UPDATE FROM MAINZ ON $a_\mu^{\rm hvp}$

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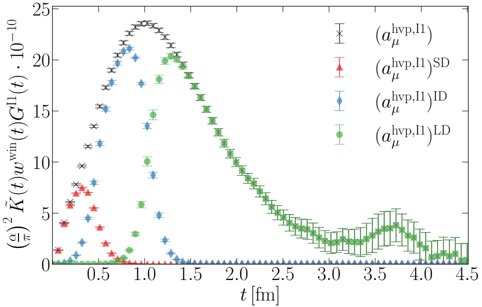
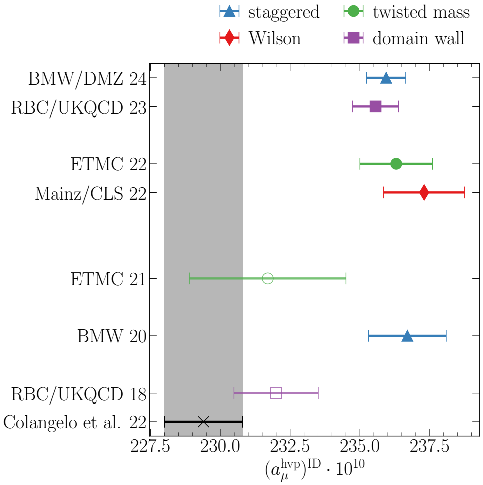


- Use windows in the time-momentum representation to compute [Blum et al., 1801.07224]

$$a_\mu^{\rm hvp} = (a_\mu^{\rm hvp})^{\rm SD} + (a_\mu^{\rm hvp})^{\rm ID} + (a_\mu^{\rm hvp})^{\rm LD}$$

$a_\mu^{\rm hvp}$ FROM LATTICE QCD



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  - Intermediate distance (✓): [Cè et al., 2206.06582]

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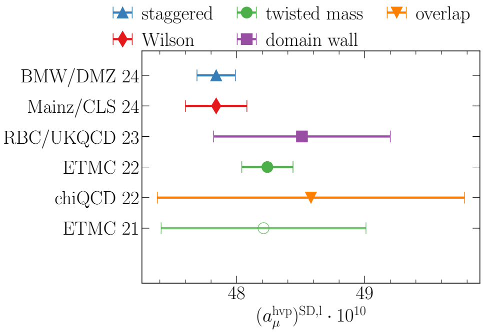

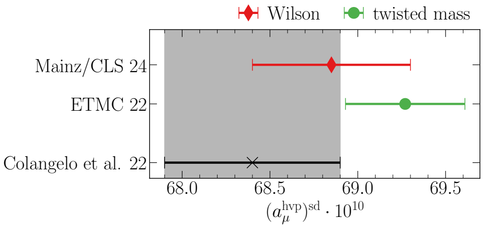

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  - Short distance (✓, this talk): [SK et al., 2401.11895]

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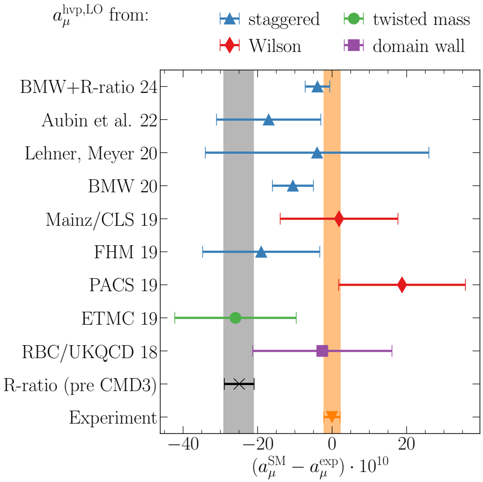
$a_\mu^{\rm hvp}$ FROM LATTICE QCD



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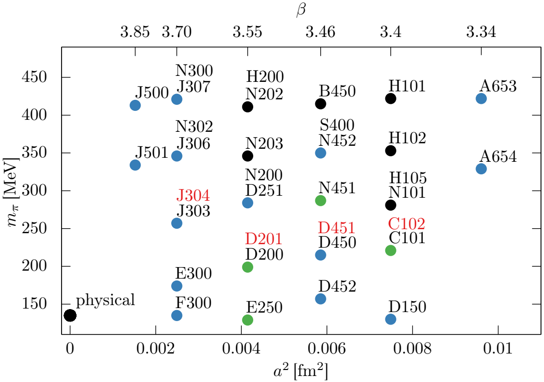
→ Our goal: update Mainz/CLS 19 [Gérardin et al., 1904.03120].

The Mainz/CLS setup

a_μ^{hvp} from $2+1$ flavors of $\mathrm{O}(a)$ improved Wilson-clover fermions

$2+1$ FLAVOR CLS ENSEMBLES



- Six values of $a \in [0.039, 0.099]$ fm.
- Open boundary conditions in the temporal direction.
- $a\mathrm{Tr}[M_\mathrm{q}] = 2am_\mathrm{l} + am_\mathrm{s} = \mathrm{const.}$ and $m_\mathrm{s} \approx m_\mathrm{s}^\mathrm{phys}$ to stabilize the strange-quark interpolation.

● New ensemble / ● significantly improved statistics since [Gérardin et al., 1904.03120].

- Generating a third ensemble with $m_\pi \approx m_\pi^\mathrm{phys}$: F300 with 256×128^3 at $0.05\,\mathrm{fm}$, $\rightarrow$ increase precision and further constrain $(am_\pi)^2$ effects.

COMPUTATIONAL SETUP

- Work in isospin decomposition of the electromagnetic current

$$j_\mu^{\text{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \ldots = j_\mu^{I=1} + j_\mu^{I=0} + \frac{2}{3}\bar{c}\gamma_\mu c + \ldots \,,$$

- $\mathrm{O}(a)$ improved correlation functions with
  - local-local (LL) and local-conserved (LC) vector currents
  - two different lines of constant physics for the $\mathrm{O}(a)$ improvement (set 1/ set 2).
- Finite-volume correction via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892] [Lellouch and Lüscher, hep-lat/0003023] pion formfactor.
- Scale setting with f_π (f_π-rescaling [1103.4818, Xu et al.], [Gérardin et al., 1904.03120])
  - f_π-rescaling reduces chiral dependence of the isovector contribution .
  - No consistent picture for the physical values of flow scales [FLAG23].
  - Avoids double counting of systematic uncertainties.
  - Small contribution of f_K enters as well - suppressed by 10^{-1} to 10^{-2} w.r.t. f_π.

The short distance contribution

[SK et al., 2401.11895]

- Cutoff effects are the main concern at short distances, especially those of $\mathrm{O}(a^2\log(a))$ [Della Morte et al., 0807.1120][Cè et al., 2106.15293] [Sommer et al., 2211.15750]:
  - removal via perturbative QCD in the spacelike regime at high energies Q^2.

Starting from the well-known formula [Bernecker and Meyer, 1107.4388]

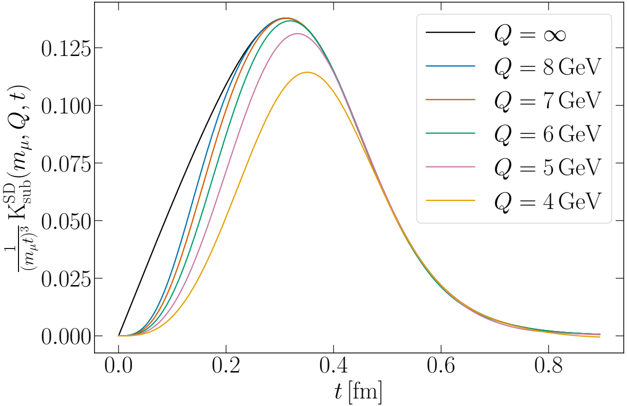
$$(a_\mu^{\rm hvp})^{\rm SD} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, w^{\rm SD}(t)\widetilde{K}(t)G(t)\,,$$

with the short-distance window $w^{\rm SD}(t)$, we change to a modified QED kernel via

$$w^{\rm SD}(t)\tilde{K}(t) \to \mathrm{K}^{\rm SD}_{\rm sub}(Q,t) = w^{\rm SD}(t)\tilde{K}(t) - w^{\rm SD}(0)\frac{16\pi^2 m_\mu^2}{9Q^2} f(Q,t)$$

where $f(Q,t) = \frac{16}{Q^2}\sin^4\left(\frac{Qt}{4}\right)$ is the kernel to compute

$$\Pi(Q^2) - \Pi((Q/2)^2) = \int_0^\infty \mathrm{d}t\, f(Q,t)G(t)\,.$$



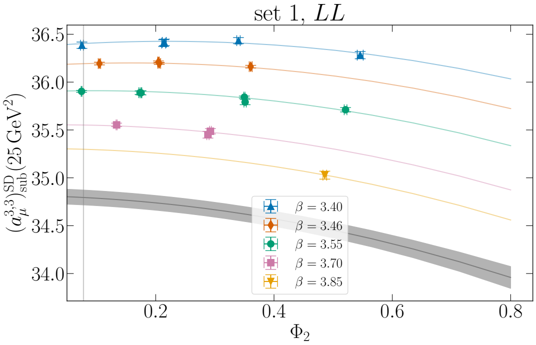
- Remove term $\propto t^4$ in kernel and thus the $a^2 \log(a)$ effects.
- New slope is Q dependent.
- We focus on $Q = 5\,\mathrm{GeV}$.
- Relevant scale for perturbation theory is $Q/2$.

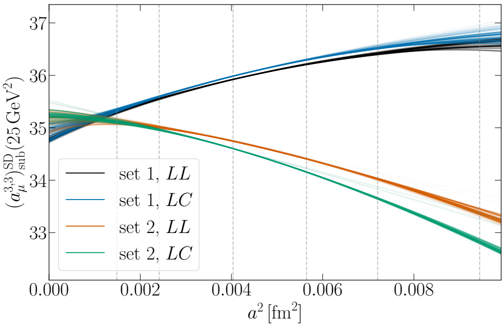
- Based on the Adler function $D(Q^2)$, we evaluate [Baikov et al., 0801.1821, 1001.3606],

$$\Pi(Q^2) - \Pi((Q/2)^2) = \frac{\pi^2}{12} \int_{(Q/2)^2}^{Q^2} \frac{\mathrm{d}Q'^2}{Q'^2} D(Q'^2)$$

and expect good convergence of the perturbative series [Jegerlehner, 2020].

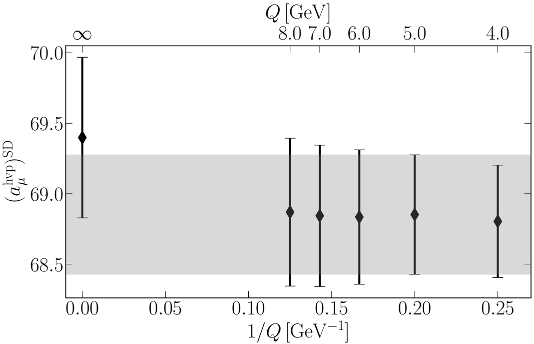
$(a_\mu^{\rm hvp})^{\rm SD}$ IN THE ISOVECTOR CHANNEL

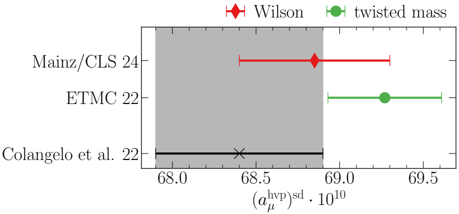




- Tiny uncertainties, benign chiral dependence, significant cutoff effects.
- Use tree-level improvement to reduce the cutoff effects.
- Combine with strange, disconnected, charm and valence connected isospin-breaking contributions for the full $(a_\mu^{\rm hvp})^{\rm SD}$.

Full result for $(a_\mu^{\mathrm{hvp}})^{\mathrm{SD}}$





- Stability under variation of the modification scale Q.
- Small but noticeable shift when $a^2 \log(a)$ effects are not removed ($1/Q = 0$).
- Final uncertainty dominated by systematics from the continuum extrapolation.

The long distance contribution

BLINDED

Our goal

Determine with $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ the last building block for the full a_μ^{hvp}.

- **Blinded analysis**
- Noise reduction techniques to get to precision in the isovector channel:
  - **Low-mode averaging** (LMA).
  - Spectral reconstruction of the $\pi\pi$ contribution.
- Finite-volume effects are sizable:
  - Correct to $Lm_\pi = 4.29$ for $a \neq 0$ prior to extrapolations.
  - Correct to $L \to \infty$ in the continuum at physical mass.
- Significant scale dependence of the long-distance tail.

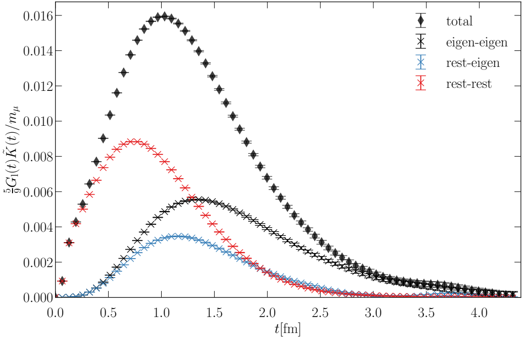
Blinding strategy for $(a_\mu^{\rm hvp})^{\rm LD}$

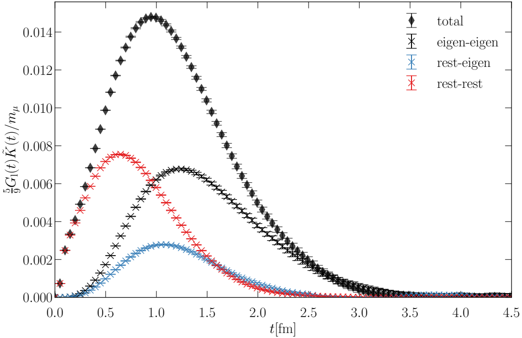
- We decided to introduce blinding at the stage of the analysis by modification of the QED kernel function $\widetilde{K}(t)$ in the integrand of the TMR:
  - Multiplicative offset.
  - Artificial cutoff effects (one kernel for each value of β).
  - …? I still don't know the details.

- Use five different sets of modified kernels.

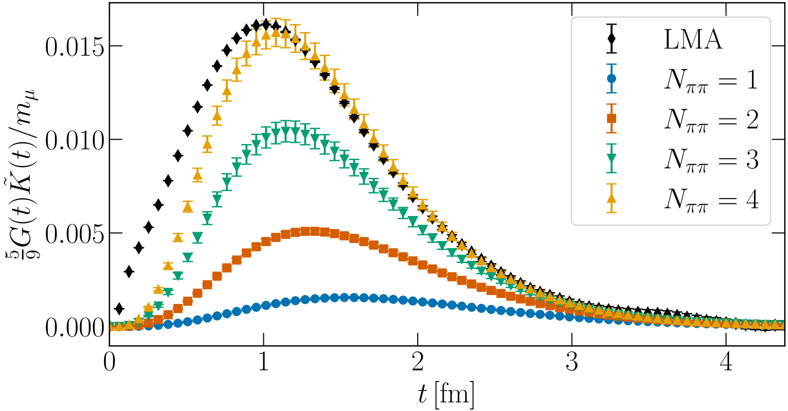
- Unblinding strategy:
  1. Cross-check each step of the blinded analysis.
  2. Agree on final analysis setup. Freeze.
  3. Relative unblinding between the five sets of kernels in the continuum.
  4. Absolute unblinding of kernels $\rightarrow$ repeat the *same* analysis with the true kernel.

Noise reduction: Low-mode averaging





- Use low-mode averaging for all ensembles where $m_\pi < 280\,\mathrm{MeV}$.
  - Left: $m_\pi = 132\,\mathrm{MeV}$, $a = 0.064\,\mathrm{fm}$ (E250)
  - Right: $m_\pi = 177\,\mathrm{MeV}$, $a = 0.049\,\mathrm{fm}$ (E300)
- Autocorrelation becomes a limiting factor at fine lattice spacing.

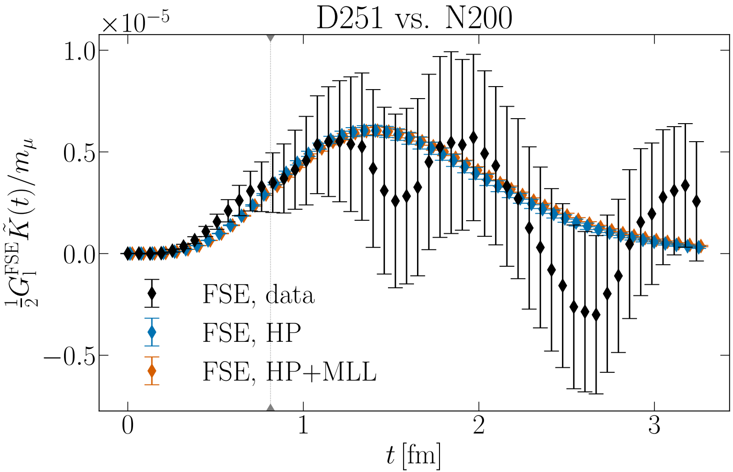


[Nolan Miller @ Lattice24]:

- Careful extraction of energies and overlaps.
- Work towards computing the timelike pion form factor.

- Spectral reconstruction of the isovector correlation function on E250 at m_π^{phys}.
- Solves the signal-to-noise problem, but LMA is more precise for $t < 2.5\,\text{fm}$.
- Reduces the uncertainty on this ensemble by another factor of 2: 0.4% for a_μ^{hvp}.

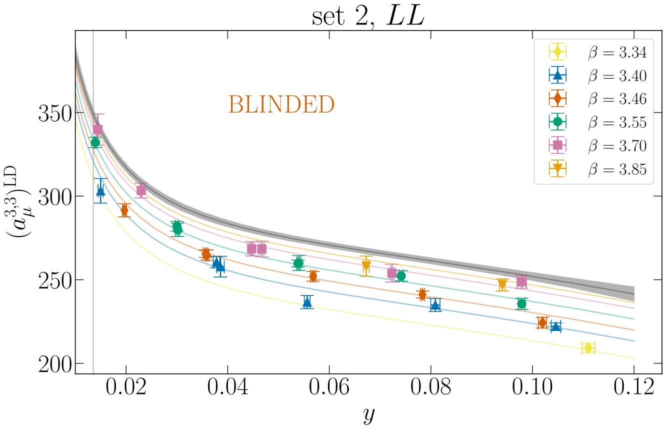
Finite-size correction: Consistency check



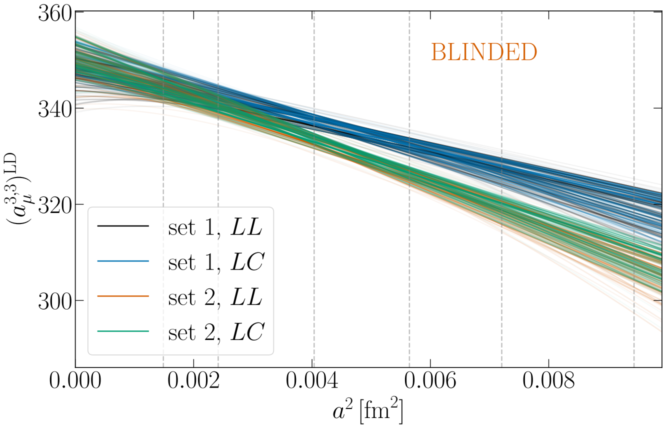
- $I = 1$ channel
- $m_\pi = 286\,\mathrm{MeV}$
- L: $3\,\mathrm{fm} \to 4.1\,\mathrm{fm}$
- $m_\pi L$: $4.4 \to 5.9$
- $a = 0.064\,\mathrm{fm}$

- Compare finite-size effects in the data with the two model predictions.
- Excellent agreement (with large statistical uncertainties).

$(a_\mu^{\rm hvp})^{\rm LD}$ IN THE ISOVECTOR CHANNEL: CHIRAL DEPENDENCE



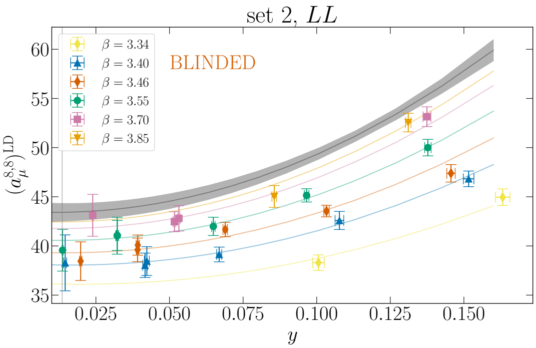
- Dependence of $(a_\mu^{3,3})^{\rm LD}$ on $y = m_\pi^2/(8\pi f_\pi^2)$.
- Data is corrected to common $Lm_\pi = 4.29$.
- Tight constraint at $m_\pi^{\rm phys}$: E250 at 0.7% precision.
- Chiral dependence well constrained across the range of pion masses.
- Need to include a term that is divergent in the chiral limit for good fit quality.
  $\rightarrow$ reduced chiral dependence when using f_π-rescaling.

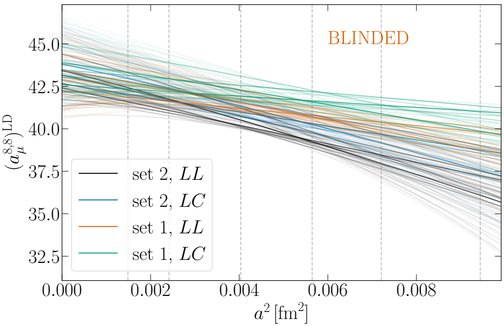


- Dependence of $(a_\mu^{3,3})^{\mathrm{LD}}$ on a^2 at physical quark masses.
- Four sets of data (colors) differ by $\mathrm{O}(a^2)$.
- Each line represents a fit in the model average.
- Include terms à la $[\alpha_s(1/a)]^{0.395}\, a^2$ [Husung, 2409.00776].

- Higher order cutoff effects have a small weight in the model average.
- After model average: statistics dominated accuracy of 1.3%.

$(a_\mu^{\text{hvp}})^{\text{LD}}$ IN THE ISOSCALAR CHANNEL

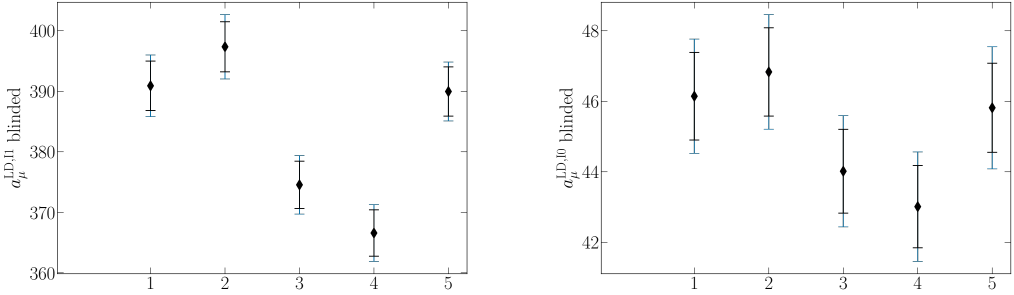




- Quark-disconnected diagram contributes significantly to noise in the isoscalar channel, despite using multiple noise reduction techniques [Cè et al., 2203.08676].
- Bounding method in the isoscalar channel to tame the long-distance tail.
- Leading finite-size effects of light-connected and disconnected cancel.

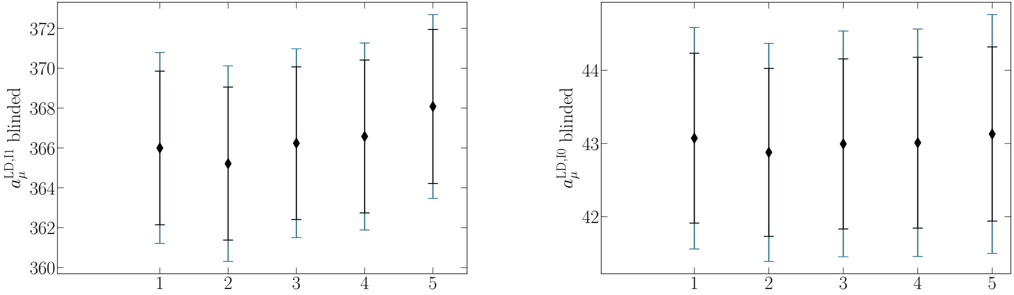
The long distance contribution

UNBLINDED



- 5 blinded kernels

Unblinding



- 5 blinded kernels agree after relative unblinding

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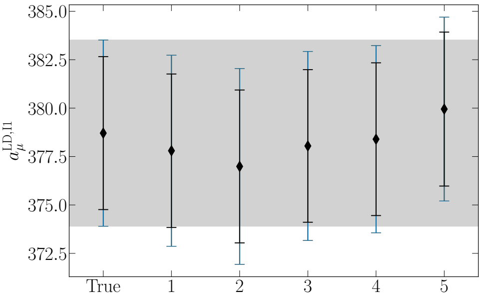
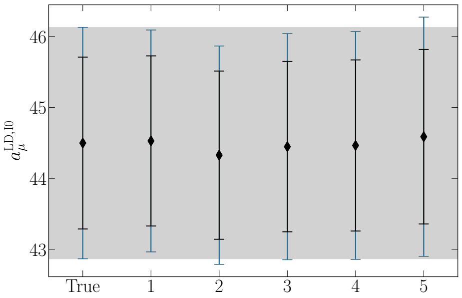
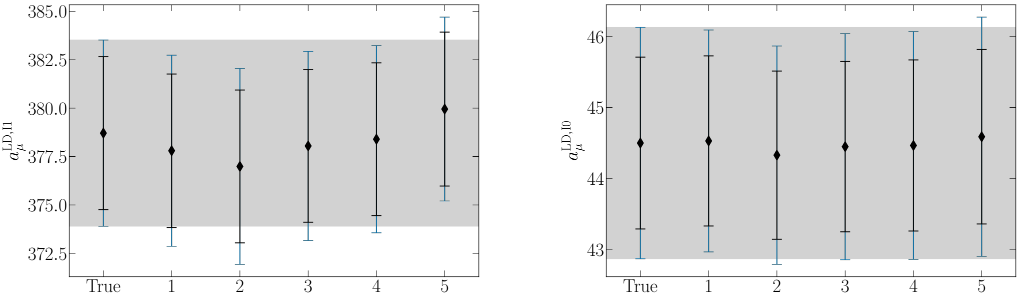




- 5 blinded kernels agree after relative unblinding and with the true kernel.

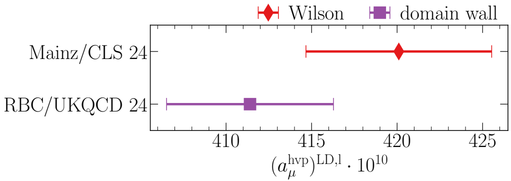
Unblinding

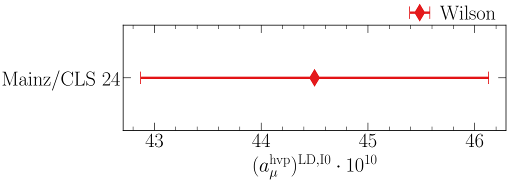


- 5 blinded kernels agree after relative unblinding and with the true kernel.
- Unblinded results in finite volume and the Mainz world

$$
\begin{aligned}
a_\mu^{\mathrm{LD,I1}}(Lm_\pi = 4.29) &= 362.0(3.7)_{\mathrm{stat}}(2.74)_{\mathrm{syst}}[4.57] \\
a_\mu^{\mathrm{LD,I0}}(Lm_\pi = 4.29) &= \;\; 44.5(1.2)_{\mathrm{stat}}(1.09)_{\mathrm{syst}}[1.63] \\
a_\mu^{\mathrm{LD,c}} &= \;\; 0.0141(4)_{\mathrm{stat}}(6)_{\mathrm{syst}}[7] \\
a_\mu^{\mathrm{LD}} - a_\mu^{\mathrm{LD}}(Lm_\pi = 4.29) &= \;\; 16.7(1.5)
\end{aligned}
$$

Overview of results for $(a_\mu^{\text{hvp}})^{\text{LD}}$





- Comparison with RBC/UKQCD 24 in different isoQCD schemes: Mainz vs. BMW20.

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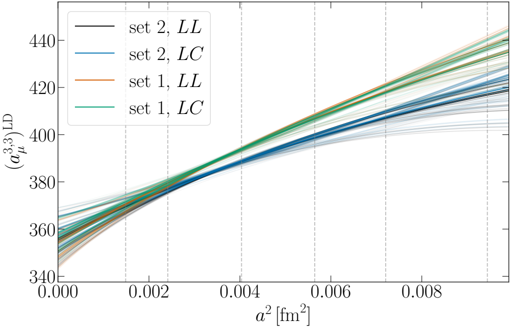

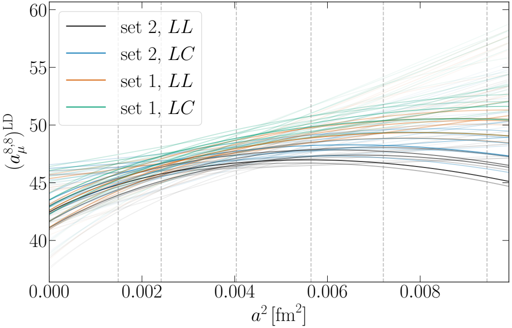

- Comparison with RBC/UKQCD 24 in different isoQCD schemes: Mainz vs. BMW20.
- Scale setting with w_0 induces large and higher-order cutoff effects $\rightarrow$ larger statistical and systematic uncertainties!

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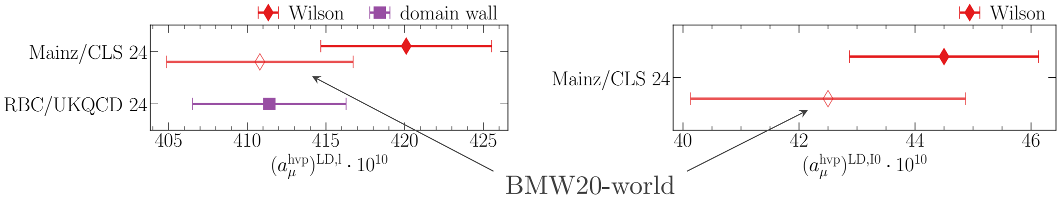
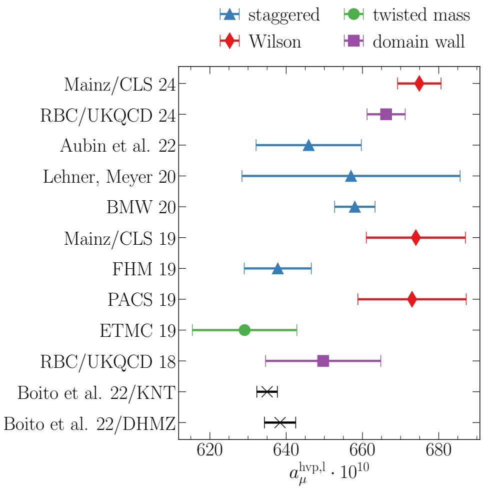
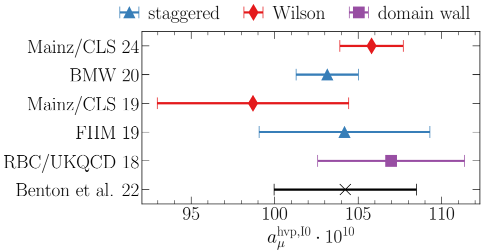


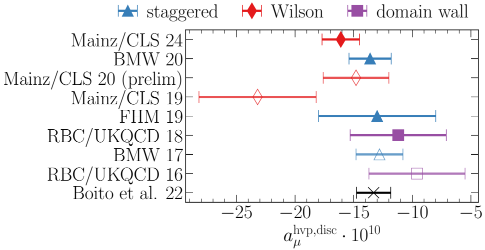
- Comparison with RBC/UKQCD 24 in different isoQCD schemes: Mainz vs. BMW20.
- Scale setting with w_0 induces large and higher-order cutoff effects
  $\rightarrow$ larger statistical and systematic uncertainties!
- Ignore scale uncertainty for w_0^{phys}(BMW20) in the comparison ◇ vs. ■.
- Shift in relation to tensions in flow scale determinations [FLAG23]?

Contributions to a_μ^{hvp} in isoQCD



- Compute contributions to a_μ^{hvp} in isoQCD (Mainz world) by combinations with $(a_\mu^{\mathrm{hvp}})^{\mathrm{SD}}$ and $(a_\mu^{\mathrm{hvp}})^{\mathrm{ID}}$.
- We (will) publish the derivatives w.r.t. the input that defines our scheme. See [Portelli] for a comparison of schemes.
- $a_\mu^{\mathrm{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.





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- $a_\mu^{\mathrm{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.
- Shift in disconnected is understood: leads to the dominant shift in a_μ^{hvp}.

$(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$: Status and outlook

Achievements

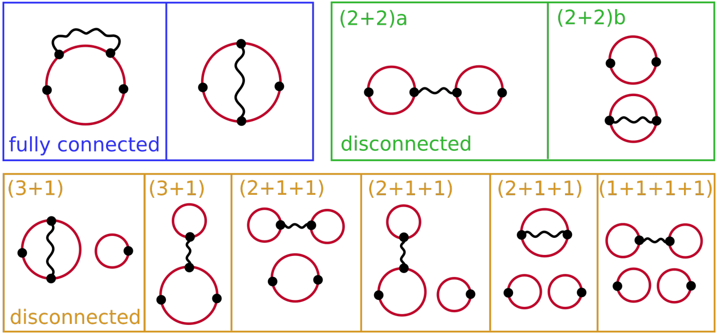
- High statistical precision at m_π^{phys} and excellent control of the m_π dependence.
- Large span of lattice spacings to control the continuum extrapolation.
- Compute full isoQCD $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ to 1.3% precision (statistics dominated).

Outlook

- More data at fine lattice spacing and m_π^{phys} is being computed.
- Strong scale dependence in the long-distance regime:
  - We observe a strong scheme dependence: due to differences in the scale setting?
  - The global status of gradient flow scales is unsatisfactory [FLAG23].
- Need to include isospin breaking effects to compute the full a_μ^{hvp}.

Isospin breaking corrections

- Compute leading-order effects on QCD gauge ensembles.
- Quark-connected contribution to a_μ^{hvp}: $\mathrm{QED_L}$ in the TMR
  [Andreas Risch @ Converging on QCD+QED prescriptions, Edinburgh]
  - Has entered in our estimates for $(a_\mu^{\mathrm{hvp}})^{\mathrm{ID}}$ and $(a_\mu^{\mathrm{hvp}})^{\mathrm{SD}}$ already.
- Quark-disconnected contribution to a_μ^{hvp}: QED_∞ in coordinate space
  [Biloshytskyi et al., 2209.02149] [Julian Parrino @ Lattice24] [Dominik Erb @ Lattice24]
- Scale setting via the baryon spectrum in $\mathrm{QCD}+\mathrm{QED}$: $\mathrm{QED_L}$
  [Alexander Segner @ MITP Workshop on Isospin-Breaking Effects]
- We aim to move beyond the electroquenched approximation in future work.

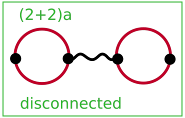


- QED_∞ : Photon propagator in the continuum in infinite volume

$$a_\mu^{\mathrm{hvp,NLO}} = -\frac{e^2}{2}\int_{x,y,z} H_{\mu\sigma}(z)\delta_{\nu\rho}\Big[G_0(y-x)\Big]_\Lambda \langle j_\mu(z)j_\nu(y)j_\rho(x)j_\sigma(0)\rangle_{\mathrm{QCD}} + \mathrm{counterterms}$$

- After renormalization take limit $\Lambda \to \infty$

The $(2+2)a$ contribution

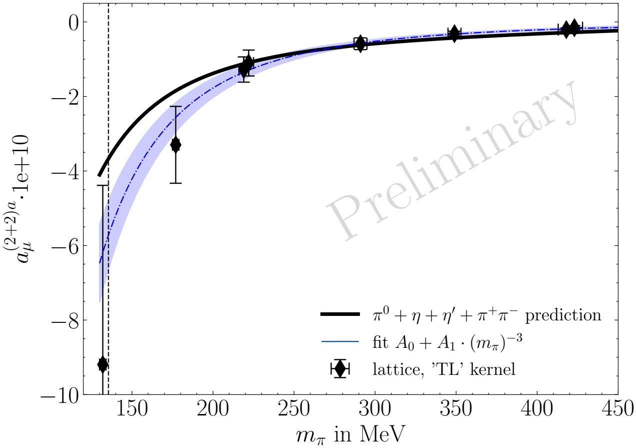


- UV finite QED correction [Julian Parrino @ Lattice24].
- Chiral extrapolation guided by pheno model including π^0, η, η' and $\pi^+\pi^-$ contribution.
- Effective chiral dependence

$$a_\mu^{(2+2)a,\pi^+\pi^-} \propto m_\pi^{-3}$$

- **Preliminary result:**

$$a_\mu^{(2+2)a-ll} = -5.94(0.99) \cdot 10^{-10}$$



Total isospin violating part at SU(3) symmetric point

- The isospin violating part:

$$a_\mu^{\text{hvp,NLO}} \to a_\mu^{\text{hvp,NLO,38}}$$

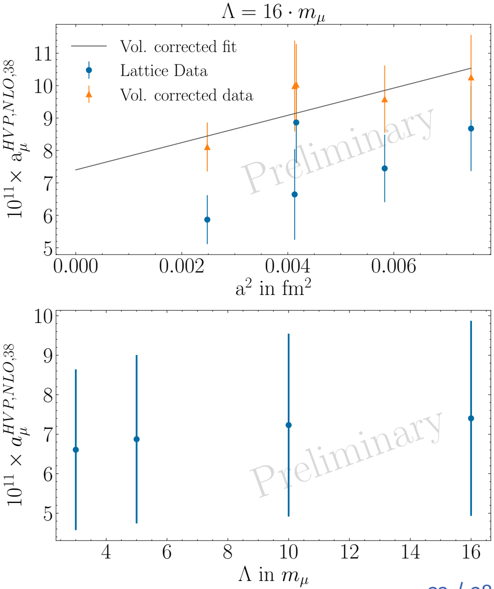
$$\langle j_\mu(z) j_\nu(y) j_\rho(x) j_\sigma(0)\rangle_{\text{QCD}}$$

$$\to \langle j^3_\mu(z) j^{em}_\nu(y) j^{em}_\rho(x) j^8_\sigma(0)\rangle_{\text{QCD}}$$

- Continuum extrapolation with volume term: $f_{\text{fit}}(a, m_\pi L) = B + C \cdot a^2 + D \cdot \mathrm{e}^{-\frac{m_\pi L}{2}}$

- Result is constant within error for different Λ $\to$ Plateau is reached

- **Preliminary result:** $a_\mu^{\text{hvp,NLO,38}} = 0.74(25) \cdot 10^{-10}$



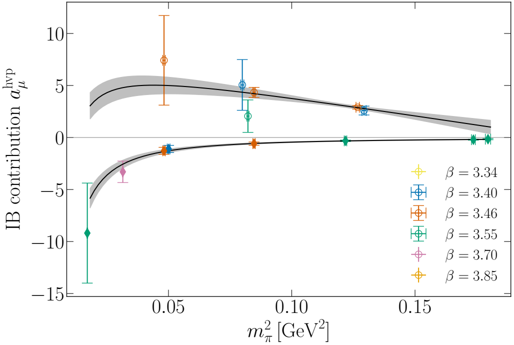


Illustration of the

- fully connected data (circles, positive)
- and the $(2+2)a$ contribution (diamonds, negative).

- Perform a **preliminary** combined fit

$$a_\mu^{\text{IB,conn}} = \frac{34}{81}\frac{A}{m_\pi^3} + b\,m_\pi^2 + c + 0.218\log\left(\frac{m_V^2}{m_\pi^2}\right)$$

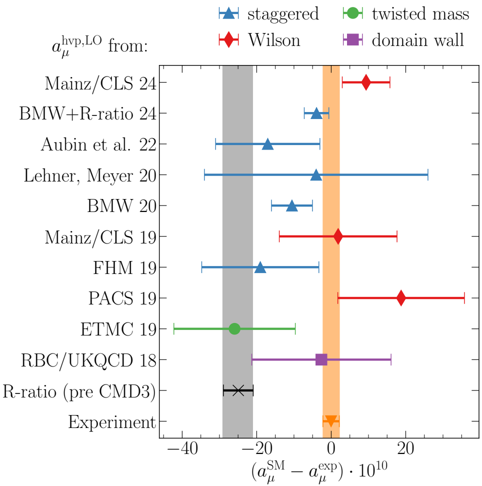
$$a_\mu^{\text{IB,(2+2a)}} = \frac{50}{81}\frac{A}{m_\pi^3} + d$$

- No cutoff effects resolved.
- Estimate the missing piece from the $\pi^+\pi^-$ loop contribution

$$a_\mu^{\text{IB,(3+1a)}} = -\frac{14}{81}\frac{A}{m_\pi^3}$$

- Combine for an estimate for IB effects in the electroquenched approximation.

The full HVP



[BNL $g-2$, hep-ex/0602035]

[FNAL $g-2$, 2104.03281, 2308.06230]

- The estimate of IB corrections allows to compute a **preliminary** a_μ^{hvp}.
- Our result supports the no new physics scenario.
- Ongoing work to compute IB corrections. So far
  - no IB in scale setting
  - electroquenched approximation
  - **preliminary estimate**

Hadronic running of the electromagnetic coupling and the electroweak mixing angle

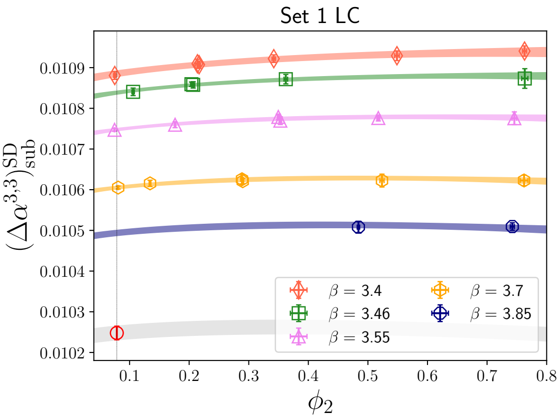
[Alessandro Conigli @ Lattice24]

Electroweak couplings

- $\alpha(-Q^2)$ and $\sin^2\theta_W(-Q^2)$ as relevant quantities for precision tests of SM $\rightarrow$ plan to update [Cè et al., 2203.08676]
- Pushing to high Q^2 to achieve increased precision at the Z pole

$$\bar{\Pi}(Q^2) = \left[\Pi(Q^2) - \Pi(Q^2/4)\right] + \left[\Pi(Q^2/4) - \Pi(0)\right]$$

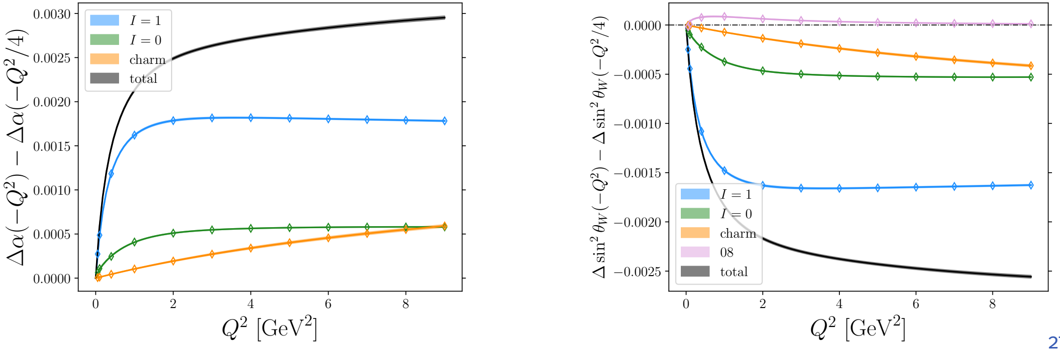
- Subtracted kernel to drop t^4 term, thus canceling $a^2\log(a)$ cutoff effects [SK et al., 2401.11895]
- Reduction of cutoff effects in the short Euclidean distance with tree-level improvement.



↑ Isovector chiral extrapolation at $Q^2 = 9\ \mathrm{GeV}^2$

- **Preliminary** results for $\bar{\Pi}(-Q^2) - \bar{\Pi}(-Q^2/4)$ in the range $0 \leq Q^2 \leq 9\ \mathrm{GeV}^2$.
- Rational approximation of the running through a multi-points Padé Ansatz [Aubin et al., 1205.3695] [Cè et al., 2203.08676].

$$\bar{\Pi}(-Q^2) \approx \frac{\sum_{j=0}^{M} a_j Q^{2j}}{1 + \sum_{k=1}^{N} b_k Q^{2k}}$$



- Determination of $(a_\mu^{\mathrm{hvp}})^{\mathrm{LD}}$ allows to update the Mainz result for a_μ^{hvp} in isoQCD:

$$(a_\mu^{\mathrm{hvp,LO}})_{\mathrm{isoQCD}} = 728.6(4.3)_{\mathrm{stat}}(3.6)_{\mathrm{syst}}[5.5][0.75\%]$$

- We plan to publish our latest result in the near future.

- Ongoing work on the isospin breaking corrections:
  Preliminary result indicates an insignificant negative contribution.

- The results support the no new physics scenario.

- Data set allows to consider related observables such as
  the running of $\Delta\alpha_{\mathrm{had}}$ or $a_\mu^{\mathrm{hvp,NLO}}$ (see the poster by Arnau Beltran).

The Mainz world

- The scheme for isoQCD [Risch and Wittig, 2112.00878]

$$m_{\pi^0} \propto m_{\rm u} + m_{\rm d}\,, \qquad m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2 \propto m_{\rm s}\,,$$
$$m_{\rm u} = m_{\rm d}\,, \qquad \alpha_{\rm em} = 0$$

corresponding to

$$m_\pi = 134.9768(5)\ \text{MeV}\,, \quad m_K = 495.011(10)\ \text{MeV}\,.$$

- The scheme for QCD + QED

$$m_{\pi^0} \propto m_{\rm u} + m_{\rm d}\,, \qquad m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2 \propto m_{\rm s}\,,$$
$$m_{K^+}^2 - m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2 \propto m_{\rm u} - m_{\rm d}\,, \qquad \alpha_{\rm em}$$

- Scale setting with the pion decay constant in the iso-symmetric theory,

$$f_\pi = 130.56(14)\ \text{MeV} \quad \text{and} \quad f_{K\pi} = \frac{2}{3}(f_K + \frac{1}{2} f_\pi) \text{ with } f_K = 157.2(5)\ \text{MeV}\,,$$

no IB in scale setting so far!