Theoretical study on Λα and Ξα correlation functions

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Akira Ohnishi (YITP)

- Introduction: Femtoscopy for baryon-baryon interactions
- Λα momentum correlation function
- Ξα momentum correlation function

AJ, Y. Kamiya, T. Hyodo, and A. Ohnishi, arXiv: 2403.09126 (accepted by PRC)
Y. Kamiya, AJ, T. Hyodo, and A. Ohnishi, in preparation.

2024/6/26-28 Reimei Workshop "Hadron interactions with strangeness and charm"

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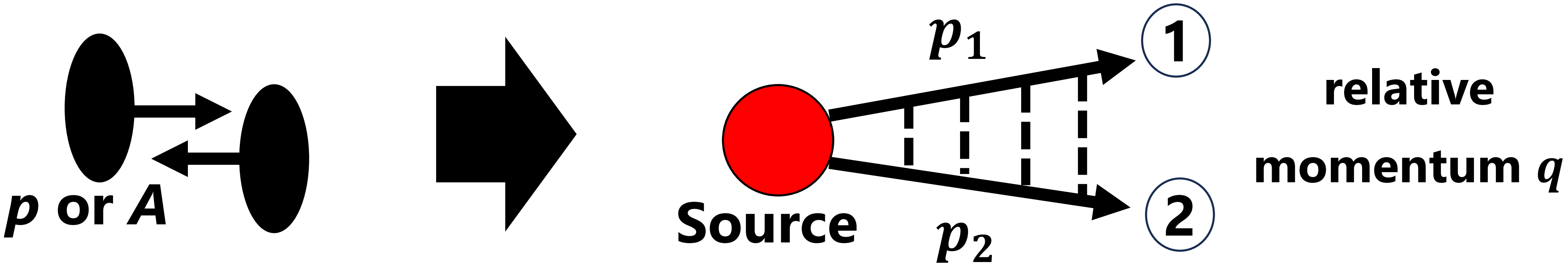
Introduction: Femtoscopy for baryon-baryon interactions

Correlation function in heavy-ion collision



- **Definition:**

$$C(q) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)}$$

If interaction and quantum statistics is none, $C(q) = 1$.

- **Historically, $C(q)$ is used to estimate <u>the size and shape of the source.</u>**
  e.g. Hanbury Brown and Twiss (1956); Goldhaber, Goldhaber, Lee, and Pais (1960); Wiedemann and Heinz, Phys. Rep, 319 (1999) 145.
- **Recently, by assuming the source properties, correlation function is used for examining <u>the hadron-hadron interactions</u>.**
  Lednicky and Lyuboshits (1982); Morita, Furumoto and Ohnishi (2015)

Theoretical model

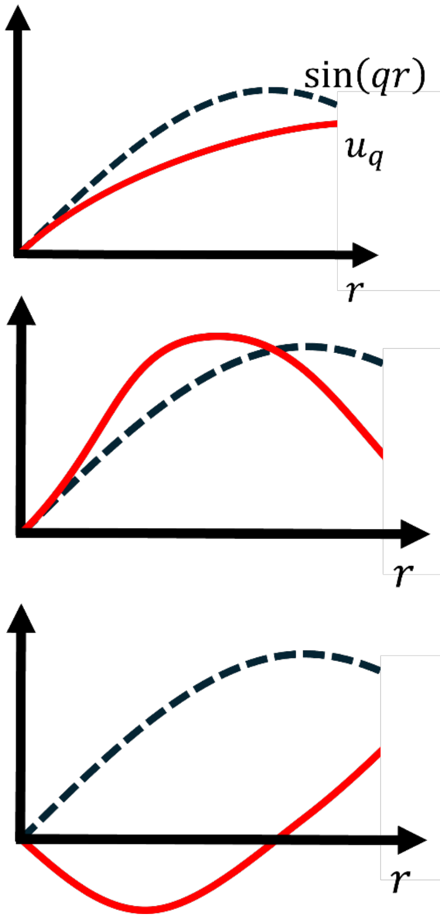
- **Basic formula: Koonin-Pratt formula**
  Koonin (1977), Pratt (1986)

$$C(\boldsymbol{q}) \cong \int d\boldsymbol{r} S(\boldsymbol{r}) \left|\Psi^{(-)}(\boldsymbol{r}, \boldsymbol{q})\right|^2$$

$S(r)$: relative source function, $\Psi^{(-)}$: outgoing relative wave function

If someone is included,

- **Repulsive potential** $\Rightarrow C < 1$
- **Attractive potential without bound state** $\Rightarrow C > 1$
- **Attractive potential with bound state** $\Rightarrow C < 1$ **(Wavefunction has a node at** $r \sim$**scattering length** a_0**.)**



Potential has non-trivial values depending on the potential properties!

Lednicky – Lyuboshits (LL) formula

Lednicky and Lyuboshits, Sov. J. Nucl. Phys. 35 (1982).

Approximation 1: Source function is a spherical Gaussian.

Approximatino 2: Asymptotic wave function is used for whole coordinate space.

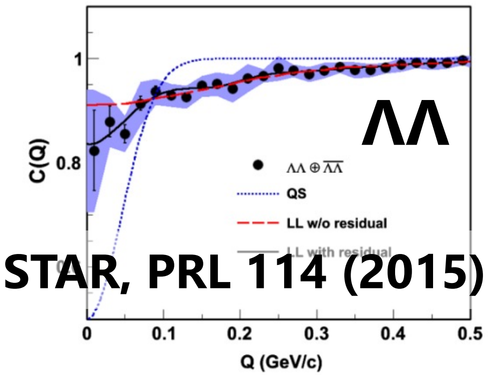
→ <u>**It can be adopted only for source size $R \gg$ (interaction range)**</u>

f: scattering amplitude

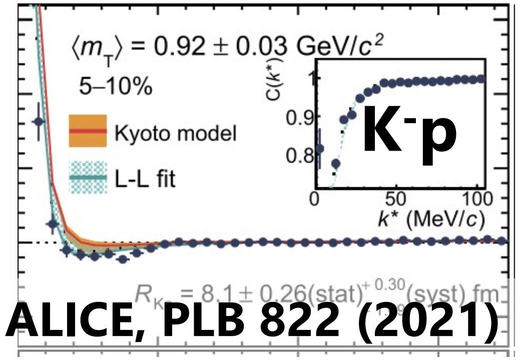
$$C_{LL}(q;f) = 1 + \int dr\, S(r)\left(\left|\psi_{\mathrm{asy}}(r;f)\right|^2 - |j_0(qr)|^2\right)$$

Expressing correlation function by only the scattering observable.

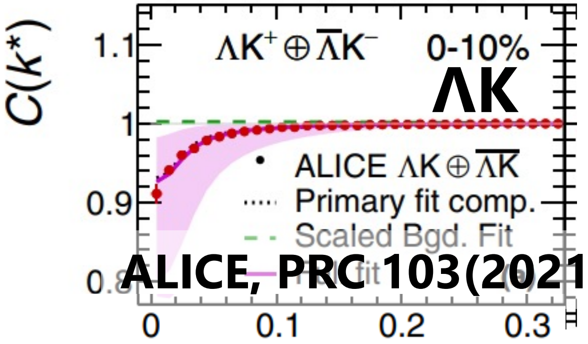
Inferring a_0 and r_{eff} from measurements



STAR, PRL 114 (2015)

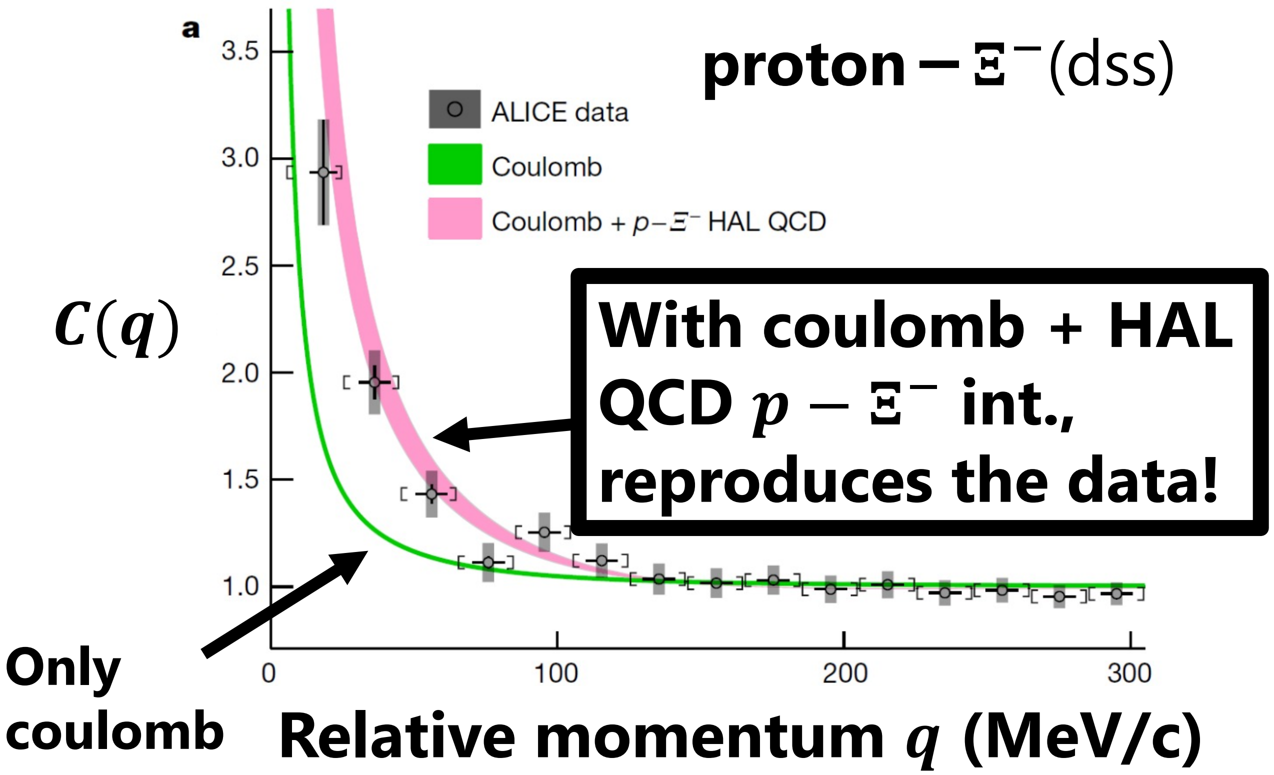


ALICE, PLB 822 (2021)



ALICE, PRC 103(2021).

Correlation function studies of BB systems



ALICE, Nature 588 (2020) 232.

HAL QCD int.: Sasaki et al., NPA 998 (2020) 121737.

- **Measured *BB* systems by ALICE and STAR** ***p*Λ, *p*Σ⁰,ΛΛ, ΛΞ, ΞΞ, and *p*Ω.** **(see AJ, Y. Kamiya, T. Hyodo, and A. Ohnishi, arXiv: 2403.09126 and references therein.)**

- **Recently, system with $A > 2$ becomes to be studied.**

measurements

***p*-deuteron(*d*): Singh, PoS EPS-HEP2021, 391 (2022).**

***pp*Λ: ALICE, EPJA 59, 145 (2023).**

theoretical studies

***pd*: Viviani *et al.*, PRC 108 (2023) 064002.**

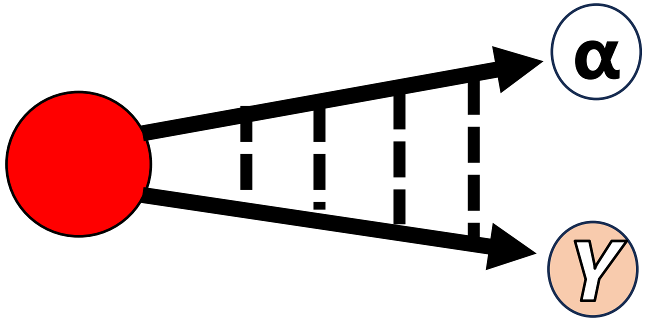
Λd: Haidenbauer, PRC 102 (2020); Kohno and Kamada, arXiv:2406.13899 (2024).

Ξd: Ogata, Fukui, Kamiya, & Ohnishi (2021).

+Λα, Ξα

hyperon-α (^{4}He) correlation function

Yα correlation function is expected to elucidate further properties of *YN* (+*YNN*) interactions!



- ✓**Two-body calculation is reasonable.**
- ✓Since the central density in α can reach $2\rho_0$, **short range** part of the *YN* (+ *YNN*) interaction could be probed.
- ✓**Enough statistics** may be obtained at the **collision energy $\sqrt{s_{NN}} < 10$ GeV**. (HADES, FAIR, J-PARC-HI)

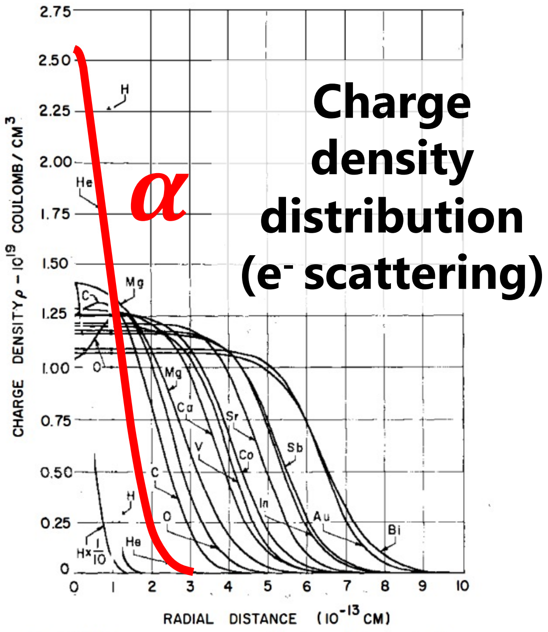
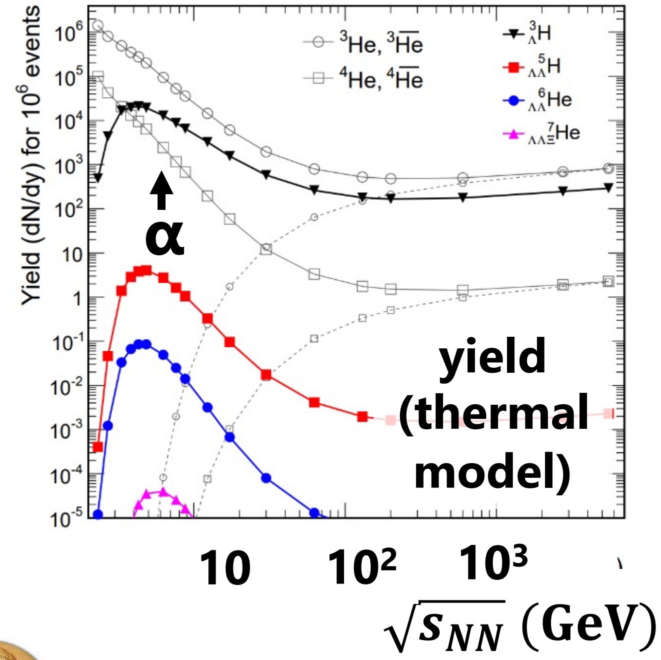


FIG. 32. This figure represents a summary of the charge distributions found for various nuclei by electron-scattering methods.

Hofstadter, Annu. Rev. Nucl. Sci. 7 (1957) 231.



Andronic et al., Phys. Lett. B 697 (2011) 203.

Λα momentum correlation function

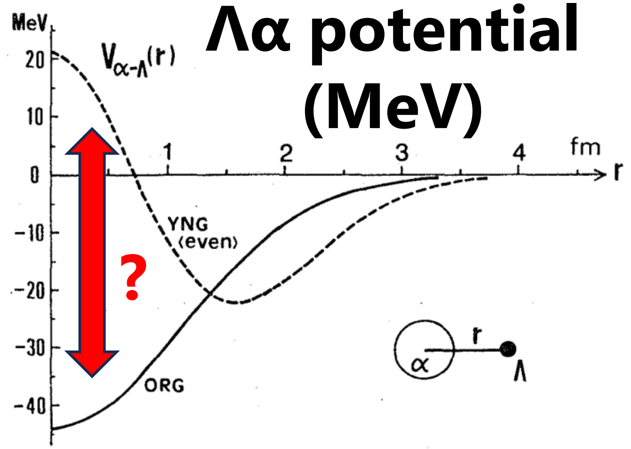
What is known/unknown for the Λα system?

- Overall attraction is constrained from ${}^{5}_{\Lambda}\mathrm{He}$ (Λ+α) Λ binding energy: 3.12 MeV
  M. Juric, et al, NPB 52 (1973) 1-30.

- Interaction range ~ (α radius + 2π exchange) ~ 2-3 fm

- **Short range behavior, or repulsive core of ΛN (and ΛNN) is unknown, although it is important in discussing dense nuclear matter!**
  - Λ binding energy in few-body hypernuclei (A<10) is not sensitive.
    Motoba, Bando, Ikeda, & Yamada, Prog. Theor. Phys. 81 (1985) 42.
  - Weak decay width of the light Λ hypernuclei is well reproduced for repulsive core case. Kumagai-Fuse, Okabe, Akaishi, PLB345 (1995) 386.

Λα potential (MeV)



➡ **Can Λα correlation function elucidate the short-range behavior?**

Λα potentials

We compare four models with different short-range behaviors.

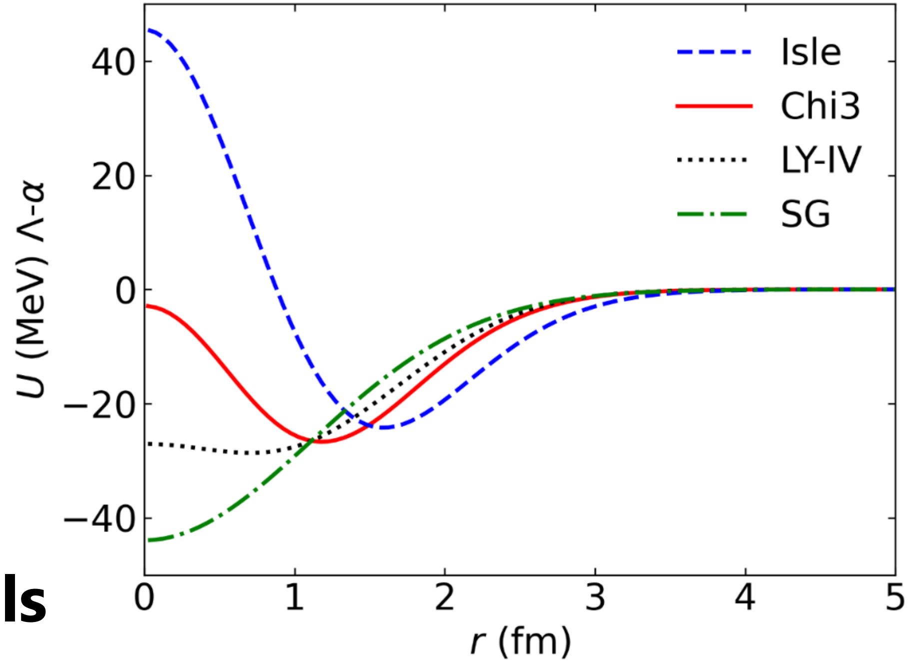
- **<u>Chi3, LY-IV</u>: Skyrme-type Λ potential substituting the density distribution in α**

(Chi3: based on Chiral EFT with $\Lambda NN - \Sigma NN$, Gerstung, Kaiser, and Weise (2020))

(LY-IV: Lanskoy and Yamamoto (1997))

It is important to distinguish them to solve the hyperon puzzle of neutron stars.

- Both reproduce the Λ hypernuclear data
  AJ, K. Murase, Y. Nara, & A. Ohnishi, PRC 108 (2023) 065803.
- (note) One parameter is tuned to reproduce the Λ binding energy data of ${}^{5}_{\Lambda}\mathrm{He}$, -3.12 MeV.



- **<u>Isle, SG</u>: conventional phenomenological models**
  Kumagai-Fuse, Okabe, Akaishi, PLB345 (1995) 386.

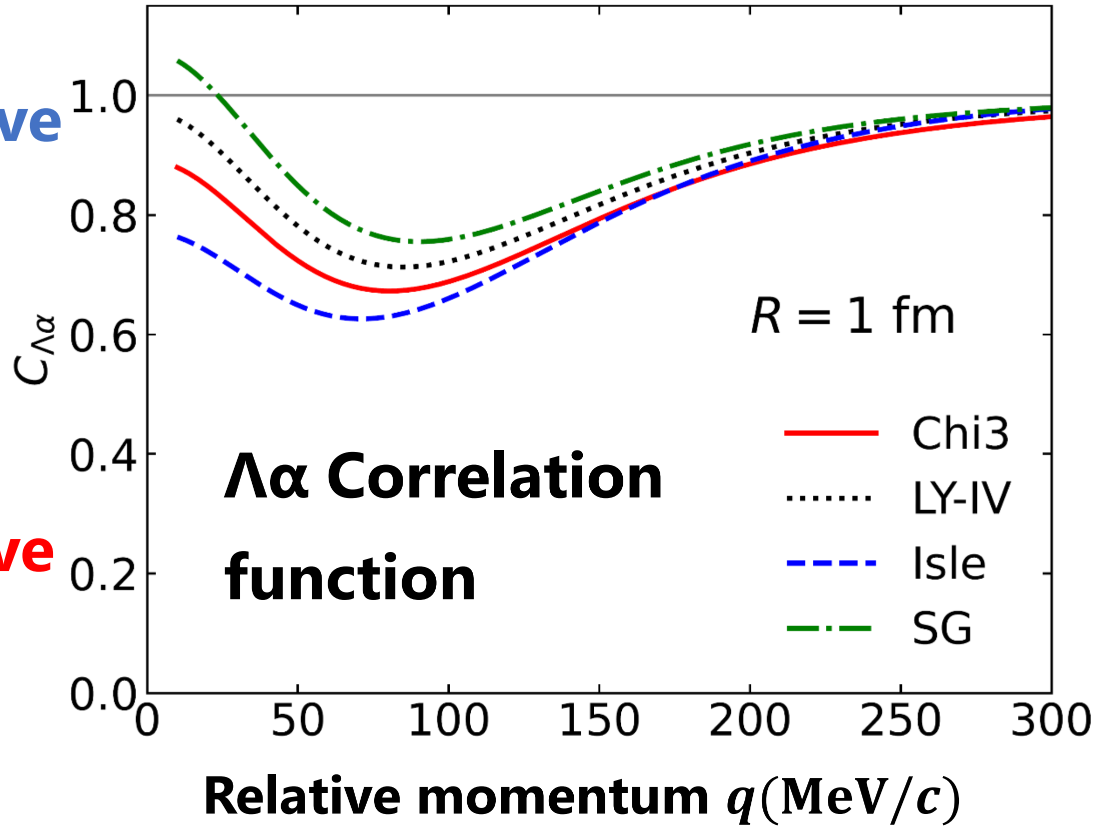
(Result) Λα correlation function

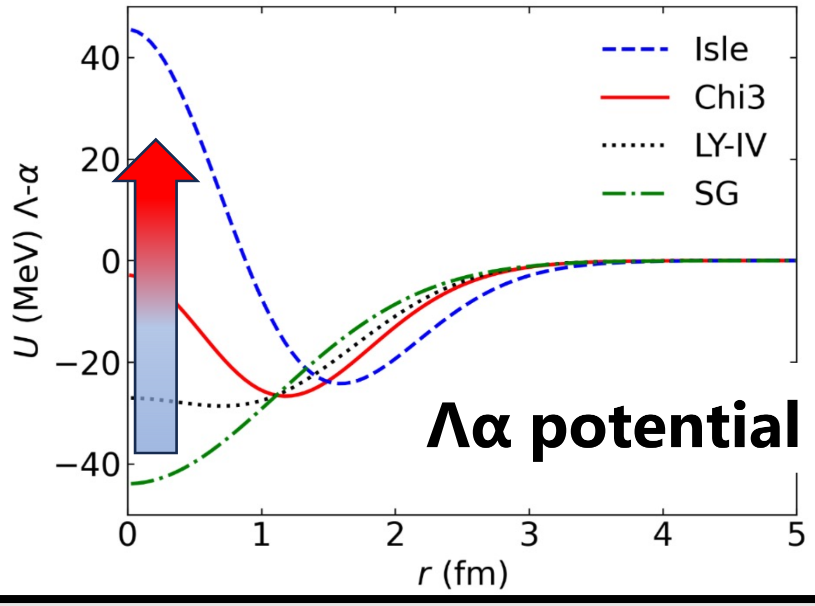
Difference among models is found at small momentum region!

Repulsive Λα potential core leads to suppression in correlation function.

Attractive core

Repulsive core





Note: The relative source is sampled by Gaussian with the width $\sqrt{2}R$, and the average distance btw. particles is $\langle r \rangle = 4R/\sqrt{\pi} \sim 2.26$ fm.

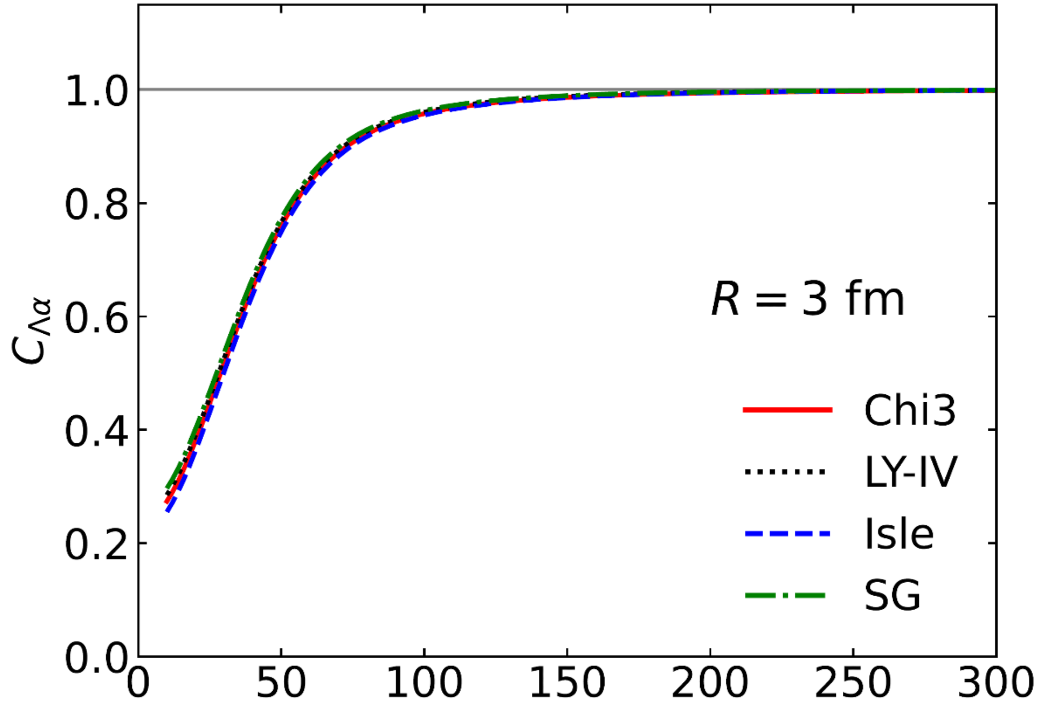
Mihaylov *et al.*, EPJC 78, 394 (2018).

AJ, Y. Kamiya, T. Hyodo, and A. Ohnishi, arXiv: 2403.09126 (accepted by PRC)

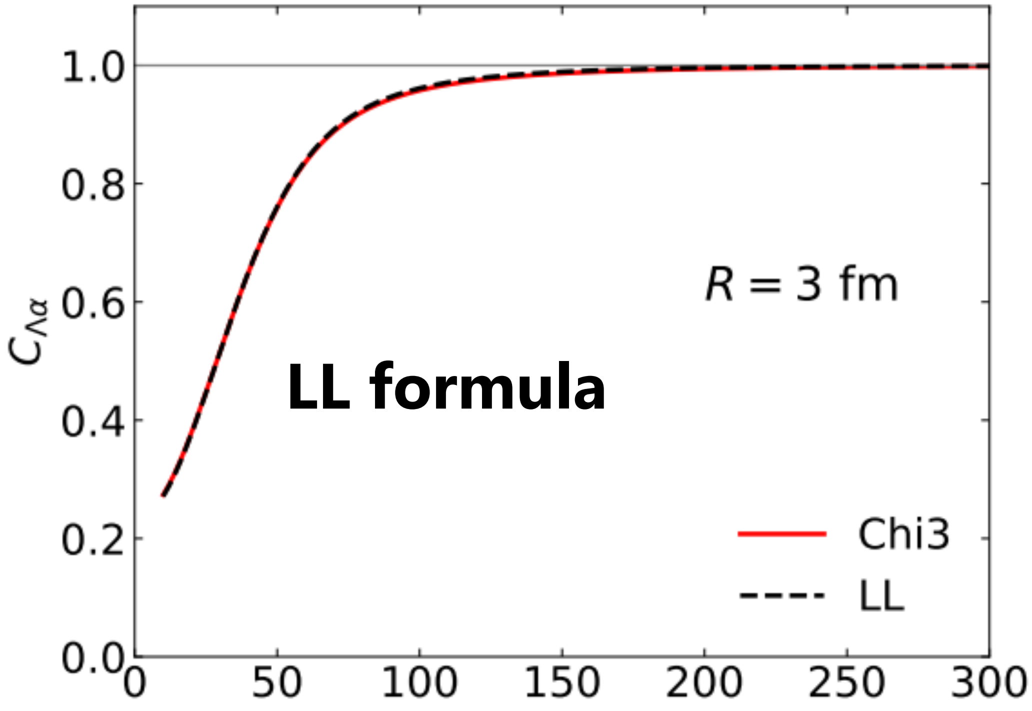
Λα correlation function (large sources)

- **No difference is found for large sources ($R \gtrsim 3$ fm).**
- **LL formula $C_{LL}(q, a_0, r_{\rm eff})$ well approximates the results.**

→ a_0 and $r_{\rm eff}$ do not differ enough to exhibit a difference.



Relative momentum q (MeV/c)



Relative momentum q (MeV/c)

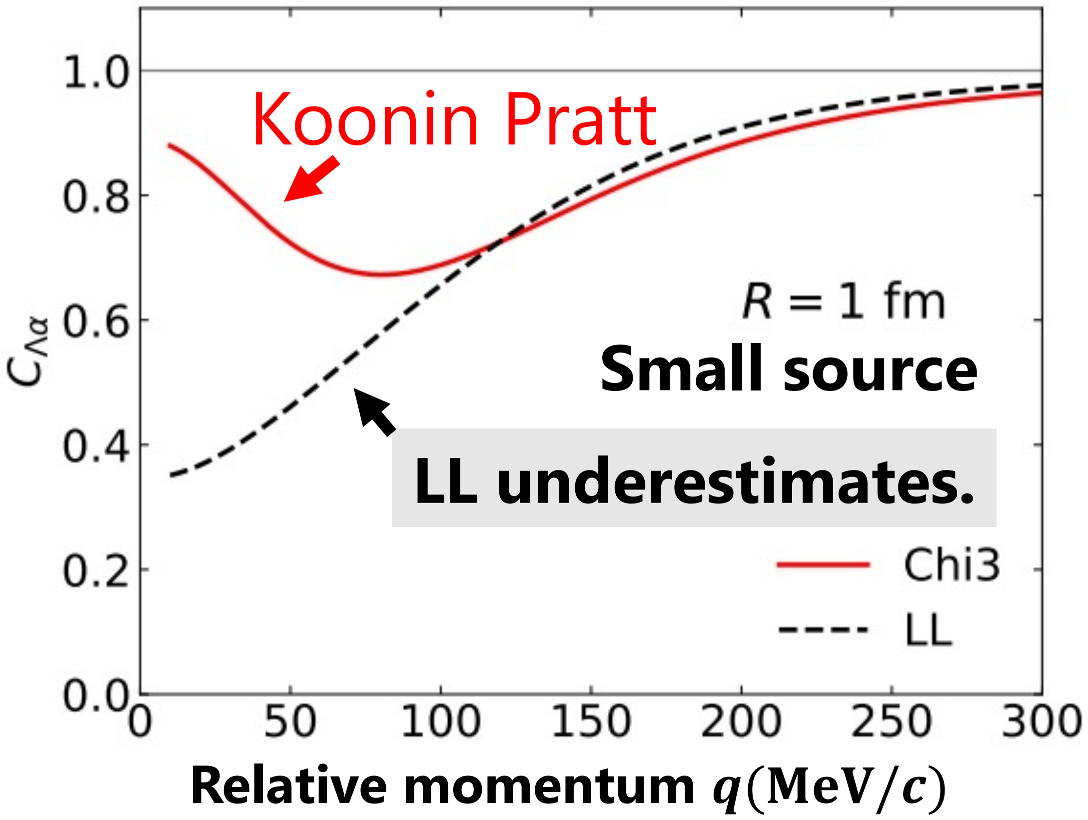
AJ, Y. Kamiya, T. Hyodo, and A. Ohnishi, arXiv: 2403.09126 (accepted by PRC)

Lednicky – Lyuboshits (LL) formula

Λα system demonstrates LL formula can deviate from Koonin-Pratt formula.

i.e. LL formula can be a good approximation only if $R \gg$ interaction range.

2-3 fm for Λα



AJ, Y. Kamiya, T. Hyodo, and A. Ohnishi, arXiv: 2403.09126 (accepted by PRC)

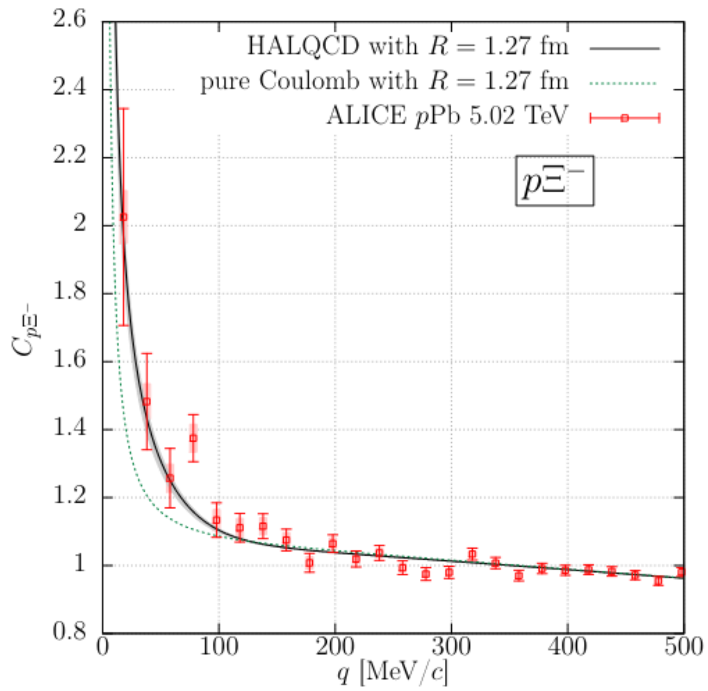
Ξα momentum correlation function

ΞN interactions and Ξα system

- ΞN system has four channels in *s* wave.
  - From $p\Xi^-$ correlation function, the $^{11}S_0$ attraction is confirmed not enough to make the ΞN bound state.

Notation: $^{2I+1,2S+1}L_J$

- Ξα potential can reveal $^{33}S_1$ channel! Relative strength in folding potential: $V(^{11}S_0) + 3V(^{13}S_1) + 3V(^{31}S_0) + 9V(^{33}S_1)$



Kamiya et al., PRC 105 (2022) 014915.

- Binding energy of $^{5}_{\Xi}\mathrm{H}$ (Ξ^-+α) is under debate.

HAL QCD *N*Ξ potential based folding potential 0.45 MeV

Hiyama, Isaka, Doi, and Hatsuda, Phys. Rev. C 106 (2022) 064318.
Sasaki et al. [HAL QCD], Nucl. Phys. A 998 (2019) 121737.

Chiral NLO using no core shell model 2.16 MeV

H. Le, J. Haidenbauer, U.-G. Meißner, and A. Nogga, EPJA 57 (2021) 339.
J. Haidenbauer. and U.-G. Meißner, EPJA 55 (2019) 23.

Unbound case also remains.

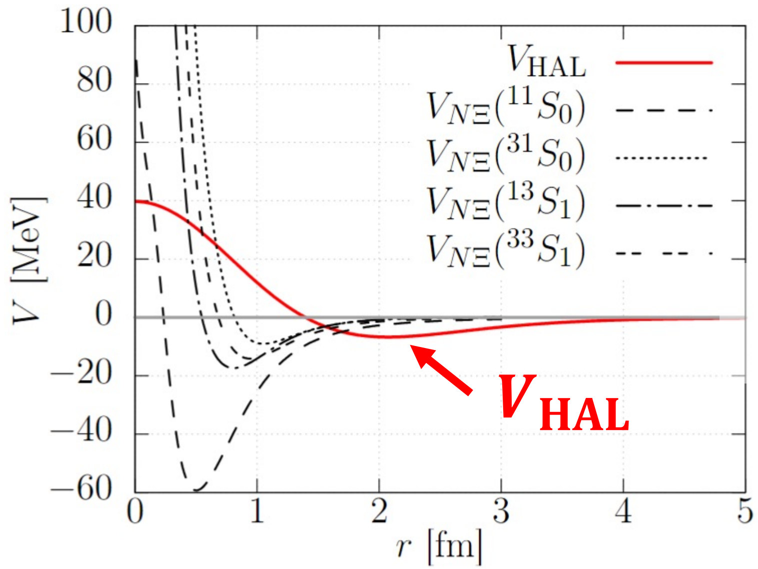
➡ **Let's discuss ΞN int. by Ξα correlation function!**

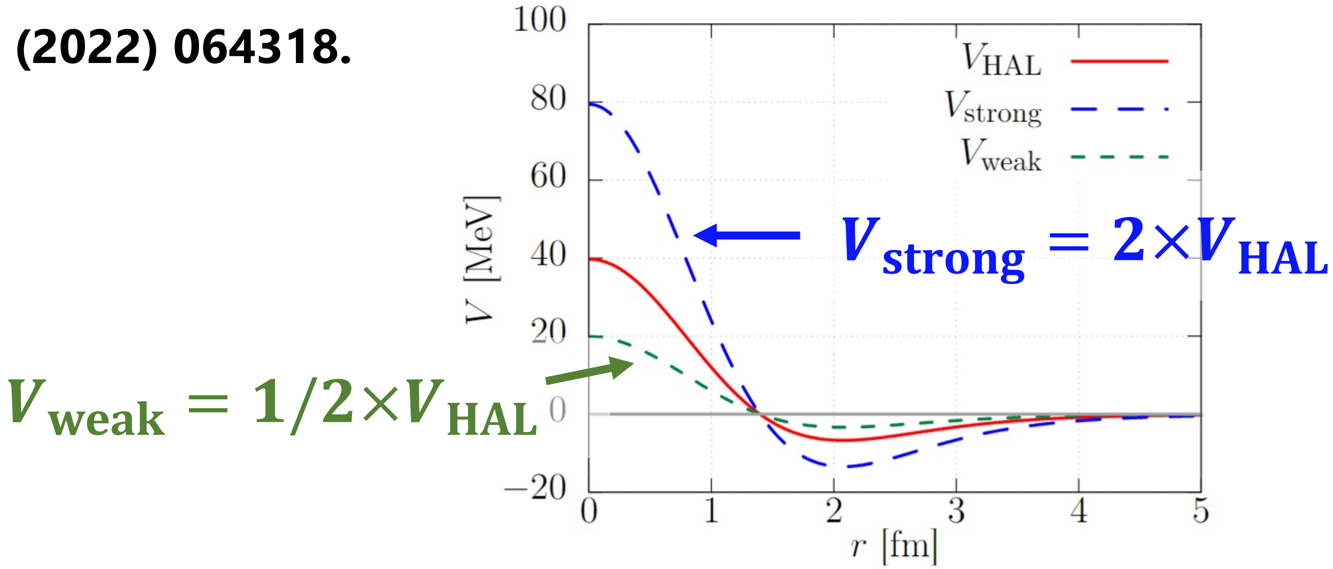
Ξα potentials

We employ the folding Ξα potential based on the HAL QCD NΞ potential,

and two variations that correspond to the deeper and shallower binding cases.

Hiyama, Isaka, Doi, and Hatsuda, PRC 106 (2022) 064318.





Relative strength: $\frac{1}{16}[V(^{11}S_0) + 3V(^{13}S_1) + 3V(^{31}S_0) + 9V(^{33}S_1)]$

$$q\cot\delta = -\frac{1}{a_0} + \frac{1}{2}r_{\rm eff}q^2 + \mathcal{O}(q^4)$$

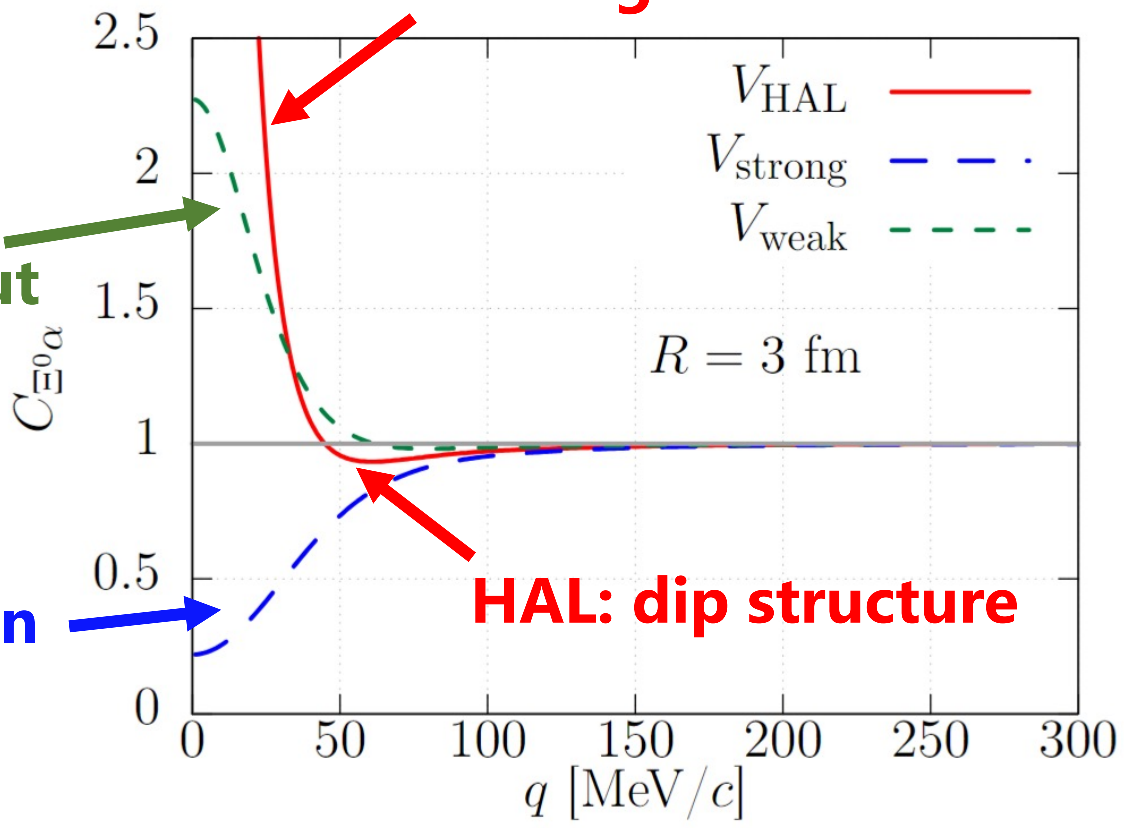
	Low-energy scattering parameters		**Binding energies**	
potential	a_0 [fm]	$r_{\rm eff}$ [fm]	$\Xi^-\alpha$ [MeV]	$\Xi^0\alpha$ [MeV]
$V_{\rm HAL}$	−522.8	4.50	0.47	unbound
$V_{\rm strong} = 2V_{\rm HAL}$	6.39	3.01	2.08	1.15
$V_{\rm weak} = V_{\rm HAL}/2$	−3.39	7.36	0.18	unbound

$\Xi^0\alpha$ correlation function (large source)

Difference among the three models is found clearly, reflecting the bound state nature!

HAL: huge enhancement ($\because |a_0| > 500$ fm)

weak: moderate enhancement ($\because$ attraction without bound state)



Qualitatively same behavior is also found for $R = 5$ fm.

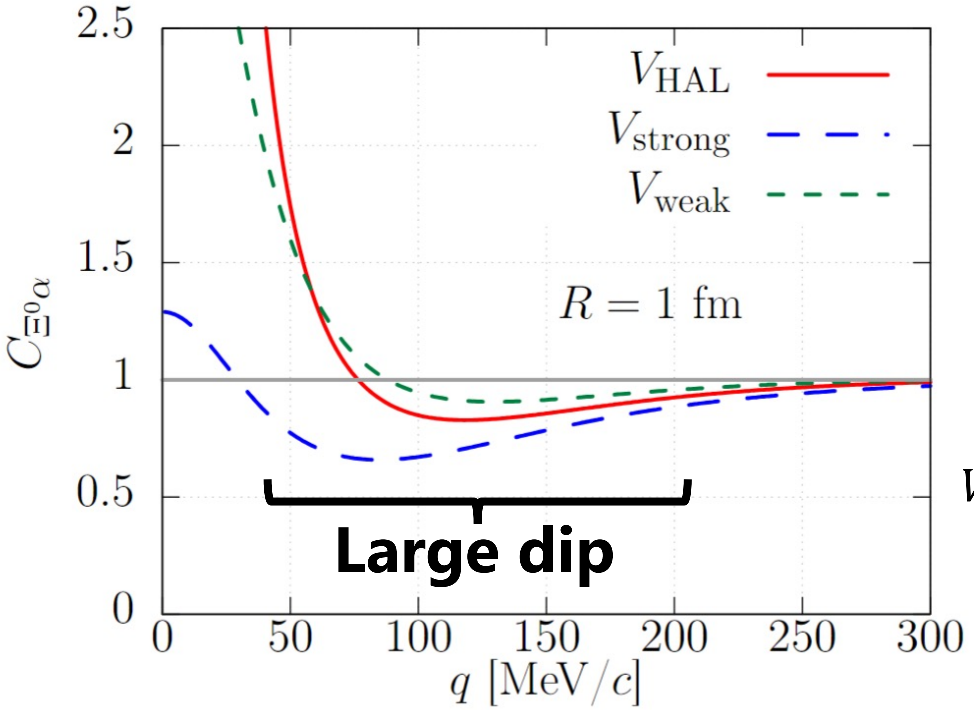
strong: suppression ($\because$ bound state)

HAL: dip structure

Y. Kamiya, AJ, T. Hyodo, and A. Ohnishi in preparation.

$\Xi^0\alpha$ correlation function (small source)

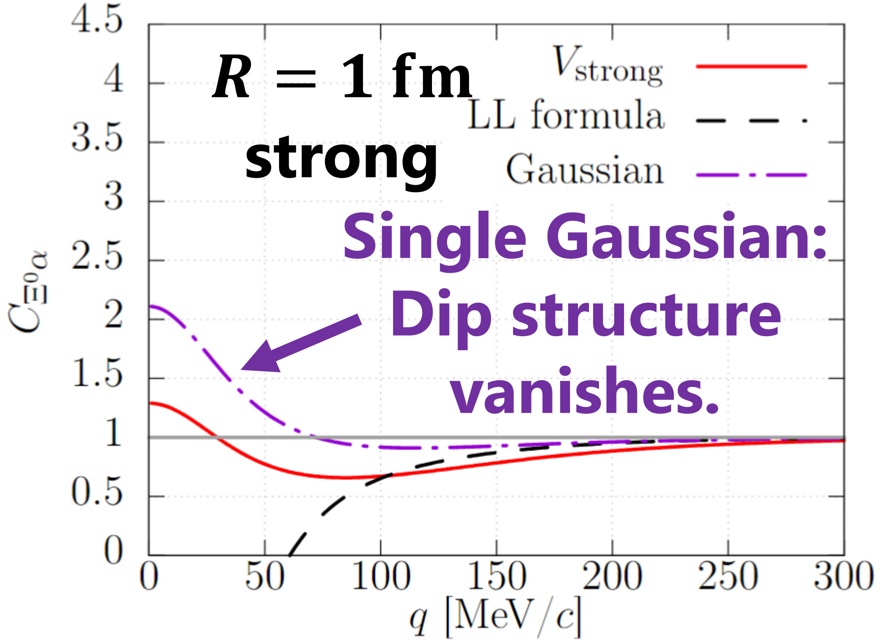
Dip structure is prominent for all models.



Comparing with the single-Gaussian potential without repulsive core

$$V_{\text{Gaussian}} = V_0 \exp\left(-\frac{r^2}{(3\ \text{fm})^2}\right)$$

V_0 tuned to reproduce the scattering length a_0



Dip structure reflects the repulsive core of the $\Xi\alpha$ potential!

Small-source measurements may be useful to investigate the existence and strength of the repulsive core.

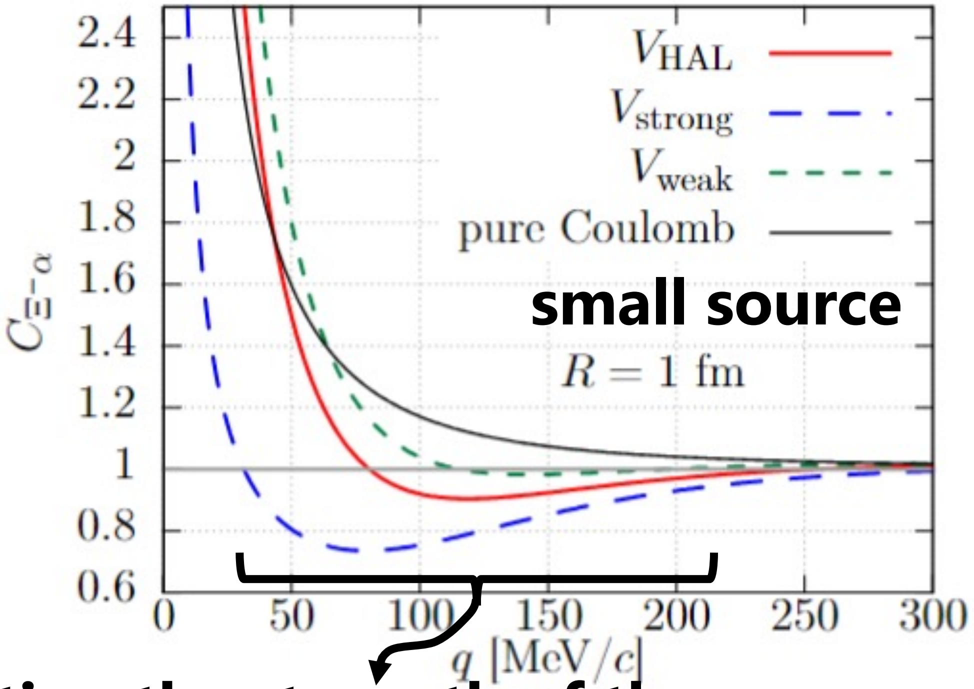
Y. Kamiya, AJ, T. Hyodo, and A. Ohnishi in preparation.

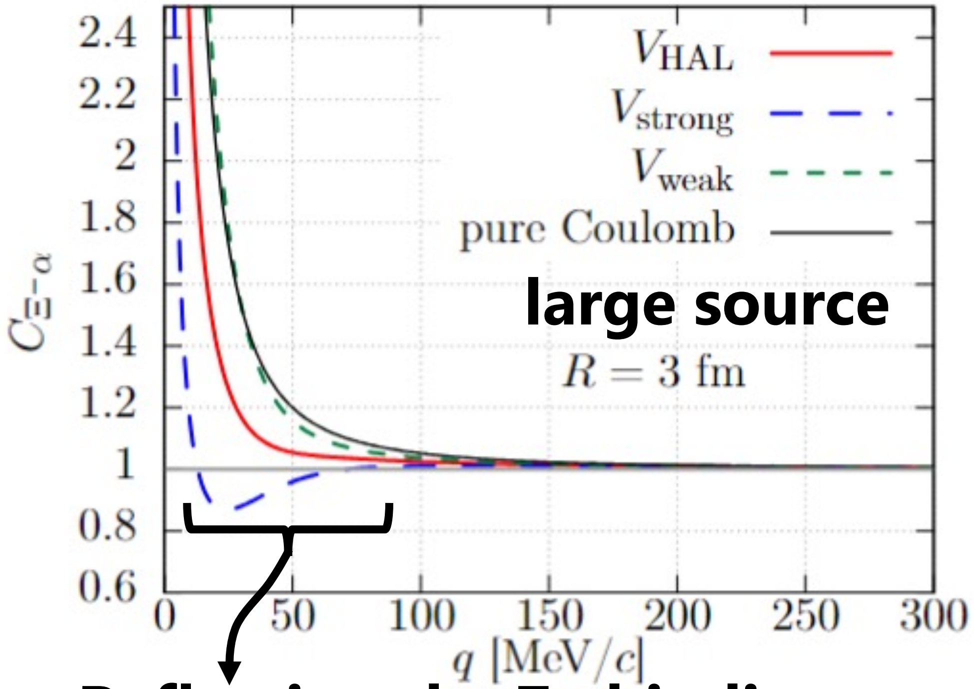
$\Xi^-\alpha$ correlation function (large source)

Bound state exists for HAL and weak.

The difference among three models is still noticeable!

potential	$\Xi^-\alpha$ [MeV]	$\Xi^0\alpha$ [MeV]
$V_{\rm HAL}$	0.47	-
$V_{\rm strong}$	2.08	1.15
$V_{\rm weak}$	0.18	–





Reflecting the strength of the repulsive core and the binding.

Reflecting the Ξ^- binding energy of $^{5}_{\Xi}$H (Ξ^- + α)

Y. Kamiya, AJ, T. Hyodo, and A. Ohnishi in preparation.

Summary

We investigated Λα and Ξα correlation function to reveal further properties of hyperon-nucleon interactions.

- **Λα correlation function ・・・for ΛN (+ΛNN) interaction at short range**
  - **We compare phenomenological Λα potentials with different strength at short range.**
  - **Difference among models is found for small size source.**
    (e.g. *pA* collision for $\sqrt{s_{NN}} < 10$ GeV (J-PARC HI, FAIR, NICA) ?)
  - **We verify that the Lednicky-Lyuboshits formula can yield erroneous results for a small source size with a potential that has large interaction range, like the Λα system.**
- **Ξα correlation function ・・・for ΞN two-body interactions**
  - **We employ the folding potential from HAL QCD ΞN potential, and their variations.**
  - **Both $\Xi^0\alpha$ and $\Xi^-\alpha$ correlation functions are sensitive to their bound state property.**
  - **The strength of the repulsive core could be accessed by the small-size source.**

We have to discuss…

- **the treatment of α as a point-like particle for small source size.**
- **the validity of the assumption of chaotic source for $\sqrt{s_{NN}} < 10$ GeV. (e.g. by JAM + coalescence)**