Review of experimental situation and prediction with tau

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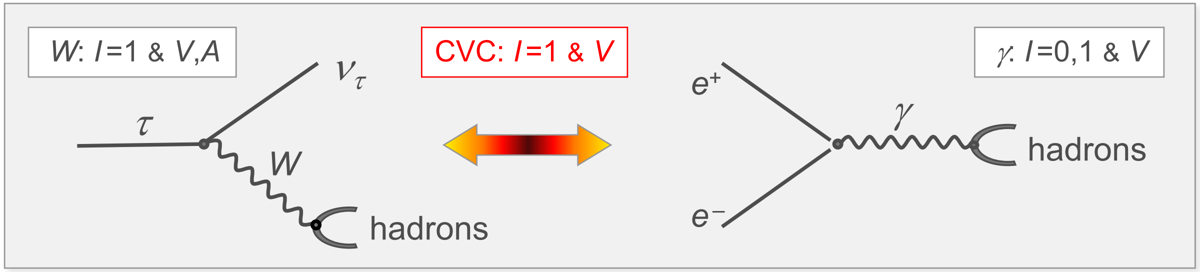
Outline

- ➢ The idea
- ➢ Measurements
- ➢ Isospin-breaking corrections
- ➢ Results and discussions
- ➢ Summary and prospects

The Idea

- The use of tau spectral functions for the LO HVP evaluation was originally proposed by R. Alemany, M. Davier and A. Hoecker (link)
  - Based on CVC (conserved vector current) relations:



$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-} = \frac{4\pi\alpha^2}{s} \upsilon_{1,\pi^-\pi^0\nu_\tau}$$

$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-\pi^+\pi^-} = 2\cdot\frac{4\pi\alpha^2}{s} \upsilon_{1,\pi^-3\pi^0\nu_\tau}$$

$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-\pi^0\pi^0} = \frac{4\pi\alpha^2}{s}\left[\upsilon_{1,2\pi^-\pi^+\pi^0\nu_\tau} - \upsilon_{1,\pi^-3\pi^0\nu_\tau}\right]$$

where υ are the relevant spectral functions in the 2π and 4π tau decay channels

  - Initial isospin-breaking corrections were discussed
    - Mass and width differences between charged and neutral rho resonances
    - EW radiative corrections

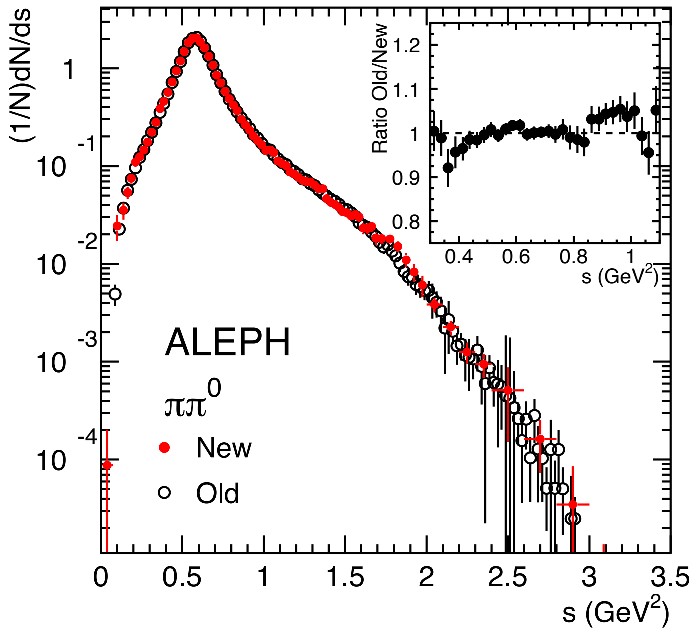
Measurements

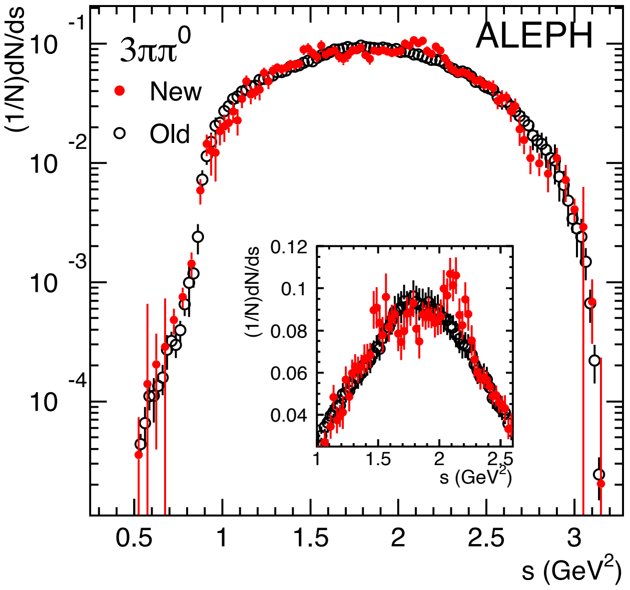
- Relevant tau spectral functions from tau decays were measured by
  - In the two-pion channel ($\tau \rightarrow \pi\pi^0\nu_\tau$):
    - by ALEPH and OPAL @ Z pole (LEP)
    - by CLEO and BELLE @ Υ(4S)
  - In the four-pion channels ($\tau \rightarrow 3\pi\pi^0\nu_\tau$, $\tau \rightarrow \pi 3\pi^0\nu_\tau$):
    - by ALEPH, CLEO and OPAL

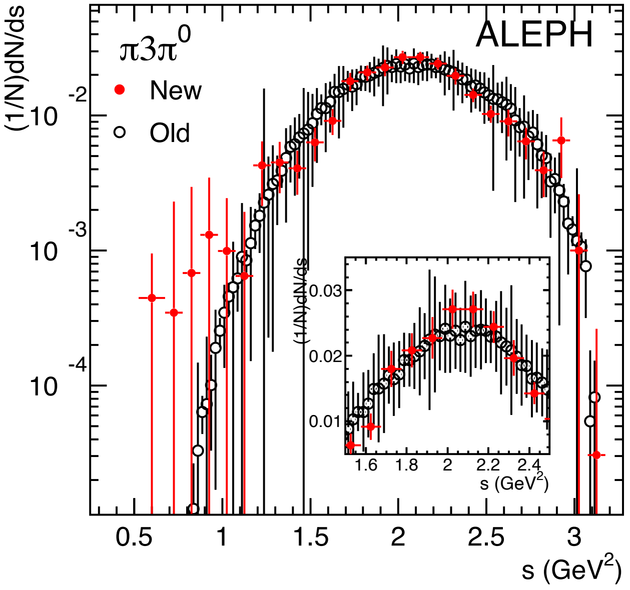
- Very different experimental conditions
  - @LEP: τ pairs can be selected with high efficiency (>90%) and small non-τ background (<1%)
  - @Υ(4S): lower efficiency and higher background but well separated final states

- ALEPH measured not only spectral functions but also branching fractions of all hadronic modes
  - ALEPH still has the best measurements for these and other channels, though performed 20 years ago

Measurements of ALEPH



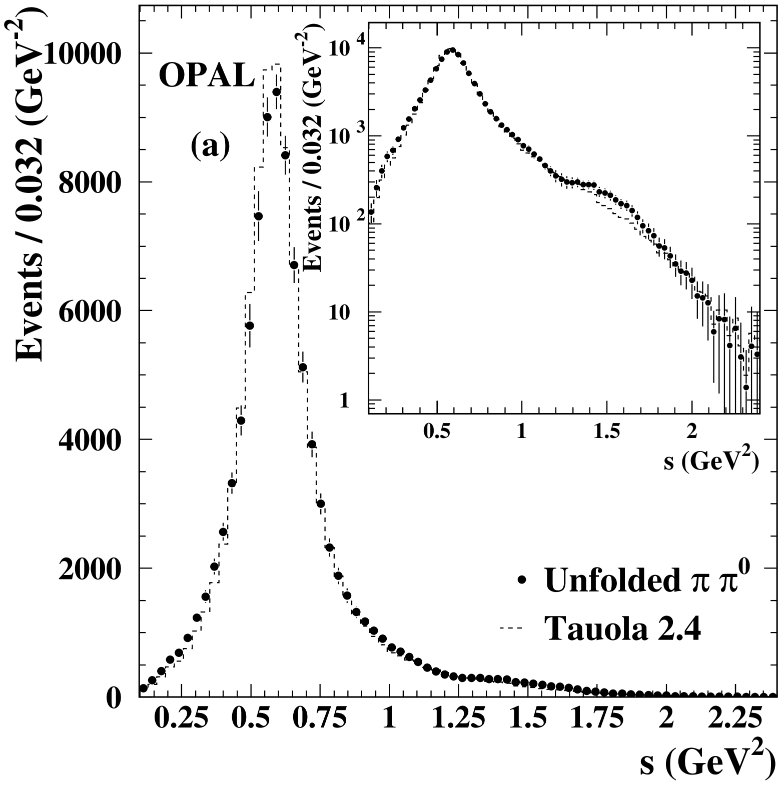


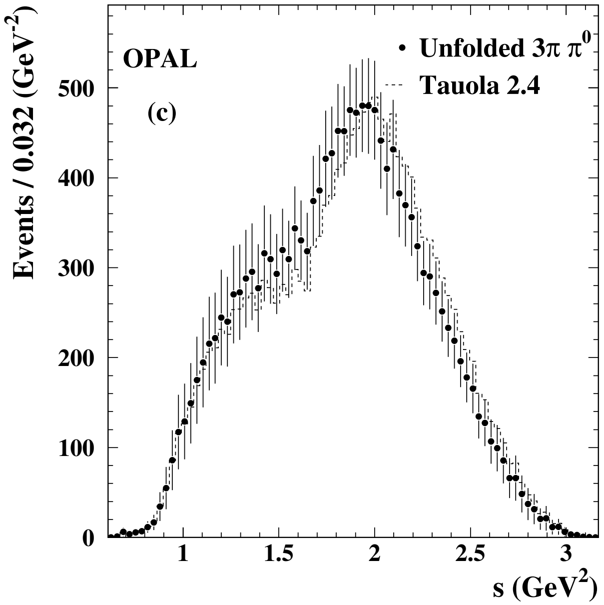


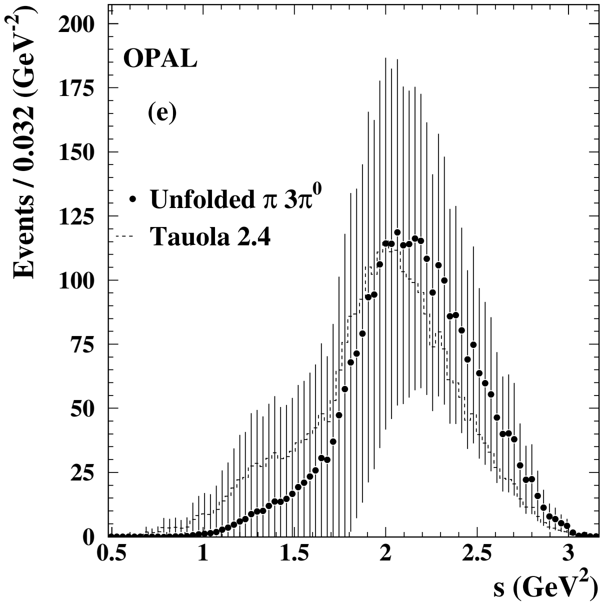
Early publication using 124 358 tau pairs with data taken in 1991-1994 was published in 1997

2005 publication (Old) based on full LEP1 data (over 300 000 measured and identified tau decays)
2013 update (New) fixed an unfolding issue having little impact on results

Measurement of OPAL

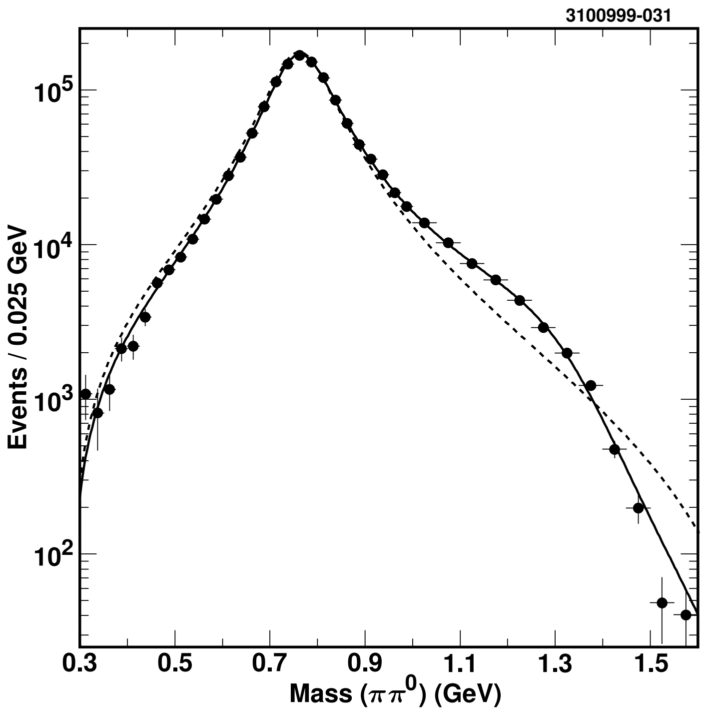


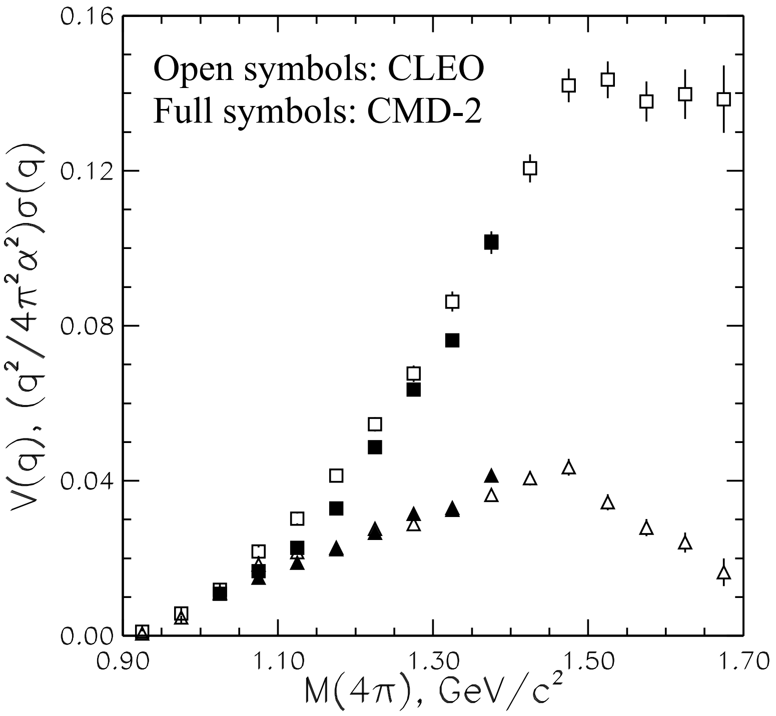




OPAL publication based on full LEP1 data
These and other spectral functions were used
to determine α_s

Measurements of CLEO





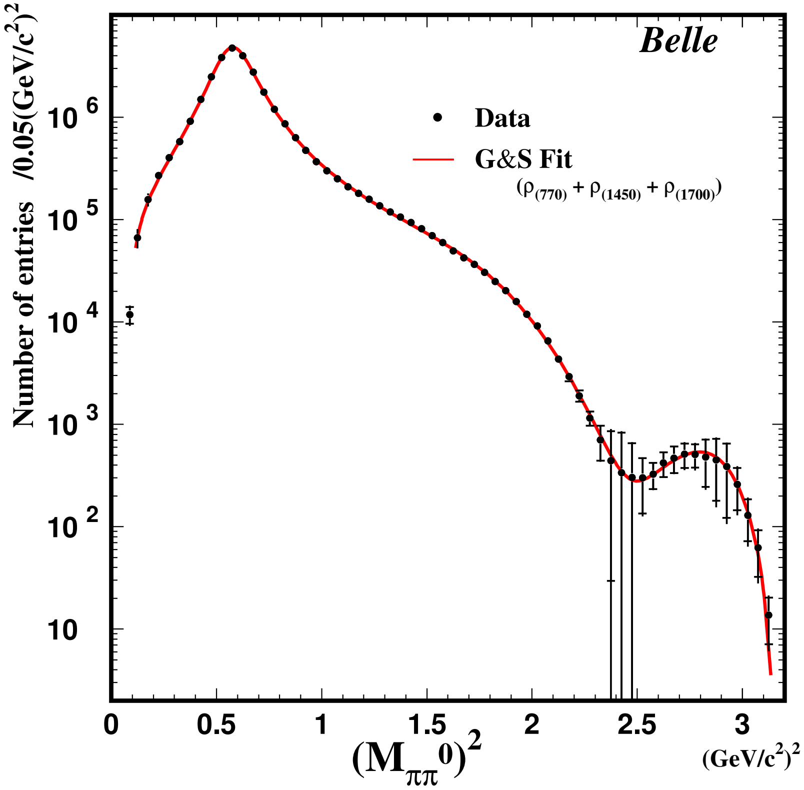
CLEO publication for $\pi\pi^0$ (no K-π separation, K subtraction using MC)

$\sqrt{s}$ = 10.6 GeV, 3.5 fb^{-1} (data 1990-1994),
103 522 tau pairs selected from 3×10^6 tau pairs

CLEO publication for $3\pi\pi^0$

Selected from a slightly larger sample (4.27×10^6 tau-pair events)

Measurement of Belle



Best shape measurement

Measured branching fraction for hπ^0 is
25.87 (0.01) (0.39) %

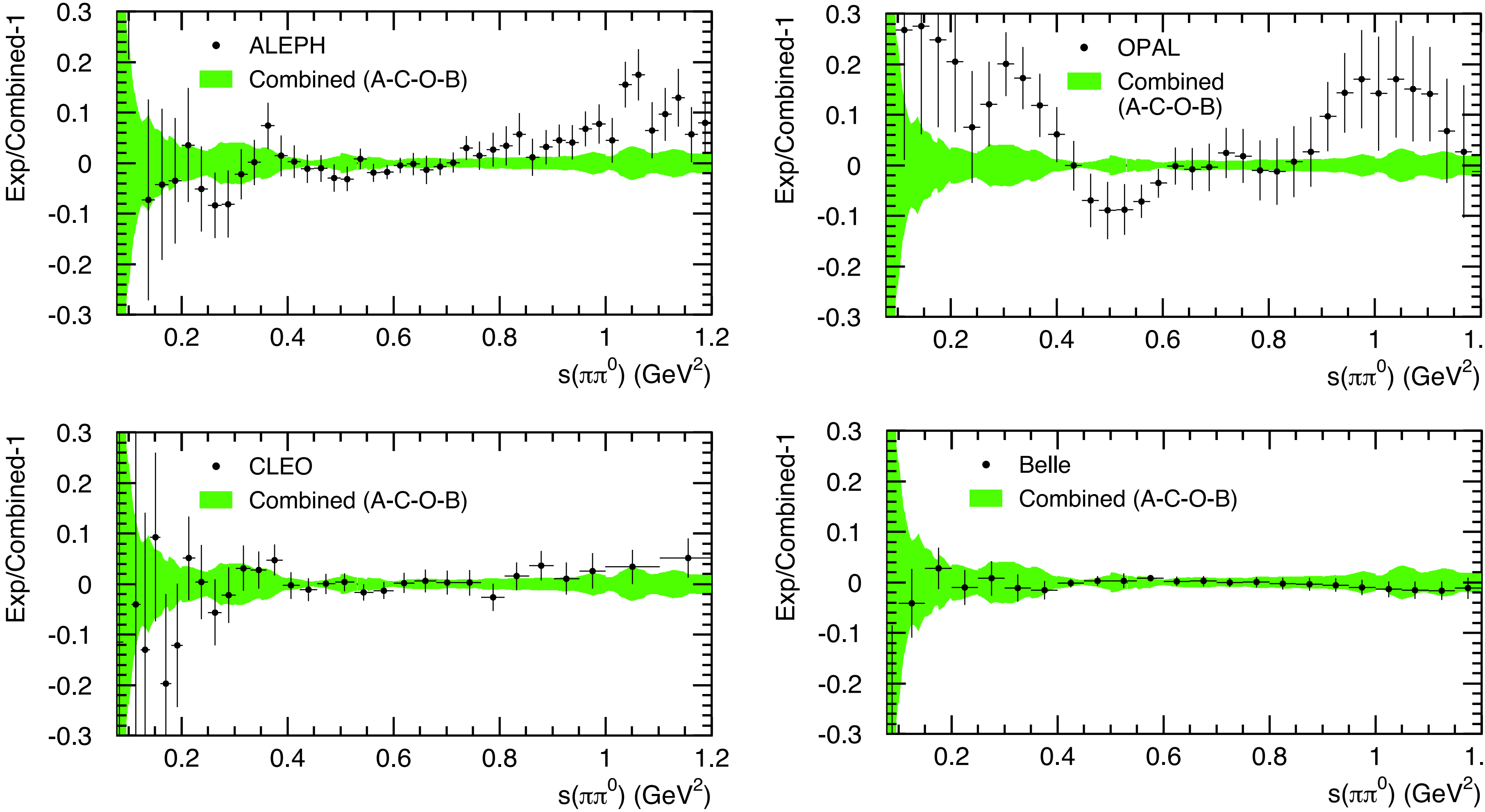
and for $\pi\pi^0$ after subtracting PDG Kπ^0
25.24 (0.01)(0.39) %

To be compared with the most precise one from ALEPH:
25.49 (0.10)(0.09) %

Belle publication for $\pi\pi^0$
$\sqrt{s}$ = 10.6 GeV, 72.2 fb^{-1}, tau sample is 50 times larger than that of the other measurements

Data Combination for the $\pi\pi^0$ Channel

Figures from ALPEH's 2013 update



The normalisation is constrained dominantly by ALEPH while the shape by Belle

Kinematic Fit at Low Energy

The measurements at low energy have limited precision

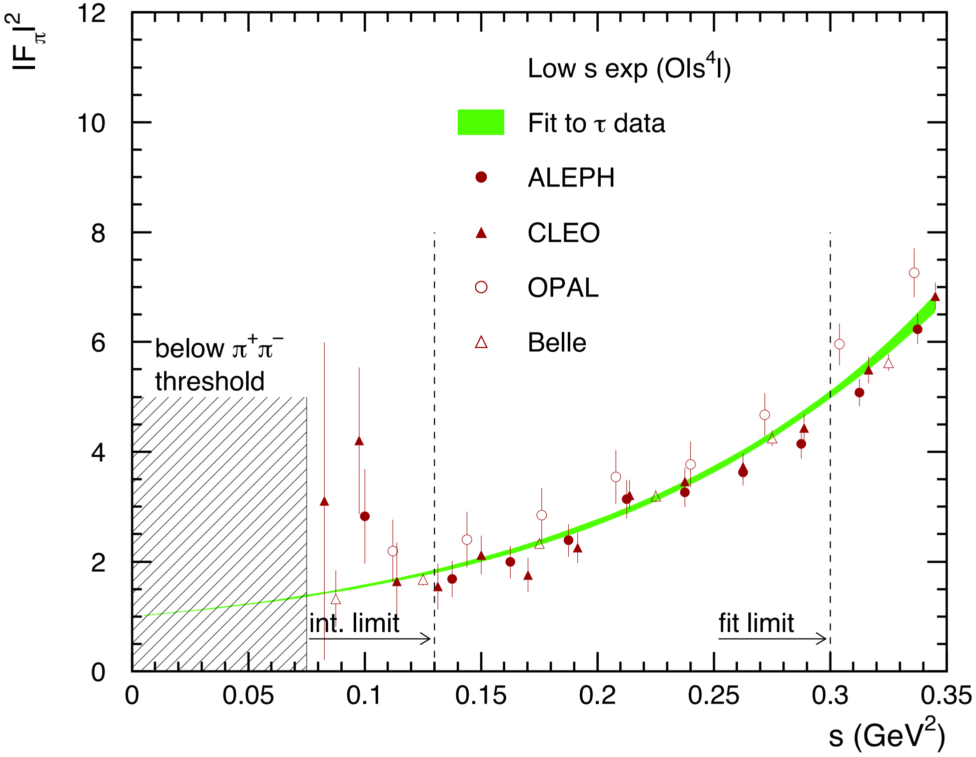
➔ Perform a fit of the pion form factor using a 3rd order expansion

$$|F_\pi| = 1 + \frac{1}{6}\langle r_\pi^2\rangle s + c_1 s^2 + c_2 s^3$$

$$\text{with } \langle r_\pi^2\rangle = (0.439 \pm 0.008)\,\text{fm}^2$$

NA7 1986

Figure from ALEPH's 2013 update



The fit is performed between threshold and 0.3 GeV^2

However the integration for a_μ is only up to 0.36 GeV

LO HVP [$\pi\pi$, τ] Results and Comparison with e+e−

Table from updated ALEPH publication

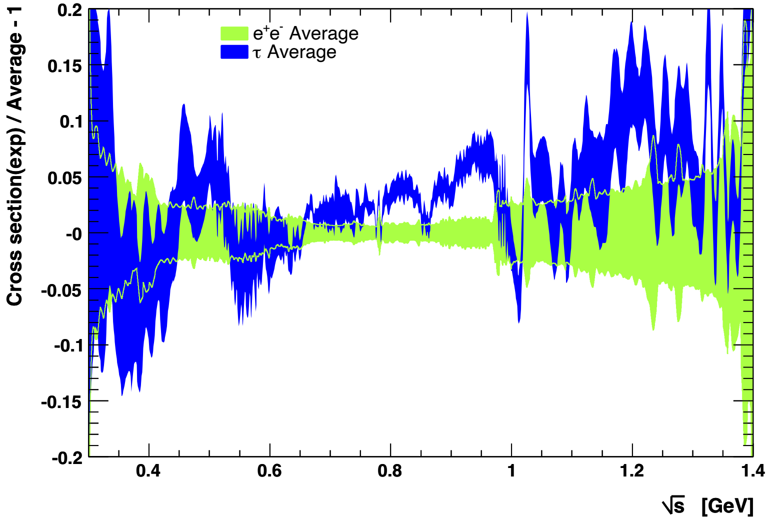
Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10})	
	$2m_{\pi^\pm}$ – 0.36 GeV	0.36 – 1.8 GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Tau result for $a_\mu[\pi\pi, \tau]$:
516.2 (1.9)(2.2)(1.9) *
Changed to:
517.3 (1.9)(2.2)(1.9)
[DHLMZ23]
⇔ $a_\mu[\pi\pi$, e+e−]:
508.4 (1.3)(2.6) **
514.1 (2.2)(3.1) [BABAR only]
[DHMYZ09]

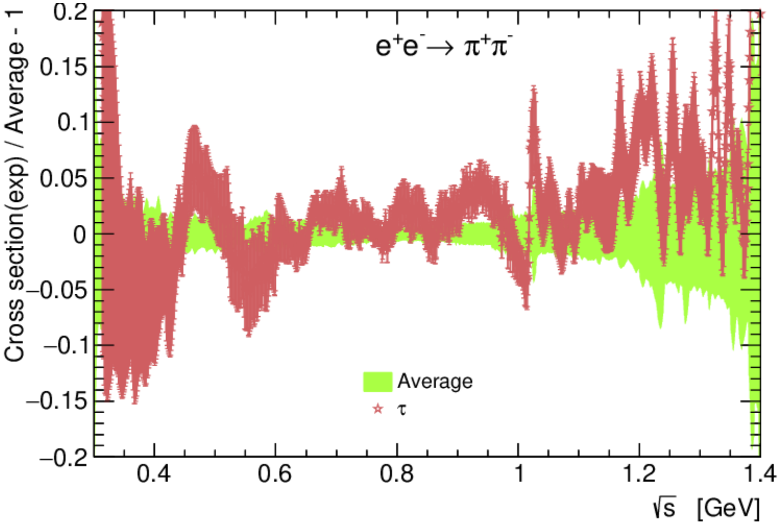
* The uncertainties correspond to stat, experimental syst and IB (isospin-breaking) corrections (slide 12)

** The uncertainties correspond to stat and sys errors

Figure taken from DHMYZ09 also in WP

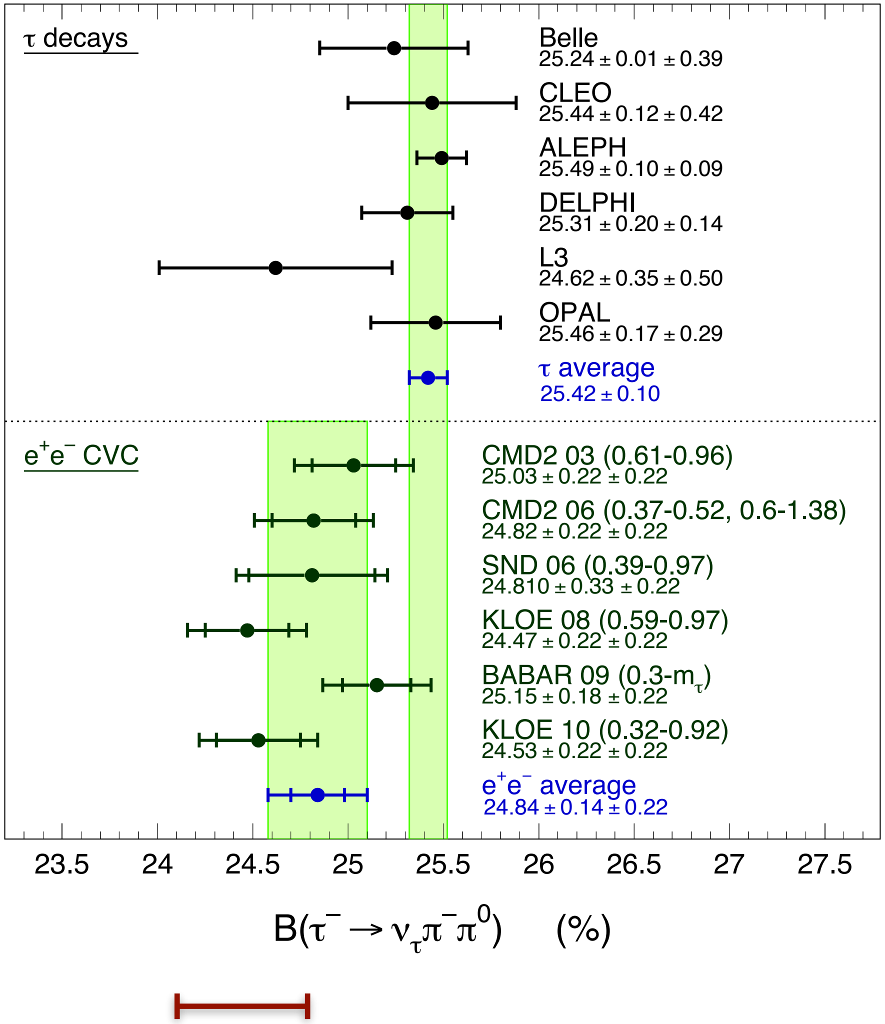


Comparison improved with CMD-3 w/o KLOE



CVC Test in terms of Branching Fractions

Figure taken from arXiv:1511.05405



+0.69±0.22 (size of the IB corrections)

Contrary to a_μ, the IB corrections are applied to e+e− data when calculating the CVC branching fractions

BABAR in fair agreement with tau data

CMD-3 (not included in the plot) also in agreement with tau data

Isospin-Breaking (IB) Corrections

IB corrections applied to $a_\mu[\pi\pi, \tau]$ are $R_{IB}(s)/S_{EW}$ with $$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Table taken from arXiv:1511.05405

Source	$\Delta a_\mu^{\rm Had,LO}[\pi\pi,\tau]$	$\Delta\mathcal{B}^{\rm CVC}_{\pi^-\pi^0}$
$S_{\rm EW}$	-12.21 ± 0.15	$+0.57 \pm 0.01$
$G_{\rm EM}$	-1.92 ± 0.90	-0.07 ± 0.17
FSR	$+4.67 \pm 0.47$	-0.19 ± 0.02
ρ–ω interference*	$+2.80 \pm 0.19$	-0.01 ± 0.01
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	$+0.19$
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	-0.22
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$+0.08 \pm 0.08$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	$+0.34 \pm 0.03$
δ(GS – KS)	-0.67	-0.03
Total	-16.07 ± 1.85	$+0.69 \pm 0.22$

* the $\rho-\omega$ interference correction +2.80 was based on $|\varepsilon_\omega|$=0.001997, arg(ε_ω)=11.6°
Changed in DHLMZ23 to +3.99 using $|\varepsilon_\omega|$=0.001990, arg(ε_ω)=3.8° (combined fit by Stoffer et al.)

S_{EW} — Short Distance EW Correction

Leading EW correction:

Marciano, Sirlin, 88

$$S_{\mathrm{EW}} = 1 + \frac{3\alpha}{4\pi}(1+2\overline{Q})\ln\frac{M_Z^2}{m_\tau^2} \simeq 1.0188 \quad \text{with} \quad \overline{Q} = \frac{1}{6} \quad \text{for semi-hadronic mode}$$

Improved by resuming all higher order logarithms using renormalisation group technique:

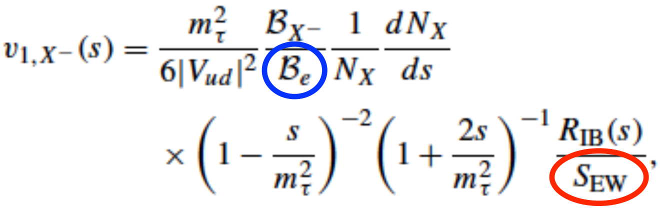
$$S_{\mathrm{EW}}^{\mathrm{had}} = \left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{\frac{9}{19}} \left[\frac{\alpha(M_W)}{\alpha(m_b)}\right]^{\frac{9}{20}} \left[\frac{\alpha(M_Z)}{\alpha(M_W)}\right]^{\frac{36}{17}} \simeq 1.0194 \xrightarrow{\text{QCD corrections}} 1.0189$$

Braaten, Narison, Pich, 92

Sirlin, 82

Taking into account sub-leading non-logarithmic short distance correction (since the spectral function is normalised to the electron mode):

$$S_{\mathrm{EW}}^{\mathrm{sub,lep}} = 1 + \frac{\alpha(m_\tau)}{\pi}\left(\frac{25}{8} - \frac{\pi^2}{2}\right) \simeq 0.9957$$



$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{X^-}}{\mathcal{B}_e}\frac{1}{N_X}\frac{dN_X}{ds} \times \left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}},$$

One has finally:

$$S_{\mathrm{EW}} = \frac{S_{\mathrm{EW}}^{\mathrm{had}}}{S_{\mathrm{EW}}^{\mathrm{sub,lep}}} \simeq 1.0233 \pm 0.0006$$

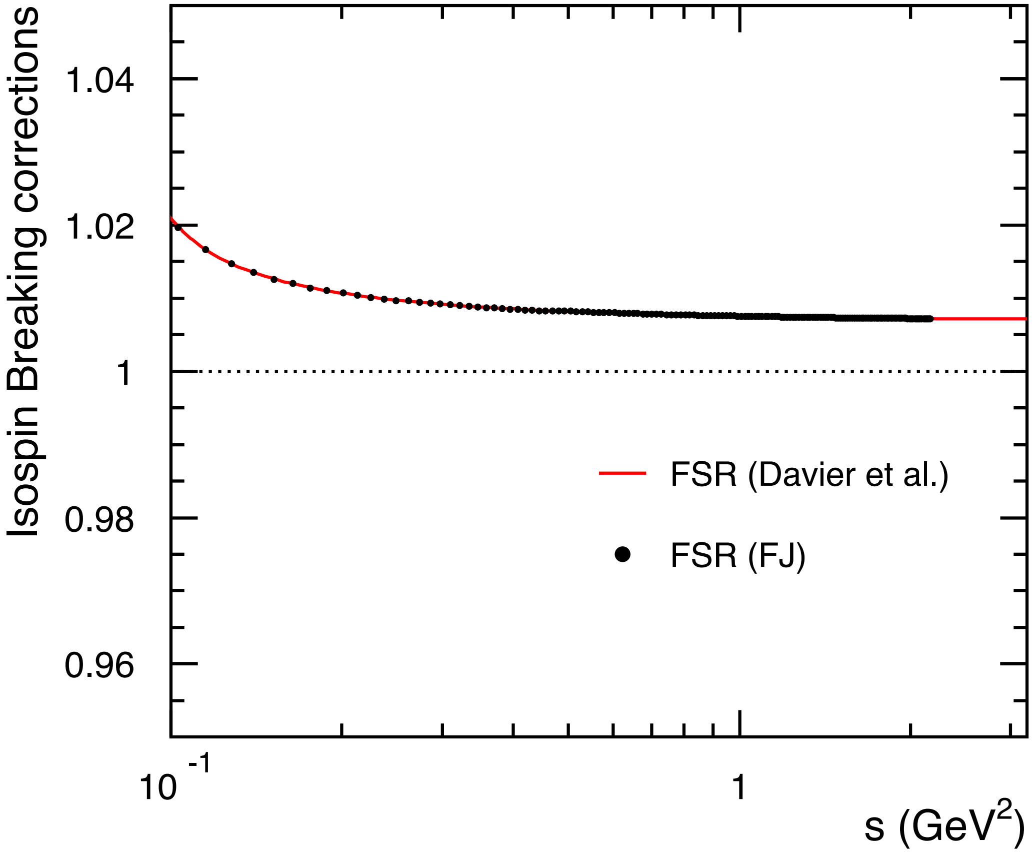
Uncertainty corresponds conservatively to the difference between the leading and resumed corrections

FSR (Final State Radiation) Correction

$$R_{\mathrm{IB}}(s) = \frac{\boxed{\mathrm{FSR}(s)}}{G_{\mathrm{EM}}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

We compared in detail with the correction from Jegerlehner
No difference found, we quoted nevertheless 10% uncertainty

Figure from WP20



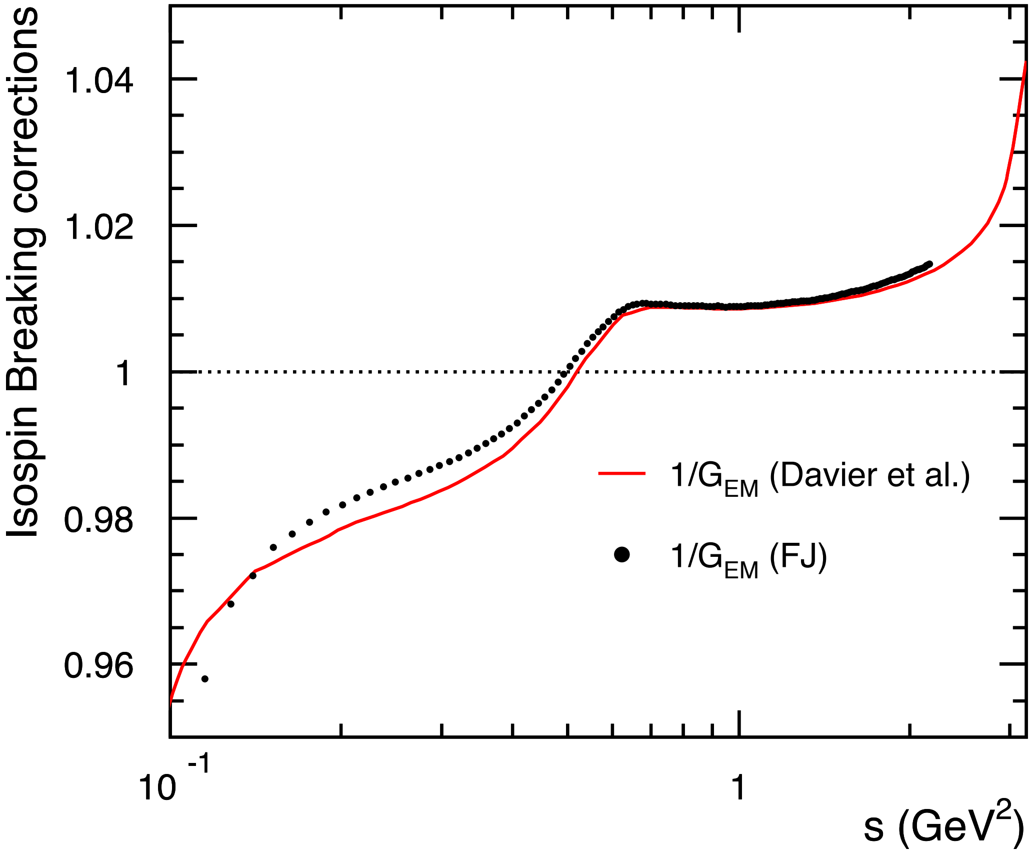
GEM — Long Distance EM Corrections

$$R_{\rm IB}(s) = \frac{\mathrm{FSR}(s)}{\boxed{G_{\rm EM}(s)}}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

Our GEM corrections based on vector meson dominance model (VBM) [1] in fair agreement with Jegerlehner

We quote the difference with corrections based on chiral perturbation theory (ChPT) [2] as uncertainty

Figure from WP20



[1] Flores-Baez et al. 06
[2] Cirigliano, Ecker, Neufeld, 01, 02

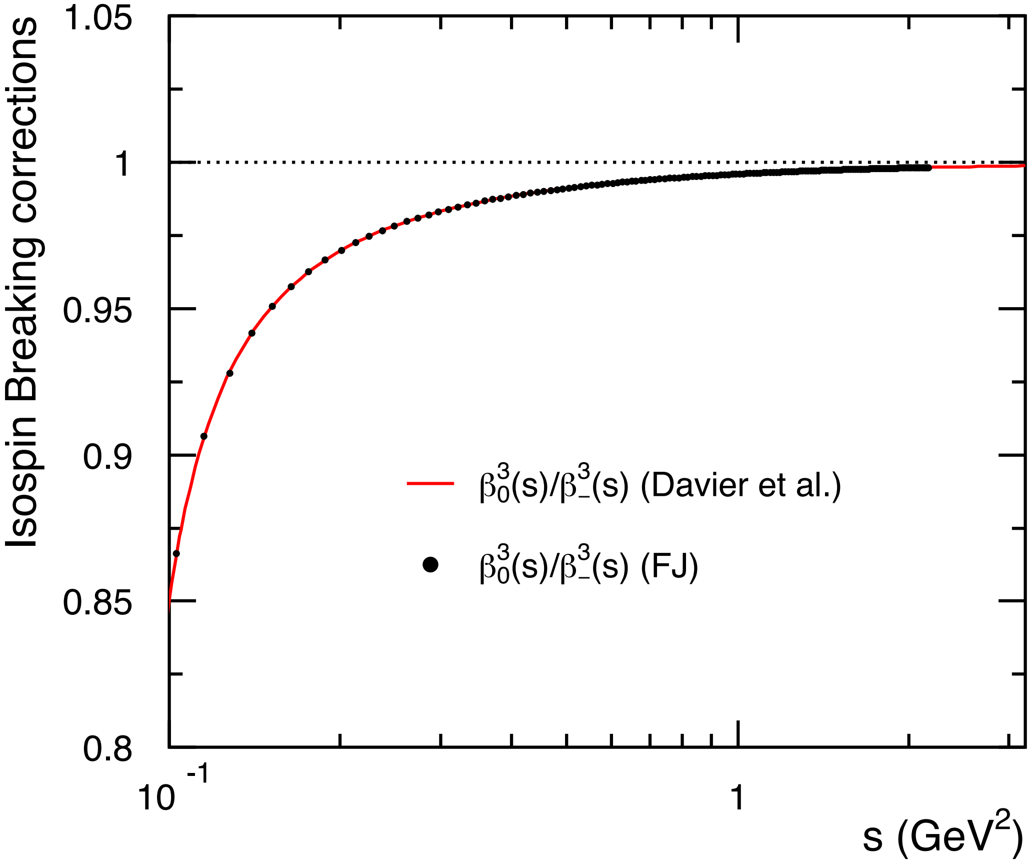
Beta Term — Phase Space Difference

$$R_{\rm IB}(s) = \frac{\mathrm{FSR}(s)}{G_{\rm EM}(s)} \boxed{\frac{\beta_0^3(s)}{\beta_-^3(s)}} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

$$\beta_{0,-} = \beta(s, m_{\pi^-}, m_{\pi^{0,-}})$$

$$\beta(s, m_1, m_2) = \left[\left(1 - \frac{(m_1+m_2)^2}{s}\right) \left(1 - \frac{(m_1-m_2)^2}{s}\right) \right]^{1/2}$$

Figure from WP20



Again no difference with Jegerlehner
No uncertainty needed

Form Factor Term

$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

One important component is the ρ–ω interference correction

It depends on parameterisation forms and its parameters (amplitude and phase)

Changed from +2.80 to +3.99 (DHLMZ23)

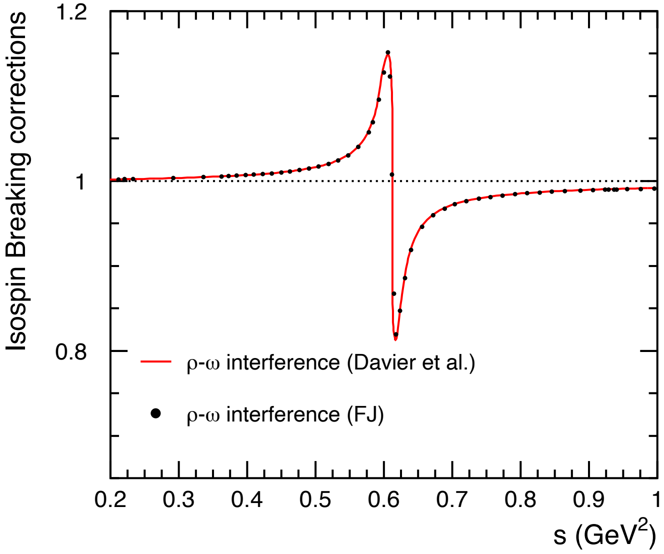
Width difference +4.09 due to neutral/charged π mass difference (partially cancel the difference in the beta term −7.88)

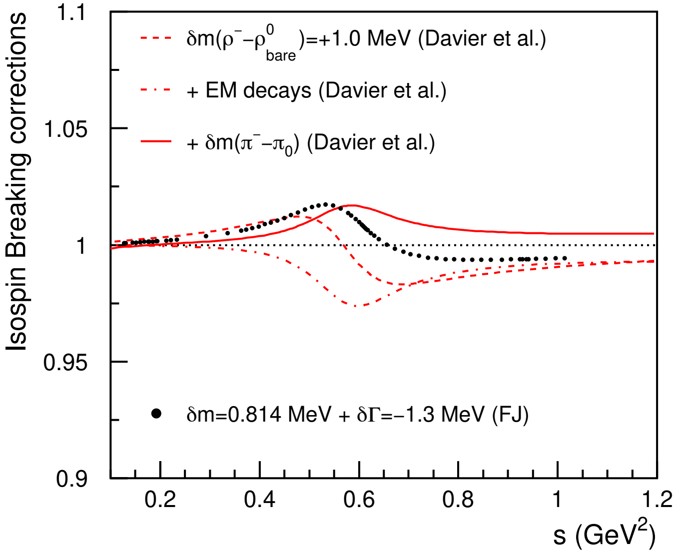
Neutral/charged ρ mass difference +0.20

EM decays −5.91

Take into account in addition difference between Gounaris–Sakurai (GS) and Kühn–Santamaria (KS) parameterisations

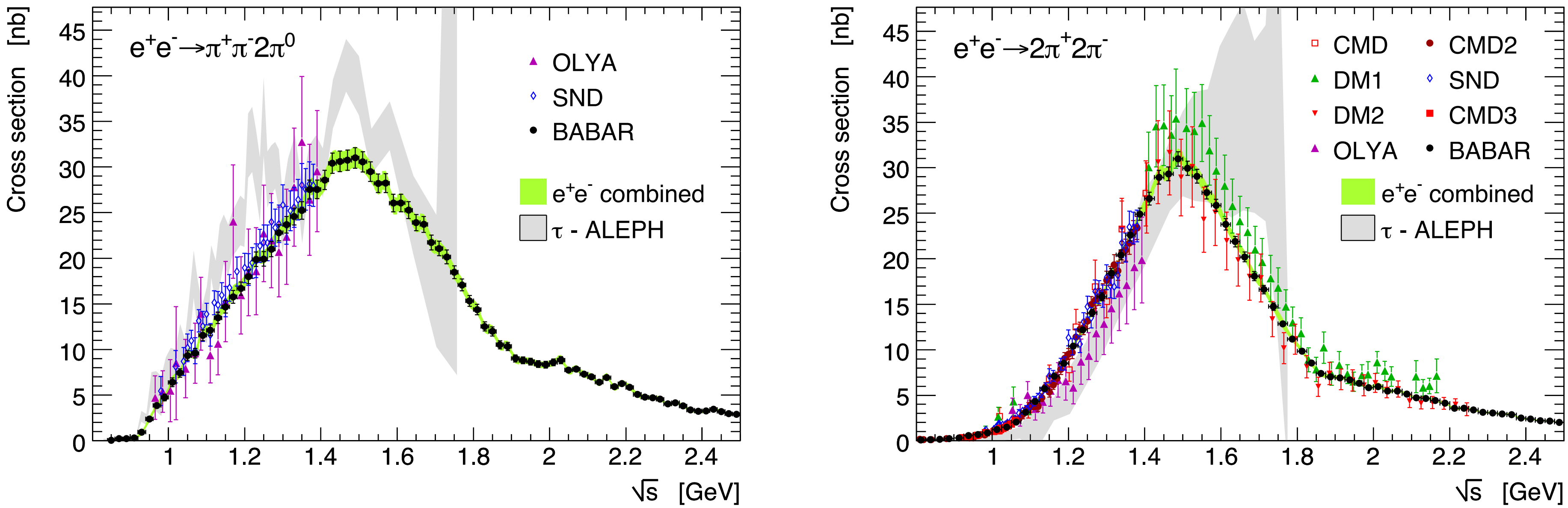
Figure from WP20





Comparison e+e− and tau for 4π Channels

Figures from DHMZ17



$\Delta a_\mu\ [10^{-10}]$	$\pi^+\pi^-2\pi^0$	$2\pi^+2\pi^-$	Reference
Tau (<1.5 GeV) Tau (<m_τ)	14.70 (0.28)(1.01)(0.40) 21.0 (1.2)(0.4)	7.07 (0.41)(0.48)(0.35) 12.8 (0.7)(0.4)	DHMYZ13 DHMZ17
e+e− (<1.8 GeV)	18.01 (0.14)(1.17)(0.29) 18.03 (0.06)(0.48)(0.26)	13.35 (0.10)(0.43)(0.29) 13.68 (0.03)(0.27)(0.14)	DHMZ10 DHMZ17

The precision of e^+e^- data increased over time, a factor of 1.7-2.3 between 2010 and 2017

There is a big room for improvement from tau side!

Prospects for New Measurements at Belle II

- Belle achieved the best (shape) measurement of the $\pi\pi^0$ spectral function with ~70 fb^{-1}

- With Belle II @SuperKEKB
  - Much higher luminosity (already recorded >500 fb^{-1} ⬌ ~0.5 billion tau pairs)
  - Introducing nonlinear collimator reduced beam background in the Belle II detector
  - Improved sub-detectors
    - Silicon pixels ➔ improve the track impact parameter & vertex resolution
    - A new large-volume central tracker
    - Powerful particle identification detectors (TOP+ARICH)
    - An updated K_L and muon detector (KLM)
    - State-of-the-art readout, trigger and DAQ systems

- New spectral function measurement in the $\pi\pi^0$ channel started
- An analysis in the $3\pi\pi^0$ channel is also planned

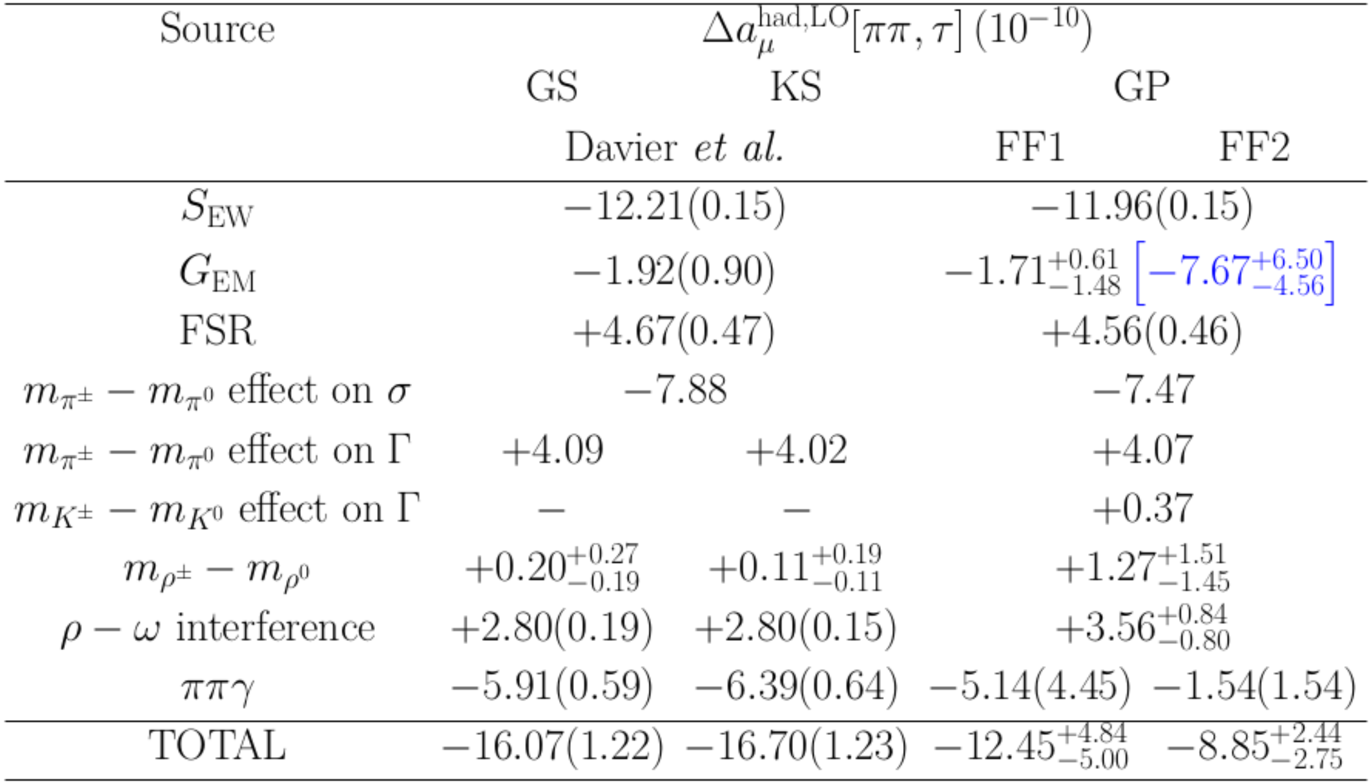
Summary and Prospects

- Tau spectral functions especially in the two-pion channel have been measured by several experiments at the Z pole and $\Upsilon(4S)$
  - The tau data valuable to compare with the data-driven prediction using the e+e− data
    - In the past when the e+e− had limited precision
    - Even now given large tensions among different e+e− measurements
  - On the isospin corrections
    - Most of the IB corrections are under control (S_{EW}, π mass difference, ρ−ω interference)
    - G_{EM} is model dependent but compatible between ChPT and VDM
    - Currently quoted IB correction uncertainty < experimental one
    - Independent approches are desirable (dispersive method, lattice)

- There are huge tau data samples with high quality to be used at Belle II

Additional Comparison with (Masjuan-)Miranda-Roig

arXiv:2007.11019
arXiv:2305.20005



Source	$\Delta a_\mu^{\text{had,LO}}[\pi\pi,\tau]$ (10^{-10})			
	GS	KS	GP	
	Davier *et al.*		FF1	FF2
S_{EW}	$-12.21(0.15)$		$-11.96(0.15)$	
G_{EM}	$-1.92(0.90)$		$-1.71^{+0.61}_{-1.48}$ $[-7.67^{+6.50}_{-4.56}]$	
FSR	$+4.67(0.47)$		$+4.56(0.46)$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88		-7.47	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ	$+4.09$	$+4.02$	$+4.07$	
$m_{K^\pm} - m_{K^0}$ effect on Γ	$-$	$-$	$+0.37$	
$m_{\rho^\pm} - m_{\rho^0}$	$+0.20^{+0.27}_{-0.19}$	$+0.11^{+0.19}_{-0.11}$	$+1.27^{+1.51}_{-1.45}$	
$\rho - \omega$ interference	$+2.80(0.19)$	$+2.80(0.15)$	$+3.56^{+0.84}_{-0.80}$	
$\pi\pi\gamma$	$-5.91(0.59)$	$-6.39(0.64)$	$-5.14(4.45)$	$-1.54(1.54)$
TOTAL	$-16.07(1.22)$	$-16.70(1.23)$	$-12.45^{+4.84}_{-5.00}$	$-8.85^{+2.44}_{-2.75}$

Our IB corrections are in agreement with those from Roig et al. though the latter have larger uncertainties in particular in some of the models

GS: Gounaris–Sakurai, KS: Kühn–Santamaria, GP: Guerrero-Pich parameterisations

G_{EM} Considered by (Masjuan-)Miranda-Roig

arXiv:2007.11019