Update on hQCD results for the HLbL contribution to $(g-2)_\mu$

Anton Rebhan
w/ Josef Leutgeb & Jonas Mager

Institute for Theoretical Physics
TU Wien, Vienna, Austria

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HLbL contributions where hQCD might be of interest

Holographic QCD (hQCD) makes interesting(*) predictions
where WP error estimates are the largest:

(*) not too model-dependent in the class of models that match m_ρ, f_π, and longitudinal short-distance constraints (LSDC)

Contribution	WP-$a_\mu^{\mathrm{HLbL}} \times 10^{11}$
π^0, η, η'-poles	$93.8(4.0)$
π, K-loops/boxes	$-16.4(0.2)$
S-wave $\pi\pi$ rescattering	$-8(1)$
scalars & tensors	$-1(3)$
axial vectors	$6(6)$
u, d, s-loops / short-distance	$15(10)$
— of which LSDC: $13(6)$	
c-loop	$3(1)$
total	$92(19)$

Chiral hQCD results

2020, at time of WP, only hQCD results for a_μ from chiral models were available

- HW1 (Erlich-Katz-Son-Stephanov 2005, but $m_q = 0$)
- HW2 (Hirn-Satz 2005, simpler, inherently chiral)
- (Witten-)Sakai-Sugimoto (2004, top-down string theory construction, inherently chiral, low energy limit only because of Kaluza-Klein circle)

Pion TFF and $a_\mu^{\pi^0}$ first fully evaluated by Leutgeb, Mager, AR, 1906.11795, following partial/hybrid evaluation of Cappiello, Cata, D'Ambrosio, 1009.1161 (m_π inserted by hand)

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Extended to axial TFF and a_μ by Leutgeb & AR, 1912.01596 and independently by Cappiello, Cata, D'Ambrosio, Greynat & Iyer, 1912.02779 (HW2 only, different extrapolation to $\eta^{(\prime)}/f_1^{(\prime)}$ sector)

LR: $a_\mu^{a_1+f_1+f_1'} = 4a_\mu^{a_1}$ (flavor symmetric) $\approx (29\ldots41)\times10^{-11}$

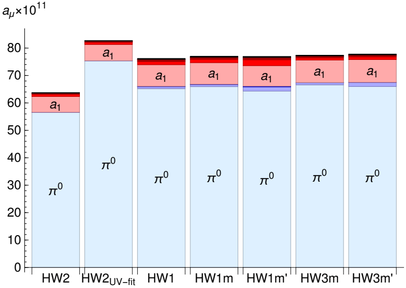
CCDGI: $a_\mu^{a_1+f_1+f_1'} \approx 3.5a_\mu^{a_1}$ (non-uniform model) $\approx 28\times10^{-11}$

model	LSDC	m_ρ	$m_{a_1(1260)}$	$a_\mu^{\pi^0}$	$a_\mu^{a_1}$	$a_\mu^{a_1}$+tower
HW1 chiral	100%	775	1375	65.2*	7.85	10.15
HW2(UV-fit)/CCDGI-Set2	100%	987	1573	75	5.75	7.2
HW2(IR-fit)/CCDGI-Set1	62%	775	1235	57	5.9	7.35
Sakai-Sugimoto	0%	775	1187	48.3	3.45	3.7

*Erratum: LMR, PRD 104 (2021) 059903 ! (included in arxiv versions)

HW models with massive pions [Leutgeb & AR: 2108.12345]

Rigorous inclusion of quark masses in HW1 and HW3 (=HW1 w/ HW2 b.c.) models:
→ little difference to chiral model with manually inserted pion mass



(darker colors: excited π^{0*}, a_1^*)

HW1m: HW1 with nonzero light quark mass and correct pion mass

HW1m': HW1m with modified scaling dimension of bifundamental scalar, additionally correct $a_1(1230)$ mass, but not mass of $\pi(1300)$

HW3m: HW1m with HW2 boundary conditions

HW3m': HW3m with modified scaling dimension of bifundamental scalar, additionally correct $\pi(1300)$ mass, but not mass of $a_1(1230)$

Excited pions: don't decouple even in chiral limit, $\sum a_\mu^{\pi^{0*}} \approx (0.8 \ldots 1.8) \times 10^{-11}$
SW model (CGN, 2301.06456): larger contribution, see below

Short distance constraints on TFFs

Crucially, hQCD models with asymptotic AdS_5 geometry reproduce
asymptotic momentum dependence of LCE [Brodsky-Lepage 1979-81]
(HW1 model exactly with $g_5 = 2\pi$; HW2 model only at 62%)

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \quad \to \quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2} - \frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2 = \frac{1}{2}(Q_1^2+Q_2^2) \to \infty$, $w = (Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2,0) = \frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2,Q^2) = \frac{2f_\pi}{3Q^2} \quad (\Leftarrow \text{OPE}).$$

- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(confirmed by pQCD result of Hoferichter & Stoffer 2004.06127):

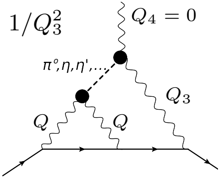
$$A_n(Q_1^2,Q_2^2) \to \frac{12\pi^2 F_n^A}{N_c Q^4}\frac{1}{w^4}\left[w(3-2w) + \frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$



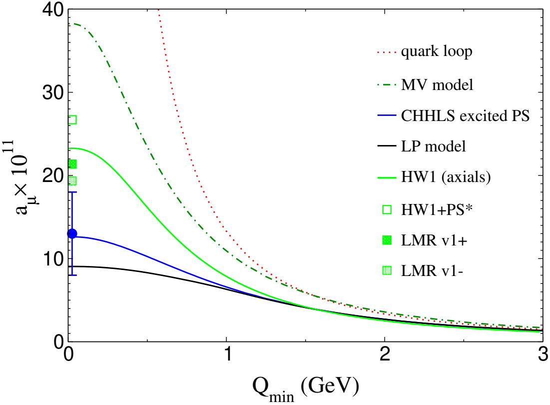
MV model: MV-SDC satisfied by replacing external TFF by constant on-shell value, leading to significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$ by $\underline{38 \times 10^{-11}}$

WP estimate for MV-SDC based on Regge model of infinite tower of excited PS states constructed to saturate MV-SDC with $\underline{\Delta a_\mu^{\rm PS} = 13(6) \times 10^{-11}}$ [Colangelo et al., 1910.11881]

HW models: infinite tower of axials saturates MV-SDC to 100% in HW1 models,
with $a_\mu^{A(L)} = 23.2 \times 10^{-11}$ in chiral model;
no contribution to MV-SDC from excited PS, $a_\mu^{\pi^{0*}} = (0.8 \ldots 1.8) \times 10^{-11}$

Comparison of results for the longitudinal SDC

Update of Fig. 69 in the WP / Fig. 5 of Colangelo et al. 2106.13222
(dropping the HW2 models which cannot fit UV and IR parameters simultaneously)



Contribution to a_μ for $Q_i \geq Q_{\rm match}$: the longitudinal part of the massless perturbative QCD quark loop (dotted red), the Melnikov-Vainshtein model (MV, dot-dashed dark green), the Lüdtke/Procura model (LP, solid black), the CHHLS Regge model of excited pseudoscalars (solid blue), and the contribution of axials in the chiral HW1 model (solid light green), with **squares indicating the final values including excited pseudoscalars** (LMR: in our 2022 model with quark masses and U(1)$_A$ anomaly)

Massive HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

$N_f = 2+1$ with $m_s \approx 24.3 m_{u,d}$ and Witten-Veneziano mechanism for η' mass
Two version of UV fits:

a) $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%
b) $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

Tuning of gluon condensate Ξ (neglected by KS) $\to$ virtually exact fit of m_η and $m_{\eta'}$:

Version a) (OPE fit)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8	f^0	f_G	$\|F(0,0)\|$	$F - F^{\rm exp}$
π^0	135	(input)	0	0	0	0.277	
η	557	+1.7%	0.101	0.027	-0.030	0.275	+1(2)% (!)
η'	950	-0.8%	-0.0385	0.113	-0.077	0.340	-0(2)% (!)
G/η''	1992	?	-0.027	0.005	0.053	0.116	
	m [MeV]	m–$m^{\rm exp}$ [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee3}(0,0)$	
a_1	1363	+11%	0	0	0	20.96	
f_1	1481	+15%	0.176	0.0365	20.77	3.857	
f_1'	1810	+27%	-0.030	0.201	-3.842	20.07	

gluon condensate parameter $|\Xi| = 0.01051\ \mathrm{GeV}^4$

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Version b) (our current "best guess" regarding a_μ)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8	f^0	f_G	$\|F(0,0)\|$	$F - F^{\rm exp}$
π^0	135	(input)	0	0	0	0.276	
η	561	+2.4%	0.103	0.030	-0.031	0.268	+2(2)%
η'	947	-1.1%	-0.039	0.121	-0.082	0.313	-8(2)%
G/η''	1943	?	-0.030	0.0076	0.048	0.111	
	m [MeV]	m–$m^{\rm exp}$ [%]	F^8_A/m_A	F^0_A/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1278	+4%	0	0	0	19.46	
f_1	1410	+10%	0.176	0.029	19.58	2.69	
f_1'	1820	+28%	-0.017	0.219	-2.56	19.00	

gluon condensate parameter $|\Xi| = 0.01416\ {\rm GeV}^4$

PS: $f^{8,0}$'s within a few % of χPT values

AV: f_1-f_1' mixing angle $\phi_f - \phi_f^{\rm ideal}$ about twice as large as indicated by L3 data

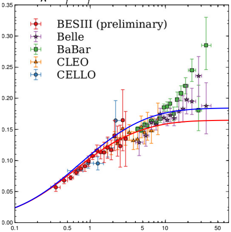
(ϕ_f strongly dependent on Ξ; but sum $a_\mu^{f_1} + a_\mu^{f_1'}$ rather insensitive)

a_μ in HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

comparing also to **Soft-Wall model of P. Colangelo, F. Giannuzzi, S. Nicotri**
with $m_s > m_{u,d}$, accurate η, η' masses, good $F(0,0)$, and correct U(1)$_A$ anomaly
(CGN 2301.06456: scalar sector with $m_s > m_{u,d}$ and U(1)$_A$ anomaly;
CGN 2402.07579: axial vector contributions, but only in a simpler, flavor symmetric set-up!)

$a_\mu^{\cdots} \times 10^{11}$	LMR(OPE fit)	LMR(F_ρ-fit)	CGN(OPE fit)	WP2020
π^0	66.1	63.4	75.2	$63.0^{+2.7}_{-2.1}$
η	19.3	17.6	21.2	16.3(1.4)
η'	16.9	14.9	12.3	14.5(1.9)
$PSGB/\eta''$	0.2	0.2	5.1	
$\sum_{PS*}$	1.6	1.4	$\gg$1.7	
PS poles total	104	97.5	>115.5	93.8(4.0)

$Q^2 F_{\pi^0\gamma^*\gamma}(Q^2, 0)$ [GeV] in LMR 2211.16562



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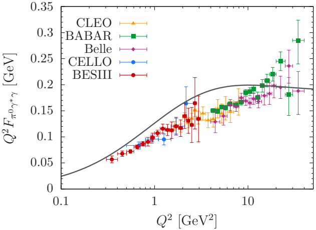

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a_1	7.8	7.1	9.0	
$f_1 + f_1'$	20.0	17.9=2.5×7.1	$3^\star \times$9.0	
$\sum_{a_1^*}$	2.2	2.4	$1.3^\dagger$	
$\sum_{f_1^{(\prime)*}}$	3.6	3.0	$3^\star \times 1.3^\dagger$	
AV+LSDC total	34	30.5	41.3	19(12)
total	138	128	>157	113(16)

*: due to U(3) flavor symmetry

†: We (LMR) can reproduce results for first few resonances, but not infinite sum

SW model actually has a fundamental problem [Kwee & Lebed, 0712.1811], to be checked!

Attempts for further improvements

Issues with LMR2022 model:

- equivalent photon decay rate of f_1, f_1' higher than L3 data indicate
- f_1-f_1' mixing angles unrealistic, too far from ideal mixing

To appear soon: LMR2024 with scalar-extended CS term (Quillen's superconnection)

(adaption of open-string-tachyon condensation model of Casero, Kiritsis & Paredes 2007)

preliminary results:

- f_1-f_1' mixing angle closer to ideal, lower equivalent photon rate:

	exp. (L3)	w/model mass	w/exp.mass
$\tilde{\Gamma}_{\gamma\gamma}(a_1)$ [keV]		1.96	1.1
$\tilde{\Gamma}_{\gamma\gamma}(f_1)$ [keV]	3.5(6)(5)	4.86	2.5
$\tilde{\Gamma}_{\gamma\gamma}(f_1')$ [keV]	$\gtrapprox$ 3.2(6)(7)	4.92	2.9

(F_ρ fit)

- $\rightarrow$ lower contribution from ground-state a_1, f_1's, but more from excited AV
- but less perfect fit of η and η', excessive π^0 TFF!

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Range of quantitatively successful $N_f = 2+1$ hQCD models:

$a_\mu^{\text{AV+LSDC}} = (34 \ldots \underline{30.5} \ldots 24.7) \times 10^{-11}$

$a_\mu^{PS*} = (1.4 \ldots \underline{1.6} \ldots 5.5) \times 10^{-11}$

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- New feature: scalar nonet naturally couples to photons, unlike minimal model, with one of the terms (ζ_+) considered by Cappiello, Cata, D'Ambrosio 2110.05962

Scalar contributions

Cappiello, Cata, D'Ambrosio 2110.05962
have calculated a_μ contributions of $\sigma(500)$, $a_0(980)$, $f_0(990)$
by non-minimal *chiral* HW1 models (using different Lagrangians for each) with results

$a_\mu^{\cdots} \times 10^{11}$	$n=1$	$n=2$	all n
σ	-8.5(2.0)	-0.07(2)	-8.7(2.0)
a_0	-0.29(13)	-0.025(10)	-0.32(14)
f_0	-0.27(13)	-0.025(9)	-0.29(14)
sum	-9(2)	-0.12(4)	-9(2)

Issues:

- if tetraquarks, qualitatively different descriptions needed (certainly at large N)
- should not be added to previous CCDGI results due to different model
- asymptotics of TFFs $\mathcal{F}_1^S(Q_1^2,Q_2^2) \sim Q^{-6}$, $\mathcal{F}_2^S(Q_1^2,Q_2^2) \sim Q^{-8}$
  instead of Q^{-2} and Q^{-4} in pQCD

LMR2024 model:

- a_0, f_0, f_0' somewhat too heavy ($a_0(1450), \ldots ?$), $a_\mu^S \neq 0$, but not yet evaluated
- asymptotics of TFFs $\mathcal{F}_1^S(Q_1^2,Q_2^2) \sim Q^{-4}$, $\mathcal{F}_2^S(Q_1^2,Q_2^2) \sim Q^{-6}$ with $m_q \neq 0$, consistent with OPE in symmetric limit, but not with LCE
  NB: Hoferichter-Stoffer result also consistent with OPE in symmetric result, since at $q_1 = -q_2$ leading terms cancel!

Tensor contributions

Following [Katz, Lewandowski & Schwartz hep-ph/0510388] CGN 2402.07579 have implemented tensor mesons in SW and HW models,

matching $f_2(1270) \to \gamma\gamma$ with $\Gamma_{\gamma\gamma} = 2.6(5)$ keV

with result $a_\mu^{f_2(1270)} = (0.61 \ldots 0.63) \times 10^{-11}$,

consistent with Pauk & Vanderhaeghen 2014

Issues:

- dual operator not only quark bilinear,
  but rather energy-momentum tensor $\leftrightarrow$ tensor glueball
- perhaps therefore: asymptotics of TFF with correct Q^{-4} behavior, but different $f(w)$ than Hoferichter & Stoffer 2020

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Tensor and (pseudo)scalar glueballs in Hechenberger, Leutgeb & AR, 2302.13379

for Witten-Sakai-Sugimoto model: $\Gamma(G^{S,P,T} \to \gamma\gamma) \sim \mathrm{few\,keV}$ but $|a_\mu^G| \lesssim 10^{-12}$

Conclusions

- Simple HW holographic QCD models as well as SHW improvements reproduce remarkably well the π^0 HLBL contribution from dispersive and lattice approaches, in particular with reduced g_5^2 to fit F_ρ (90% of OPE limit $\leftrightarrow$ typical gluonic corrections)
- Extension with strange quark and WV η_0 mass (LMR2022): nice fit of η, η' data
- Melnikov-Vainshtein constraint naturally satisfied by tower of axial vector mesons
- Axial vector and LSDC contribution estimated together (≈58% of AV is longitudinal) with good agreement among various (flavor-symmetric) models
    - U(3)-symmetric models with OPE fit: $a_\mu^{\text{AV+LSDC}} = 40(3) \times 10^{-11}$
    - Best guess (LMR2022): $\boxed{a_\mu^{\text{AV+LSDC}} = 30.5^{+3.2(\text{OPE})}_{-6(\text{Quillen})} \times 10^{-11}}$

      around upper end of WP20 estimate $a_\mu^{\text{AV+LSDC}} = 19(12) \times 10^{-11}$
- Excited pseudoscalars (in WP20 contained in LSDC estimate)
    - U(3)-symmetric HW models with OPE fit: $a_\mu^{P*} = 4a_\mu^{\pi^*} = 5(2) \times 10^{-11}$
    - Best guess (LMR2022): $\boxed{a_\mu^{P*} = 1.6^{+4(\text{Quillen})}_{-0.2(\text{OPE})} \times 10^{-11}}$
- Scalar and tensor contributions very model dependent

  BL short-distance behavior of scalar and tensor TFFs not reproduced

Appendix

If issues of SW model [Kwee & Lebed, 0712.1811] can be resolved, potentially interesting as starting point for further improvements. . .

Improving the SW model: **S**emi-**H**ard **W**all model

Leutgeb, AR, Stadlbauer: HVP contribution in hQCD, 2203.16508

Simple Wood-Saxon like interpolation of HW and SW model with only one additional free parameter [Kwee & Lebed 0708.4054] yields asymptotically linear Regge trajectories with good fit of first few vector mesons $\rho(770), \rho(1450), \rho(1900)$, and $\rho(2150)$, $\rightarrow$ reduces strong SW mismatch in HVP contribution:

	HW		SW		SHW	
n	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$
1	*775*	329.1	*775*	260.0	*775*	314.0
2	1779	615.8	1096	309.2	*1465*	458.5
3	2789	863.3	1342	342.2	1903	498.7
4	3800	1089	1550	367.7	2230	540.0
5	4812	1300	1733	388.8	2511	570.7

	$a_{\mu(N_f=2)}^{\text{LO-HVP}} \times 10^{10}$	mismatch
HW1,HW3	476.9	0.86
HW2(IR|UV-fit)	773.9|304.0	1.39|0.55
SW	276.4	0.50
SHW	415.4	0.75

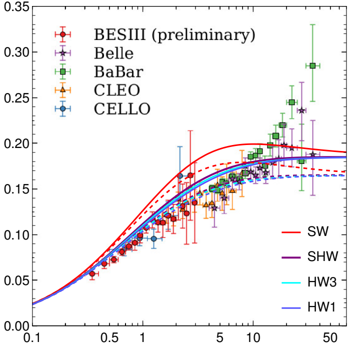
Improving the SW model: **S**emi-**H**ard **W**all model

Leutgeb, AR, Stadlbauer: HVP contribution in hQCD, 2203.16508

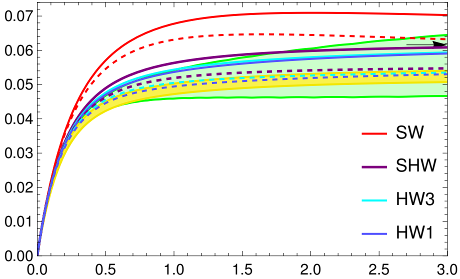
Simple Wood-Saxon like interpolation of HW and SW model with only one additional free parameter [Kwee & Lebed 0708.4054] yields asymptotically linear Regge trajectories with good fit of first few vector mesons

Leutgeb, Mager, AR: HLbL contribution in SHW model, 24xx.xxxxx

Bump in $Q^2 F_{\pi^0\gamma^*\gamma^{(*)}}$ disappears in SHW model: dashed lines: reduced g_5 coupling



singly virtual Q^2 [GeV^2]



doubly virtual Q^2 [GeV^2]

$$a_\mu^{\pi^0}(\text{SW}) = 75.2 \times 10^{-11} \quad \rightarrow \quad a_\mu^{\pi^0}(\text{SHW}) = 67.7 \times 10^{-11} \text{ (OPE fit)}$$

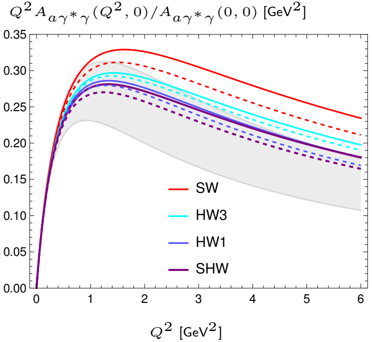
(reduced by ∼4% when g_5^2 reduced by 10%)

Improving the SW model: **S**emi-**H**ard **W**all model

Leutgeb, Mager, AR: HLbL contribution in SHW model, 24xx.xxxxx

Axial vector meson mass in SW model too high, reduced in SHW: 1679 $\to$ 1454 MeV

better agreement with Q^2 dependence of L3 data on $f_1(1285)$:



differences in mass and TFF cancel out approximately: (preliminary)

$$4 \times a_\mu^{a_1}(\text{SW}) = 36.0 \times 10^{-11} \quad \to \quad 4 \times a_\mu^{a_1}(\text{SHW}) = 36.2 \times 10^{-11} \text{ (OPE fit)}$$

however sum over (less overweight) AV tower somewhat larger:

$$4 \times \sum_{n=1}^{7} a_\mu^{a_1^{(n)}}(\text{SW}) = 41.3 \times 10^{-11} \quad \to \quad 4 \times \sum_{n=1}^{7} a_\mu^{a_1^{(n)}}(\text{SHW}) = 43.3 \times 10^{-11}$$