Updates on QED Contributions since WP2020

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7th Plenary Workshop of
the Muon g-2 Theory Initiative
KEK, Tsukuba, Japan

Talk Outline

New experimental results

- Rb atom-recoil measurement
- Electron g-2 measurement

Updates on input values for the theory of muon g-2

- The fine-structure constant α
- The electron-muon mass ratio me/mμ

Status of the QED g-2 calculation

- QED 10th-order mass-independent universal term $A_1^{(10)}$

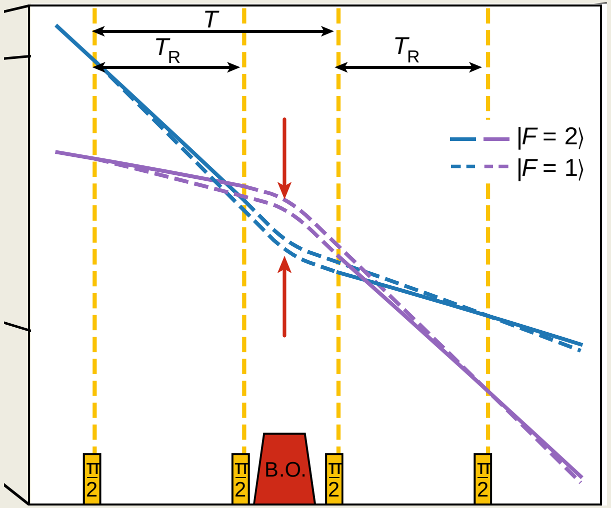
New atom recoil measurement of Rb

A New result of h/M

Leo Morel, Zhibin Yao, Pierre Clade, Saida Guellati-Khelifa,
Laboratoire Kastler Brossel (LKB),
Nature 588, 61-65 (2020)

$$h/M_{\mathrm{Rb}} = 4.591\,359\,258\,90\,(65) \times 10^{-9}\,\mathrm{m^2 s^{-1}}$$

Rb(rubidium) atom interferometer   h/M_{Rb}



$$\Phi = T_R[4N_B \frac{\hbar k_R k_B}{m_{\mathrm{Rb}}} - \delta\omega_R]$$

Since 2019, the Planck constant *h* is the defined constant, this is the most precise direct atomic mass measurement.

atom mass = ion mass + electron mass − ionization energy
Penning Trap   Optical expt.

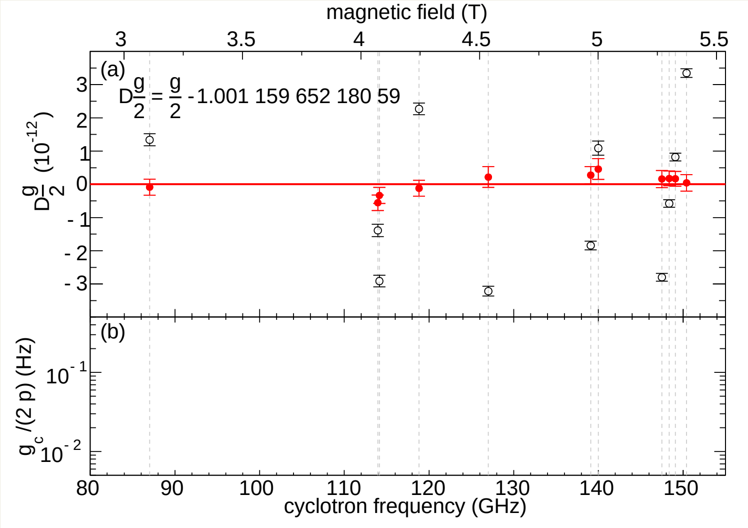
New Electron g-2 measurement

arXiv: 2209.13084, PRL130,071801(2023)

X. Fan, T. G. Mayers, B.A.D. Sukra, and G. Gabrielse,   Northwestern U

$$a_e(\mathrm{NW22}) = 1\,159\,652\,180.59\,(13) \times 10^{-12}$$
$$a_e(\mathrm{HV08}) = 1\,159\,652\,180.72\,(28) \times 10^{-12}$$

2.2 times better than before
different values of Magnetic fields
less systematic uncertainty



the fine-structure constant α

- Atomic Recoil h/M of Cs or Rb

$$\alpha = \left[\frac{h}{M} \times \frac{A_r(M)}{A_r(m_e)} \times \frac{2R_\infty}{c}\right]^{1/2}$$

least precise

CODATA2022, May2024

R_∞	$= 10\,973\,731.568\,157(12)\ \mathrm{m}^{-1}$	1.1ppt
$A_r(e)$	$= 5.485\,799\,090\,441\,(97) \times 10^{-4}\ \mathrm{u}$	18ppt
$A_r(M_{\mathrm{Rb}})$	$= 86.909\,180\,5310\,(60)\ \mathrm{u}$	69ppt
$A_r(M_{\mathrm{Cs}})$	$= 132.905\,451\,9615\,(86)\ \mathrm{u}$	65ppt

- Electron g-2

$$a_e(\mathrm{expt.}) = a_e(\mathrm{theory}; \alpha)$$

WP2022 α values

h/M_Cs18

$$\alpha^{-1}(\mathrm{Cs18}) = 137.035\,999\,046\,(27)$$

$$\alpha^{-1}(a_e 08 \times 19) = 137.035\,999\,1496\,(13)(14)(330)$$

QED19, Hadron VP, HV08

Expt theory

08×19 means HV08 and QED19

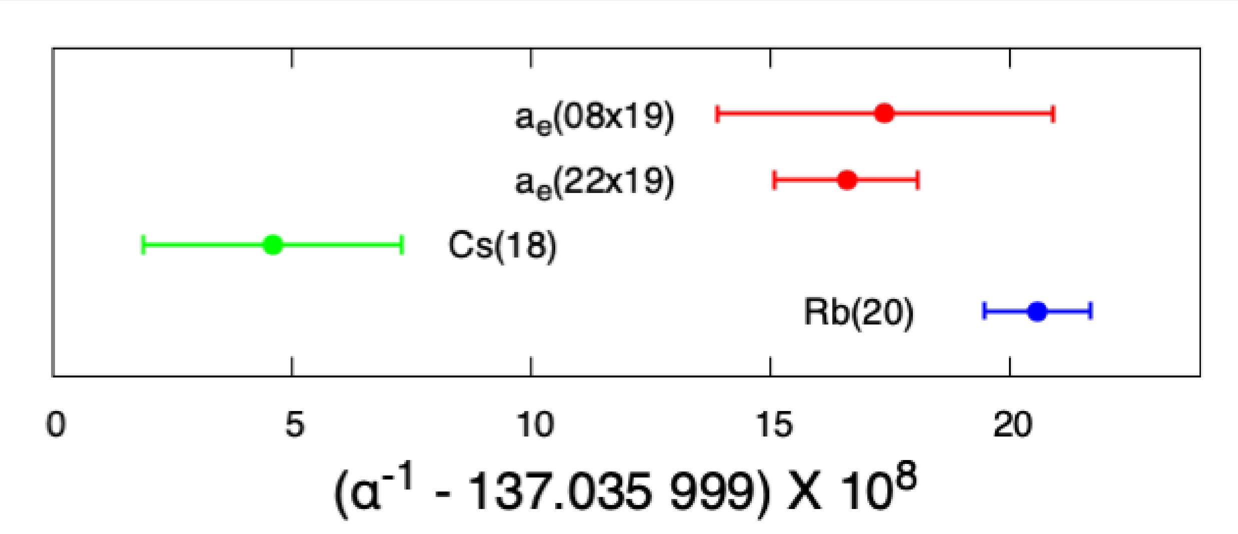
Values of α for WP2025

Updates

Cs02 (not shown)→ Cs18

Rb11 (not shown)→ Rb20

ae08x19 → ae22x19



		difference	
$\alpha^{-1}(a_e 22\times 19)$	$= 137.035\,999\,166\,(15)$	0	
$\alpha^{-1}(\mathrm{Cs18})$	$= 137.035\,999\,046\,(27)$	$-120(31)\times 10^{-9}$	$-3.9\,\sigma$
$\alpha^{-1}(\mathrm{Rb20})$	$= 137.035\,999\,206\,(11)$	$+40(19)\times 10^{-9}$	$+2.1\,\sigma$

Effects on lepton g-2 values from Δα

The leading theory term of g-2 $a_l = \frac{\alpha}{2\pi} + \cdots$

a_l value relative to the a_l w/ $\alpha(a_e 22 \times 19)$

If $\alpha(\mathrm{Cs18})$ is used, a_l shifts $+1.0 \times 10^{-12}$

If $\alpha(\mathrm{Rb20})$ is used, a_l shifts -0.34×10^{-12}

any of the three α's can be used for the theory of muon g-2

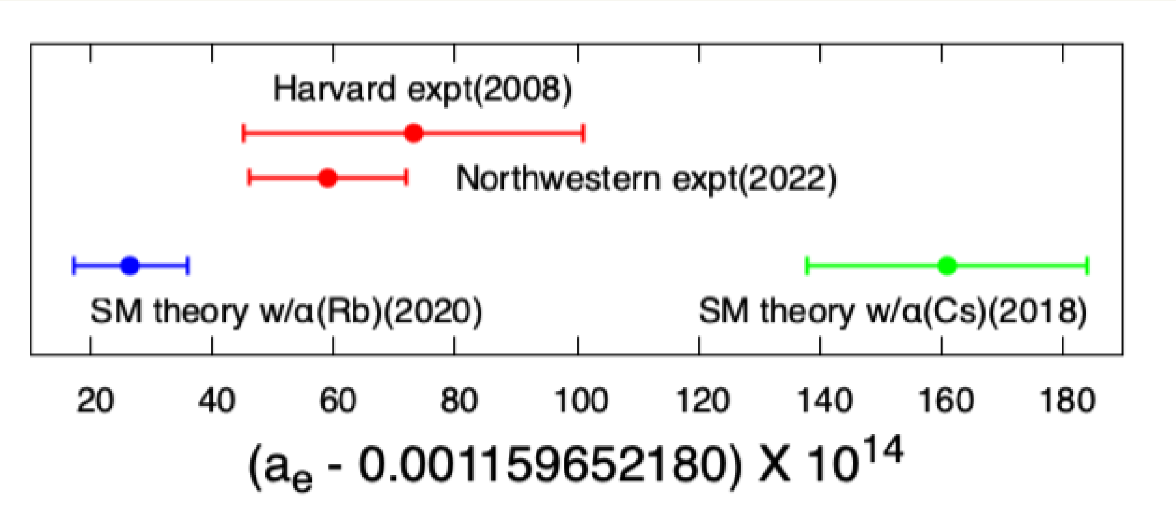
Muon g-2 World exp average in 2023

$a_\mu(\mathrm{exp}) = 116\,592\,059\,(22) \times 10^{-11}$

future uncertainty will be of order

100×10^{-12}

Electron g-2 Theory v.s. Experiment



$$a_e(\mathrm{exp}) = 1\,159\,652\,180.59\,(13) \times 10^{-12}$$

$$a_e(\mathrm{th}_{\alpha(\mathrm{Rb})}) = 1\,159\,652\,180.265\,(93)(11)(9)[94] \times 10^{-12}$$

α QED hadron total

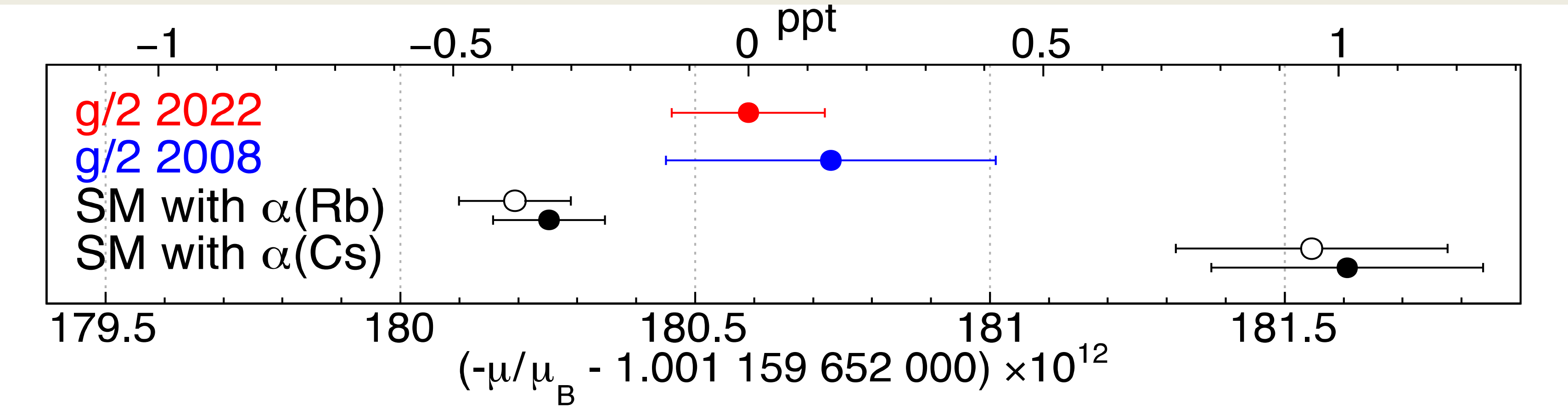
$$a_e(\mathrm{th}_{\alpha(\mathrm{Cs})}) = 1\,159\,652\,181.616\,(229)(11)(9)[229] \times 10^{-12}$$

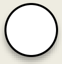
Improvement of Hadron contribution due to

A. Keshavarzi, D. Nomura, T. Teubner, arXiv:1911.00367, PRD101,014029 (2020)

Effects of two QED results of the 10th-order term

X. Fan, et. al. arXiv: 2209.13084,PRL130,071801(2023)





QED 10th-order term from AHKN 2019

QED 10th-order term from S. Volkov 2019

Different is Not crucial right now, but must be resolved. Discuss later

Electron-Muon mass ratio CODATA values

Electron-muon mass ratio

determined from Muonium 1S HFS (μ+e-) no update since 1999

w/ α (~0.1 ppb) and the Rydberg constant (~1 ppt)

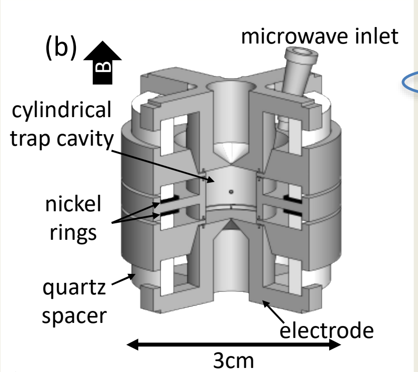
CODATA	m_e/m_μ	
2010	$4.836\,331\,66\,(12) \times 10^{-3}$	
2014	$4.836\,331\,70\,(11) \times 10^{-3}$	Used for WP2020 $\alpha(a_e 08 \times 12)$
2018	$4.836\,331\,69\,(11) \times 10^{-3}$	Effect of $\alpha(\mathrm{Cs18})$
2022	$4.836\,331\,70\,(11) \times 10^{-3}$	Effect of $\alpha(\mathrm{Rb22})$

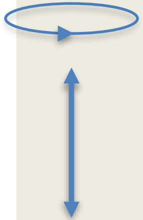
CODATA years are deadlines. Publications are 1 or 2 years later.

Further improvement of Electron g-2

G. Gabrielse @ Northwestern U   keeps working

- positron g-2 measurement

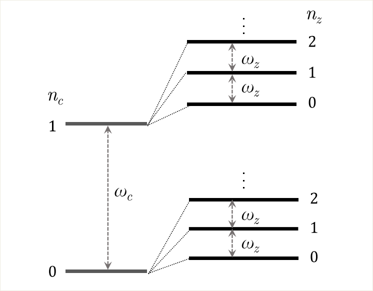




Cyclotron motion is in the ground state

Axial motion is NOT in the ground state

→ Source of the systematic error



- cooling of axial motion, quantum measurement

X. Fan and G. Gabrielse
arXiv:2008.01898

A factor 10 ~ 20 improvement are expected

Improvement of Atomic Recoil Measurements

- H. Mueller@UC Berkeley

  Cs a factor of 20 improvement is aimed

- S. Guellati-Khélifa@LKB

  Rb new result in a few years?

  New challenge w/ Yb (ytterbium)

- C. J. Foot@Oxford new!

  new proposal arXiv:2403.10225

  Sr (strontium) or Yb

  an atom interferometer of height 3m

  a factor of 10 improvement in α is possible

  $$\Delta M_{\rm A}/M_{A} \sim 1 \times 10^{-11}$$

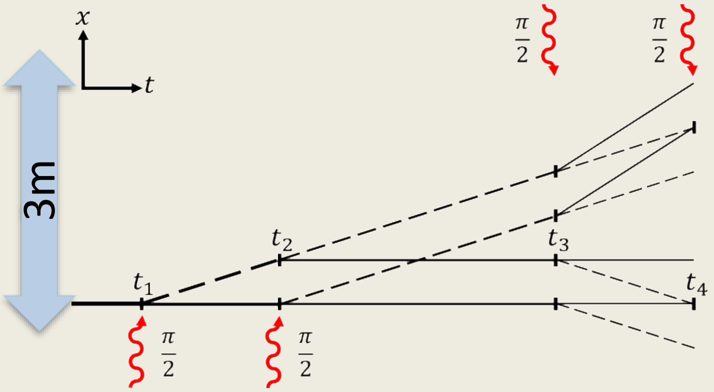


FIG. 1. Schematic spacetime diagram of a Ramsey-Bordé atom interferometer in the absence of external acceleration. The ground (excited) state is indicated by a solid (dashed) line with thickness indicative of the probability density. The timings of the laser pulses are such that $t_4 - t_3 = t_2 - t_1$, thus two pairs of trajectories result in interference.

arXiv:2403.10225

QED contribution to g-2

QED contribution to the lepton g-2

$$a_l(\mathrm{QED}) = A_1 + A_2\left(\frac{m_l}{m_{l_1}}\right) + A_2\left(\frac{m_l}{m_{l_2}}\right) + A_3\left(\frac{m_l}{m_{l_1}}, \frac{m_l}{m_{l_2}}\right)$$

QED perturbation

$$A_i = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n A_i^{(2n)}$$

l=*e*, *μ*, and *n*=1, 2, 3, and 4 , well established

We focus on the tenth-order terms, *n*=5

10^{th}-order QED g-2 A_1

S. Volkov's calculation of the 10^{th}-order mass-independent A_1 term

- arXiv:1909.08015 Phys. Rev. D 100, 096004 (2019)

  Diagrams of Set V w/o fermion loop

  discrepancy 4.8 σ from the AHKN 2019 result

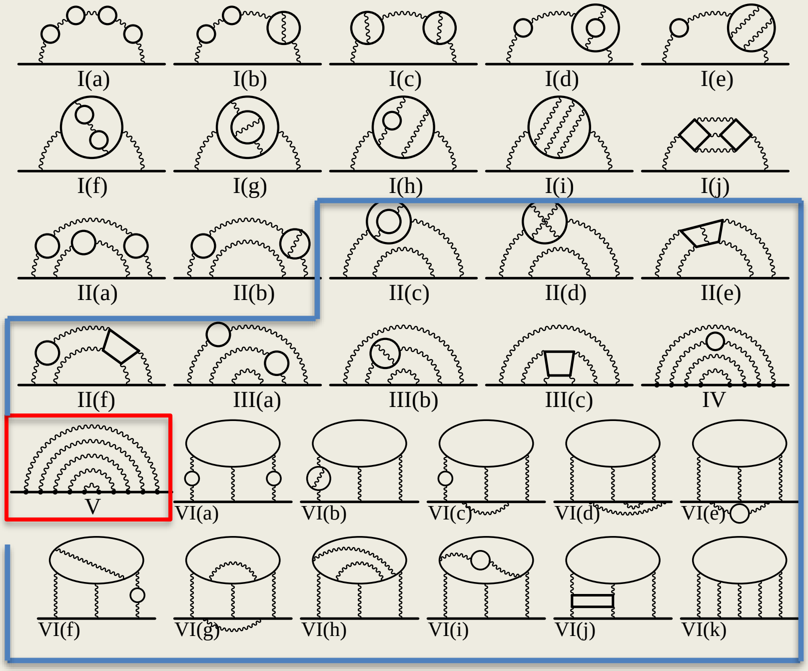
- arXiv:2404.00649 Phys. Rev. D 110, 036001 – Published 2 August 2024

  Diagrams w/ fermion loop agree with the AHKN 2012 results

  Set V discrepancy 4.6 σ from the AHKN 2019 result

Details will be described by S. Volkov's talk in this session

10^{th}-order diagrams



Confirmed the first time

- 12,672 vertex diagrams over 32 Sets
- 31 Sets w/ fermion loop (6,318 diagrams)

  I(a-j), II(a,b) were already confirmed

  S. Lapota (1994)

  P. A. Baikov, A. Maier, and P. Marquard (2013)

  agreement between AHKN and Volkov

  AHKN Volkov

  $$-0.933\,(17) - \{-0.9377\,(35)\} = 0.005\,(17)$$

- Set V w/o fermion loop (6,354 diagrams)

  AHKN Volkov

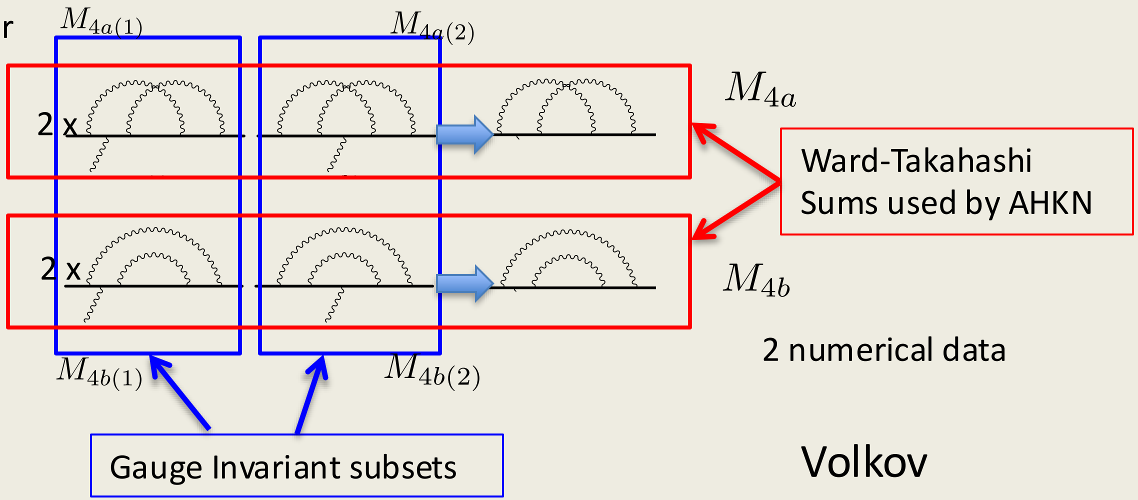
  $$7.668\,(159) - \begin{cases} 6.793\,(90) & = 0.875\,(182), \quad 4.8\sigma\,(2019) \\ 6.857\,(81) & = 0.811\,(178), \quad 4.6\sigma\,(2024) \end{cases}$$

Set V: Ward-Takahashi sum v.s. Vertices

AHKN: 389 Ward-Takahashi sums of v-diagrams.

Volkov: 3,213 vertex diagrams.

4th-order



Every vertices are calculated by Volkov
4 numerical data

Volkov AHKN

$$\sum_{i=1}^{2n-1} \Lambda^{\nu}_{\mathcal{G}(i)}(p,q) \simeq -q_{\mu} \sum_{i=1}^{2n-1} \left[\frac{\partial \Lambda^{\mu}_{\mathcal{G}(i)}(p,q)}{\partial q_{\nu}} \right]_{q=0} - \frac{\partial \Sigma_{\mathcal{G}}(p)}{\partial p_{\nu}}$$

Different renormalization scheme

On-shell renormalization constants for a self-energy diagram *G*:

$L_{G(i)}$ for vertex renormalization

B_G for wave-function renormalization

Volkov used IR-free and Ward-Takahashi identity holds:

$$BV_G + \sum_{i=1}^{2n-1} LV_{G(i)} = 0$$

AHKN used IR free, easy-to-determine, but breaking WT-identity:

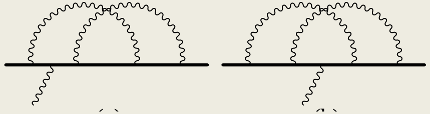
$$BK_G + \sum_{i=1}^{2n-1} LK_{G(i)} + \Delta LB_G = 0$$

Finite renormalization

Restore the gauge invariance

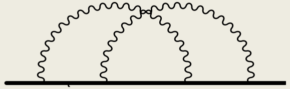
Connection bw Volkov and AHKN

Volkov



$$a_{4a} = M_{4a(1)} + 2M_{4a(2)} - 2\ L_2\ M_2$$
$$= \Delta M_{4a(1)} + 2\Delta M_{4a(2)} - 2(L_2 - LV_2)\ M_2$$

AHKN



$$a_{4a} = M_{4a} - 2\ L_2\ M_2$$
$$= \Delta M_{4a} - 2(L_2 - LK_2)\ M_2$$

The same physical contribution, but IR divergent

Numerical and finite numbers

$$\Delta M_{4a} - (\Delta M_{4a(1)} + 2\Delta M_{4a(2)}) = 2\ \delta L_2\ M_2$$

AHKN' s integral Volkov's integral Gap equation

where $$\delta L_2 \equiv LV_2 - LK_2$$

What to do

To compare AHKN and Volkov's numerical results of integrals,

- Obtain the symbolic expressions of the gap equations
  expressed by $\delta L \text{ and } \Delta M_G$ of the $2^{nd} \sim 8^{th}$ order quantities

- Calculate the values of δL .
  ΔM_G are known from AHKN's old publications.

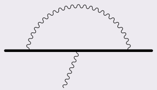
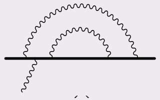
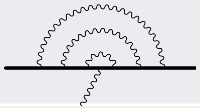
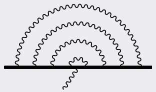
- The difference of numerical integrals is compared to the numerical values of the gap equation.

New calculation of $\delta L_{G(i)}$

Difference of vertex renormalization constants

numerically calculated for n=1, 2, 3, 4 loops

(#) # of independent diagrams

Order 2n	2	4	6	8
# of vertex diagrams	1	6 (4)	50 (38)	518 (269)
				

$$\delta L_{G(i)} = \left(1 + \sum_f \left[\prod_{S_i \in f} (-\mathbb{K}_{S_i})\right]\right)(LV_{G(i)} - LK_{G(i)})$$

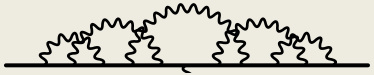
UV subtraction w/ AHKN's K-operation

269 x 1 hour x 40 core ~ 10,000 core x hours

easy calculation compared to the 10th-order g-2

A. Hirayama, JPS 2021 spring meeting

X001 as an example



abacbdcede

gap equation

$$\Delta M_{X001} - \sum_{i=1}^{9} \Delta M_{X001(i)} = \Delta M_2 \ (-3(\delta L_{4a1})^2 - 6\delta L_2 \delta L_{6f1} + 12(\delta L_2)^2 \delta L_{4a1} - 5(\delta L_2)^4 + 2\delta L_{01v1}) + \Delta M_{01} \ (2\delta L_2) + \Delta M_{6f} \ (2\delta L_{4a1} - 3(\delta L_2)^2) + \Delta M_{4a} \ (2\delta L_{6f1} - 6\delta L_2 \delta L_{4a1} + 4(\delta L_2)^3)$$

AHKN

Volkov2019

Numerical integrals

Symbolic expression

$$\begin{aligned} \text{l.h.s} &= -0.16083\ (334) - 0.58095\ (534) \\ &= -0.74178\ (630) \\ \text{r.h.s} &= -0.73854\ldots \\ \text{l.h.s} - \text{r.h.s} &= -0.00324\ (630) \end{aligned}$$

Substitute numerical values for lower-order symbols

A. Hirayama, JPS 2021 spring meeting

X001 safely passes the numerical check.

Nothing wrong in AHKN and in Volkov

All X001 – X389 pass the check

almost 0

Diagram $\mathcal{G}$	Expression	$\Delta M_{\mathcal{G}} - \sum_i \Delta M_{\mathcal{G}(i)}$	Gap Equation	Difference
X001	*abacbdcede*	−0.7418	−0.7385	−0.0033 (63)
X002	*abaccddebe*	8.0130	8.0253	−0.0123 (139)
X003	*abacdbcede*	2.0226	2.0221	0.0006 (29)
X004	*abacdcdebe*	−6.5041	−6.5146	0.0104 (130)
X005	*abacddbece*	−0.2680	−0.2789	0.0110 (106)
X006	*abacddcebe*	0.5522	0.5522	−0.0000 (125)
X007	*abbcadceed*	−0.2365	−0.2250	−0.0115 (128)
X008	*abbccddeea*	−3.8164	−3.8115	−0.0050 (168)
X009	*abbcdaceed*	0.1962	0.2050	−0.0089 (69)
X010	*abbcdcdeea*	−1.4020	−1.4014	−0.0007 (137)
X011	*abbcddaeec*	−0.6609	−0.6645	0.0035 (110)
X012	*abbcddceea*	−0.4561	−0.4717	0.0156 (129)
X013	*abcabdecde*	1.7891	1.7879	0.0012 (14)
X014	*abcacdedbe*	0.2015	0.2018	−0.0003 (32)
X015	*abcadbecde*	1.1125	1.1141	−0.0016 (6)
X016	*abcadcedbe*	−0.4574	−0.4567	−0.0007 (5)
X017	*abcaddebce*	−0.7935	−0.7966	0.0030 (15)
X018	*abcaddecbe*	−0.4154	−0.4194	0.0040 (17)
X019	*abcbadeced*	2.3984	2.3968	0.0016 (31)
X020	*abcbcdedea*	7.6668	7.6799	−0.0131 (131)
X021	*abcbdaeced*	0.3403	0.3408	−0.0005 (16)
X022	*abcbdcedea*	−1.1144	−1.1238	0.0094 (114)
X023	*abcbddeaec*	−0.4987	−0.5074	0.0087 (58)
X024	*abcbddecea*	4.5369	4.5595	−0.0226 (131)
X025	*abccadeebd*	2.4091	2.4333	−0.0242 (119)
X026	*abccbdeeda*	1.2078	1.2042	0.0035 (110)
X027	*abccdaeebd*	−0.2238	−0.2272	0.0034 (46)

Preliminary

Both AHKN and Volkov correctly formulated the Set V integrals.

Why discrepancy arise?

- The differences range from -0.03 to $+0.03$
- Not randomly distributed → Bias in numerical integration

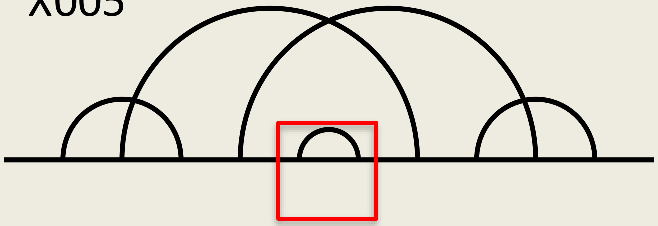
  # of negative differences << # of positive differences

- We divided 389 self-energy diagrams into 4 classes

Diagrams w/o a self-energy subdiagram	XL	135
Diagrams w/ one 2nd-order self-energy subd.	XB1B2	98
Diagrams w/ two 2nd-order self-energy subd	XB2B2	33
Others	others	123

XB1B2 98 S.E. diagrams

X005



2^{nd}-order s.e. subdiagram

Differences are positive for 90 diagrams
negative for 8 diagrams

$$\sum_G \left(\Delta M_G - \sum_{i=1}^{9} \Delta M_{G(i)}\right) = -49.095\,(73)$$

$$\sum_G \text{gap equation for } G = -50.089$$

$$\sum_G \text{differences} = +0.994\,(73)$$

AHKN is bigger than the answer

OR

Volkov is smaller than the answer

This accounts for the discrepancy between AHKN and Volkov of Set V

$$7.668\,(159) - \begin{cases} 6.793\,(90) & = 0.875\,(182), \quad 4.8\sigma\,(2019) \\ 6.857\,(81) & = 0.811\,(178), \quad 4.6\sigma\,(2024) \end{cases}$$

Re-evaluation of the 98 integrals of AHKN

Numerical calculations since December 2023 @RIKEN

We used the same integration program VEGAS used for previous works.

- Quasi double-double-precision calculation

shift

$$\sum_G \Delta M_G = 57.806\,(64) \longrightarrow 57.002\,(33),\ \boxed{-0.803\,(72)}$$

old new

Preliminary

- Double-precision calculation

$$\sum_G \Delta M_G = 57.806\,(64) \longrightarrow 56.868\,(18),\ \boxed{-0.937\,(66)}$$

old new

Updated Set V result

If XB1B2 is replaced by the quasi-double-double result ,

AHKN Volkov

$$6.865\,(149) - \begin{cases} 6.793\,(90) & = 0.072\,(174), \quad 0.41\sigma\,(2019) \\ 6.857\,(81) & = 0.008\,(169), \quad 0.47\sigma\,(2024) \end{cases}$$

Preliminary Preliminary

The discrepancy may be resolved

The 98 integrals of XB1B2 are relatively easy to evaluate
Calculated around 2008 ~ 2012, more than 10 years ago
The # of samplings for Monte Carlo integration is not sufficient

Summary & Prospectives

- Input values of QED

  α from 0.1 ppb to 0.01 ppb in 10 years

  me/mμ Muonium H.F.S. J-PARC MUSEUM

  1S-2S J-PARC Okayama U and PSI Mu-MASS
- 10th-order QED

  the discrepancy on the universal term A_1 is almost resolved

  further independent checks are needed for the mass-dependent term A_2
  preparation of the 12th-order QED is in progress

- No significant change in the QED contribution to the muon g-2,

  but the uncertainty is halved due to the improvement of α

Thank you for your patience