# Memories of Toichiro Kinoshita

1925-2023

Makiko Nio (RIKEN)

September 12, 2024

$7^{th}$ Plenary Workshop of

the Muon g-2 Theory Initiative

KEK, Tsukuba, Japan

# Toichiro Tom Kinoshita (1925-2023)

木下　東一郎　　　　　(Japan→ U.S.A.)
under a tree,　the No.1 man in the East

| | | |
|---|---|---|
| 1952 | Ph. D. | University of Tokyo |
| 1952 | | Institute of Advanced Study |
| 1954 | | Columbia University |
| 1955 | | Cornell University |
| | | Goldwin-Smith Professor, |
| | | Cornell University |
| 1995 | Retirement | |
| ~2019 | active in physics research | |

Known for his QED calculations of the lepton g-2

# Personality

## Toichiro “Tom” Kinoshita

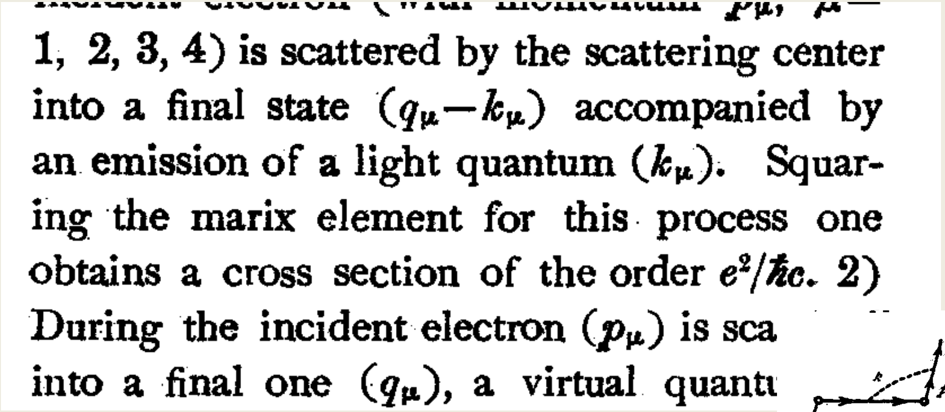
- arrived at the office at 9:00 a.m. sharp and left at 5:00 p.m. sharp.
- kept the regular schedule from Monday to Friday.
- maintained this habit until his mid-80s.
- had an exceptional ability to remain calm and warm.
- never became emotionally angry.
- had been skeptical of others' results until he achieved the same results.
- never bragged about his accomplishments.
- always looked forward and remained curious about new things.

# His works for 70 years

| | |
|---|---|
| • Mass singularity | 1950 – 1962 |
| • Helium atom | 1954 – 1957 |
| • Muon decay | 1957 – 1959 |
| • Muon g-2: $6^{th}$-order contribution | 1967 |
| • Muon g-2: $6^{th}$-order light-by-light | 1967 – 1969 |
| • Electron g-2: $6^{th}$-order | 1970 – 1995 |
| • Cornell Potential for Charmonium | 1978 – 1979 |
| • Electron g-2: $8^{th}$-order | 1975 – 2015 |
| • Muon g-2: hadronic light-by-light | 1984 – 2002 |
| • Lepton g-2: $10^{th}$-order | 1995 – 2020 |

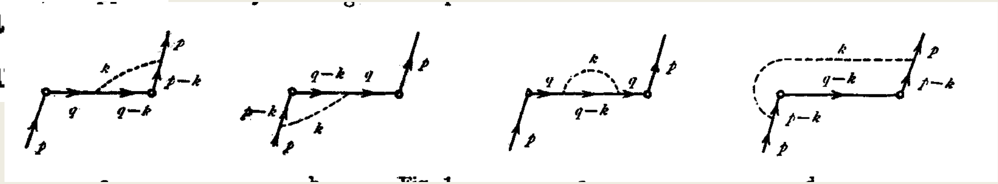
I focus only on QED-related works.

# Mass singularity



1, 2, 3, 4) is scattered by the scattering center into a final state ($q_\mu - k_\mu$) accompanied by an emission of a light quantum ($k_\mu$). Squaring the marix element for this process one obtains a cross section of the order $e^2/\hbar c$. 2) During the incident electron ($p_\mu$) is sca into a final one ($q_\mu$), a virtual quantu

*Progress of Theoretical Physics*, Volume 5, Issue 6, December 1950, Pages 1045–1047




Tom Kinoshita's supervisors

K. Kodaira
1954 Fields Medalist

S. Tomonaga
1965 Nobel laureate in Physics

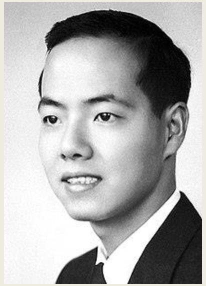
A Feynman amplitude often contains a divergent term

$$\lim_{m\to 0} \ln\left(\frac{q^2}{m^2}\right)$$

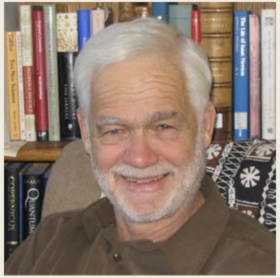
IR divergence disappears when all processes are summed up

Why? How?

# Infrared finite theorem



T. D. Lee



M. Nauenberg

## Mass Singularities of Feynman Amplitudes

Toichiro Kinoshita

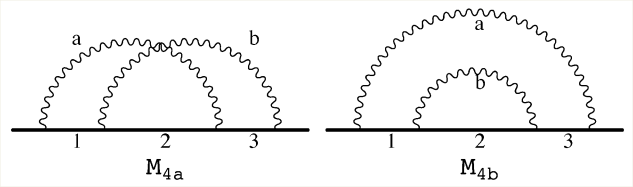
Check for updates

*J. Math. Phys.* 3, 650–677 (1962)

https://doi.org/10.1063/1.1724268 **Article history** &

It is found that, although partial transition probabilities may have divergences associated with the vanishing of masses of particles in the final state, they always cancel each other in the calculation of total probability.

## Kinoshita and Lee–Nauenberg theorem




$$z_1 + z_2 + z_3 + z_a + z_b = 1$$

$$0 \le z_i \le 1$$

Need to analyze complex Feynman diagrams

Momentum representation → Feynman parameters

The systematic description was invented.

# Helium atom


T. Nambu


A. Einstein

Kinoshita and Nambu went to the U.S.
as postdocs at IAS, Princeton (1952-1954).

Einstein asked Kinoshita what he(Kinoshita) was studying.
"Helium atom," Kinoshita answered.
Einstein lost interest in Kinoshita.
(When I asked him about it, Kinoshita was smiling and amused.)

The ionization energy of the ground state He atom
Sizeable discrepancy b/w theory and measurement
Fermi conjectured "new force" b/w electron and nucleus

# Numerical calculation of Helium atom

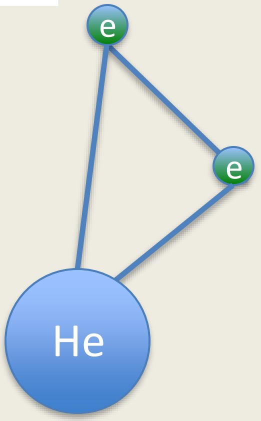
ments of the relativistic correction and showed that the discrepancy between theory and experiment would be even larger ($\sim$30 cm$^{-1}$) if all relativistic corrections of order $\alpha^2$ ry are correctly taken into account. In order to identify the source of such a large discrepancy, Chandrasekhar and Herzberg[4] have extended their com-

* Supported by the joint program of the Office of Naval

Hylleraas is −4.95 cm$^{-1}$.[6] It is of course desirable to evaluate this with more accurate wave functions. (3) It has been pointed out by Wilets and Cherry[7] that the mathematical lower bound evaluated with the 18-parameter function of Chandrasekhar and Herzberg is about 400 cm$^{-1}$ lower than the upper bound determined

[5] Those trial functions which can be written in the general form (2.7) will be said to be of Hylleraas type.

our calculation with 10- and 22-parameter functions may be regarded as an indication that the negative terms are in fact useful for our purpose (see Table II). On the other hand, the remarkable improvement of accuracy in the cases of 34 or more terms might have been obtained simply by the large flexibility of trial functions resulting from the tremendous number of terms involved. It would be interesting to notice,




Fermi's conjecture, "new force" is denied

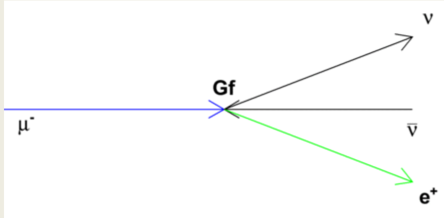
3-body non-relativistic Coulomb-Schroedinger calculation

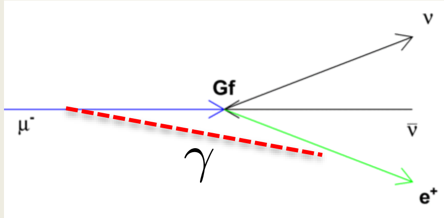
Variation with 39 basis + Relativistic corrections + Lamb-shift corrections ( by others )

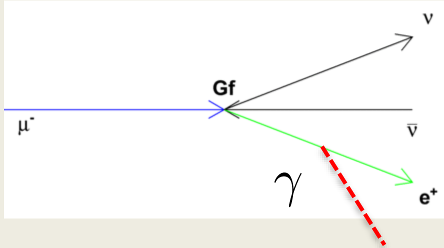
Use the 1$^{st}$ commercial mainframe computer UNIVAC

FORTRAN (1954, manual available 1956 )

# Muon Life




tree

Wikipedia
Fermi Interaction

1 loop
IR divergent

cancel

Bremsstrahlung
Phase space integration
IR divergent

finite small photon mass λ for IR regularization

Possibility of a large enhancement factor ?

$\alpha = 1/137.035\cdots$

Fine-structure constant

$$\alpha \ln^2\left(\frac{m_\mu}{m_e}\right) \sim 0.207$$

# Muon Life discussion w/ Feynman

V. AC

The authors are
Wheeler, Dr. Avron
for helpful suggest

Everyone Makes Mistakes – Including Feynman


Toichiro Kinoshita
Newman Laboratory, Cornell University,
Ithaca, New York 14853


November 8, 2018

arXiv:hep-ph/0101197

is connection, we note that
ıgularities occurring in the
of nonlinear wave equation
Keller, Comm. Pure Appl.

No. No dependence of $\ln\left(\frac{m_\mu}{m_e}\right)$

Photon polarization

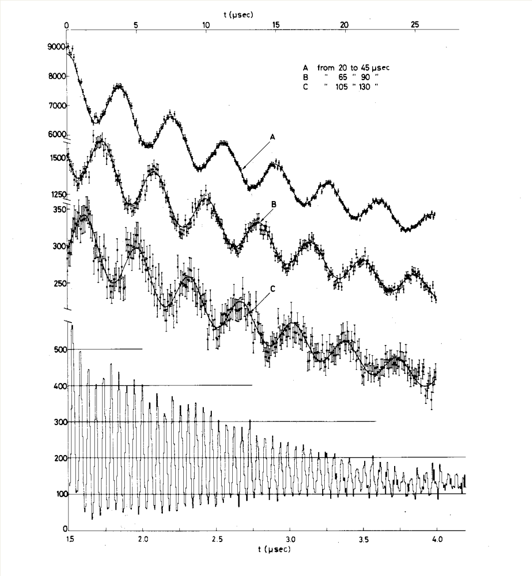
| | | |
|---|---|---|
| massless λ =0 | transverse only | degrees of freedom 2 |
| massive λ | transverse + longitudinal | degrees of freedom 3 |

The additional freedom does not vanish even if λ -> 0 limit is taken

# Muon g-2: $6^{th}$ order contribution 1

Kinoshita visited CERN in 1967

A funny plot was on the wall

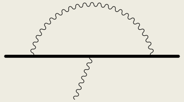



F. J. M. Farley and E. Picasso, *Ann.Rev.Nucl.Part.Sci.* 29 (1979) 243

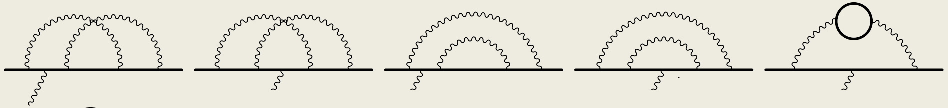
Muon g-2 expt. precision in 1966

$$0.016\% = 0.016 \times 10^{-2}$$
$$\alpha^2 \simeq 0.005 \times 10^{-2}$$

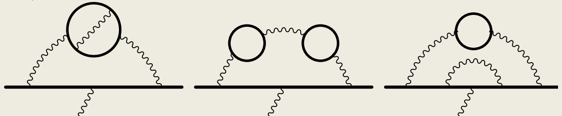
2nd



4th



6th



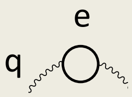
…

Three loop ($6^{th}$-order) contribution is needed

# Muon g-2: 6th order contribution 2

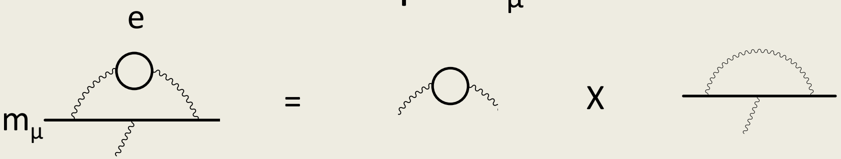
Mass singularity!

The enhancement factor $\ln\left(\frac{m_\mu}{m_e}\right) \simeq 5.33$ arises




Vacuum polarization $\propto \ln\left(\frac{q^2}{m_e^2}\right)$ in any order

$q \to m_\mu$



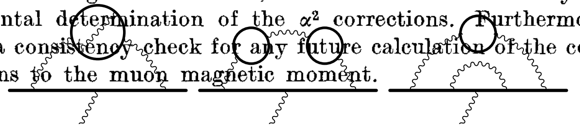

+ non-logarithmic terms

known known

Quick Estimate, No calculation is needed

# Muon g-2: $6^{th}$ order contribution 3

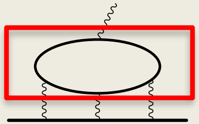
second- and fourth-order results of $g_e$ and $Z_3$ (charge renormalization constant). Our result shows that the $\alpha^3$ corrections to $\frac{1}{2}(g_\mu - g_e)$ will not be much larger than $2 \cdot 10^{-8}$, and thus will not seriously affect the experimental determination of the $\alpha^2$ corrections. Furthermore it will provide a consistency check for any future calculation of the complete $\alpha^3$ corrections to the muon magnetic moment.



The coefficient of the double log term is small
The coefficient of the single log term has an opposite and relatively large coefficient.
None of the above gives the leading contribution.

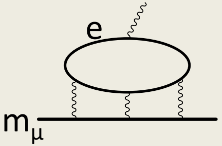
An estimate using the vp functions is now called
an estimate using the "renormalization group" of QED
Efficient to pick up LO terms in the higher-order QED contributions to muon g-2

## Hou about a new type of the $6^{th}$-order diagrams?



6 diagrams including a light-by-light scattering diagrams
Euler–Heisenberg Lagrangian can approximate the off-shell photons?

# Muon g-2: $6^{th}$-order light-by-light 1




Too lengthy and Too complicated to analyze by hand

Kinoshita had experience in

- Numerical calculations on a computer in his He-atom study
- Expressing a complicated Feynman diagram suitable for numerical integration in his mass-singularity study

| | |
|---|---|
| momentum space integration | 3 loop x 4dim = 12 dim |
| Feynman parameter space integration | 9 lines – 1 constraint = 8 dim |

Ready to go to the numerical evaluation

# Muon g-2: 6$^{th}$-order light-by-light 2

have been performed at CERN. The most recent $a_{exp} = (116\,616 \pm 31) \times 10^{-8}$. (1)

, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AU

$S(\Lambda_-^0) - \sqrt{3}S(\Sigma_0^+) = 0$. For pc decays, on hand, the $\Delta I = \frac{1}{2}$ rule remains valid for $\Sigma \to N\pi$, and $K \to 3\pi$, but we have not been rive the same rule for $\Lambda$ and $\Xi$ decays sence of SU(3) breaking.

e this note with a few remarks: (a) As ore, the amplitude corresponding to (1) ned to satisfy the duality principle that

[3]E. R. McCliment and K. Nishijima, Phys. R 1970 (1962); R. E. Cutkosky and P. Tarjanne, Rev. 132, 1355 (1963); R. Dashen and S. Frau Phys. Rev. 137, B1331 (1965), and Phys. Rev. B698 (1965).

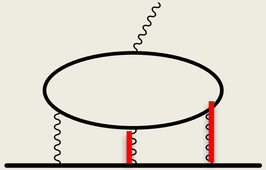
[4]R. Dolen, D. Horn, and C. Schmid, Phys. R 1768 (1968).

[5]J. L. Rosner, Phys. Rev. Letters 21, 950 (1 D. P. Roy and M. Suzuki, Phys. Letters 28B.

among the diagrams of Fig. 1 for the logarithmic terms are disproved.

We have calculated all integrands contributing to the logarithmic term in (9) by hand. The complete integrand was obtained by two separate, dissimilar methods with the help of REDUCE,[13] an

The sum of them is THE leading order term of the 6$^{th}$ order



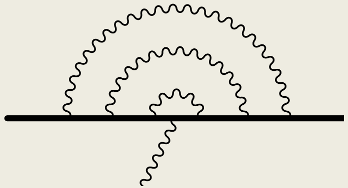
Coulomb photon exchange

$$= \left(\frac{\alpha}{\pi}\right)^3 \frac{2}{3}\pi^2 \ln\left(\frac{m_\mu}{m_e}\right)$$

+ non-logarithmic terms

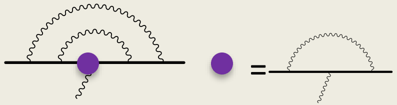
The leading term of the 6$^{th}$ order

# Electron g-2: 6th-order diagram w/o f-loop



| | | |
|---|---|---|
| All 6th-order vertex diagrams | 72 | |
| Diagrams w/o a fermion loop | 50 | |
| independent diagrams | 28 | by time-reversal symmetry |

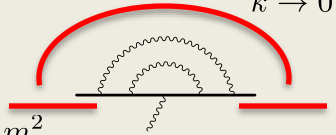
Ultraviolet (UV) divergence ← Loop momentum goes to infinity




$$M_{4b(2)} \times \int^{\Lambda} d^4k \frac{1}{k^2}\frac{1}{k}\frac{1}{k} \propto M_{4b(2)} \times \ln\Lambda$$

Infrared (IR) divergence ← Outer photon loop can go infinitely far from the vertex

$k \to 0$



$p^2 = m_e^2$

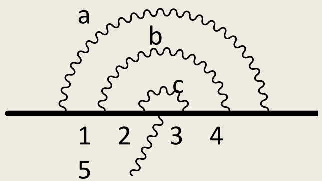
On-shell

$$M_{4b(2)} \times \int d^4k \frac{1}{k}\frac{1}{k}\frac{1}{k^2+\lambda^2} \propto M_{4b(2)} \times \ln\lambda$$

# Electron g-2: UV and IR subtractions

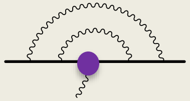
How to treat UV and IR divergence?

pointwise subtraction




Unrenormalized amplitude from Feynman rules

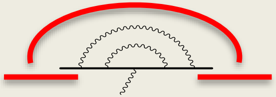
$$M_G = \int (dz) J_G(z_1, \cdots, z_c)$$



K-operation for UV pointwise subtraction

$$z_3 \to 0, \ z_c \to 0 \quad \text{for } J_G$$

$$\mathbb{K}_S M_G = L_S^{\mathrm{UV}} M_{G/S}$$



I-operation for IR pointwise subtraction

$$z_1 \to 0, \ z_5 \to 0, \ z_a \to 1 \quad \text{for } J_G$$

$$\mathbb{I}_R M_G = L_R^{\mathrm{IR}} M_S$$

Finite integral
Numerically calculable

$$\Delta M_G = (1 - \mathbb{K}_S - \mathbb{I}_R) M_G$$

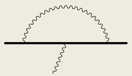
# Electron g-2: two step renormalization

On-shell renormalization must be realized

s.t. *e* is an elementary charge

*m* is an observed fermion mass

$$\begin{aligned} M_G - L_2 M_{4b(2)} &= M_G - L_2^{\mathrm{UV}} M_{4b(2)} - L_2^{\mathrm{R}} M_{4b(2)} \\ &= (M_G - L_2^{\mathrm{UV}} M_{4b(2)} - L_2^{\mathrm{IR}} M_{4b(2)}) + (-L_2^{\mathrm{R}} + L_2^{\mathrm{IR}}) M_{4b(2)} \\ &= \Delta M_G + \Delta L_2 M_{4b(2)} \end{aligned}$$



$L_2$ : 2nd-order vertex renormalization constant w/ on-shell scheme

Two Step renormalization=

numerical subtraction + finite renormalization

# Electron g-2: $6^{th}$-order 4

grams belong to this group. A typical diagram is shown in Fig. 1(b).

tion subgraph. There are altogether fifty diagrams of this type.


n, J. Geophys. Res. 76, 7470 (1971).
Proszler, G. M. Simnett, and R. S. White,
v. Lett. 28, 982 (1972).
Cyles, A. D. Linney, and G. K. Rochester, in

[14]A. J. Dragt, M. M. Austin, and R. S. White,
phys. Res. 71, 1293 (1966).
[15]T. A. Farley and M. Walt, J. Geophys. Res.
(1971).


## Sixth-Order Radiative Corrections to the Electron Magnetic Moment*


T. Kinoshita and P. Cvitanovic
*Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850*



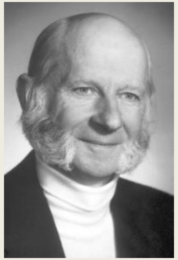
(Received 9 October 1972)



We have calculated the contribution of fifty Feynman diagrams of the order $\alpha^3$ to the electron magnetic moment. Our result, $(1.02 \pm 0.04)(\alpha/\pi)^3$, agrees with the result of


Numerical calculation of the $6^{th}$-order continued until 1995.
In 1996, the analytic result was obtained by S. Laporta and E. Remiddi.

# Electron g-2: 8$^{th}$-order 1



H. G. Dehmelt
From the Nobel
Foundation archive

## T. Kinoshita in mid 1970's

- A feeling of accomplishment from the 6$^{th}$-order g-2 calculation
- Looking for something new and interesting
- 8$^{th}$-order g-2 was out of his scope, too difficult

## H. G. Dehmelt in mid 1970's

- a single electron measurement w/ a Penning Trap he had invented
- Electron g-2 can be measured 100 or 1000 times precisely

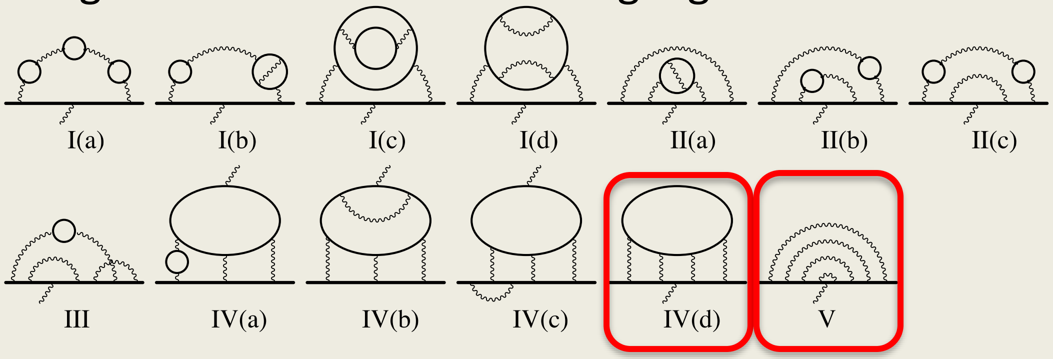
ppm (10^-6) → ppb (10^-9), 1989 Nobel Prize in Physics

## T. Kinoshita

- Decided to work for the 891 diagrams of the 8$^{th}$-order

# Electron g-2: 8th-order 2

891 diagrams are classified into 13 gauge-invariant subsets




IV(d) internal light-by-light vertex diagrams
need a new way to handle UV divergences

4 vertex diagrams out of 518 vertex diagrams of V
need a new way to handle IR divergences

# Electron g-2: 8$^{th}$-order 3

The first three coefficients have been calculated[3]:

$$C_1 = 0.5,$$
$$C_2 = -0.328\,478\,966\ldots, \quad (3)$$
$$C_3 = 1.176\,5(13).$$

If one uses the best current value[4] of the fine-structure constant

$$\alpha^{-1} = 137.035\,963(15), \quad (4)$$

the QED prediction (3) gives

$$a_e^{\text{QED}} = 1\,159\,652\,478 \times 10^{-12}. \quad (5)$$

Comparing (1) and (5) we see that the experi-

the hadronic contribution, and the effect of weak interaction (we assume the standard Weinberg-Salam model)[5]:

$$a_e(\text{muon}) = 2.8 \times 10^{-12},$$
$$a_e(\text{tauon}) = 0.1 \times 10^{-12},$$
$$a_e(\text{hadronic}) = 1.6(2) \times 10^{-12}, \quad (8)$$
$$a_e(\text{weak}) \simeq 0.05 \times 10^{-12}.$$

Besides the uncertainties in the values (1) and (4), we thus find three possible sources for the

$$1981 \qquad (-0.8 \pm 2.5)\left(\frac{\alpha}{\pi}\right)^4$$

$$2015 \qquad (-1.91298 \pm 0.00084)\left(\frac{\alpha}{\pi}\right)^4$$

Aoyama, Hayakawa, Kinoshita, Nio

$$2017 \qquad -1.91224576492\cdots\left(\frac{\alpha}{\pi}\right)^4$$

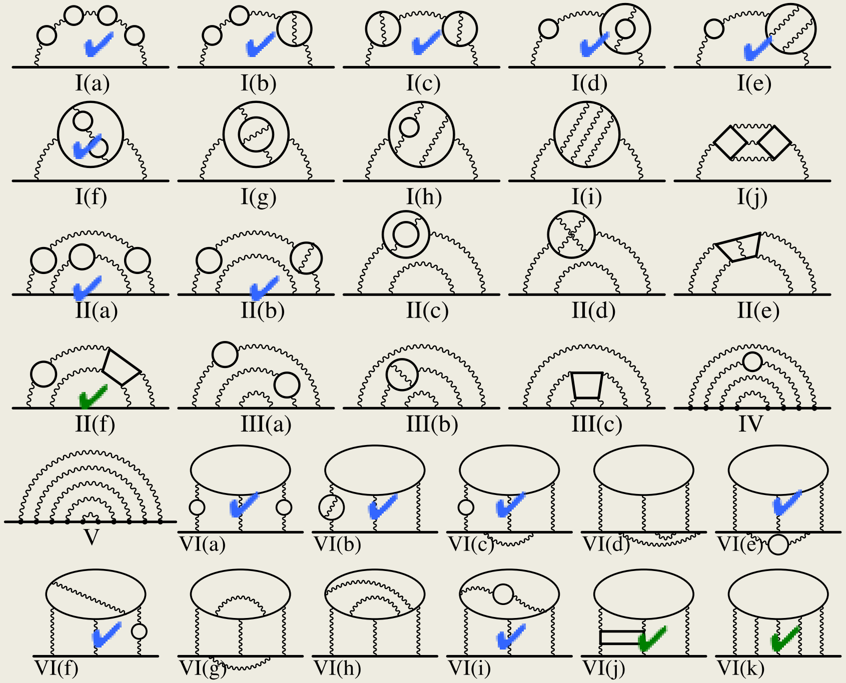
S. Laporta

1,100 digits are known

# Lepton g-2: 10th-order 1

Kinoshita retired from teaching in 1995

He began to try the tenth-order terms around 1995-1996?



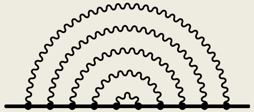

12,672 diagrams, 32 subsets

✔ Until 2002, he calculated 14 subsets

✔ I joined in 2002 and worked on 3 more subsets w/ him

10th-order LO term for muon g-2 is obtained in 2006

# Lepton g-2: 10$^{th}$-order 2



Set V 6,354 diagrams

X072 representing 9 vertex diagrams
to make it finite, we need 134 UV and IR subtraction terms
the integrand consists of about 100,000 lines

Kinoshita insists necessity of automation of code-generation

M. Hayakawa and T. Aoyama joined the project

# Lepton g-2: $10^{th}$-order 3

PRL **109,** 111807 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

**Tenth-Order QED Contribution to the Electron $g-2$ and an Improved Value**

[1]Kobayashi-M

PRL **109,** 111808 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

**Complete Tenth-Order QED Contribution to the Muon $g-2$**

[1]Kobayashi-

PHYSICAL REVIEW D **91,** 033006 (2015)

**Tenth-order electron anomalous magnetic moment: Contribution of diagrams without closed lepton loops**

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**Erratum: Tenth-order electron anomalous magnetic moment: Contribution of diagrams without closed lepton loops [Phys. Rev. D 91, 033006 (2015)]**

Tatsumi Aoyama, Masashi Hayakawa, Toichiro Kinoshita, and Makiko Nio

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2017 value of Electron g-2 and derived α
is used to determine
the defined value of the Planck Constant *h*
in the SI unit.

# Summary

T. Kinoshita pioneered

- Numerical means to theoretical studies of particle physics

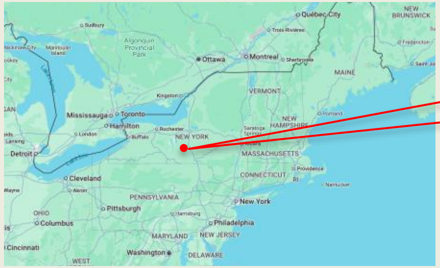
  K. Wilson's speech at Kinoshita's retirement party

- Precision tests b/w theory and experiment to explore new physics
- Determining the value of the fine-structure constant and other derived physical constants

His contributions

to the foundation of fundamental physics

# Toichiro and Masako Kinoshita rest here




Cornell University
Ithaca, NY, U.S.A.

Pleasant Grove Cemetery


Master of loop
Calculation
Upper diagram

Master of loop
Braiding
Lower diagram

Thank you, Tom,

from all of us,

all researchers involved in the lepton g-2 in the past, the present, and the future.

back up

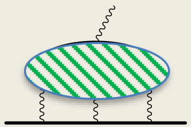
# Muon g-2: hadronic light-by-light 1

Muon g-2 expt:

CERN final precision  7 ppm in 1979

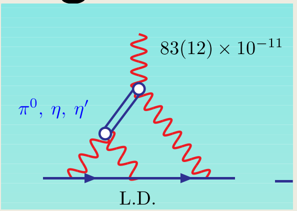
BNL took over in 1984

New type of hadronic correction,  light-by-light



Chiral Anomaly coupling

$$\pi^0 \longrightarrow \gamma + \gamma$$




T. Izubuchi, talk 2016

# Muon g-2: hadronic light-by-light 2

metric theories.[3]

The most accurate measurements of the muon anomaly thus far are those obtained at the CERN muon storage ring:[4]

$$a_{\mu^-}^{\exp} = 11\,659\,370(120)\times 10^{-10}\,, \tag{1.1a}$$

$$a_{\mu^+}^{\exp} = 11\,659\,110(110)\times 10^{-10}\,, \tag{1.1b}$$

where the numerals enclosed in parentheses represent the uncertainties in the final digits of the measured values. The best theoretical estimate reported prior to this article is[5,6]

$$a_{\mu}^{\text{th}} = 11\,659\,213(100)\times 10^{-10}\,, \tag{1.2}$$

in good agreement with (1.1).

While the electron anomaly is dominated by the QED effect, the muon anomaly is much more sensitive to phys-

cal error in (1.2) comes mostly from the uncertainty in hadronic contributions, while it also contains a non-negligible QED component. We have tried to improve both contributions substantially over the last three years. Our results are summarized in a recent publication.[10] In this article we report in detail the result of our work on the hadronic contribution to $a_\mu$. It arises from two types of diagrams: Hadronic vacuum polarization diagrams and hadronic light-by-light scattering diagrams shown in Figs. 1 and 2, respectively. The dominant contribution, which also has the largest error, comes from the diagram of Fig. 1(a), and it is this error that is the most serious obstacle for further improvement on the theoretical side.

$$\text{Hadronic LbL} \quad 49\ (5) \times 10^{-11}$$

The vector-Meson-dominance model by J. J. Sakurai was used
The model does not satisfy the Ward Identity of QED
breaking the gauge symmetry of QED.

# Muon g-2: hadronic light-by-light 3

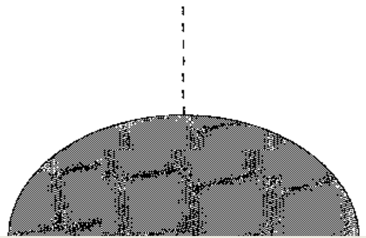
$$a_\mu(\text{weak-1}) = 195(1) \times 10^{-11}, \qquad (2)$$

and of the same order of magnitude as the leading logarithmic term of the two-loop electroweak correction [5], $a_\mu(\text{weak-2}) = -42 \times 10^{-11}$, offering an exciting opportunity to test the quantum effect of the electroweak theory.

Before comparing theory with the forthcoming measurement, however, it is necessary to reduce further the uncertainties in the theoretical prediction for the hadronic contribution. The largest uncertainty comes from the hadronic vacuum-polarization contribution, $a_\mu(\text{had.v.p.})$ [6]. Fortunately, this contribution can be expressed as a convolution of a known function with the experimentally measurable quantity $R$, the ratio of the hadron production cross section to the $\mu^+\mu^-$ production cross section in $e^+e^-$ collisions. Recent measurements of $R$ at VEPP-

it is not an easy job to carry out such a calculation from first principles.

Fortunately, this energy region is populated mostly by pions, and considerable information is available about low-energy pion dynamics. Chiral symmetry governs most of it. However, higher energy regions may also



$$\text{Hadronic LbL} \quad -36\ (16) \times 10^{-11}$$

Nambu–Jona-Lasinio model, Hidden Local Chiral symmetry

# Muon g-2: hadronic light-by-light 4

PHYSICAL REVIEW D VOLUME 57, NUMBER 1 1 JANUARY 1998

**Pseudoscalar pole terms in the hadronic light-by-light scattering contribution to muon $g-2$**

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(Received 4 August 1997; published 8 December 1997)

The pseudoscalar pole contribution is the dominant source of the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering effect in muon $g-2$. We have examined this contribution, taking account of the off-shell structure of the pseudoscalar-photon-photon anomaly vertex deduced from available experimental data. Our work leads to an improved estimate $-79.2\ (15.4)\times 10^{-11}$ for the total hadronic light-by-light scattering contribution to the muon $g-2$. [S0556-2821(98)06601-6]

CLEO @ Cornell Univ
Measurement of the pion form factor

$$\pi^0 \to \gamma + \gamma^*$$

* off-shell photon

$$\text{Hadronic LbL} \quad -79\ (16) \times 10^{-11}$$

# Muon g-2: hadronic light-by-light 5

hep-ph/0112102

Comment on the sign
of the pseudoscalar pole contribution
to the muon g − 2

Masashi Hayakawa and Toichiro Kinoshita

Muon g-2 result from BNL in 1999 –
aimed uncertainty $40 \times 10^{-11}$

In 2001, Knecht and Nyffeler wrote Kinoshita and Hayakawa
" Is the sign of a pion pole contribution correct? "

Hayakawa found an algebraic manipulation system FORM cannot correctly handle the antisymmetric tensors in the Minkowski space.

$$\text{Hadronic LbL} \quad 89.6\ (15.4) \times 10^{-11}$$

$$\text{Hadronic LbL} \quad 92\ (19) \times 10^{-11}$$

Worldwide consensus in the White Paper on muon g-2 2020