}{3}p_1p_2 + \frac{p_2^2}{3}$$

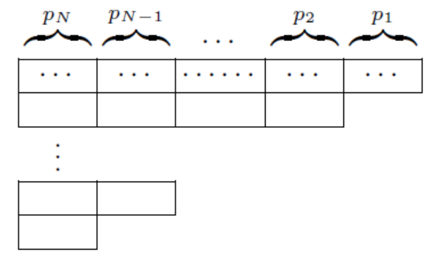
$$C_2^{\mathrm{SU}(4)} = \frac{3p_1}{2} + \frac{3p_1^2}{8} + 2p_2 + \frac{1}{2}p_1p_2 + \frac{p_2^2}{2} + \frac{3p_3}{2} + \frac{1}{4}p_1p_3 + \frac{1}{2}p_2p_3 + \frac{3p_3^2}{8}$$

$$C_2^{\mathrm{SU}(5)} = 2p_1 + \frac{2p_1^2}{5} + 3p_2 + \frac{3}{5}p_1p_2 + \frac{3p_2^2}{5} + 3p_3 + \frac{2}{5}p_1p_3 + \frac{4}{5}p_2p_3 + \frac{3p_3^2}{5} + 2p_4 + \frac{1}{5}p_1p_4 + \frac{2}{5}p_2p_4 + \frac{3}{5}p_3p_4 + \frac{2p_4^2}{5}$$

$$C_2^{\mathrm{SU}(6)} = \frac{5p_1}{2} + \frac{5p_1^2}{12} + 4p_2 + \frac{2}{3}p_1p_2 + \frac{2p_2^2}{3} + \frac{9p_3}{2} + \frac{1}{2}p_1p_3 + p_2p_3 + \frac{3p_3^2}{4} + 4p_4 + \frac{1}{3}p_1p_4 + \frac{2}{3}p_2p_4 + p_3p_4 + \frac{2p_4^2}{3} + \frac{5p_5}{2} + \frac{1}{6}p_1p_5 + \frac{1}{3}p_2p_5 + \frac{1}{2}p_3p_5 + \frac{2}{3}p_4p_5 + \frac{5p_5^2}{12}$$

Casimir invariants does not depend on p_N.

Casimir Invariants



For SU(N),

total number of box : $n = p_1 + 2p_2 + \cdots + Np_N = \sum_{k=1}^{N} kp_k$

$$C_3^{\mathrm{SU}(3)} = \frac{1}{18}(p_1 - p_2)(p_1 + 2p_2 + 3)(2p_1 + p_2 + 3)$$

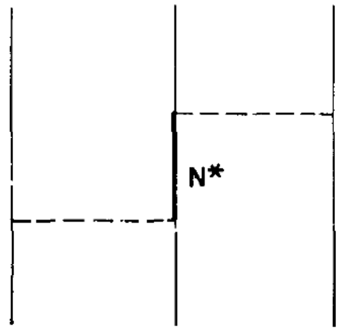
$$C_3^{\mathrm{SU}(4)} = \frac{3}{2}p_1 + \frac{9}{8}p_1^2 + \frac{3}{16}p_1^3 + \frac{3}{4}p_1p_2 + \frac{3}{8}p_1^2p_2 - \frac{3}{2}p_3 + \frac{3}{16}p_1^2p_3 - \frac{3}{4}p_2p_3 - \frac{9}{8}p_3^2 - \frac{3}{16}p_1p_3^2 - \frac{3}{8}p_2p_3^2 - \frac{3}{16}p_3^3$$

$$\begin{aligned}C_3^{\mathrm{SU}(5)} &= 3p_1 + \frac{9p_1^2}{5} + \frac{6p_1^3}{25} + \frac{3p_2}{2} + \frac{9}{5}p_1p_2 + \frac{27}{50}p_1^2p_2 + \frac{9p_2^2}{10} + \frac{9}{50}p_1p_2^2 + \frac{3p_2^3}{25} - \frac{3p_3}{2} + \frac{3}{5}p_1p_3 + \frac{9}{25}p_1^2p_3 + \frac{6}{25}p_1p_2p_3\\
&+ \frac{6}{25}p_2^2p_3 - \frac{9p_3^2}{10} - \frac{3}{25}p_1p_3^2 - \frac{6}{25}p_2p_3^2 - \frac{3p_3^3}{25} - 3p_4 + \frac{9}{50}p_1^2p_4 - \frac{3}{5}p_2p_4 + \frac{3}{25}p_1p_2p_4 + \frac{3}{25}p_2^2p_4 - \frac{9}{5}p_3p_4\\
&- \frac{3}{25}p_1p_3p_4 - \frac{6}{25}p_2p_3p_4 - \frac{9}{50}p_3^2p_4 - \frac{9p_4^2}{5} - \frac{9}{50}p_1p_4^2 - \frac{9}{25}p_2p_4^2 - \frac{27}{50}p_3p_4^2 - \frac{6p_4^3}{25}\end{aligned}$$

$$\begin{aligned}C_3^{\mathrm{SU}(6)} &= 5p_1 + \frac{5p_1^2}{2} + \frac{5p_1^3}{18} + 4p_2 + 3p_1p_2 + \frac{2}{3}p_1^2p_2 + 2p_2^2 + \frac{1}{3}p_1p_2^2 + \frac{2p_2^3}{9} + \frac{3}{2}p_1p_3 + \frac{1}{2}p_1^2p_3 + \frac{3}{2}p_2p_3 + \frac{1}{2}p_1p_2p_3\\
&+ \frac{1}{2}p_2^2p_3 - 4p_4 + \frac{1}{2}p_1p_4 + \frac{1}{3}p_1^2p_4 + \frac{1}{3}p_1p_2p_4 + \frac{1}{3}p_2^2p_4 - \frac{3}{2}p_3p_4 - 2p_4^2 - \frac{1}{6}p_1p_4^2 - \frac{1}{3}p_2p_4^2 - \frac{1}{2}p_3p_4^2 - \frac{2p_4^3}{9}\\
&- 5p_5 + \frac{1}{6}p_1^2p_5 - \frac{1}{2}p_2p_5 + \frac{1}{6}p_1p_2p_5 + \frac{1}{6}p_2^2p_5 - \frac{3}{2}p_3p_5 - 3p_4p_5 - \frac{1}{6}p_1p_4p_5 - \frac{1}{3}p_2p_4p_5 - \frac{1}{2}p_3p_4p_5 - \frac{1}{3}p_4^2p_5\\
&- \frac{5p_5^2}{2} - \frac{1}{6}p_1p_5^2 - \frac{1}{3}p_2p_5^2 - \frac{1}{2}p_3p_5^2 - \frac{2}{3}p_4p_5^2 - \frac{5p_5^3}{18}\end{aligned}$$

Casimir invariants does not depend on p_N.
However, note that interaction factor can depend on total number of quarks.

Three-body forces in nuclear physics

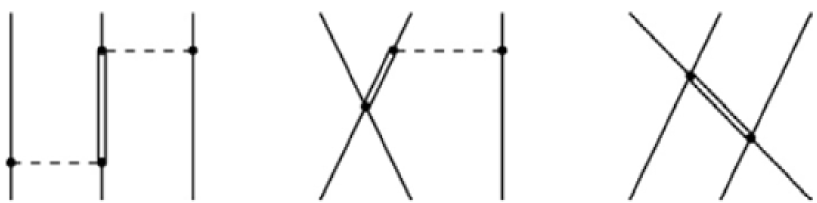


$$
\begin{aligned}
V^{(3)} &= V(123) + V(231) + V(312), \\
V(123) &= -\big[\{A(\tau^1\tau^2)(\tau^2\tau^3) + B(\tau^3\tau^2)(\tau^2\tau^1)\}(\sigma^1\nabla^1)(\sigma^2\nabla^1)(\sigma^2\nabla^3)(\sigma^3\nabla^3) \\
&\quad + \{B(\tau^1\tau^2)(\tau^2\tau^3) + A(\tau^3\tau^2)(\tau^2\tau^1)\}(\sigma^3\nabla^3)(\sigma^2\nabla^3)(\sigma^2\nabla^1)(\sigma^1\nabla^1) \\
&\quad + 2D(\tau^1\tau^3)(\sigma^1\nabla^1)(\sigma^3\nabla^3)\big]\, Y(12)\, Y(23),
\end{aligned}
$$

G.E.Brown et al, Nucl. Phys. A 115, 435(1968)

J.Fujita and H.Miyazawa, Prog. Theor. Phys. 17, 360(1957)

The middle nucleon goes virtually to some excited state.



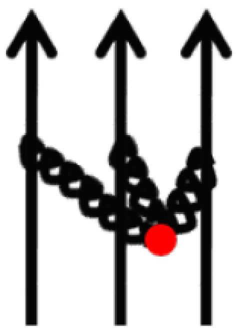
Leading contributions to the 3NF due to explicit Δ's.

E.Epelbaum, Prog. Part. Nucl. Phys. 57, 654(2006)

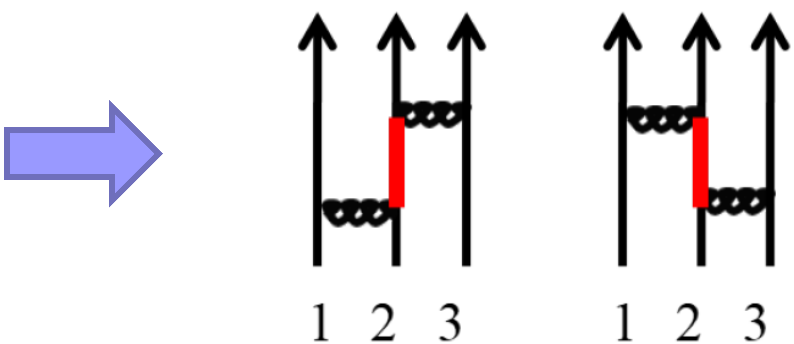
How about applying this concept to the three-quark potential?

New three-quark potentials

New three-quark potentials: color-color

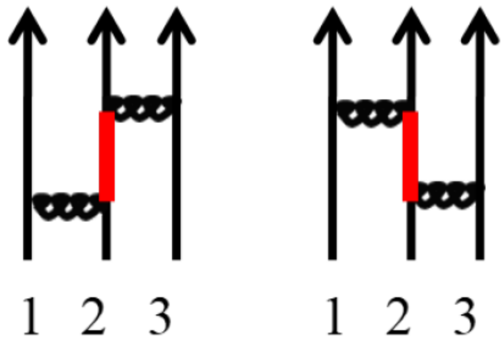


$d^{abc}F_1^a F_2^b F_3^c$,
$i f^{abc}F_1^a F_2^b F_3^c$,



$+ \quad (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3)$

New three-quark potentials: color-color



$+ \quad (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3)$

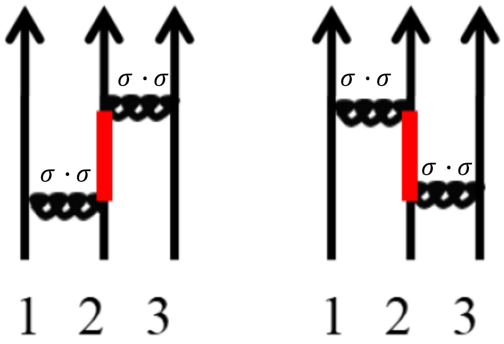
$$\{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c$$

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

$$\begin{aligned}
(\lambda_1^a\lambda_2^a)(\lambda_2^b\lambda_3^b) + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) &= \lambda_1^a\left(-\lambda_2^b\lambda_2^a + \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda_2^c\right)\lambda_3^b + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= -\lambda_1^a\lambda_2^b\lambda_2^a\lambda_3^b + \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^c\lambda_3^b + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= -(\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) + \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^b\lambda_3^c + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^b\lambda_3^c
\end{aligned}$$

$$L_{123}^{C-C} = \frac{4}{3}\left(\frac{\lambda_2^c\lambda_3^c}{m_1} + \frac{\lambda_1^c\lambda_3^c}{m_2} + \frac{\lambda_1^c\lambda_2^c}{m_3}\right) + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right)$$

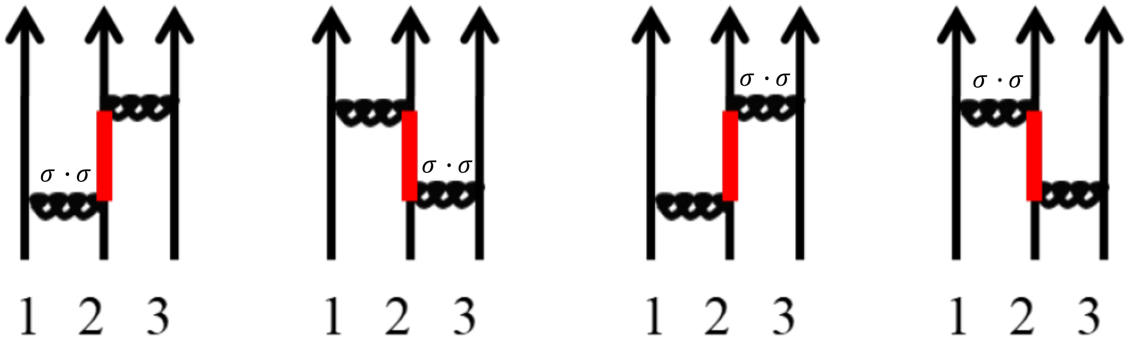
New three-quark potentials: color-spin



$+ \ (1 \leftrightarrow 2) \ + \ (2 \leftrightarrow 3)$

$$L_{123}^{S-S} = \frac{1}{m_1 m_2 m_3}\Bigg[\frac{4}{3}\left(\frac{(\sigma_2\cdot\sigma_3)(\lambda_2^c\lambda_3^c)}{m_1^2} + \frac{(\sigma_1\cdot\sigma_3)(\lambda_1^c\lambda_3^c)}{m_2^2} + \frac{(\sigma_1\cdot\sigma_2)(\lambda_1^c\lambda_2^c)}{m_3^2}\right) + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{\sigma_2\cdot\sigma_3}{m_1^2} + \frac{\sigma_1\cdot\sigma_3}{m_2^2} + \frac{\sigma_1\cdot\sigma_2}{m_3^2}\right) - 2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2}\right)\Bigg]$$

New three-quark potentials: color-spin hybrid



$$+ \ (1 \leftrightarrow 2) \ + \ (2 \leftrightarrow 3)$$

$$L_{123}^{C-S} = \frac{4}{3}\left[\frac{(\lambda_1^c\lambda_3^c)}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)+\frac{(\lambda_1^c\lambda_2^c)}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right)+\frac{(\lambda_2^c\lambda_3^c)}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right]$$
$$+ 2d_{abc}(\lambda_1^a\lambda_2^b\lambda_3^b)\left[\frac{1}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)+\frac{1}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right)+\frac{1}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right]$$

New three-quark potentials: antiquarks

For antiquarks, $\{\bar{\lambda}^a, \bar{\lambda}^b\} = \frac{4}{3}\delta^{ab} - 2d^{abc}\bar{\lambda}^c$

$[\bar{\lambda}^a, \bar{\lambda}^b] = 2if^{abc}\bar{\lambda}^c$

$$L_{C-C} = \left[\sum_{i<j<k} \frac{4}{3}\left(\frac{\lambda_i^c\lambda_j^c}{m_k} + \frac{\lambda_i^c\lambda_k^c}{m_j} + \frac{\lambda_j^c\lambda_k^c}{m_i}\right) + 2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)}{m_i} + \frac{\mathrm{sign}(j)}{m_j} + \frac{\mathrm{sign}(k)}{m_k}\right)\right]$$

$$\begin{aligned} L_{S-S} = \sum_{i<j<k} \frac{1}{m_i m_j m_k} &\left[\frac{4}{3}\left(\frac{\lambda_j^c\lambda_k^c\sigma_j\cdot\sigma_k}{m_i^2} + \frac{\lambda_i^c\lambda_k^c\sigma_i\cdot\sigma_k}{m_j^2} + \frac{\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j}{m_k^2}\right)\right. \\ &+ 2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)\sigma_j\cdot\sigma_k}{m_i^2} + \frac{\mathrm{sign}(j)\sigma_i\cdot\sigma_k}{m_j^2} + \frac{\mathrm{sign}(k)\sigma_i\cdot\sigma_j}{m_k^2}\right) \\ &\left. - 2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2}\right)\right], \end{aligned}$$

$$\begin{aligned} L_{C-S} = \sum_{i<j<k} \frac{4}{3}&\left\{\frac{\lambda_j^c\lambda_k^c}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right) + \frac{\lambda_i^c\lambda_k^c}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right) + \frac{\lambda_i^c\lambda_j^c}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\right] \\ &+ 2d^{abc}\lambda_i^c\lambda_j^c\lambda_k^c\left[\frac{\mathrm{sign}(i)}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right) + \frac{\mathrm{sign}(j)}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right) + \frac{\mathrm{sign}(k)}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\right]\right\} \end{aligned}$$

where $\mathrm{sign}(i) = -1$ for antiquarks.

Results(Baryons, Tetraquarks)

Baryon fitting

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{4}\sum_{i<j}^{n}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}\left(V_{ij}^C + V_{ij}^{CS}\right),$$

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij})r_{ij}}\sigma_i \cdot \sigma_j .$$

$$r_{0ij} = (\alpha + \beta m_{ij})^{-1},\ \kappa' = \kappa_0(1 + \gamma m_{ij}) \text{ and } m_{ij} = \frac{m_i m_j}{m_i + m_j}$$

$$V^{3-body} = AL_{C-C} + BL_{S-S} + CL_{C-S}$$

$$A = -3.67522 \times 10^{-4}\,\mathrm{GeV}^2,\ B = -2.85156 \times 10^{-7}\,\mathrm{GeV}^6,\ C = -7.68351 \times 10^{-6}\,\mathrm{GeV}^4$$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameter (fm^{-2})
η_c	2983.6	2996.9	$a = 13.1$
$J\Psi$	3096.9	3089.6	$a = 11.1$
D	1864.8	1864.1	$a = 4.5$
D^*	2010.3	2010.7	$a = 3.7$
π	139.57	139.39	$a = 4.6$
ρ	775.11	775.49	$a = 2.2$
K	493.68	494.62	$a = 4.6$
K^*	891.66	888.82	$a = 2.8$

The standard deviation is $\sigma = 5.86$.

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm^{-2})
Λ_c	2286.5	2266.7 (2281.6)	$a_1 = 2.9,\ a_2 = 3.7$
Σ_c	2452.9	2441.6 (2480.9)	$a_1 = 2.1,\ a_2 = 3.8$
Λ	1115.7	1113.6 (1134.1)	$a_1 = 2.8,\ a_2 = 2.7$
Σ	1192.6	1196.5 (1231.6)	$a_1 = 2.1,\ a_2 = 3.1$
Σ_c^*	2518.5	2522.9 (2567.7)	$a_1 = 2.0,\ a_2 = 3.4$
Σ^*	1383.7	1398.9 (1455.2)	$a_1 = 1.9,\ a_2 = 2.4$
p	938.27	980.47 (1005.3)	$a_1 = 2.4,\ a_2 = 2.4$
Δ	1232	1272.1 (1346.8)	$a_1 = 1.8,\ a_2 = 1.8$

$\sigma = 24.44(63.10)$

With 3-quark potential

Three-body color matrices for a tetraquark

$|3_{12}\bar{3}_{34}\rangle, |\bar{6}_{12}6_{34}\rangle$ color basis $T_{cc}: \bar{u}\bar{d}cc$

$$f^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_3^c = \begin{pmatrix} -\frac{40}{9} & 0 \\ 0 & \frac{20}{9} \end{pmatrix}$$

$$f^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_4^c = \begin{pmatrix} -\frac{40}{9} & 0 \\ 0 & \frac{20}{9} \end{pmatrix}$$

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$$f^{abc}\bar{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} \frac{40}{9} & 0 \\ 0 & -\frac{20}{9} \end{pmatrix}$$

$|1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle$ color basis $\chi_{c1}: \bar{c}\bar{q}cq$

$$f^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

$$f^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_1^a\bar{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

$$f^{abc}\bar{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -\frac{20\sqrt{2}}{9} \\ -\frac{20\sqrt{2}}{9} & \frac{20}{9} \end{pmatrix}$$

$$f^{abc}\bar{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\bar{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -\frac{20\sqrt{2}}{9} \\ -\frac{20\sqrt{2}}{9} & \frac{20}{9} \end{pmatrix}$$

Three-body potentials in a tetraquark

T_{cc} $\quad (|\mathbf{3}_{12}\bar{\mathbf{3}}_{34}\rangle, |\bar{\mathbf{6}}_{12}\mathbf{6}_{34}\rangle)$

$$L_{C-C} = \begin{pmatrix} \frac{32}{9m_u} + \frac{32}{9m_c} & 0 \\ 0 & -\frac{208}{9m_u} - \frac{208}{9m_c} \end{pmatrix}$$

$$L_{S-S} = \begin{pmatrix} -\frac{224}{9m_u^3 mc^2} + \frac{224}{3m_u^2 m_c^3} & -\frac{224\sqrt{2}}{3m_u^4 m_c} + \frac{32\sqrt{2}}{m_u^3 m_c^2} - \frac{32\sqrt{2}}{m_u^2 m_c^3} + \frac{160\sqrt{2}}{3m_u m_c^4} \\ -\frac{224\sqrt{2}}{3m_u^4 m_c} + \frac{32\sqrt{2}}{m_u^3 m_c^2} - \frac{32\sqrt{2}}{m_u^2 m_c^3} + \frac{160\sqrt{2}}{3m_u m_c^4} & -\frac{112}{3m_u^3 m_c^2} + \frac{112}{9m_u^2 m_c^3} \end{pmatrix}$$

$$L_{C-S} = \begin{pmatrix} -\frac{256}{3m_u^3} + \frac{256}{9m_c^3} & \frac{32\sqrt{2}}{3m_u^2 m_c} + \frac{32\sqrt{2}}{3m_u m_c^2} \\ \frac{32\sqrt{2}}{3m_u^2 m_c} + \frac{32\sqrt{2}}{3m_u m_c^2} & -\frac{320}{9m_u^3} + \frac{320}{3m_c^3} \end{pmatrix}$$

$$V^{3-body} = AL_{C-C} + BL_{S-S} + CL_{C-S}$$

$$A = -3.67522 \times 10^{-4}\ \mathrm{GeV}^2$$
$$B = -2.85156 \times 10^{-7}\ \mathrm{GeV}^6$$
$$C = -7.68351 \times 10^{-6}\ \mathrm{GeV}^4$$

$\chi_{c1}(3872)$ $\quad (|\mathbf{1}_{13}\mathbf{1}_{24}\rangle, |\mathbf{8}_{13}\mathbf{8}_{24}\rangle)$

$$L_{C-C} = \begin{pmatrix} -\frac{128}{9m_u} - \frac{128}{9m_c} & -\frac{80\sqrt{2}}{9m_u} - \frac{80\sqrt{2}}{9m_c} \\ -\frac{80\sqrt{2}}{9m_u} - \frac{80\sqrt{2}}{9m_c} & -\frac{16}{3m_u} - \frac{16}{3m_c} \end{pmatrix}$$

$$L_{S-S} = \begin{pmatrix} -\frac{128}{9m_u^3 mc^2} - \frac{128}{9m_u^2 m_c^3} & -\frac{80\sqrt{2}}{9m_u^3 m_c^2} - \frac{80\sqrt{2}}{9m_u^2 m_c^3} \\ -\frac{80\sqrt{2}}{9m_u^3 m_c^2} - \frac{80\sqrt{2}}{9m_u^2 m_c^3} & \frac{32}{3m_u^3 m_c^2} + \frac{32}{3m_u^2 m_c^3} + \frac{16}{m_u m_c^4} + \frac{16}{m_u^4 m_c} \end{pmatrix}$$

$$L_{C-S} = \begin{pmatrix} \frac{256}{9m_u^2 m_c} + \frac{256}{9m_u m_c^2} & \frac{160\sqrt{2}}{9m_u^2 m_c} + \frac{160\sqrt{2}}{9m_u m_c^2} \\ \frac{160\sqrt{2}}{9m_u^2 m_c} + \frac{160\sqrt{2}}{9m_u m_c^2} & -\frac{16}{m_u^3} - \frac{16}{3m_u^2 m_c} - \frac{16}{3m_u m_c^2} - \frac{16}{m_c^3} \end{pmatrix}$$

Particle	Measured mass (MeV)	Mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$	Variational parameters (fm^{-2})
T_{cc}	3875	3972.06 (3955.18)	-4.84236	0.0319013	20.9444	$a_1 = 2.8,\ a_2 = 7.3,\ a_3 = 2.6$
$\chi_{c1}(3872)$	3872	3884.23 (3866.20)	19.3694	0.0427164	-1.36541	$a_1 = 11.1,\ a_2 = 2.2,\ a_3 = 0.01$

With 3-quark potential

Three-body potentials in a tetraquark

T_{cc} $\quad \left(\boxed{|\mathbf{3}_{12}\bar{\mathbf{3}}_{34}\rangle}, |\bar{\mathbf{6}}_{12}\mathbf{6}_{34}\rangle\right)$

$$L_{C-C} = \begin{pmatrix} \boxed{\frac{32}{9m_u}} + \frac{32}{9m_c} & 0 \\ 0 & -\frac{208}{9m_u} - \frac{208}{9m_c} \end{pmatrix}$$

$$L_{S-S} = \begin{pmatrix} \boxed{-\frac{224}{9m_u^3 mc^2}} + \frac{224}{3m_u^2 m_c^3} & -\frac{224\sqrt{2}}{3m_u^4 m_c} + \frac{32\sqrt{2}}{m_u^3 m_c^2} - \frac{32\sqrt{2}}{m_u^2 m_c^3} + \frac{160\sqrt{2}}{3m_u m_c^4} \\ -\frac{224\sqrt{2}}{3m_u^4 m_c} + \frac{32\sqrt{2}}{m_u^3 m_c^2} - \frac{32\sqrt{2}}{m_u^2 m_c^3} + \frac{160\sqrt{2}}{3m_u m_c^4} & -\frac{112}{3m_u^3 m_c^2} + \frac{112}{9m_u^2 m_c^3} \end{pmatrix}$$

$$L_{C-S} = \begin{pmatrix} \boxed{-\frac{256}{3m_u^3}} + \frac{256}{9m_c^3} & \frac{32\sqrt{2}}{3m_u^2 m_c} + \frac{32\sqrt{2}}{3m_u m_c^2} \\ \frac{32\sqrt{2}}{3m_u^2 m_c} + \frac{32\sqrt{2}}{3m_u m_c^2} & -\frac{320}{9m_u^3} + \frac{320}{3m_c^3} \end{pmatrix}$$

$$V^{3-body} = AL_{C-C} + BL_{S-S} + CL_{C-S}$$

$$A = -3.67522 \times 10^{-4}\ \mathrm{GeV}^2$$

$$B = -2.85156 \times 10^{-7}\ \mathrm{GeV}^6$$

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$\chi_{c1}(3872)$ $\quad \left(\boxed{|\mathbf{1}_{13}\mathbf{1}_{24}\rangle}, |\mathbf{8}_{13}\mathbf{8}_{24}\rangle\right)$

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$$L_{S-S} = \begin{pmatrix} \boxed{-\frac{128}{9m_u^3 mc^2}} - \frac{128}{9m_u^2 m_c^3} & -\frac{80\sqrt{2}}{9m_u^3 m_c^2} - \frac{80\sqrt{2}}{9m_u^2 m_c^3} \\ -\frac{80\sqrt{2}}{9m_u^3 m_c^2} - \frac{80\sqrt{2}}{9m_u^2 m_c^3} & \frac{32}{3m_u^3 m_c^2} + \frac{32}{3m_u^2 m_c^3} + \frac{16}{m_u m_c^4} + \frac{16}{m_u^4 m_c} \end{pmatrix}$$

$$L_{C-S} = \begin{pmatrix} \boxed{\frac{256}{9m_u^2 m_c}} + \frac{256}{9m_u m_c^2} & \frac{160\sqrt{2}}{9m_u^2 m_c} + \frac{160\sqrt{2}}{9m_u m_c^2} \\ \frac{160\sqrt{2}}{9m_u^2 m_c} + \frac{160\sqrt{2}}{9m_u m_c^2} & -\frac{16}{m_u^3} - \frac{16}{3m_u^2 m_c} - \frac{16}{3m_u m_c^2} - \frac{16}{m_c^3} \end{pmatrix}$$

Particle	Measured mass (MeV)	Mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$	Variational parameters (fm^{-2})
T_{cc}	3875	3972.06 (3955.18)	-4.84236	0.0319013	20.9444	$a_1 = 2.8$, $a_2 = 7.3$, $a_3 = 2.6$
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With 3-quark potential

Three-body potentials in a tetraquark

$|1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle$ color basis

$\chi_{c1}: \bar{c}\bar{q}cq$ $\qquad (\boxed{|\mathbf{1}_{13}\mathbf{1}_{24}\rangle}, |\mathbf{8}_{13}\mathbf{8}_{24}\rangle)$

$$d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

$$d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

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$$B = -2.85156 \times 10^{-7}\,\mathrm{GeV}^6$$

$$C = -7.68351 \times 10^{-6}\,\mathrm{GeV}^4$$

$$L_{C-C} = \left[\sum_{i<j<k} \frac{4}{3}\left(\frac{\lambda_i^c\lambda_j^c}{m_k} + \frac{\lambda_i^c\lambda_k^c}{m_j} + \frac{\lambda_j^c\lambda_k^c}{m_i}\right) + 2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)}{m_i} + \frac{\mathrm{sign}(j)}{m_j} + \frac{\mathrm{sign}(k)}{m_k}\right)\right]$$

$$\lambda_1^c\lambda_3^c = -\frac{16}{3} \qquad\qquad 0$$

AL_{C-C} ⇒ Repulsive

Particle	Measured mass (MeV)	Mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$	Variational parameters (fm^{-2})
T_{cc}	3875	3972.06 (3955.18)	-4.84236	0.0319013	20.9444	$a_1 = 2.8$, $a_2 = 7.3$, $a_3 = 2.6$
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With 3-quark potential

Summary

- Inspired by the three-body nuclear force, the three-quark potentials are newly constructed.
- We found the fitting parameters for three-quark potentials using meson and baryon spectrum.
- When the three-quark potential is applied to tetraquark, it shows repulsive interaction.

Thank you