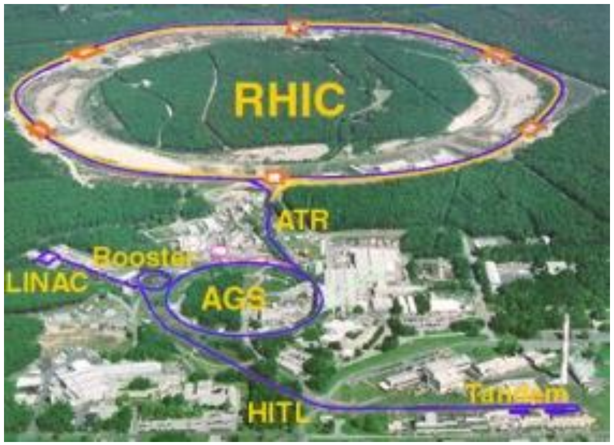
Hadron interactions in high-energy heavy-ion collisions

Koichi Murase
Tokyo Metropolitan University

High-energy heavy-ion collisions (HIC)

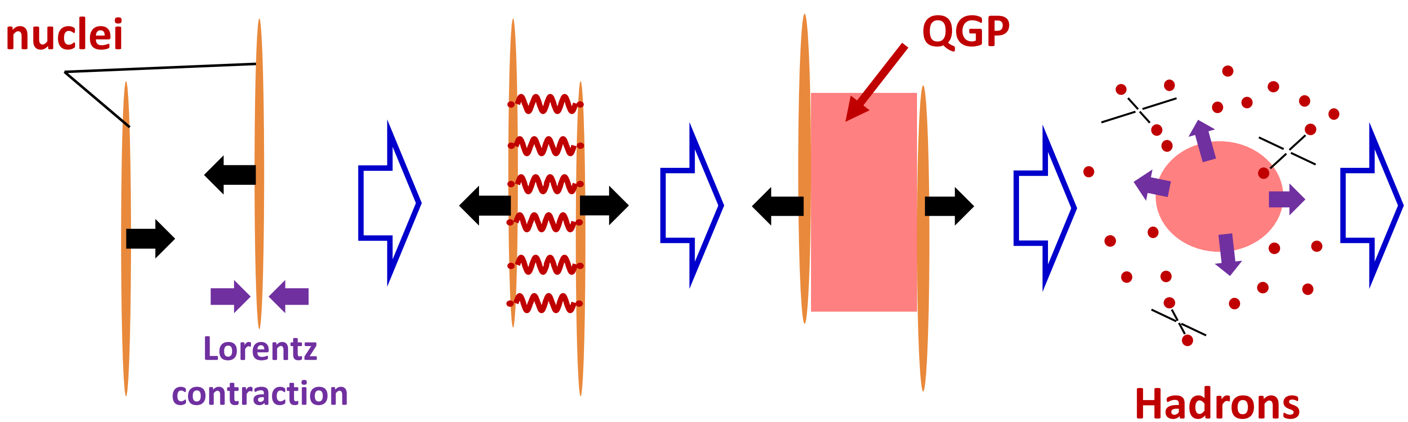
Colliders: LHC, RHIC, FAIR, NICA, J-PARC-HI, HIAF, …

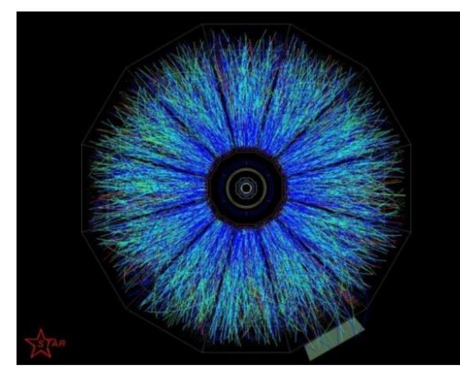


RHIC $\sqrt{s_{NN}} \sim 3$—200 GeV

LHC $\sqrt{s_{NN}} = 2.76$—5.44 TeV

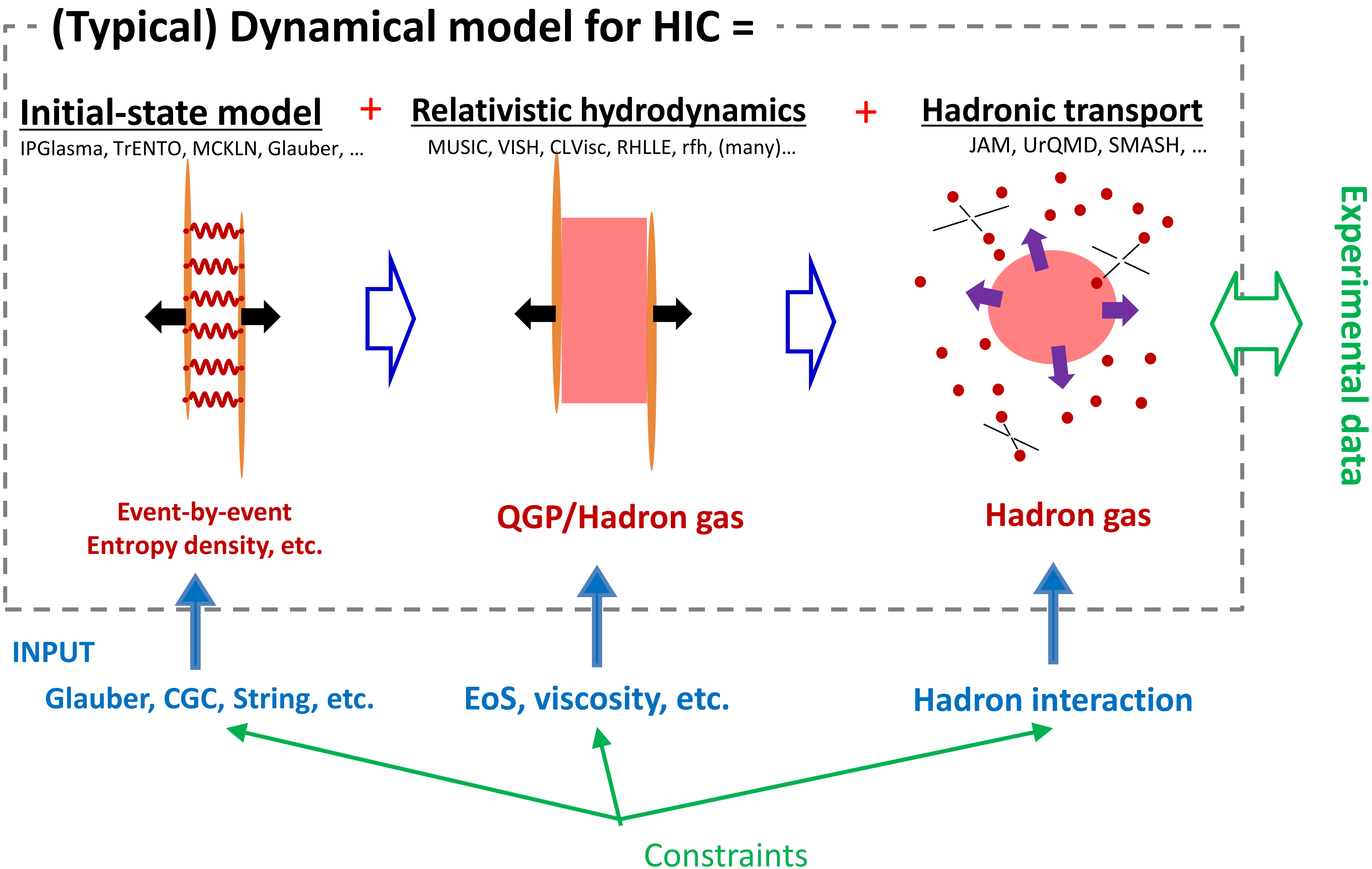
Heavy ions (Nuclei) collide at relativistic speed





STAR Collaboration

High-energy heavy-ion collisions (HIC)



Hadron interactions from HIC

ExHIC, PPNP **95**, 279 (2017)

Production of
Hadrons & Resonances

statistical model,
quark recombination,
rescatterings, …

Production of
Light nuclei & hypernuclei

statistical model,
(hadron) coalescence model,
dynamical formation, …

Anisotropic flow v_1, v_2, …

hadron transport,
cascade / mean-field interaction
@ low energy collisions
@ small systems
@ central collisions

Femtoscopy
two-particle momentum correlation

$S(r)$: source function,
$\varphi(r)$: relative wave function
← $V(r)$: hadron potential
\+ nontrivial assumptions

Λ DIRECTED FLOW V1
IN HEAVY-ION COLLISIONS

WITH
Y. NARA (AKITA INTERNATIONAL UNIV),
A. JINNO (KYOTO UNIV),
A. OHNISHI† (YITP, KYOTO UNIV)

ΛNN potential and NS/HIC/hypernuclei

Hyperon puzzle in neutron stars

"EoS with hyperons is typically too soft to explain the observed massive neutron stars"

Scenario: *Λ baryons do not appear in NS* because of 3-body repulsive forces of Λ

ΛNN force from χEFT:
Gerstung, Kaiser, Weise (2020) (GKW), Kohno (2018)

ΛNN force

Directed flow v_1 in heavy-ion collisions

Repulsion →
positive v1 slope @ initial stage

negative v1 slope @ later stage

Y. Nara, A. Jinno, KM, and A. Ohnishi,
PRC **106**, 4, 044902 (2022)

Binding energies of hypernuclei

Λ binding energy

$$B_\Lambda = -\left(E_{\mathrm{Hyper}} - E_{\mathrm{Core}}\right)$$

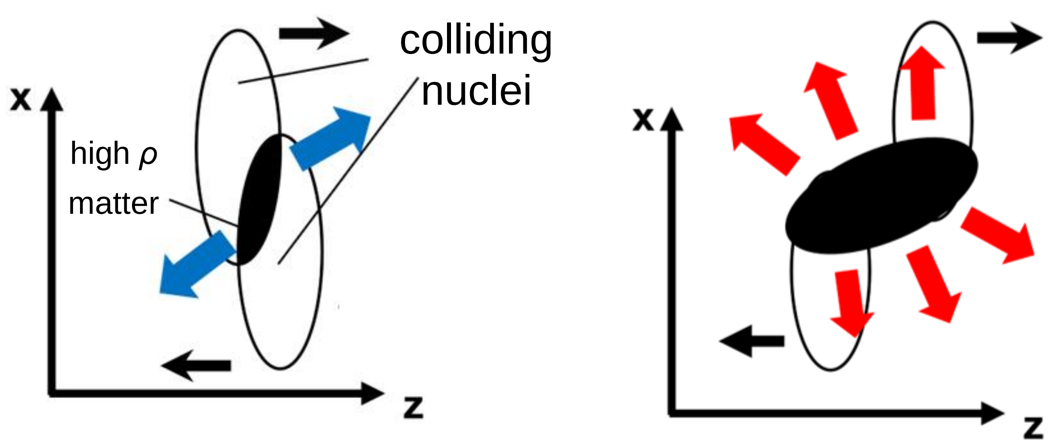
with Skyrme-Hartree-Fock calc

A. Jinno, Y. Nara, KM, A. Ohnishi,
PRC **108,** 6, 065803 (2023)

Test Λ potentials using experimental data

Lower-energy collisions $\sqrt{s_{NN}} \sim$ 3—20 GeV

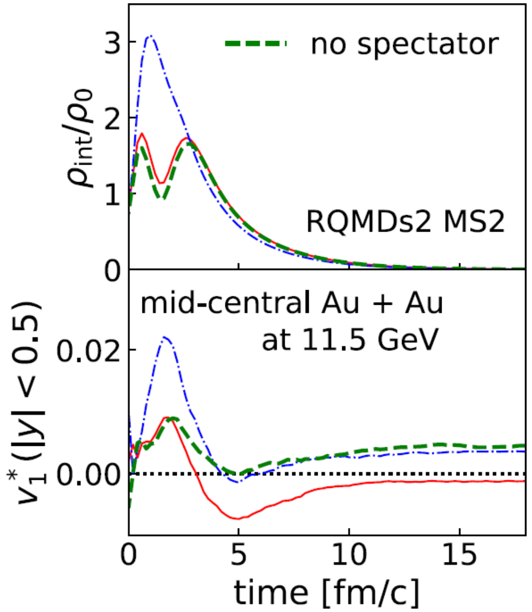
Collision reaction (schematic)



Flows shown by ➡ (positive v1), ➡ (negative v1) at η>

→Repulsion by ΛNN potential affects Λ-flow: v_1(PID=Λ)!

Time evolution of ρ & observable v_1



Nara & Ohnishi (2022)

We expect Λ potential affects Λ v_1

v_1 in heavy-ion collisions

Model for collision reaction: JAM/RQMDv

Nara & Ohnishi, PRC 105 (2022) 014911

Relativistic quantum molecular dynamics (RQMD)

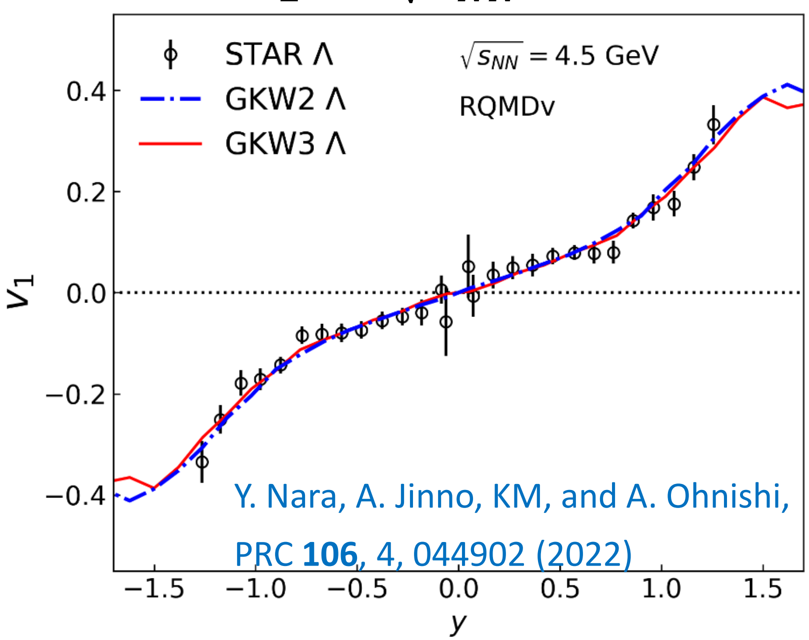
with Lorentz vector type potential (RQMDv)

★JAM2 available at

https://gitlab.com/transportmodel/jam2

Result

$\boldsymbol{v_1}$ **at** $\boldsymbol{\sqrt{s_{NN}} = 4.5}$ **GeV**



STAR Λ: exp. data; MS2: mom.-dep. soft (K=210 MeV) potential for N; GKW3: χEFT w/2+3-body force, w/o mom.-dep.

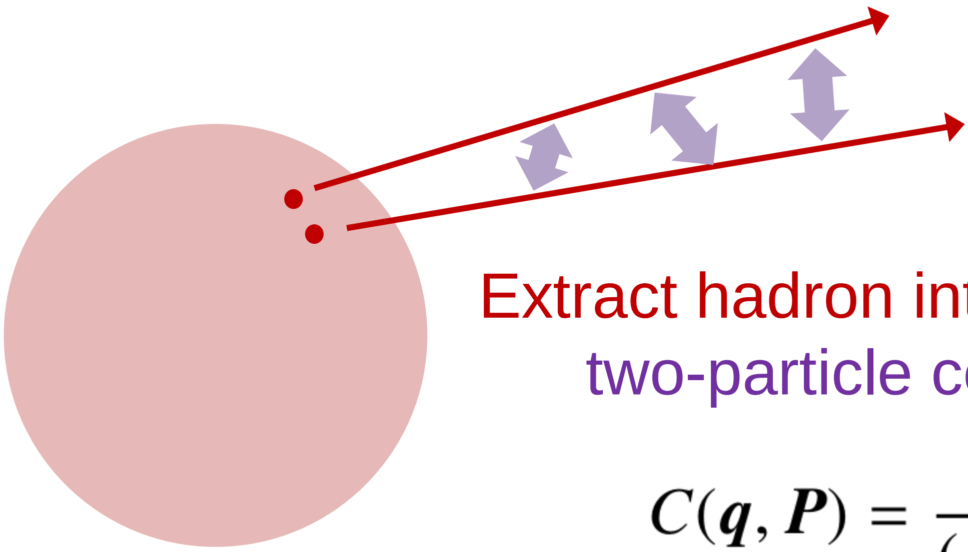
- proton v_1 slope as a function of $\sqrt{s_{NN}}$ is reproduced by the model with a single EOS
- *The Λ result also reproduces the data*
- *No significant dependence on the potential*

Future: potentiaals of Σ and other hyperons by A. Jinno

FEMTOSCOPY

Femtoscopy

See e.g. ExHIC, Prog. Part. Nucl. Phys. 95 (2017) 279-322



Extract hadron interactions from two-particle correlations:

$$C(\boldsymbol{q}, \boldsymbol{P}) = \frac{E_1 E_2 dN_{12}/d\boldsymbol{p}_1 d\boldsymbol{p}_2}{(E_1 dN_1/d\boldsymbol{p}_1)(E_2 dN_2/d\boldsymbol{p}_2)}$$

P & q: Total & relative momentum

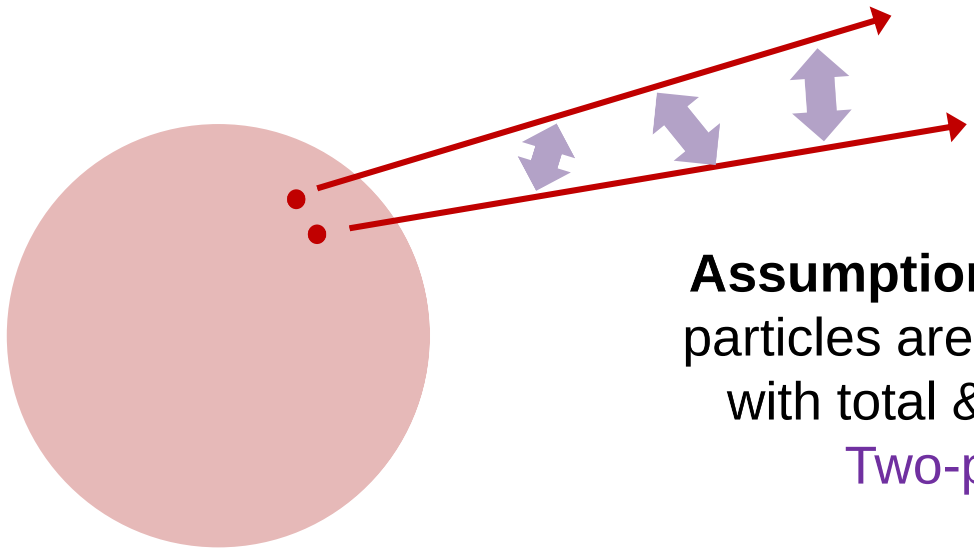
Koonin-Pratt formula (widely used to relate it to physics)

$$C(\boldsymbol{q}, \boldsymbol{P}) = \int d\boldsymbol{r} S_{12}(\boldsymbol{r}) |\varphi^{(-)}(\boldsymbol{q}, \boldsymbol{r})|^2.$$

$\varphi^{(-)}$: *Relative wave function*, S_{12}: two-particle source

(-) = “inverse” of standard scattering experiment

Koonin-Pratt (KP) formula



Emitted = isolated from the rest of the system at time *t* and evolve independently. *(Well-defined?)*

Assumption 1: Two (or a few number of) particles are emitted from source at r_1 & r_2 with total & relative momentum q' & P.
Two-particle source function
$S_{12}(q', r_1 - r_2)$

Assumption 2: After interaction with $V(r)$, *no change in relative momentum*: $q' = q$ *between emission point* q' *and final state* q

Assumption 3: Two particles become distant from each other and observed as asymptotic plane waves p_1 & p_2

Wave fn: $\varphi^{(-)} \sim \varphi^{*}_{\mathrm{scatt}}$

interaction → plane waves
(*cf* plane wave → interaction in scattering)

$$C(\boldsymbol{q}, \boldsymbol{P}) = \int d\boldsymbol{r} S_{12}(\boldsymbol{r}) |\varphi^{(-)}(\boldsymbol{q}, \boldsymbol{r})|^2.$$

KP with spherical source & *S*-wave

Only *S*-wave

Plane wave without S-wave　　S-wave wave fn

$$\varphi_{\boldsymbol{q}}(\boldsymbol{r}) = \boxed{e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)} + \boxed{\chi_q(r)}$$

Assumption 1: Spherical two-particle source *S*(*r*)

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(r) |\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2$$
$$= 1 + \int d\boldsymbol{r} S(r) \left\{ |\chi_q(r)|^2 - |j_0(qr)|^2 \right\}$$

j_0 : contribution from the plane wave

X_q : S-wave component of scattered wave fn

Lednicky-Lyuboshitz (LL) formula (S-wave)

Assumption 2: Gaussian source

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} e^{-\frac{r^2}{4R^2}}.$$

Assumption 3: φ is asymptotic form in the entire range

$$\varphi = e^{ikz} + f(\theta) \frac{e^{ikr}}{r}$$

Result: Correlation function is analytically calculated

$$\begin{aligned} C_{\mathrm{LL}}(q) =& 1 + \int d\boldsymbol{r} S_{12}(r) \left(|\psi_{\mathrm{asy}}(r)|^2 - |j_0(qr)|^2 \right) \\ =& 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\mathrm{eff}}}{R}\right) + \frac{2\mathrm{Re} f(q)}{\sqrt{\pi} R} F_1(2x) - \frac{\mathrm{Im} f(q)}{R} F_2(2x) \end{aligned}$$

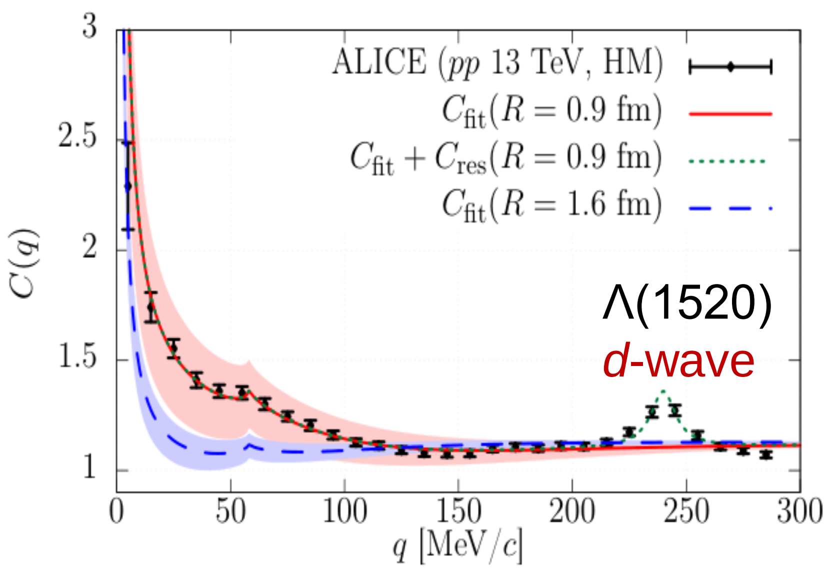
Conventionally used to fit the experimental data

FEMTOSCOPY
HIGHER PARTIAL WAVES

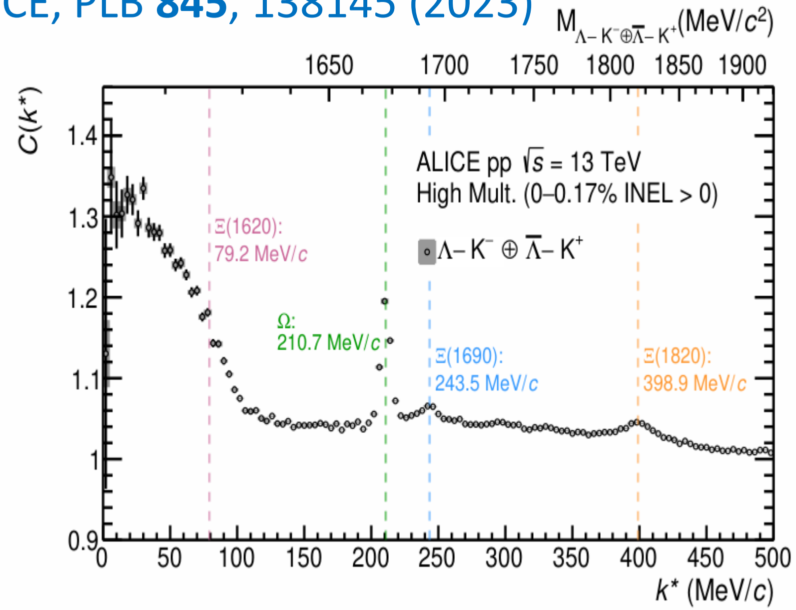
WITH **T. HYODO** (TOKYO METROPOLITAN UNIV)

Contributions from *higher partial waves* (*l* > 0)

Y. Kamiya, et al, PRL **124**, 132501 (2020)



ALICE, PLB **845**, 138145 (2023)



→Understanding of the contributions from higher partial waves?

Experimental data for the resonance peak are fitted by the Breit-Wigner form

→ *How is this justified/can be improved?*

Spherical source w/ *higher partial wave*

Full partial-wave expansion

$$\varphi_q(r,\theta) = \sum_{l=0}^{\infty}(2l+1)i^l R_l(r) P_l(\cos\theta)$$

Correlation function

$$C(q) = 1 + \sum_{l=0}^{\infty}(2l+1)\int dr 4\pi r^2 S(r) \times \left[|R_l(r)|^2 - |j_l(qr)|^2\right]$$

Contributions from partial waves: sum of each wave

Note: if source is not spherical, there arise
all the mixtures of different partial waves *l ≠ l'*

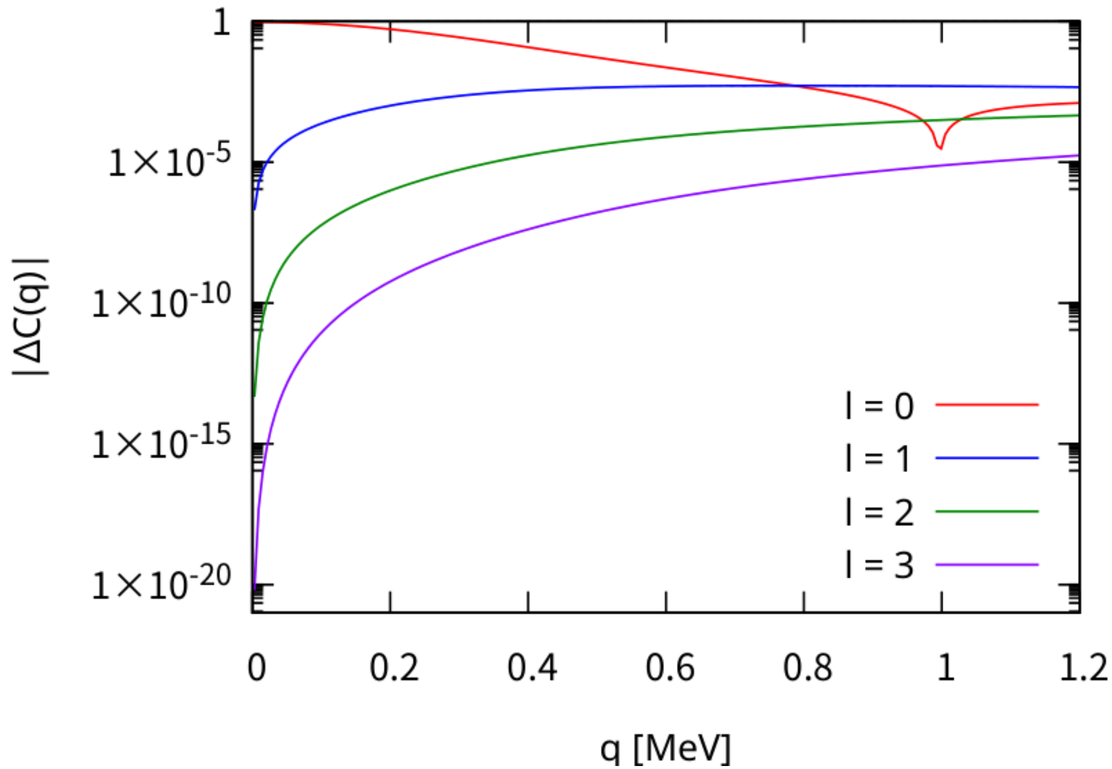
Example: Partial-wave contributions to C(q)

Potential well

$$V(r) = \begin{cases} -V_0, & (r < b), \\ 0, & (r > b). \end{cases}$$

μ = 600 MeV
V_0 = 50 MeV
b = 1 fm

$\Delta C_l(q)$: contribution to C(q) from the l-th partial wave



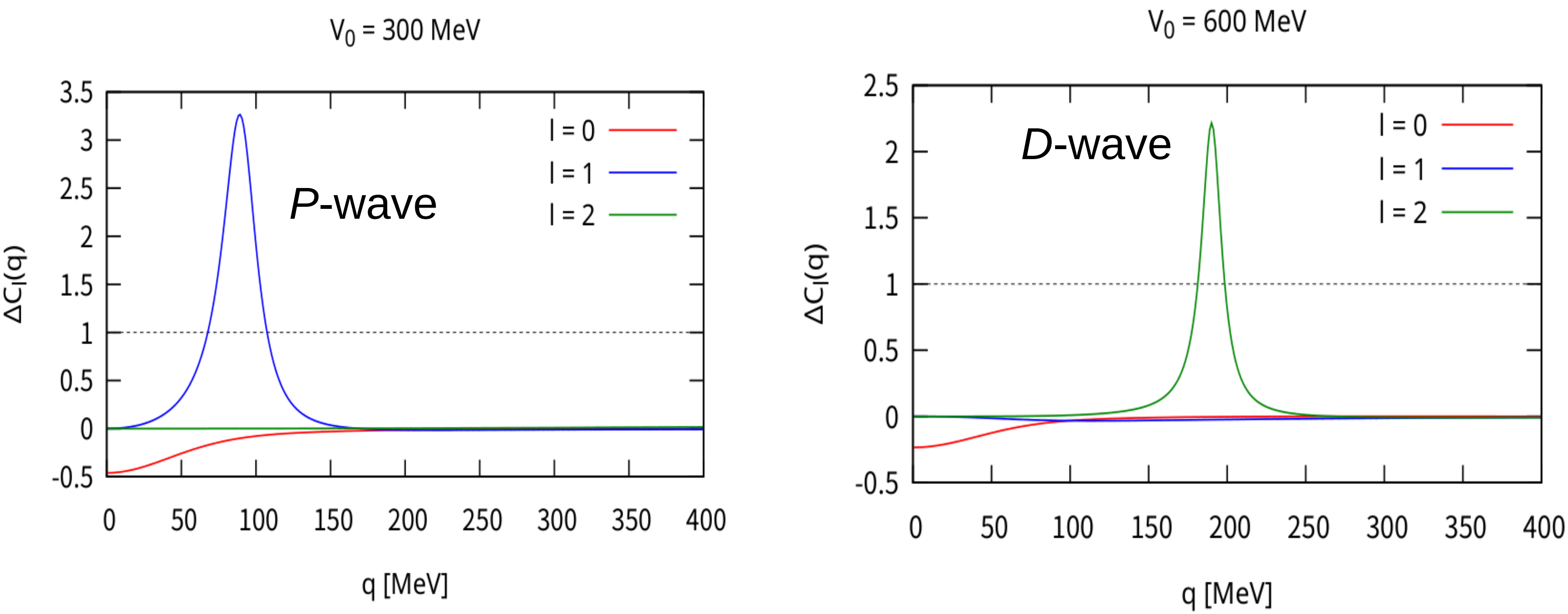
At low momentum, higher partial-waves are by orders of magnitudes suppressed

At high momentum, higher partial-wave typically have more contributions: $f_l \sim -a_l q^{2l}$

Example: Partial-wave contributions to C(q)

$\Delta C_l(q)$: contribution to C(q) from the *l*-th partial wave

Change potential parameter V_0



With resonances, higher partial waves can also have significant contributions.

LL formula with higher-partial waves

With the same assumptions

Assumption: Gaussian source

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} e^{-\frac{r^2}{4R^2}}.$$

Assumption: φ is asymptotic form in the entire range

$$\varphi \approx e^{iqz} + \frac{f(\theta)e^{ikr}}{r} + \mathcal{O}\left(\frac{l(l+1)}{r^2}\right)$$

ignore

Note: With spherical Bessel, KP divergent

Result: Correlation function

$$C(q) = 1 + \sum_{l=0}^{\infty} \frac{(2l+1)(-1)^l e^{-4x^2} q}{2\sqrt{\pi}x^2} \times \left(\sqrt{\pi}\Im f_l + 2\Re f_l \int_0^{2x} dt e^{t^2}\right)$$

f_l: $f(\theta) =: \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) f_l.$

For l=0 (S-wave), this reproduces the original LL formula (w/o r_{eff} correction)

Q. Only two terms?

A. With the optical theorem

$|f_0|^2 = q \text{ Im } f_0$,

LL reduces to only two terms

Simpler expression for LL formula

In correlation function

$$C(q) = 1 + \sum_{l=0}^{\infty} \frac{(2l+1)(-1)^l e^{-4x^2} q}{2\sqrt{\pi}x^2} \times \left(\sqrt{\pi}\Im f_l + 2\Re f_l \int_0^{2x} dt e^{t^2}\right)$$

The *l*-dependent part

$$\sum_{l=0}^{\infty}(2l+1)(-1)^l f_l = f(\theta=\pi)$$

Note: $(-1)^l = P_l(-1) = P_l(\cos\pi)$

Simpler representation of LL formula

$$C(q) = 1 + \frac{4\pi}{q}\Im\left[f(\pi)\int_0^{\infty} dr S(r) e^{2iqr}\right]$$

$\sim$ Im f Re S^ + Re f Im S^

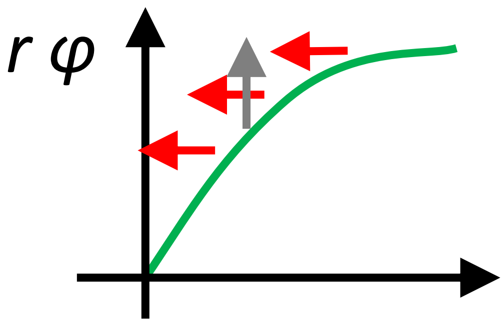
Discussion

Why alternating sign $(-1)^l$?

Asymptotic form: $j_l \sim \sin(kr - (l+1)\pi/2)/kr$

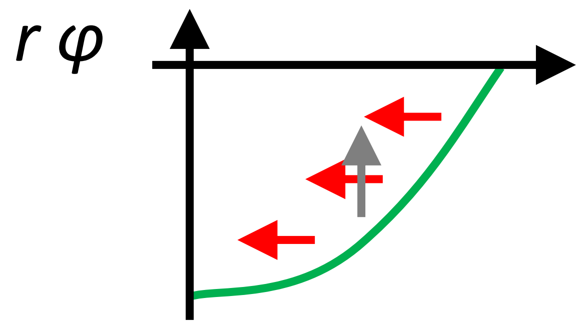
We extend this form to r → 0

$l = 0$



Increases $|\varphi|^2$ and thus C(q)

$l = 1$



Decreases $|\varphi|^2$

This is unphysical

(i.e., doesn't happen in reality).

Needs centrifugal corrections

Discussion

Why backward amplitude f(π)?

The plane waves contains
the delta functions at forward and backward
when expanded by the power of centrifugal force $[l(l+1)/r^2]^p$

$$e^{iqz} = \frac{2}{iqr}[\delta(1-\cos\theta)e^{iqr}$$

outgoing θ=0

$$+\delta(1+\cos\theta)e^{-iqr}] + \mathcal{O}\left(\frac{1}{r^2}\right)$$

Incoming $\theta=\pi$

Correlation function ΔC(q) is generated by interference between outgoing $f(\theta)/r$ and incoming plane wave $\delta(\theta-\pi)$
→ $f(\theta = \pi)$

Note: **Optical theorem** comes from the normalization of the outgoing wave $|f(\theta)/r + \delta(\theta=0)/iqr|^2$ and thus $f(\theta=0)$ plays a role

Summary

Heavy-ion collisions can be used to constrain interactions

Production of
Hadrons & Resonances

Production of
Light nuclei & hypernuclei

Anisotropic flow v_1, v_2, …

Femtoscopy
two-particle momentum correlation

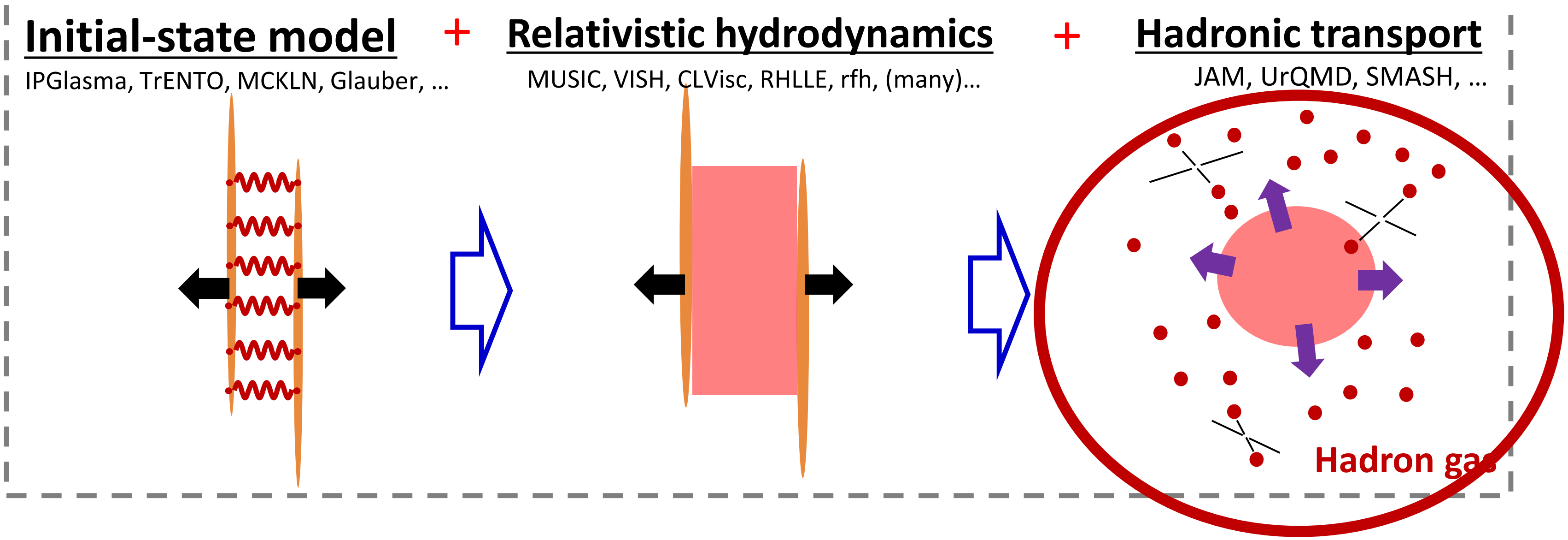
Femtoscopy

Contribution of higher partial waves are also contained in the correlation function

- What is a good fitting form of the contribution?
- What is its understanding?

Future

Dynamical model for HIC =



Further development & understanding in hadronic stage

- Hadronic transport model
  Covariant formulation of RQMD & RAMD,
  Dynamical integration with hydro,
  Dynamical formation of light nuclei, etc.
- Underlying assumptions and understanding of KP formula
  (Re-)Validation of assumptions
  What is the source function $S(r)$ in dynamical model?

BACKUP

Maximum NS mass and hyperon puzzle

Observed massive NSs

$M > 2\ M_{\odot}$

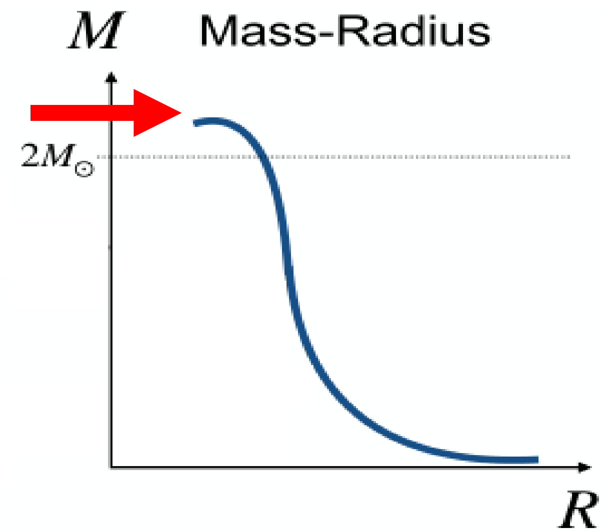
PSR J0952–0607: $M = 2.35 \pm 0.17\ M_{\odot}$
PSR J0740+6620: $M = 2.08 \pm 0.07\ M_{\odot}$
PSR J0348+0432: $M = 2.01 \pm 0.04\ M_{\odot}$
…

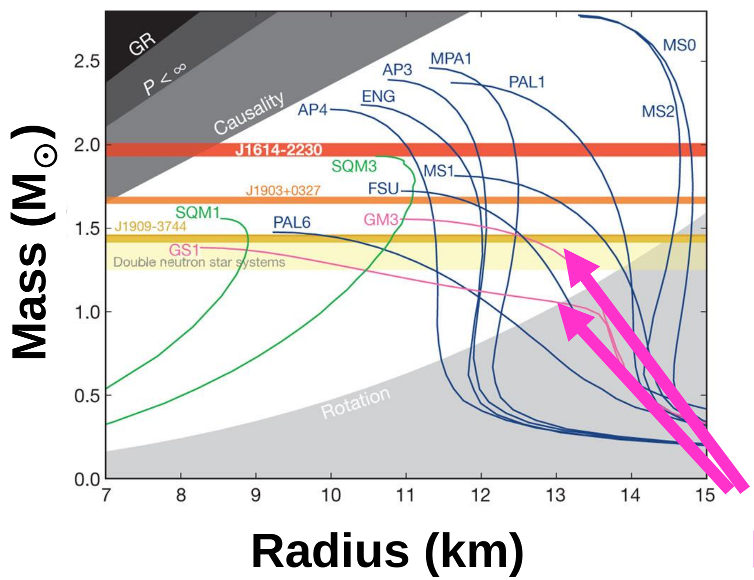
Romani et al. (2022), Miller et al. (2021), Antoniadis et al. (2013).

vs

Maximum mass by EOS

M_{max}





Demorest et al. (2010)

Nucleons + strange hadrons

"hyperon puzzle"

With strange hadrons (e.g. Λ baryon) being considered in EOS,

massive NSs ($M > 2\ M_{\odot}$) are unlikely? (in most EOSs)

One scenario of solving the puzzle

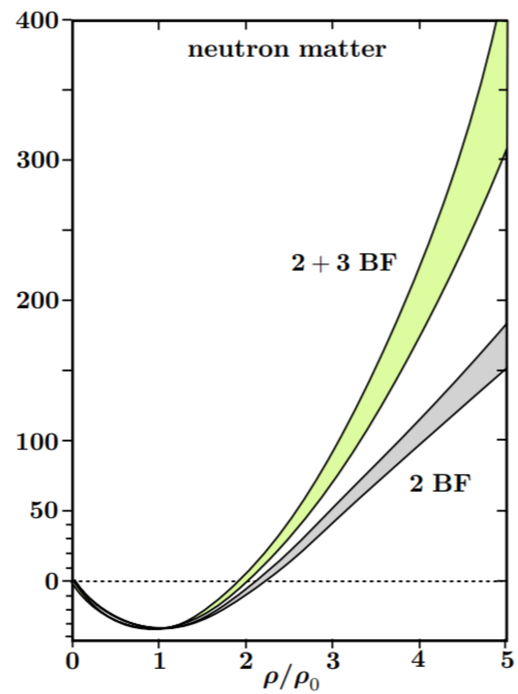
Scenario *Λ baryons do not appear in NS*

because of 3-body repulsive forces of Λ

ΛNN force from χEFT: Gerstung, Kaiser, Weise (2020) **(GKW), Kohno** (2018)

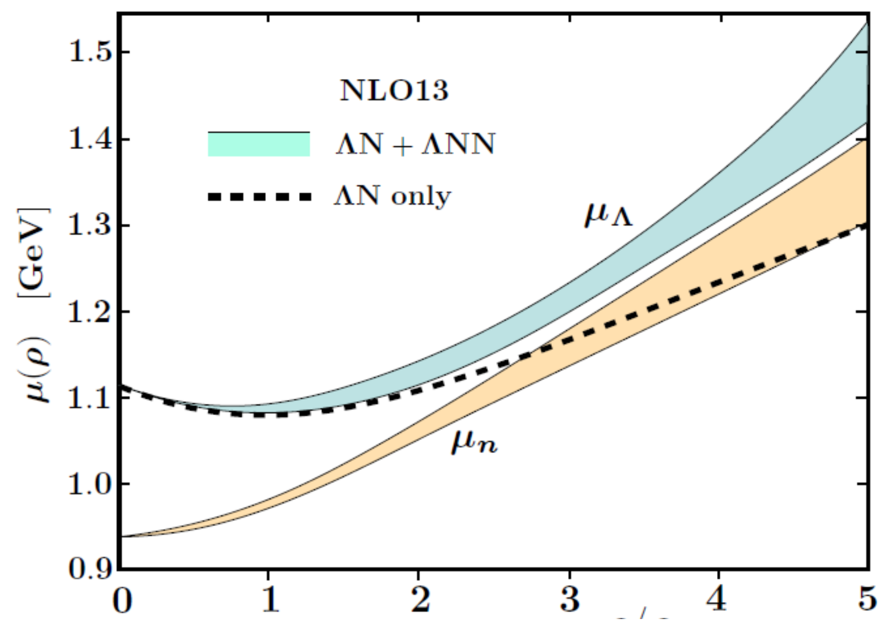
→ Repulsive at high ρ & consistent with the scenario

1-particle (mean-field) Λ potential $U_\Lambda(\rho)$



Density ρ/ρ_0

Chemical potential $\mu_\Lambda(\rho)$



Density ρ/ρ_0