Dense QCD equation of state:
aspects of conformality and duality

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(University of Washington)

INSTITUTE for NUCLEAR THEORY

References:
[1] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022) [2207.06753]
[2] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024) [2306.04304]; in preparation

June 28, 2024 — Reimei Workshop "Hadron interactions with strangeness and charm”

Equation of state (EoS) from first-principles QCD

Freedman,McLerran(1978);
Baluni(1979);
Kurkela,Romatschke,Vuorinen,
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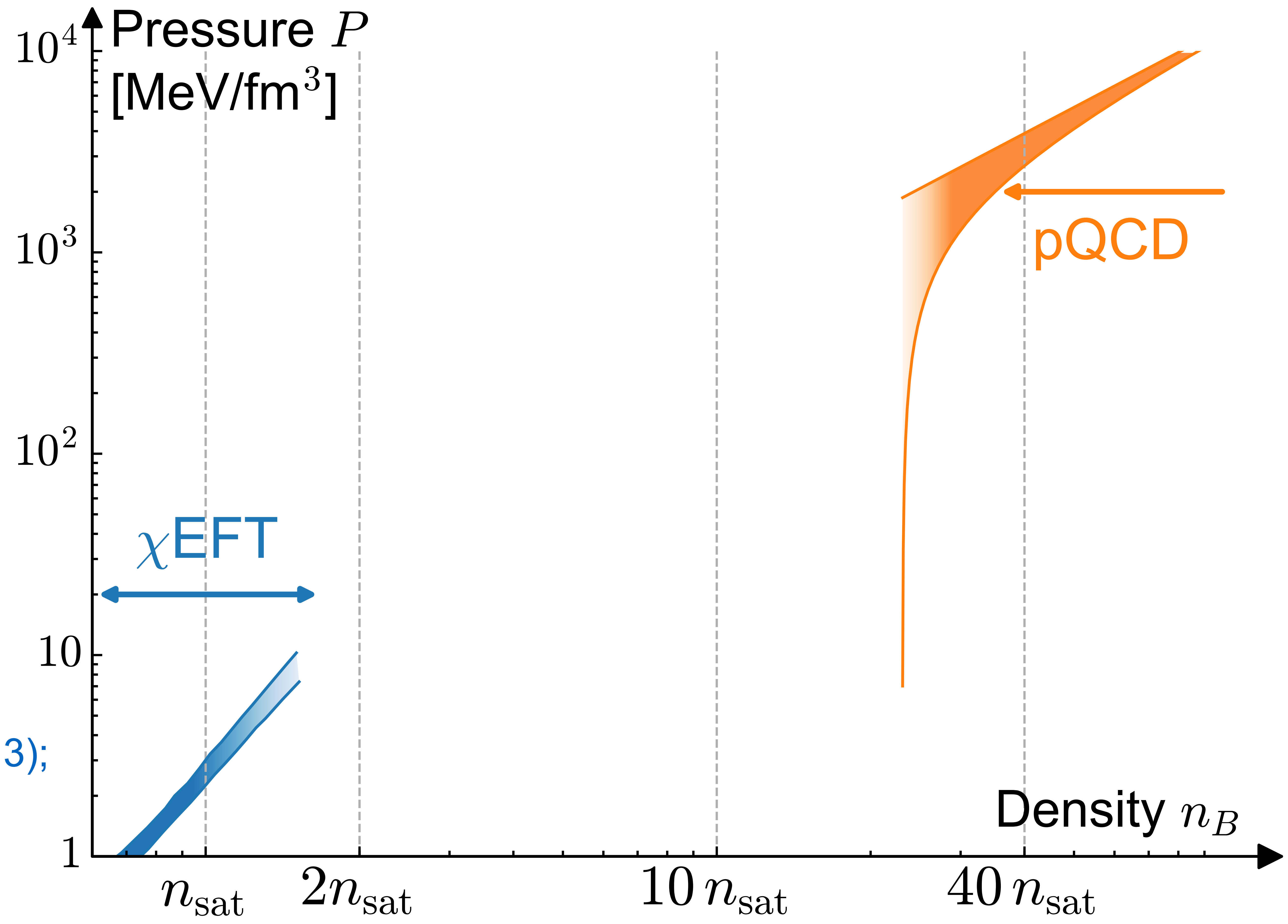

Tews,Krüger,Hebeler,Schwenk(2013);
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& many others

Nuclear density: $n_{\rm sat} = 0.16\ \text{fm}^{-3}$

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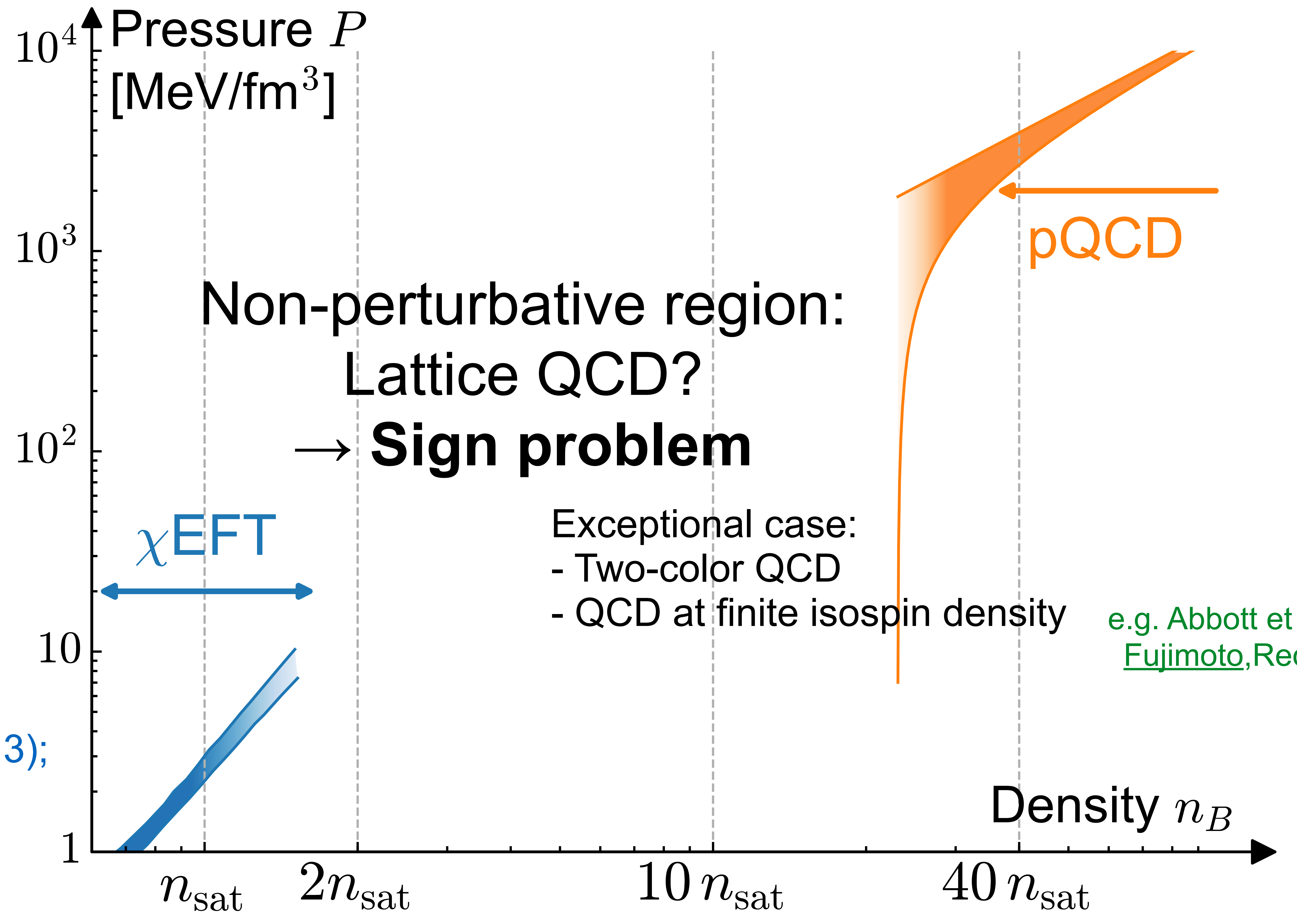

e.g. Abbott et al. (2023);
Fujimoto,Reddy (2023)

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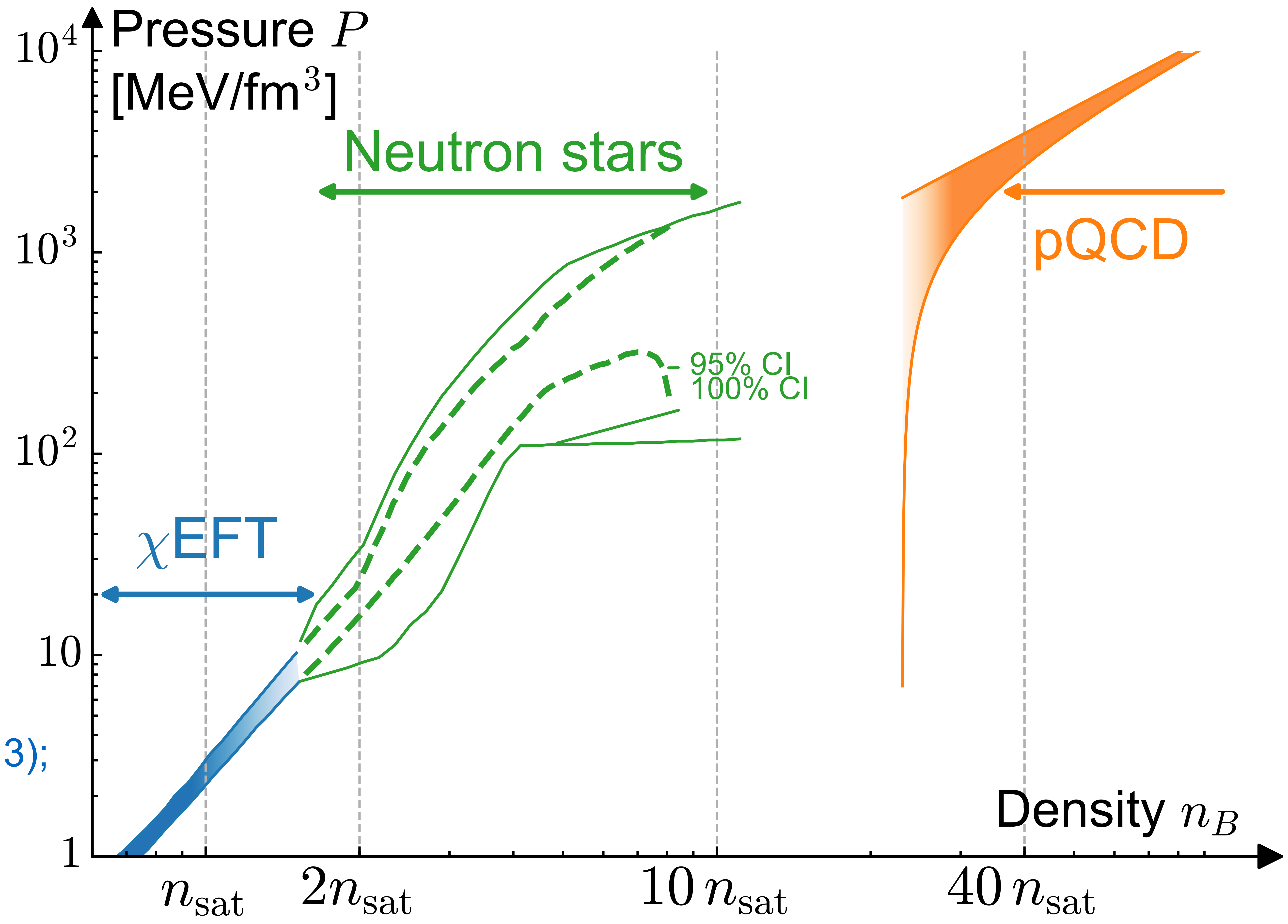

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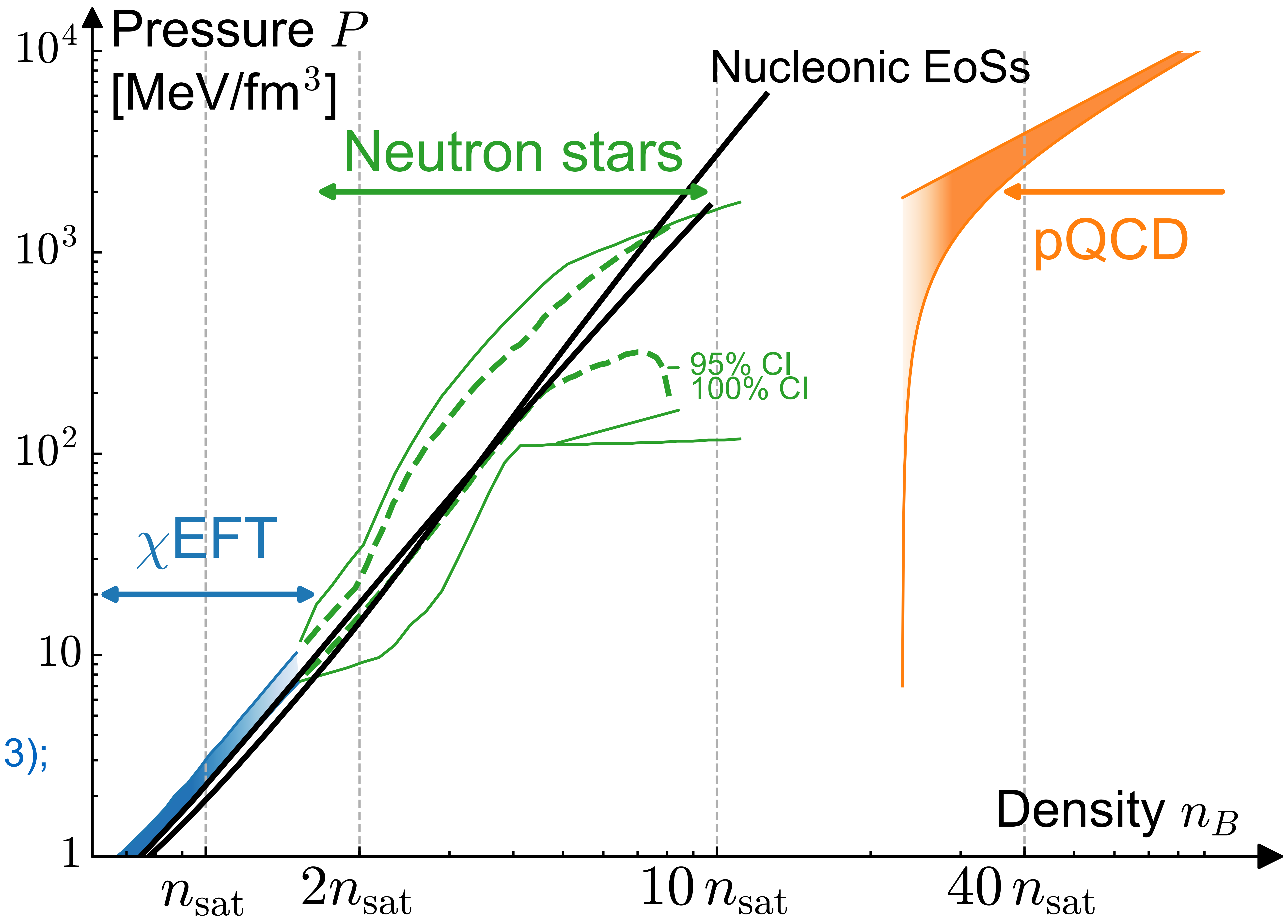

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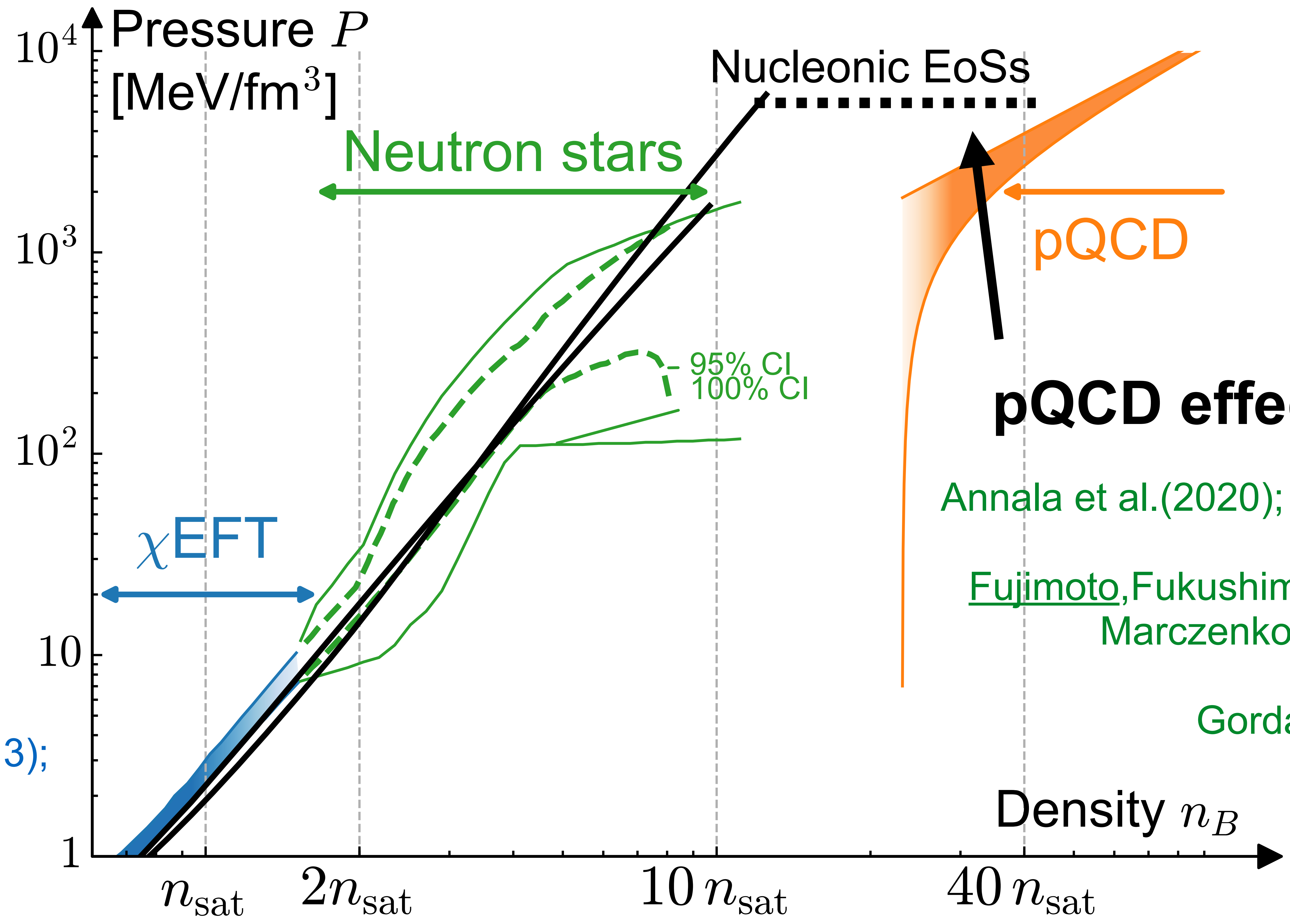

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Fujimoto,Fukushima,McLerran,Praszalowicz(2022);
Marczenko,McLerran,Redlich,Sasaki(2022);
Komoltsev,Somasundaram,
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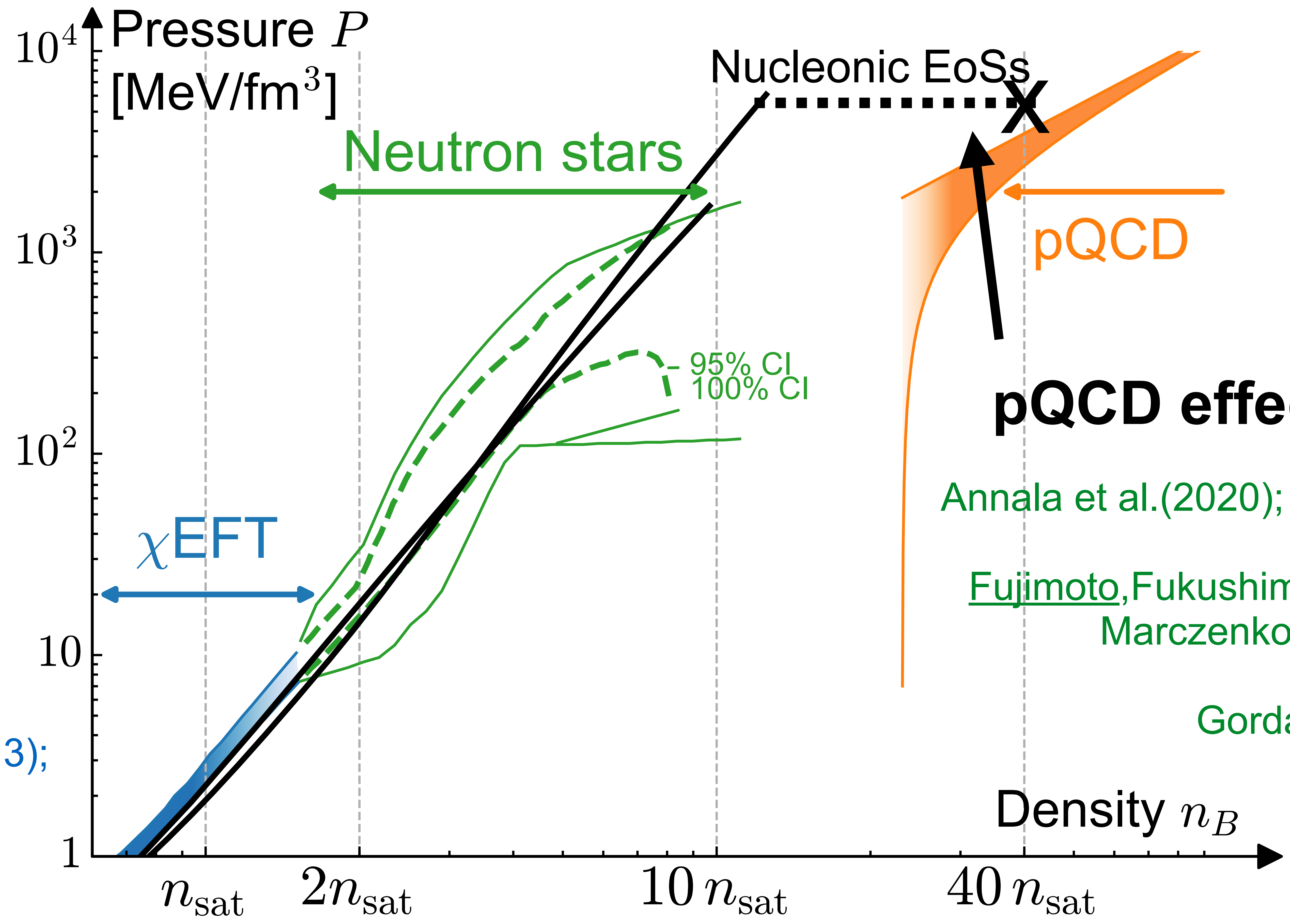

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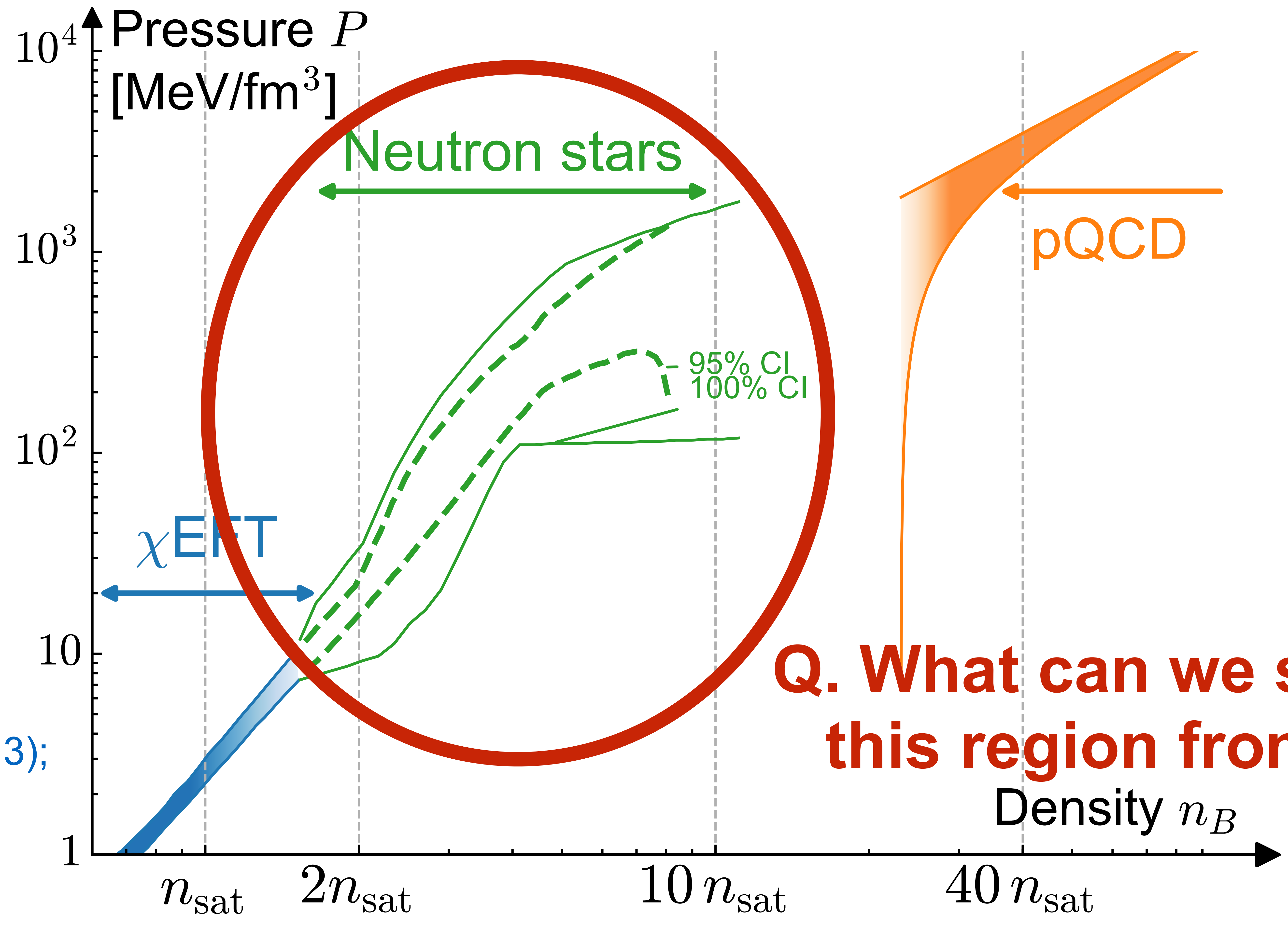

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Outline

1. **Conformality from pQCD: Neutron-star favors conformal EoS**

   Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

2. **Duality implied from large-Nc QCD: Quarkyonic matter**

   Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024); in preparation

   a) confinement-deconfinement at high baryon density

   b) Quarkyonic shell structure from duality

   c) Implications to hyperon puzzle

Conformal limit

Weak coupling limit $\alpha_s \to 0$ is achieved when $\varepsilon \to \infty$.

The pQCD EoS has properties in this **conformal limit** as:

Trace anomaly: $\varepsilon - 3P \sim \beta_0 \mu^4 \left(\frac{\alpha_s}{\pi}\right)^2 \to 0$

Sound speed: $v_s^2 = \frac{dP}{d\varepsilon} \sim \frac{1}{3} \frac{1}{1 + \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2} \to \frac{1}{3}$

NB: at the intermediate density, below the perturbative regime,
$\varepsilon - 3P = 0$ and $v_s^2 = 1/3$ are **different conditions**

Trace anomaly and effective d.o.f. in NSs

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

- Trace anomaly:
  related to the changes in the effective degrees of freedom ν

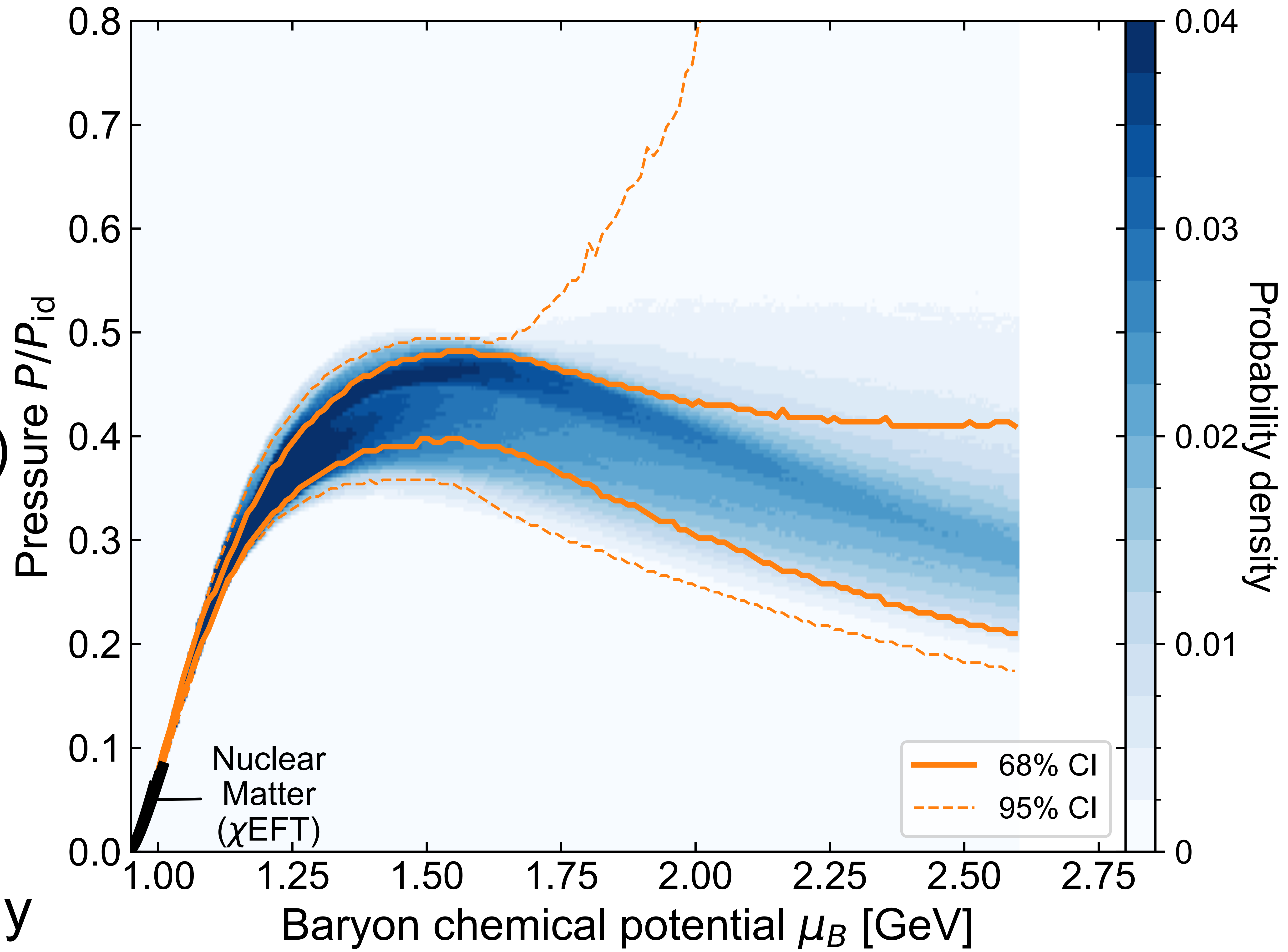
$$\frac{\varepsilon - 3P}{P_{\text{ideal}}} = \frac{d\nu}{d\ln\mu}$$

$$(\nu = P/P_{\text{ideal}},\ P_{\text{ideal}} = N_c N_f \frac{\mu^4}{12\pi^2})$$

- $\nu \sim 1$ in quark matter regime

- If ν increases: positive trace anomaly
  if ν decreases: negative trace anomaly

Bayesian inference from NS data w/o pQCD input:



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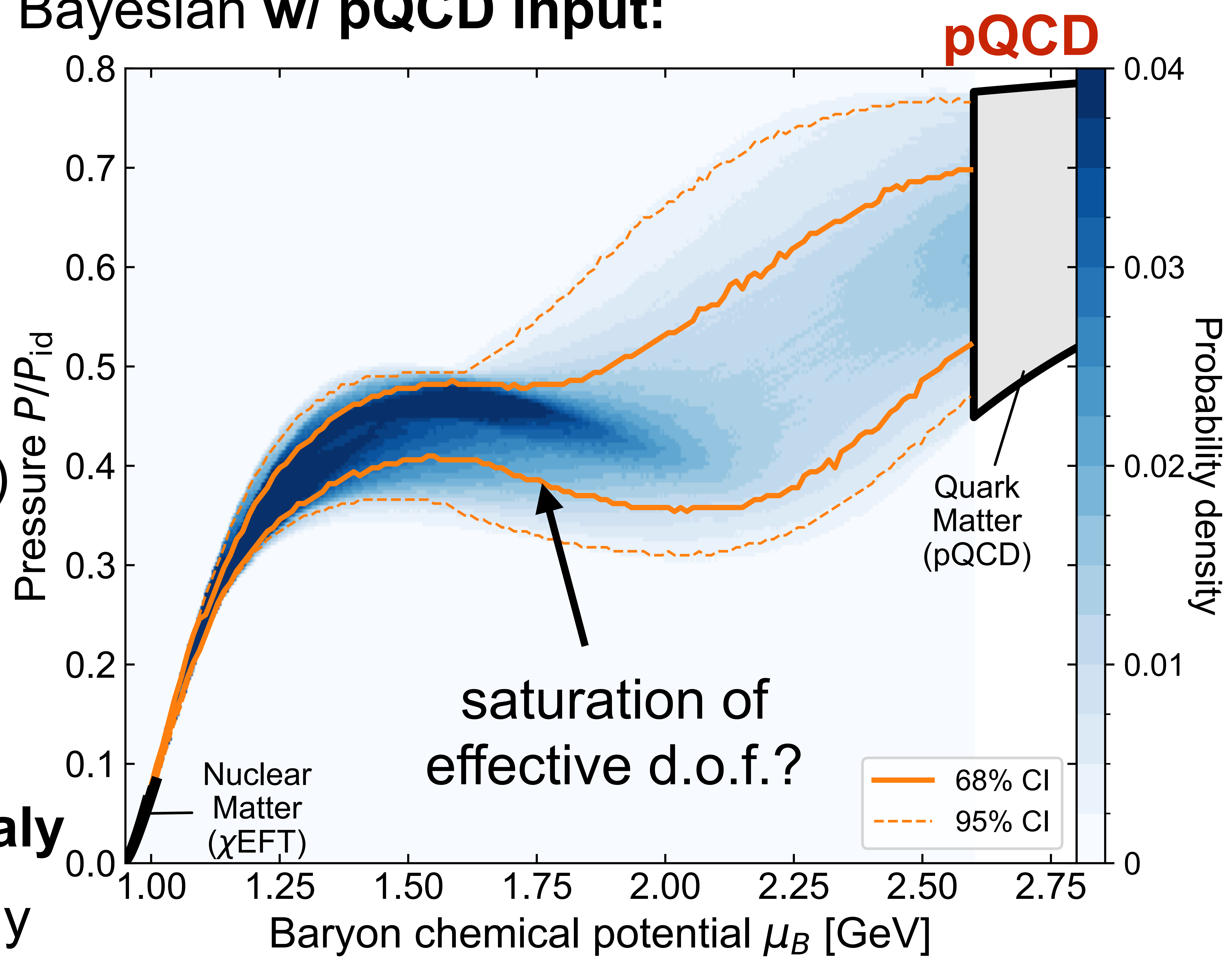


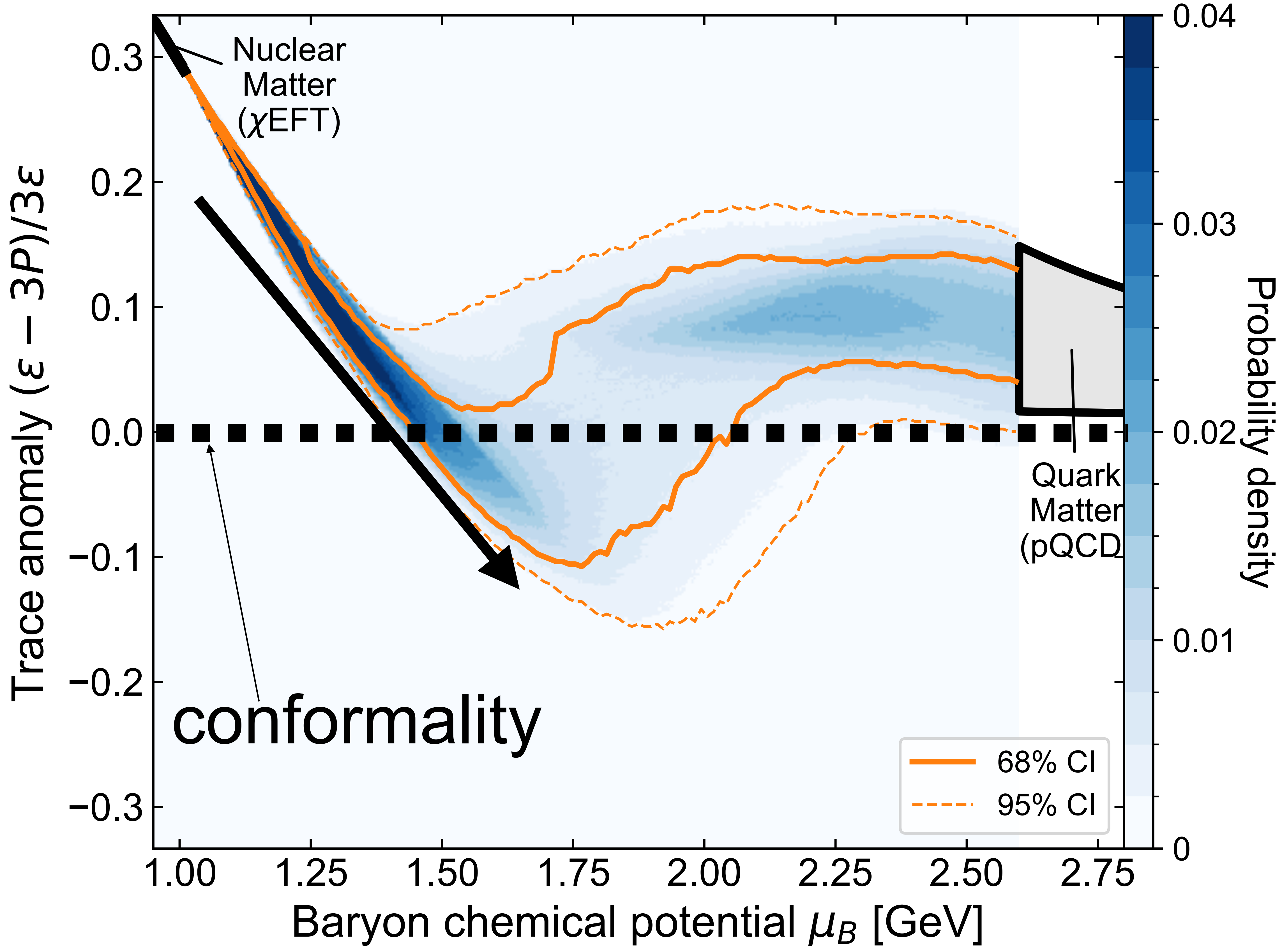
Positive trace anomaly favored by QCD effect

Trace anomaly and peak in sound speed

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

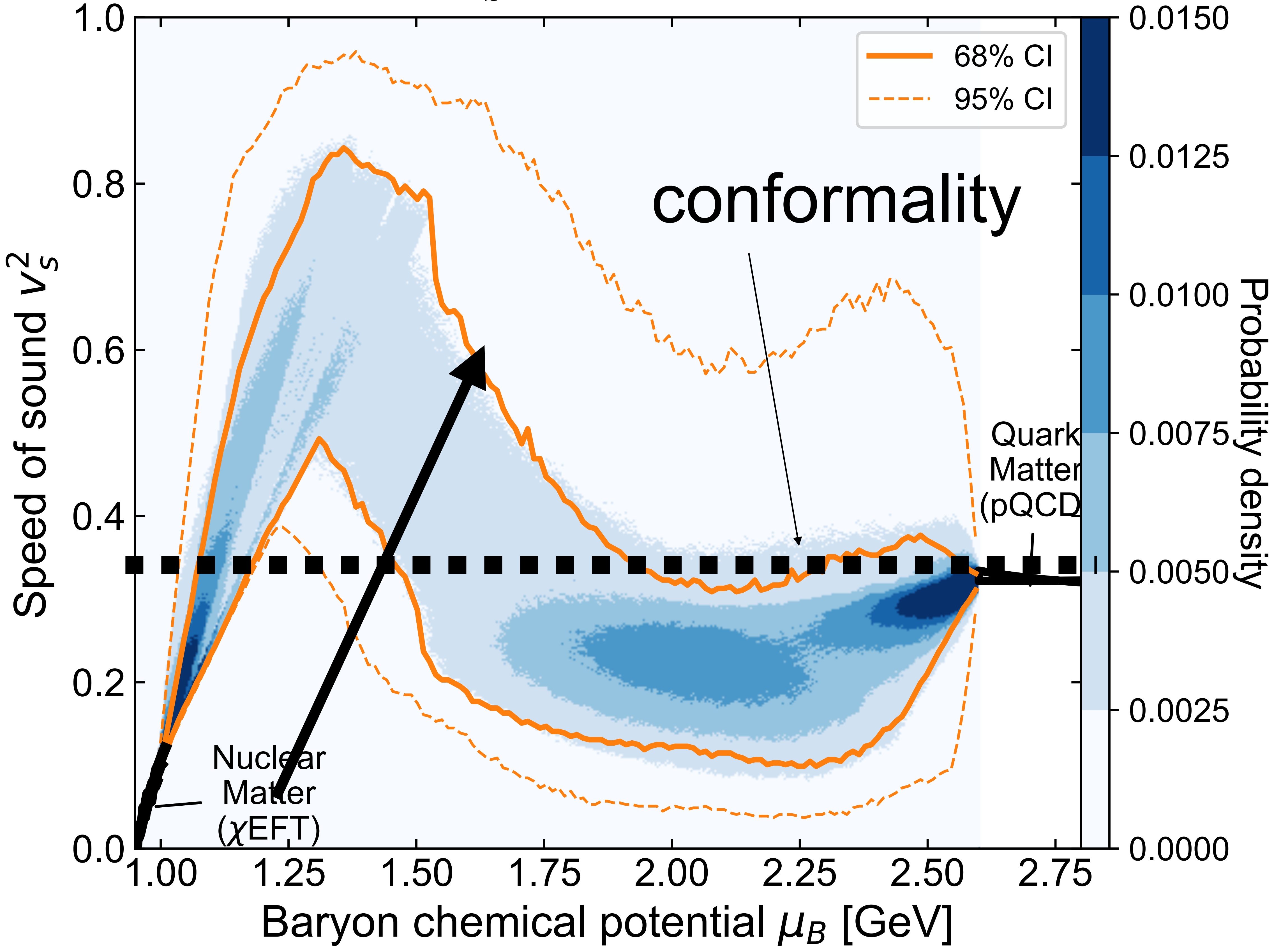
Normalized trace anomaly:

$(\varepsilon - 3P)/3\varepsilon$



Sound speed:

$v_s^2 = dP/d\varepsilon$



Rapid approach to $\varepsilon - 3P \to 0$ drives the peak in v_s^2

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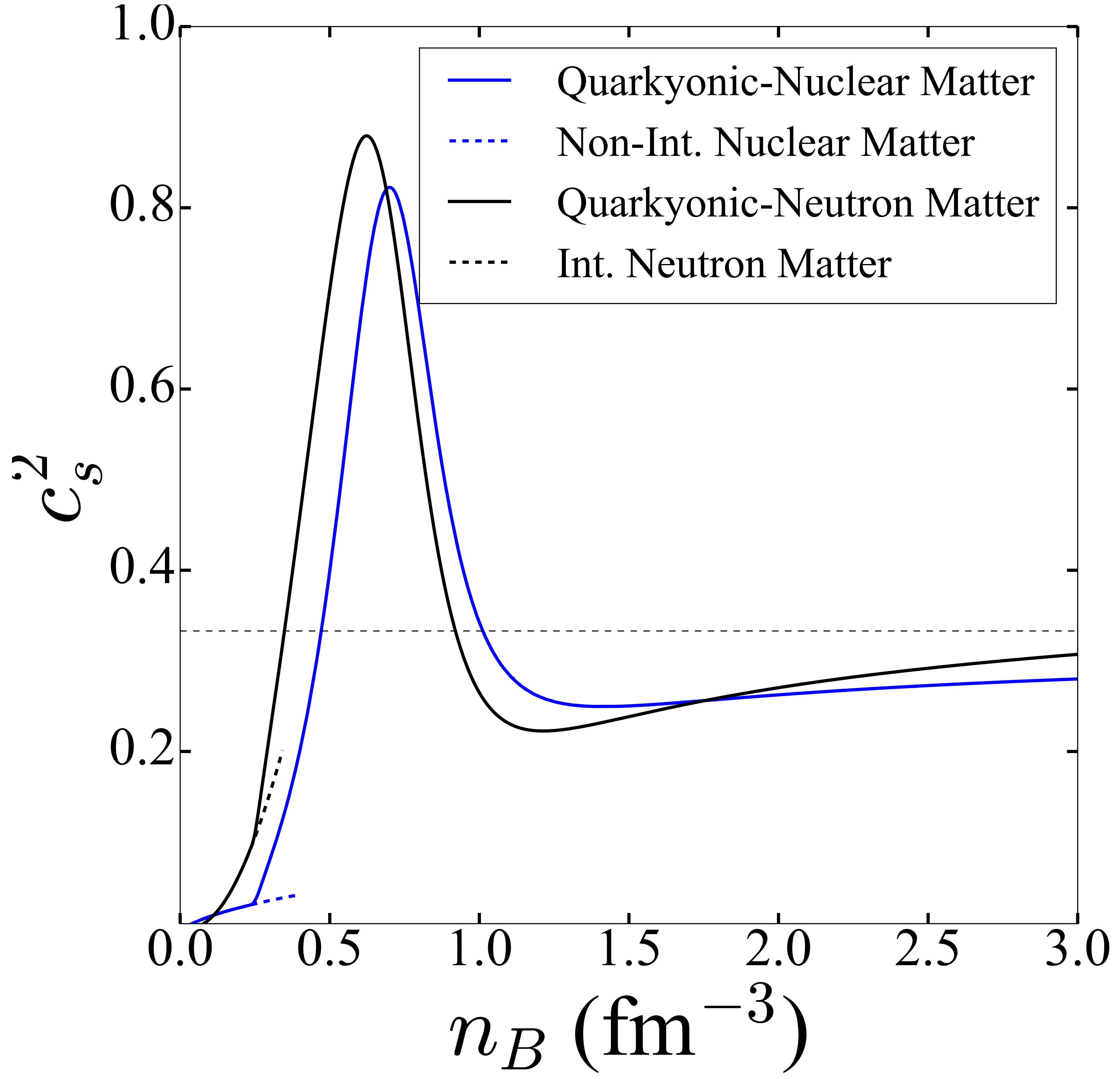
Quarkyonic matter

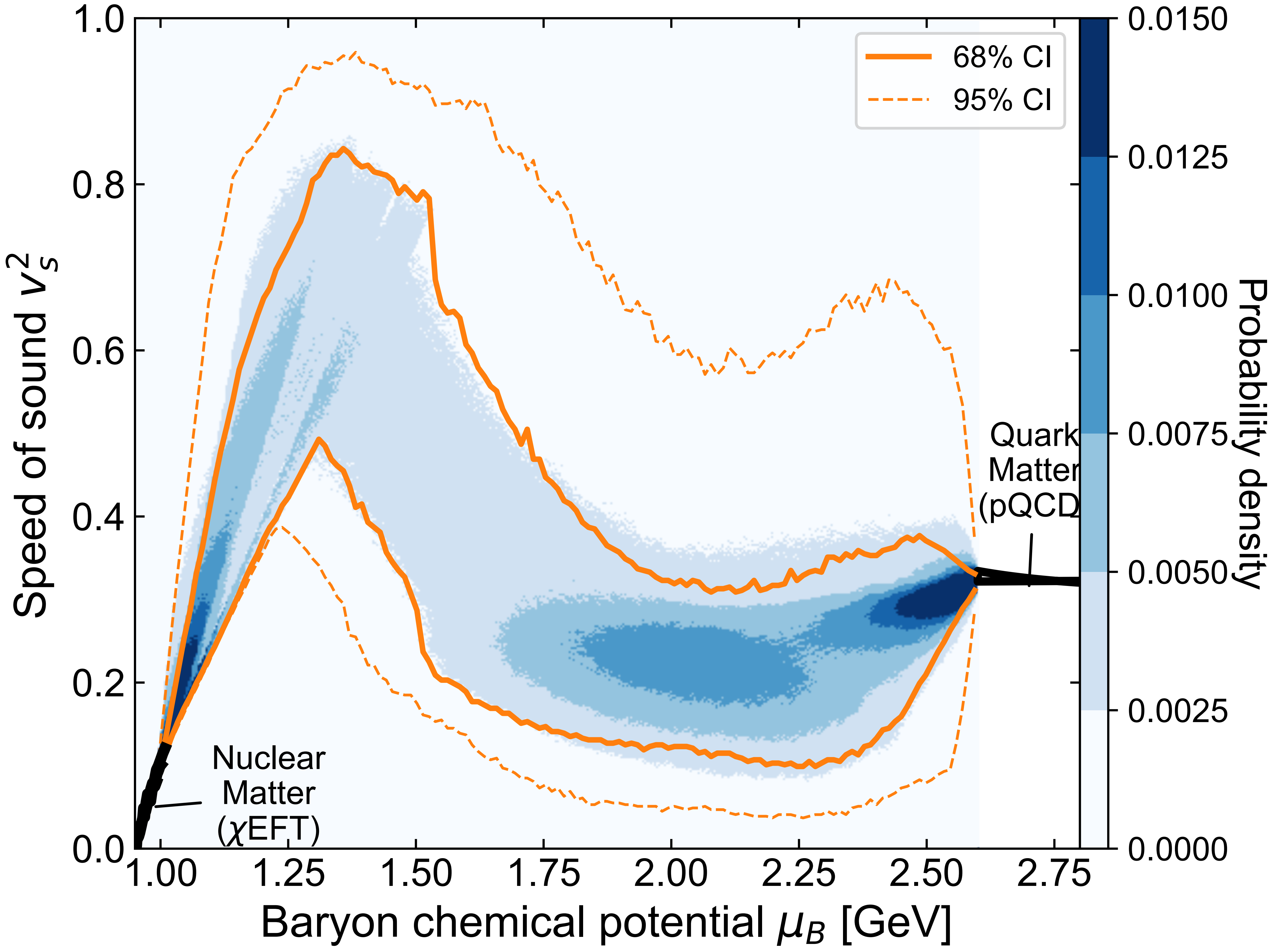
Looks very similar!?

Model EoS of Quarkyonic matter



McLerran,Reddy (2018)

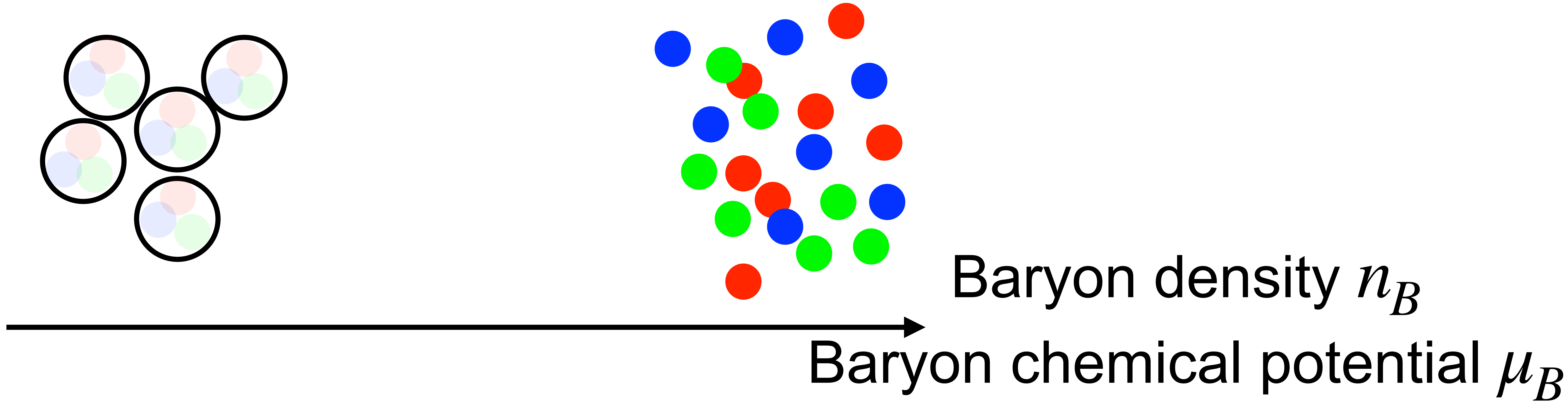
Bayesian



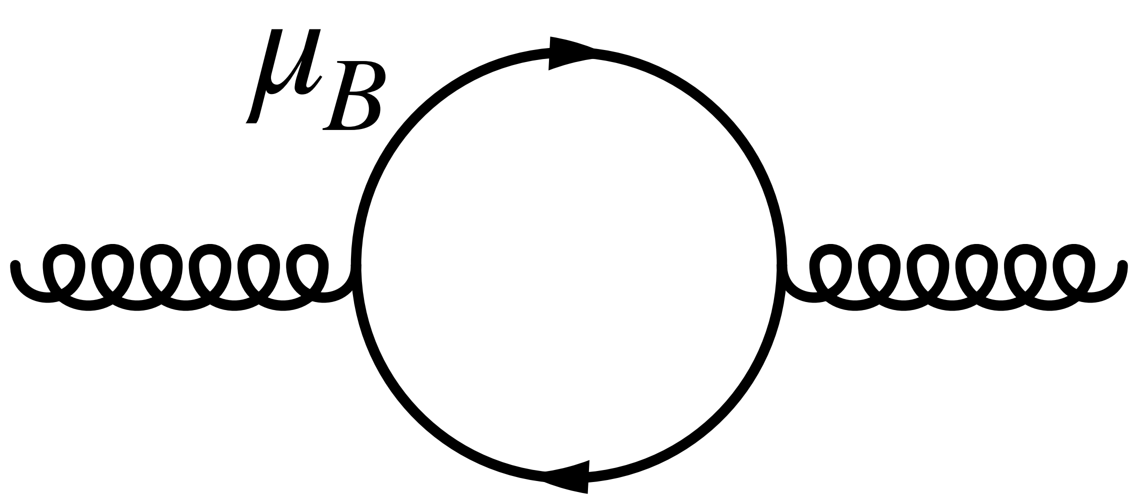
Confinement at high baryon densities

Collins & Perry (1974): **Naive picture of quark deconfinement at high density**

In weak-coupling regime at high density, quarks liberate



This is led by screening of the confinement potential

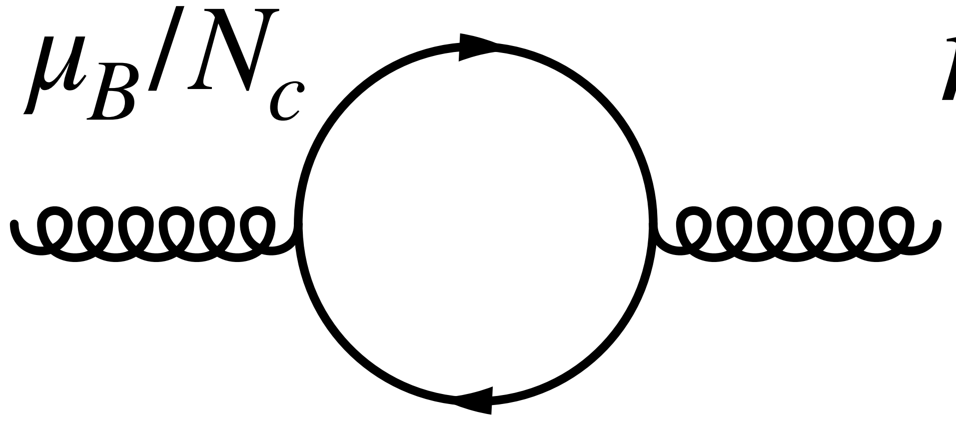


Confinement at high baryon densities

McLerran & Pisarski (2007): **Quarkyonic duality**

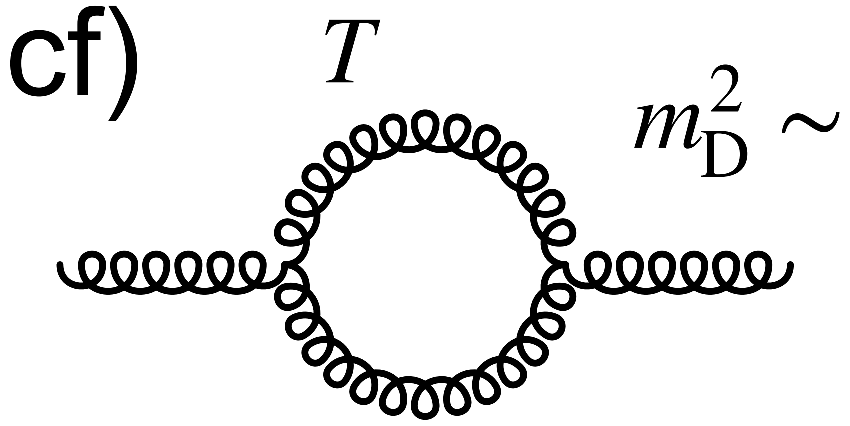
In Large-N_c QCD…

$\mu = \mu_B / N_c$



$$m_{\rm D}^2 \sim g^2 \mu^2 \sim \lambda_{'t\,Hooft} \mu^2 / N_c \to 0$$

cf)



$m_{\rm D}^2 \sim g^2 N_c T^2$

$\sim \lambda_{'t\,Hooft} T^2$

… confinement is never affected by quarks!

Dense QCD matter can be described either as

- Confined baryons (because confining interaction is never screened)
- (Weakly-coupled) Quarks

→ implies duality (paradox?) between quark and confined baryonic matter

Quark yonic

Confinement at high baryon densities

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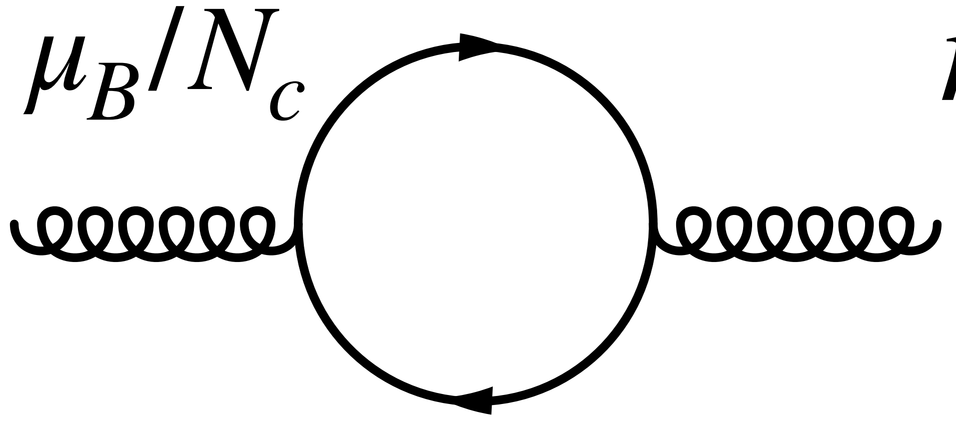

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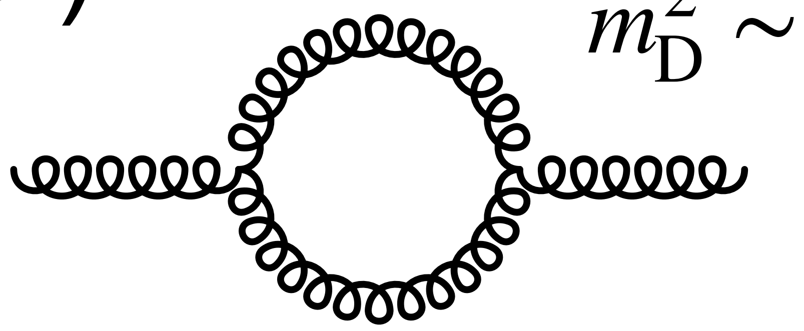


$$m_D^2 \sim g^2 N_c T^2 \sim \lambda_{'t\ Hooft} T^2$$

… confinement is never affected by quarks!

Dense QCD matter can be described either as $\quad m_D^2 \ll \Lambda_{QCD}^2 \to \mu \ll \sqrt{N_c}\Lambda_{QCD}$

- Confined baryons (because confining interaction is never screened)
- (Weakly-coupled) Quarks $\quad \mu \gg \Lambda_{QCD}$

→ **implies duality (paradox?) between quark and confined baryonic matter**

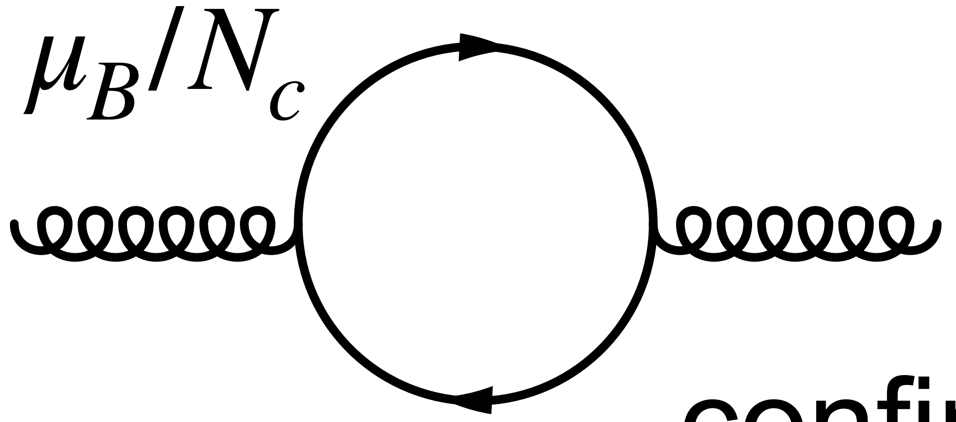
Quark yonic

Confinement at high baryon densities

McLerran & Pisarski (2007): **Quarkyonic duality**

In $N_c = 3$ QCD…

$\mu = \mu_B / N_c$



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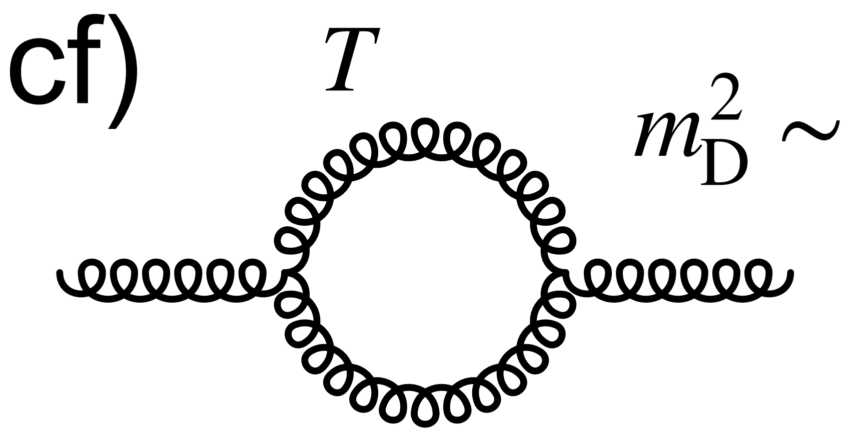


$m_D^2 \sim g^2 N_c T^2$

$\sim \lambda_{'t\ Hooft} T^2$

… confinement is **less** affected by quarks!

inefficient deconfinement compared to finite-T case

Speculation:

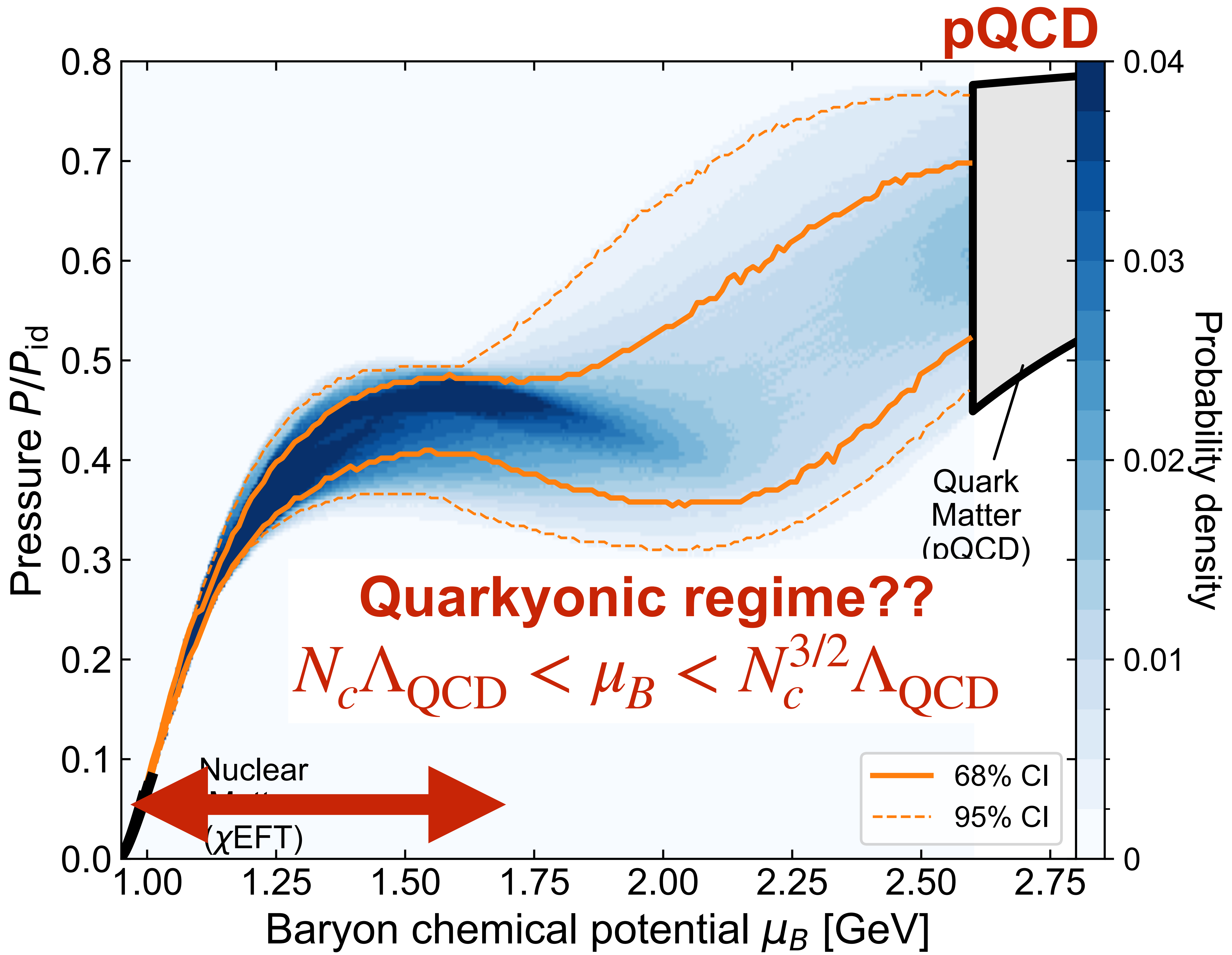
Duality persists in finite-Nc, i.e.,

Dense QCD matter can be described either as baryons and quarks at

$$\Lambda_{QCD} < \mu < \sqrt{N_c}\Lambda_{QCD}$$ … Quarkyonic regime

Trace anomaly and effective d.o.f.

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)



$(\Lambda_{QCD} \simeq 310\,\mathrm{MeV})$

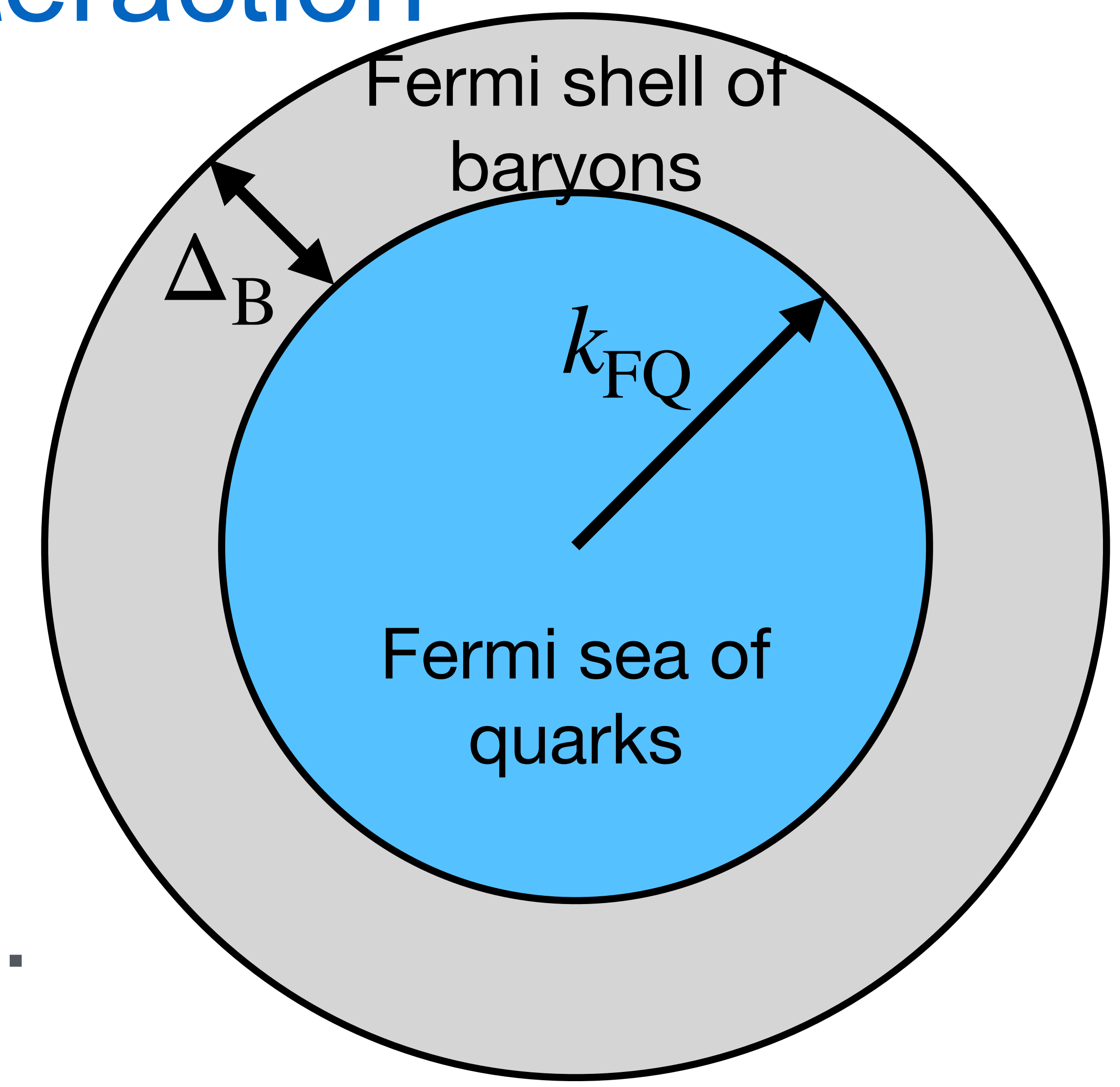
Quarkyonic "shell" model

McLerran & Pisarski (2007):

To resolve the duality "paradox", the following picture of Fermi shell of baryons is proposed:

Fermi sea: dominated by interaction that is less sensitive to IR
→ quarks

Fermi shell: interaction sensitive to IR d.o.f.
→ baryons, mesons, glues…



see also: Jeong,McLerran,Sen (2019)

Quarkyonic model for neutron stars

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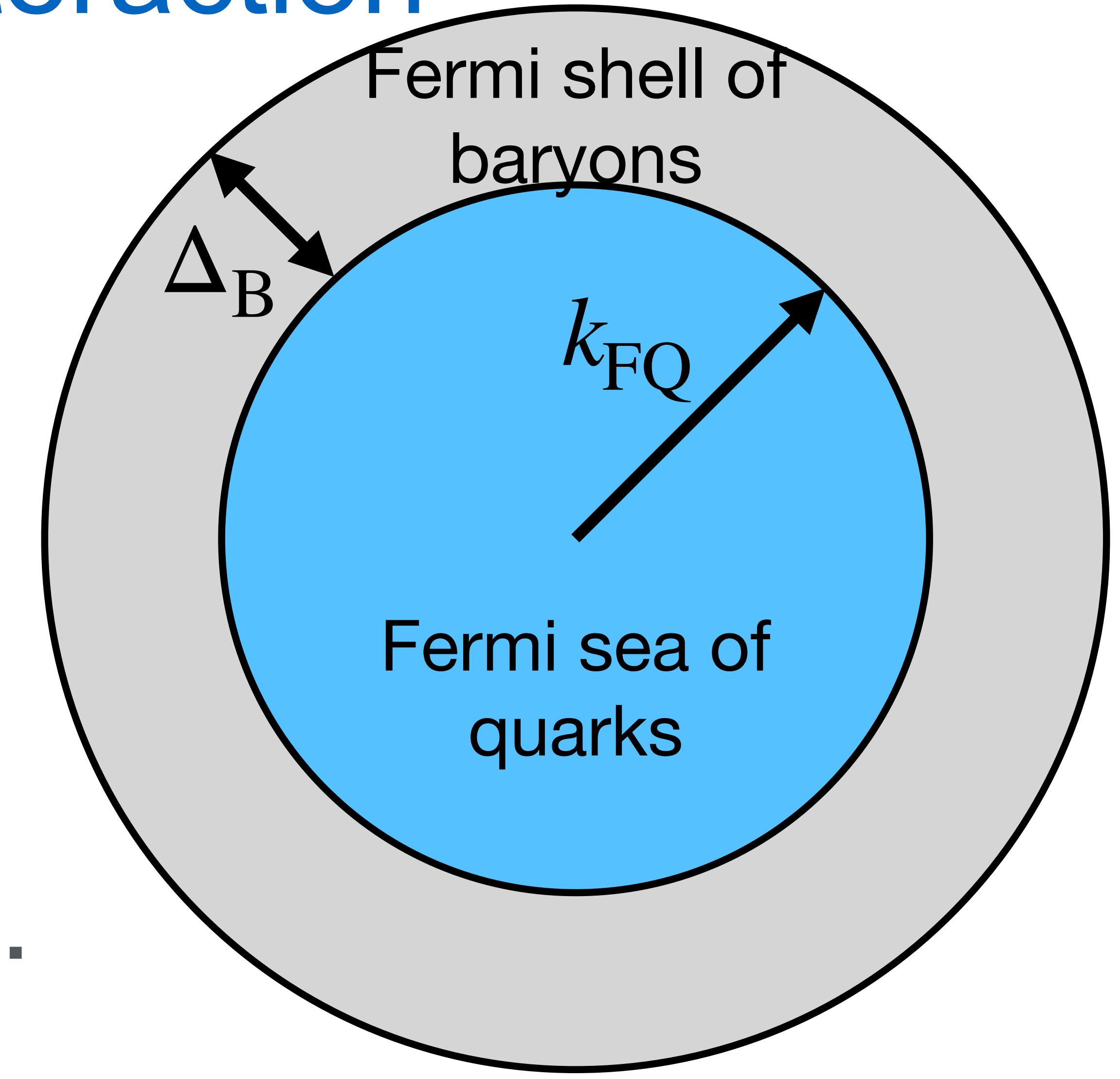
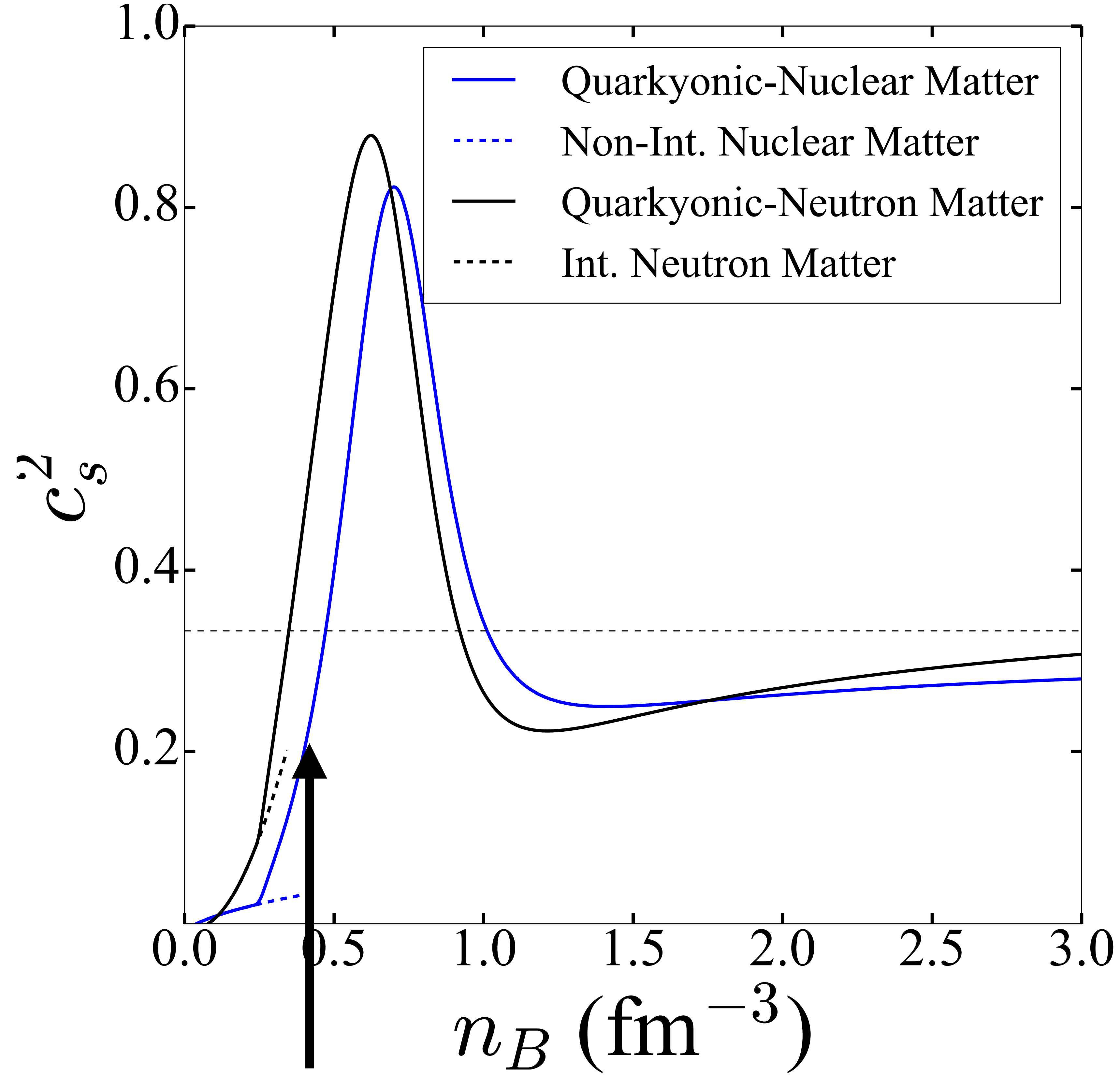


see also: Jeong,McLerran,Sen (2019)

McLerran,Reddy (2018):

Quarkyonic model applied to NS EoS:



can reproduce rapid stiffening in EoS
(the only robust feature confirmed in NS EoS)

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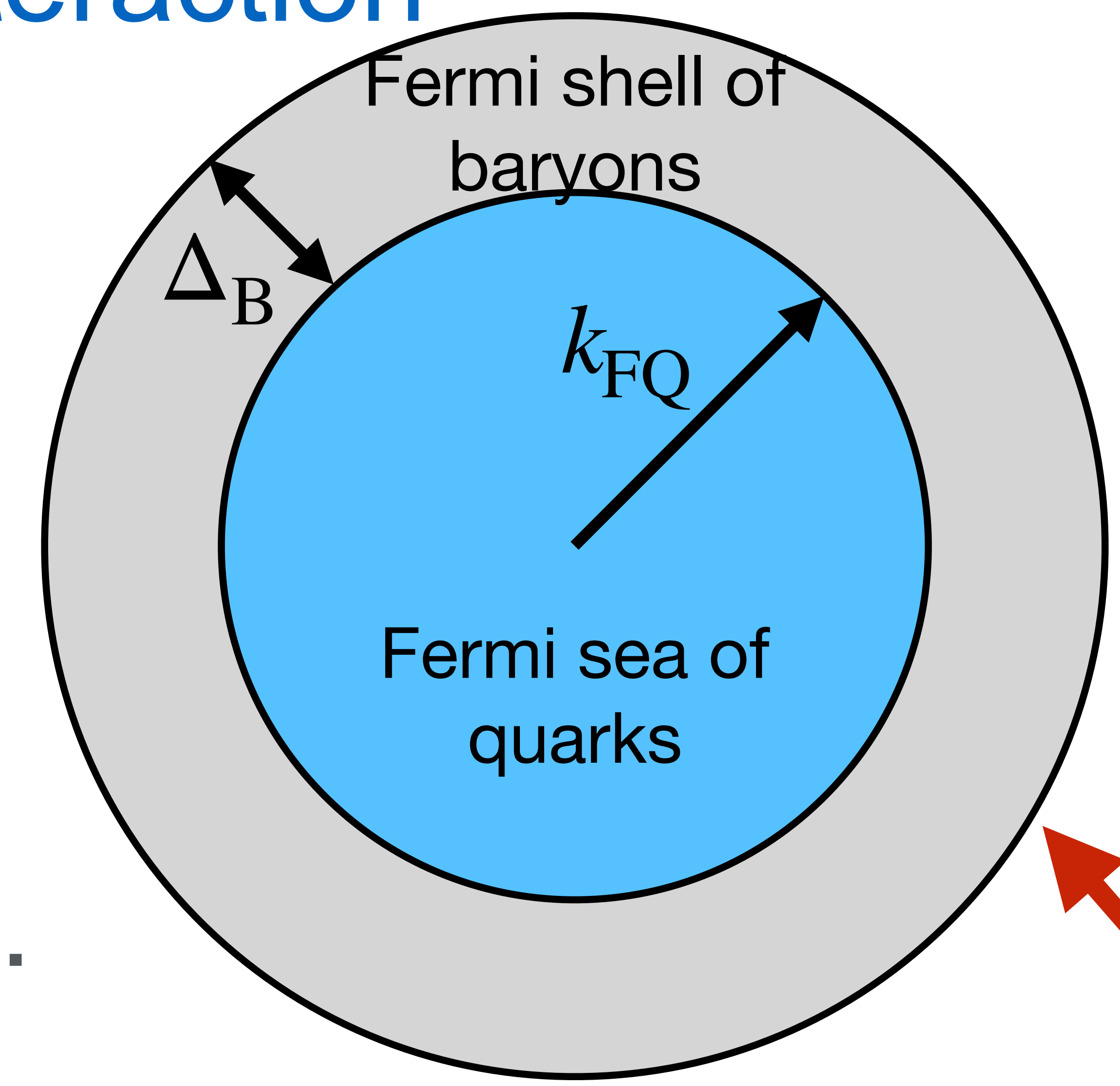

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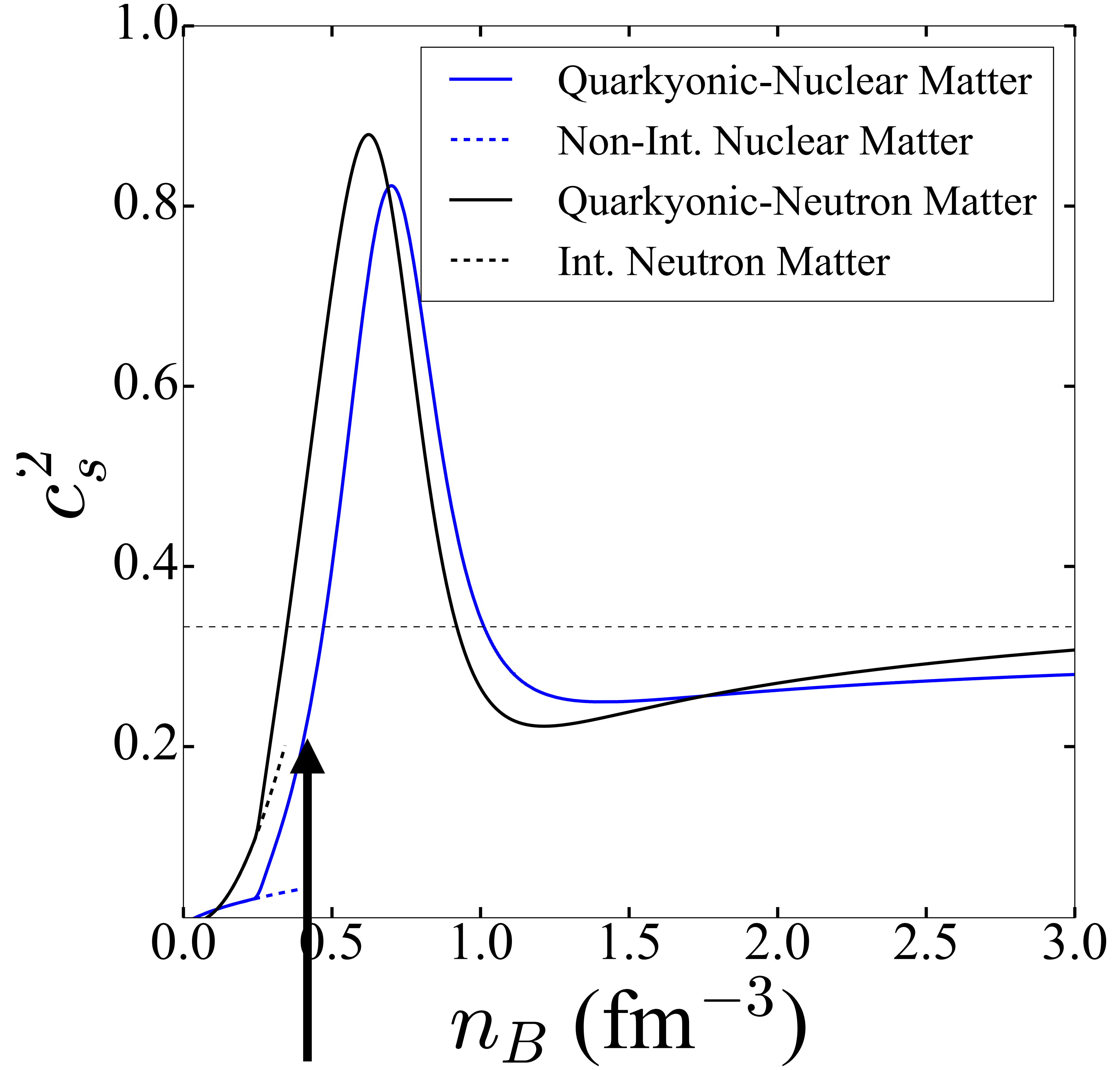


can reproduce rapid stiffening in EoS

nfirmed in NS EoS)

This talk: reinterpretation of this baryon shell

Duality in Fermi gas model

Kojo (2021); Fujimoto,Kojo,McLerran (2023)

Implement duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

Fermi gas model w/ an explicit duality:

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{k}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

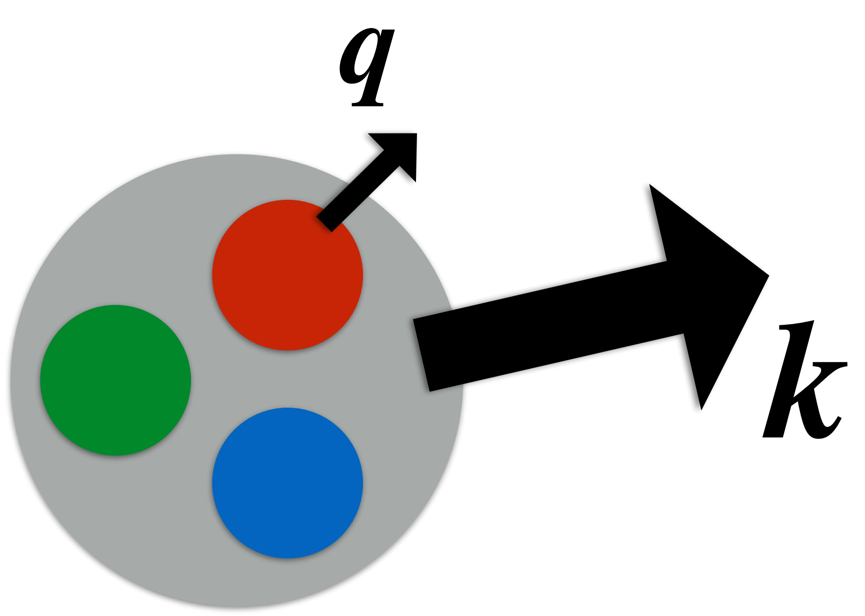
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

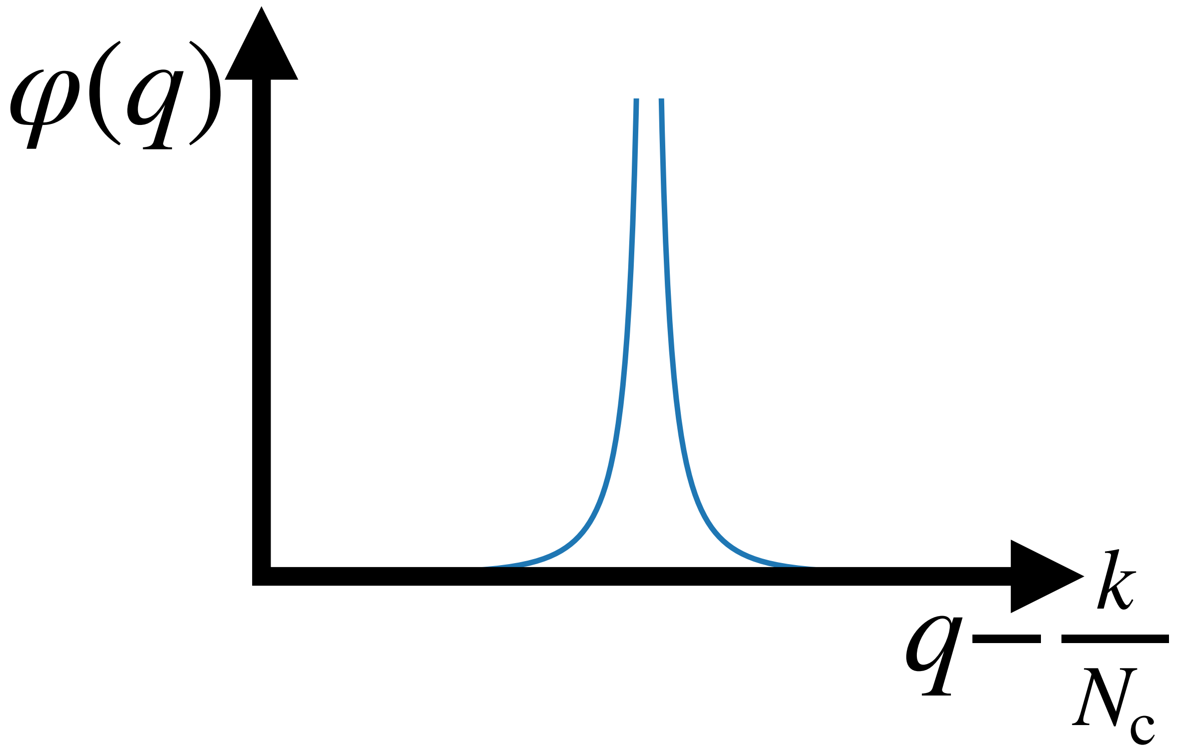
$0 \leq f_{\mathrm{B,Q}} \leq 1$: Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$: ideal baryon dispersion relation

Modeling of confinement:

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$





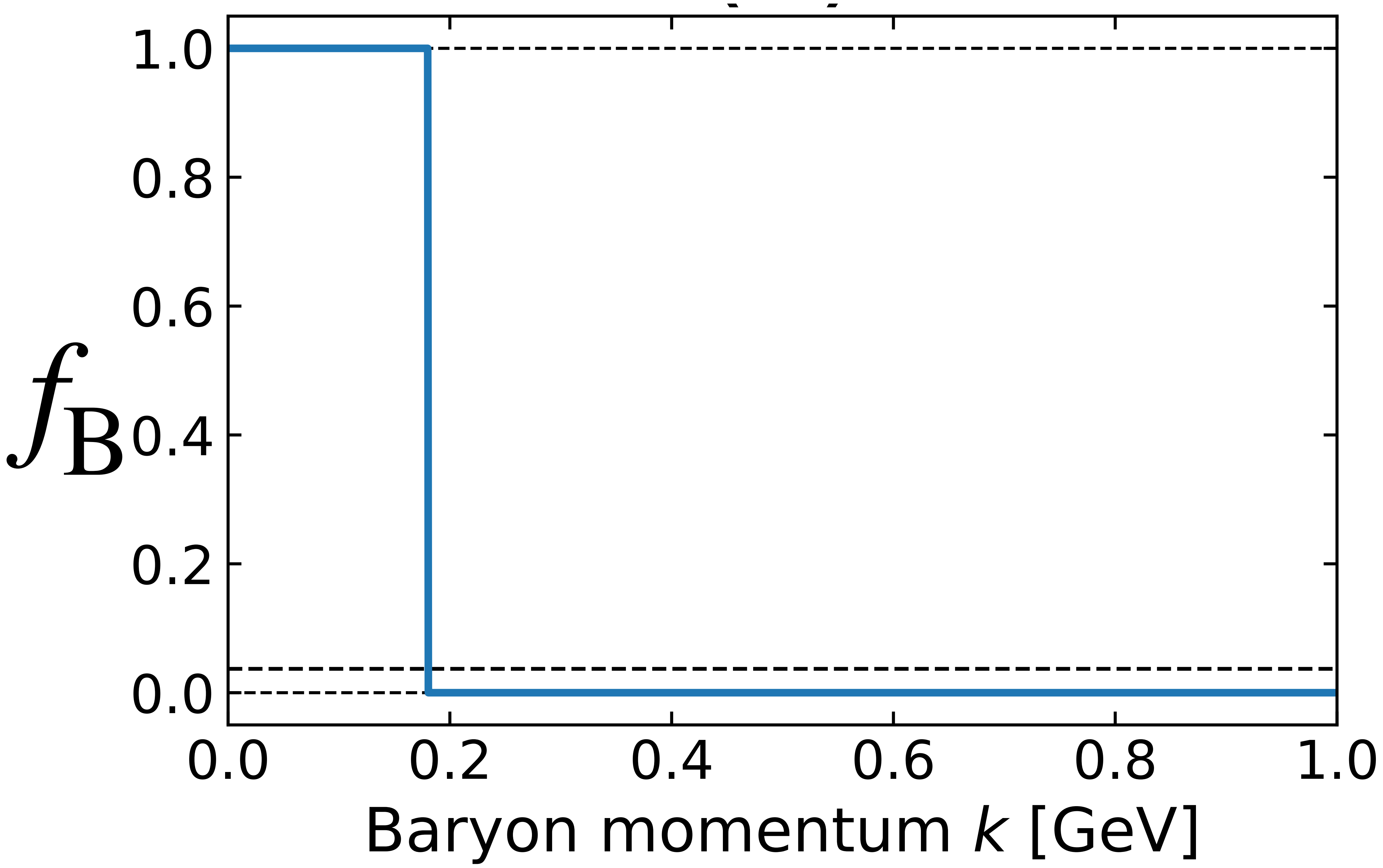
Ideal dual Quarkyonic model (**IdylliQ** model)
→ Find a solution for f_{B} and f_{Q} with minimum ε at a given n_B

Solution of IdylliQ model

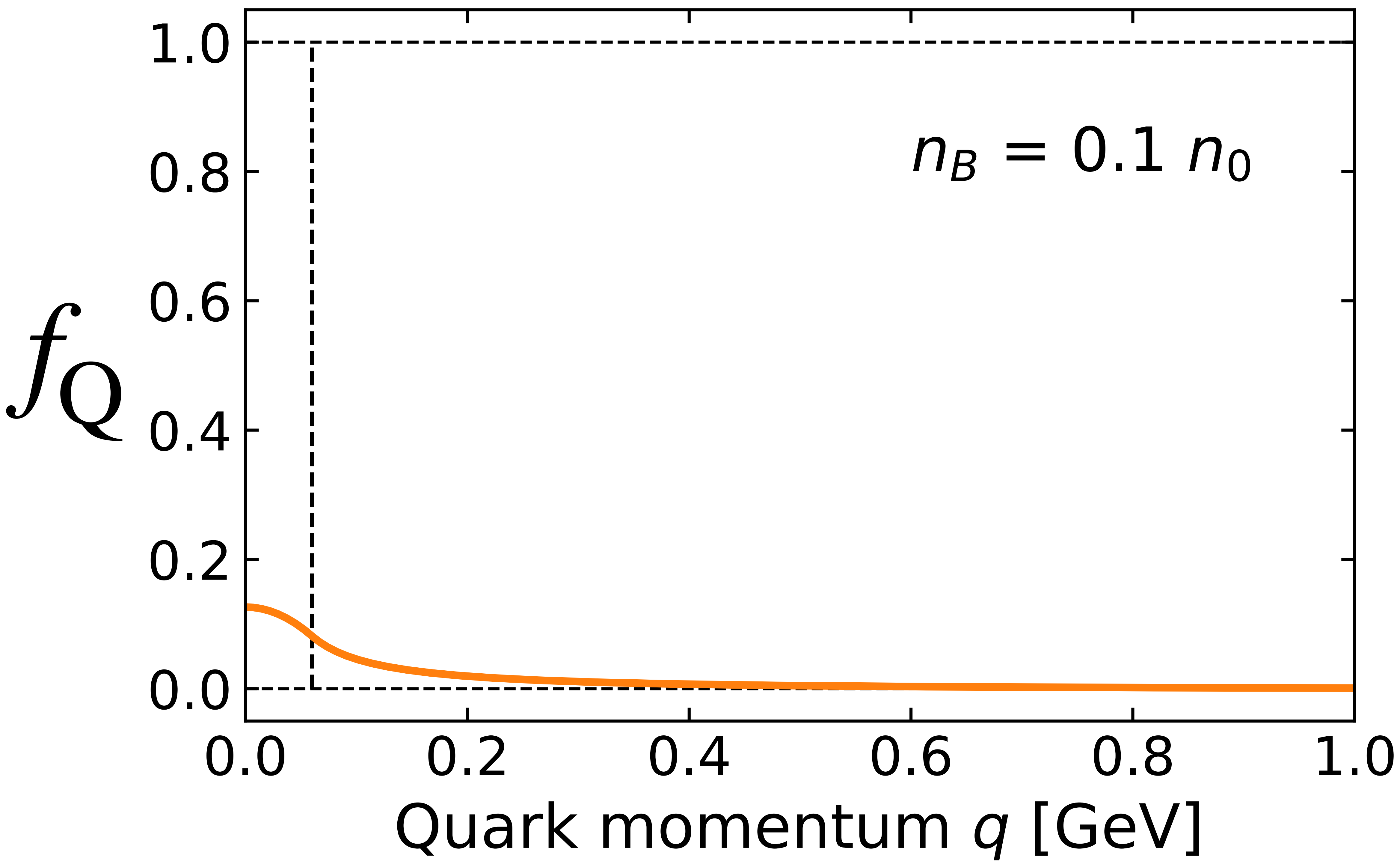
Kojo (2021); Fujimoto,Kojo,McLerran (2023)

At low density…

Fermi-Dirac distribution for baryons



Quarks do not fill up the Fermi sea yet

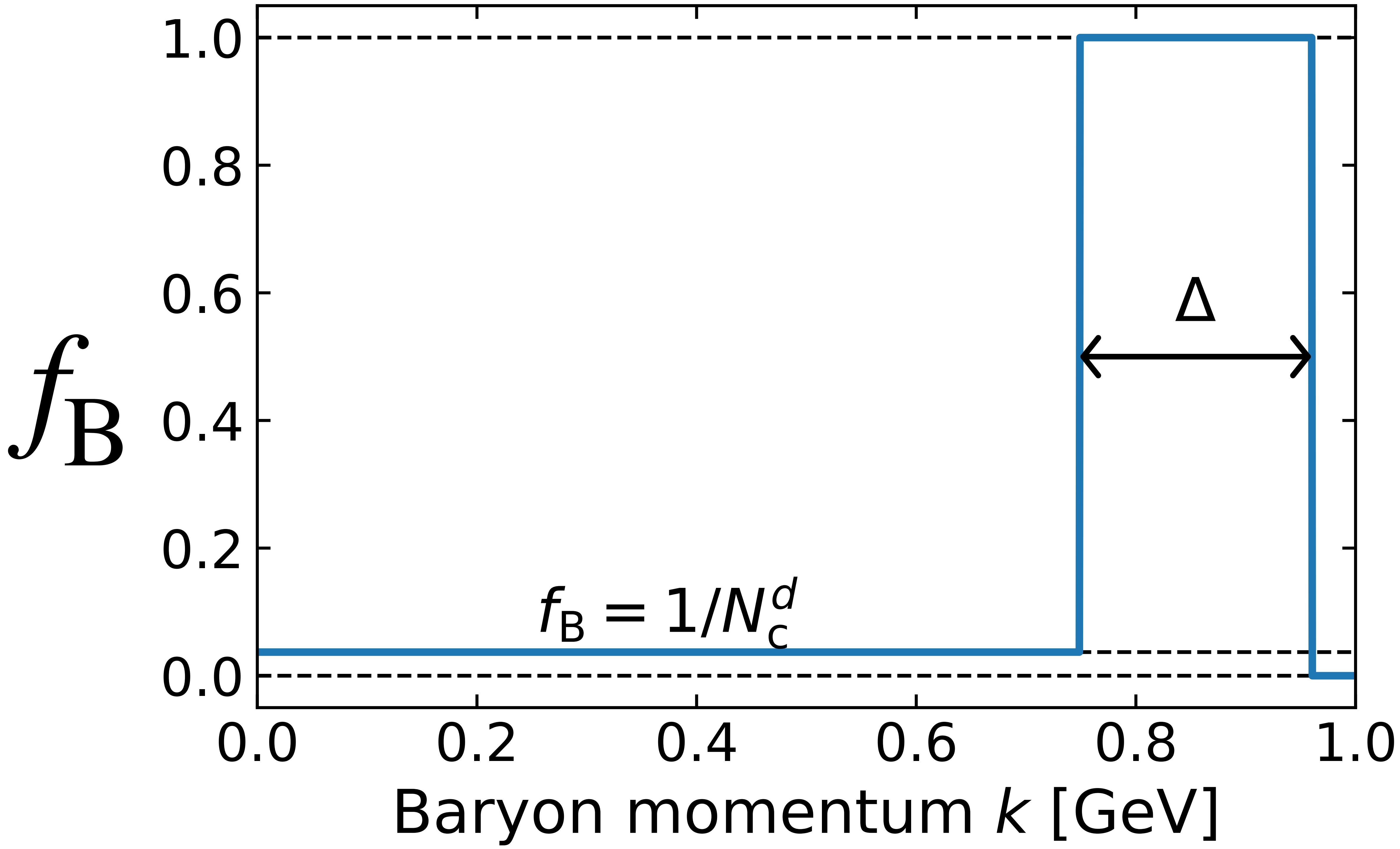


Solution of IdylliQ model

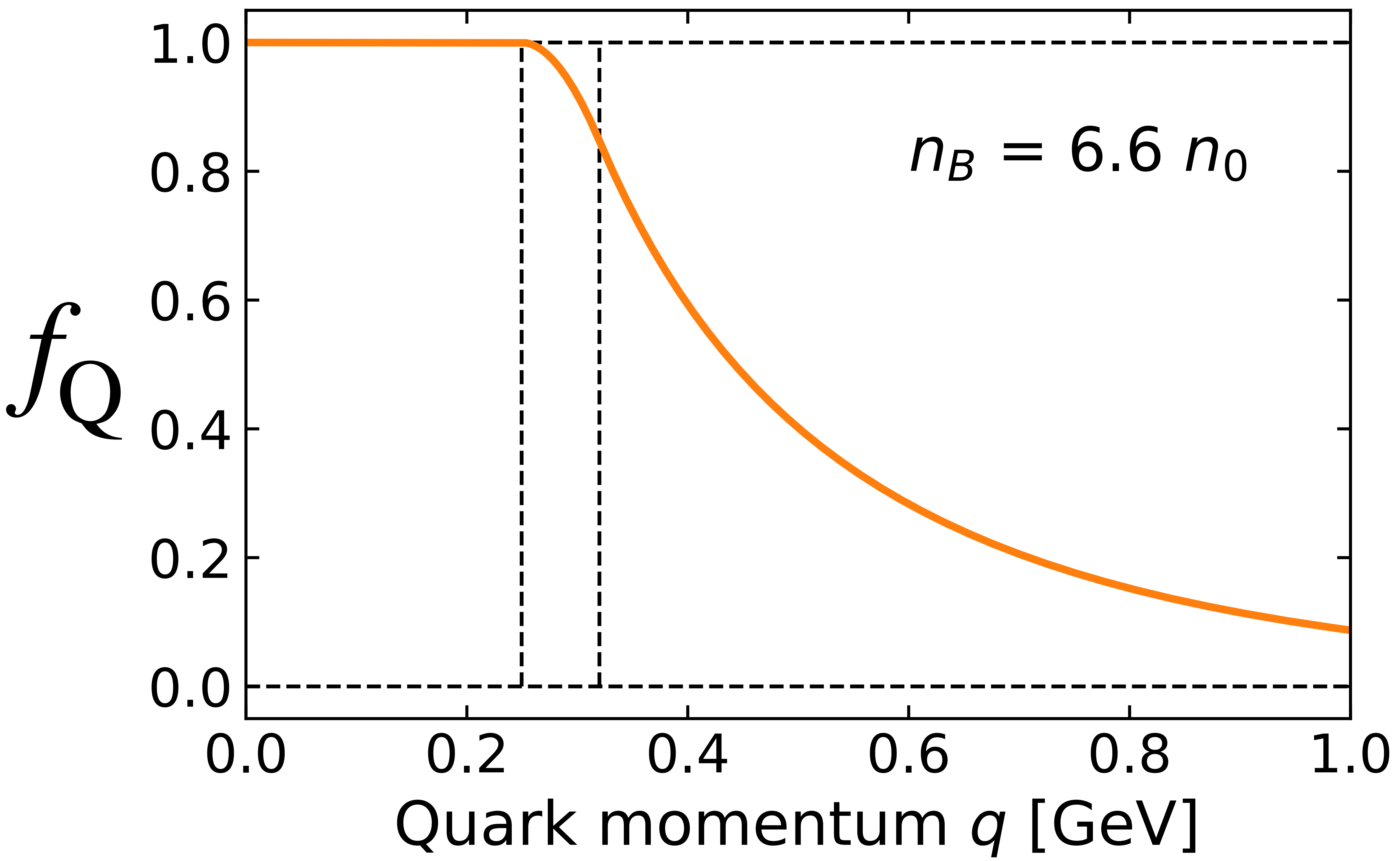
Fujimoto,Kojo,McLerran (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



Quark obeys the FD distribution (with a tail from confinement)



Equivalence to Quarkyonic model

Fujimoto,Kojo,McLerran (2023)

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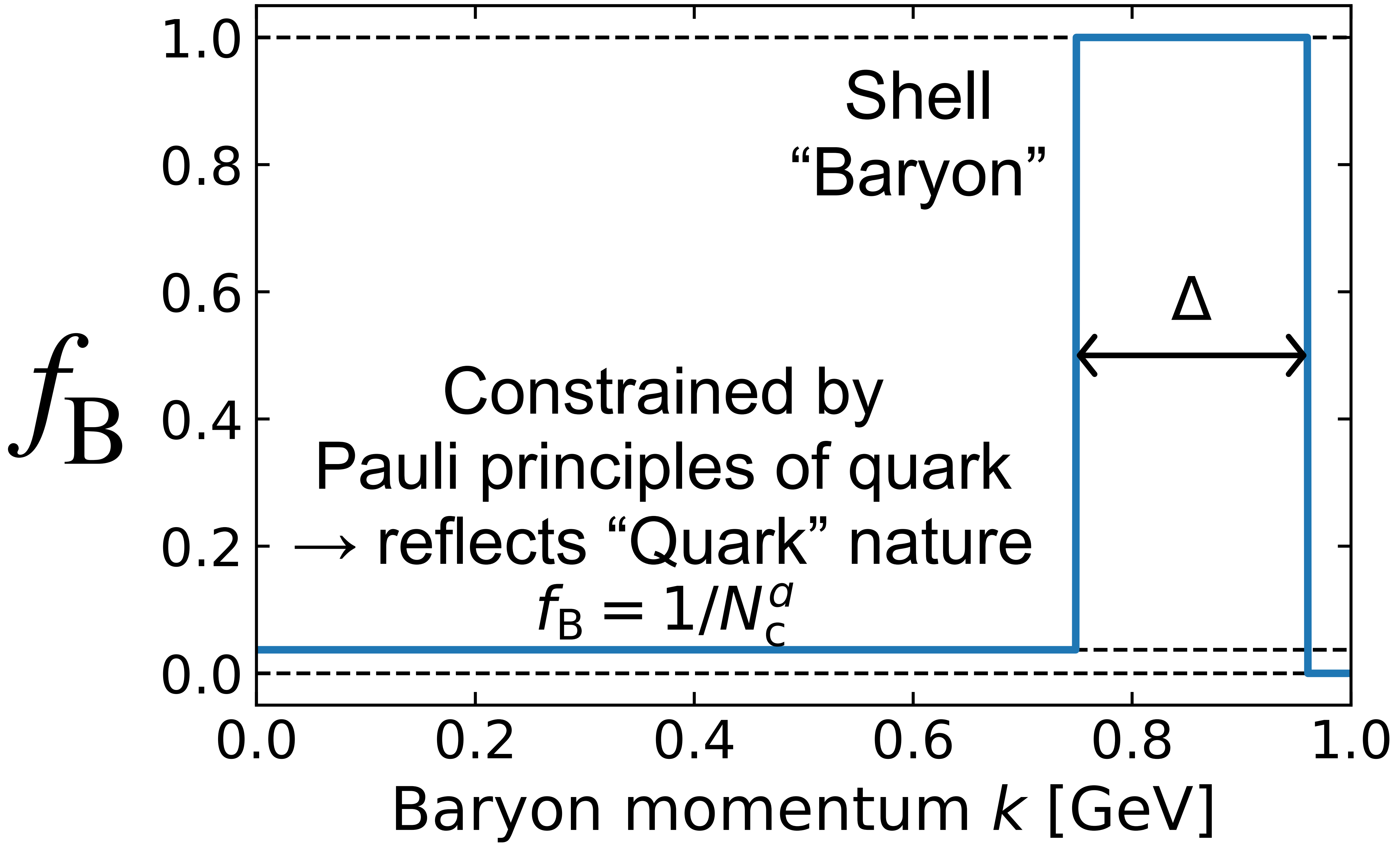
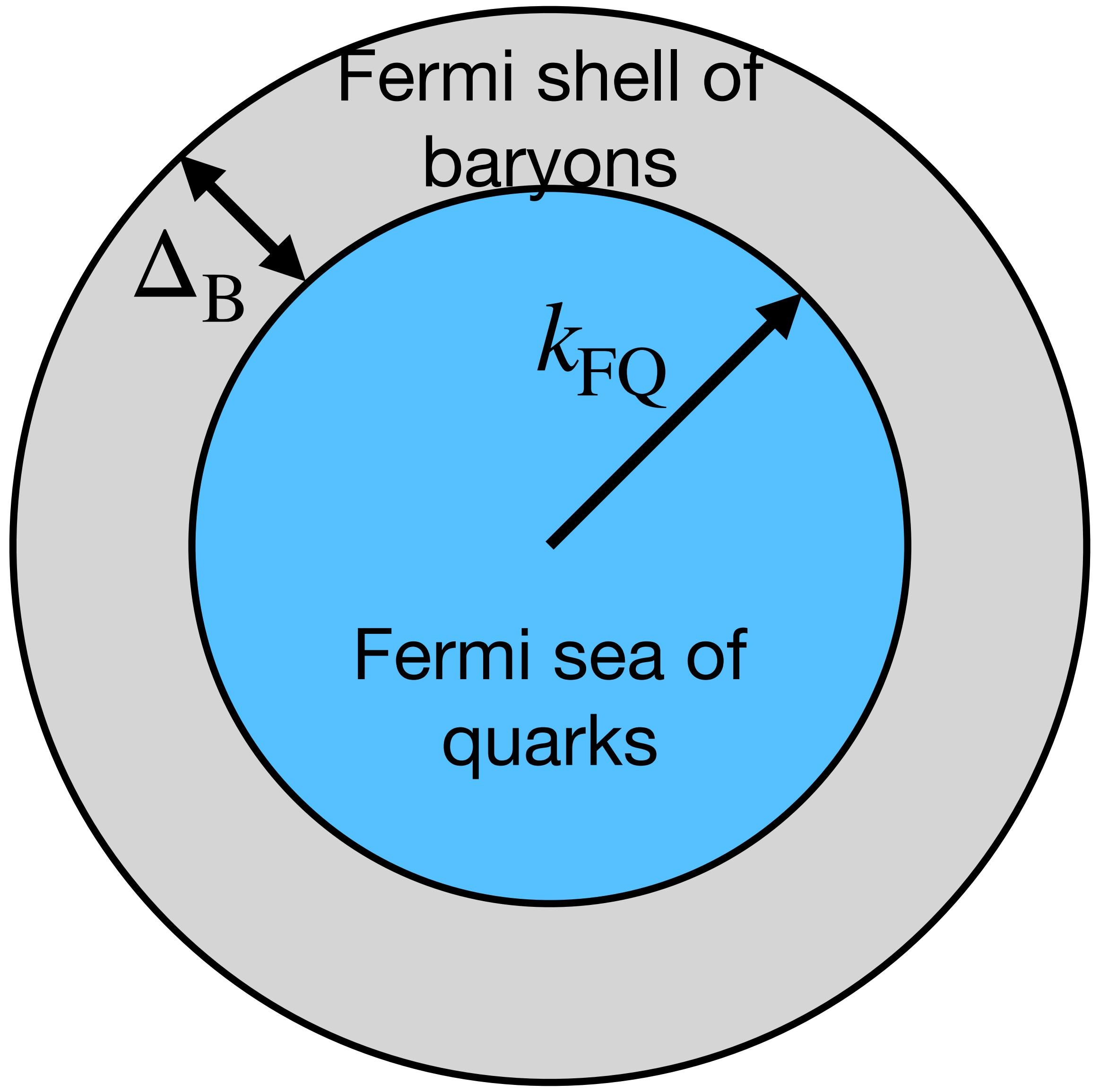
Fermi-Dirac distribution for baryons is modified





McLerran,Pisarski (2007)
McLerran,Reddy (2018)
Jeong,McLerran,Sen (2019)

Fermi shell structure arises in f_B
(Note: this is **purely baryonic description**)

This picture is equivalent to McLerran-Reddy model of the NS based on the McLerran-Pisarski picture

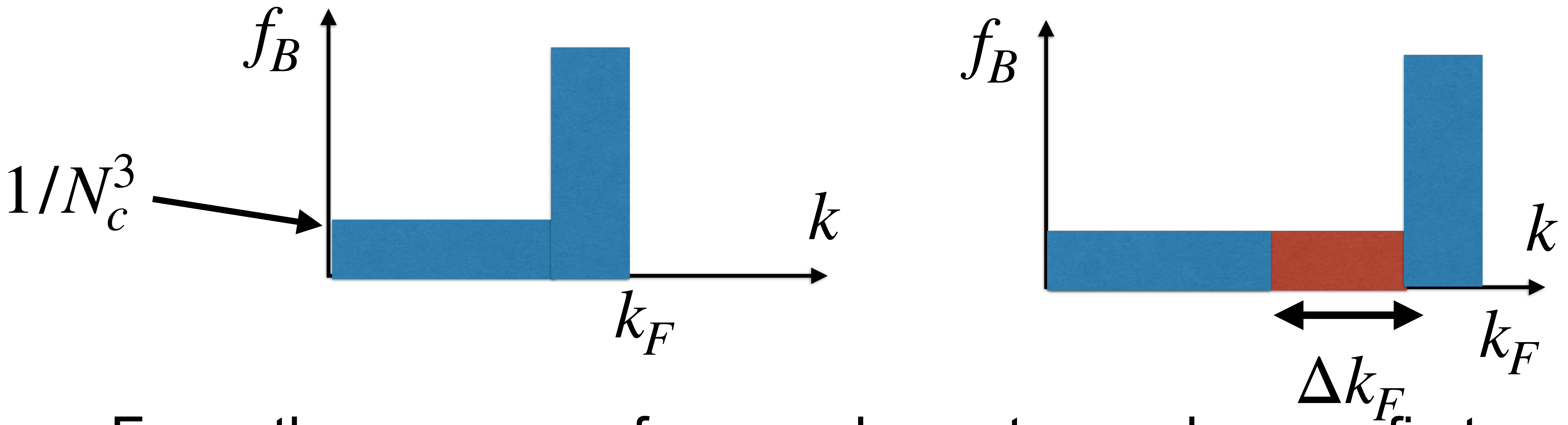
Rapid stiffening in the EoS

Fujimoto,Kojo,McLerran (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_B}{\mu_B dn_B/d\mu_B} \rightarrow \frac{\delta\mu_B}{\mu_B} \sim v_s^2 \frac{\delta n_B}{n_B}$$

If baryons have underoccupied state, the change in density is small while the change in Fermi energy ($\sim k_F$) is large



→ Favor the crossover from nucleons to quarks over first-order phase transition

Summary

- **Conformality of the EoS:** Neutron-star data suggests EoS may be conformal (signature of quarks?), leading to peak in the speed of sound

- **Quarkyonic matter:** duality between confined baryons and weakly-coupled quarks

- **Saturation of quark momentum distribution**
  → under-occupied states in baryonic momentum distribution
  (modification from Fermi-Dirac distribution)

- **Implication to hyperon puzzle:**
  Because of the saturation in d-quark states,
  1) The threshold of hyperons shifted to a higher μ_B
  2) The softening in the EoS is milder