Lattice HVP WP discussion

Agenda

- Which windows to review?
  short/intermediate/long, one-sided, ...

- Adoption of the Edinburgh Consensus for isospin-breaking effect separation?
  Final stages within FLAG, scheme itself not questioned

- How well do we know the a_μ QED corrections
  electro-quenched approximation, non-physical point estimates

- Can we compare IB and disconnected contributions with data-driven analysis?

Edinburgh Consensus

	QCD	isoQCD
M_{π^+}	135.0 MeV	135.0 MeV
M_{K^+}	491.6 MeV	494.6 MeV
M_{K^0}	497.6 MeV	494.6 MeV
$M_{D_s^+}$	1967 MeV	1967 MeV
$M_{B_s^0}$	5367 MeV	5367 MeV
f_{π^+}	130.5 MeV	130.5 MeV

	QCD	isoQCD
M_{π^+}/f_{π^+}	1.034	1.034
M_{K^+}/f_{π^+}	3.767	3.790
M_{K^0}/f_{π^+}	3.813	3.790
$M_{D_s^+}/f_{\pi^+}$	15.07	15.07
$M_{B_s^0}/f_{\pi^+}$	41.13	41.13

Table 5: Edinburgh Consensus for the definition of pure QCD and isospin-symmetric QCD. The rightmost table is redundant and provided for convenience.

One is assumed to know from lattice data

$\tilde{M}^{\mathrm{sim}}, \tilde{f}^{\mathrm{sim}}_{\pi^+}$, and $\tilde{M}^{\mathrm{sim}}_{\Omega^-}$

$\left(\frac{\partial \tilde{M}}{\partial \tilde{m}}\right)^{\mathrm{sim}}$ $: N_f \times N_f$ matrix $\quad \left(\frac{\partial \tilde{M}}{\partial e^2}\right)^{\mathrm{sim}}$ $: N_f \times 1$ matrix

$\left(\frac{\partial \tilde{f}_\pi}{\partial \tilde{m}}\right)^{\mathrm{sim}}$ & $\left(\frac{\partial \tilde{M}_{\Omega^-}}{\partial \tilde{m}}\right)^{\mathrm{sim}}$ $: 1 \times N_f$ matrices

$\left(\frac{\partial \tilde{M}_{\Omega^-}}{\partial \tilde{e^2}}\right)^{\mathrm{sim}}$ $: 1 \times 1$ matrix

Therefore one also knows

$\left(\frac{\partial \rho}{\partial \tilde{m}}\right)^{\mathrm{sim}}$ & $\left(\frac{\partial R}{\partial \tilde{m}}\right)^{\mathrm{sim}}$ with $\rho = M/f_{\pi^+}$ and $R = M/M_{\Omega^-}$ ($N_f \times N_f$ matrices)

$M = (M_{\pi^+}, M_{K^+}, M_{K^0}, \ldots)$

m : bare quark masses

$\tilde{X}$: lattice units

X^{sim} : simulation point (close to physical point)

Step 1: find Edinburgh Consensus QCD point

Solve for $\Delta\tilde{m}^{\mathrm{EC}}$ $(N_f \times N_f$ linear system)

blue = known

$$\rho^{\mathrm{EC}} = \rho^{\mathrm{sim}} + \left(\frac{\partial \rho}{\partial \tilde{m}}\right)^{\mathrm{sim}} \Delta\tilde{m}^{\mathrm{EC}}$$

On can now predict observables in lattice units at the EC point

$$\tilde{X}^{\mathrm{EC}} = \tilde{X}^{\mathrm{sim}} + \left(\frac{\partial \tilde{X}}{\partial \tilde{m}}\right)^{\mathrm{sim}} \Delta\tilde{m}^{\mathrm{EC}}$$

Step 2: QCD scale setting

Predict $\tilde{f}_{\pi^+}^{\mathrm{EC}}$, then $a_{\mathrm{QCD}} = \dfrac{\tilde{f}_{\pi^+}^{\mathrm{EC}}}{f_{\pi^+}^{\mathrm{EC}}}$

One can now predict dimensionful $e = 0$ quantities $X^{\mathrm{EC}} = a_{\mathrm{QCD}}^{-[X]}\tilde{X}$

Step 3: find QCD physical (ϕ) point

Solve for $\Delta\tilde{m}^{\phi}$ ($N_f \times N_f$ linear system) **blue** = known

$$R^{\mathrm{PDG}} = R^{\mathrm{sim}} + \left(\frac{\partial R}{\partial \tilde{m}}\right)^{\mathrm{sim}} \Delta\tilde{m}^{\phi} + e^2 \left(\frac{\partial R}{\partial e^2}\right)^{\mathrm{sim}}$$

Step 4: Full (QCD+QED) scale setting

Predict $\tilde{M}^{\phi}_{\Omega^-}$, then $a_{\mathrm{QCD+QED}} = \dfrac{\tilde{M}^{\phi}_{\Omega^-}}{M^{\mathrm{PDG}}_{\Omega^-}}$

One can now predict physical quantities $X^{\phi} = a^{-[X]}_{\mathrm{QCD+QED}} \tilde{X}^{\phi}$

NB

- After step 2 one will know $M^{\mathrm{EC}}_{\Omega^-}$ in MeV, and does not need to refer to f_{π^+} anymore
- Derivatives in physical quantities (not bare masses) can be obtained through Jacobian product