Update on the electroweak contribution

Martin Hoferichter

Albert Einstein Center for Fundamental Physics,
Institute for Theoretical Physics, University of Bern

UNIVERSITÄT BERN

AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

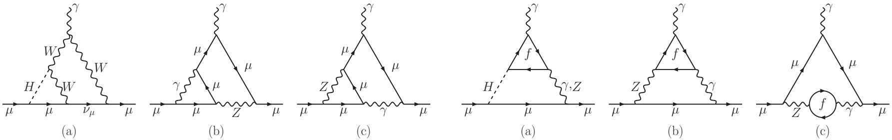
Sep 13, 2024

Seventh Plenary Workshop of the Muon $g-2$ Theory Initiative

KEK, Tsukuba

MH, J. Lüdtke, L. Naterop, M. Procura, P. Stoffer, work in progress

Electroweak contribution: status

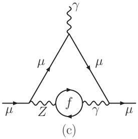


representative EW two-loop diagrams from WP20

- Current number $a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11}$ Gnendiger, Stöckinger, Stöckinger-Kim 2013 uses **outdated estimate of hadronic uncertainties** from Czarnecki, Marciano, Vainshtein 2003

  $\hookrightarrow$ diagrams (*b*) (VVA) and (*c*) (γZ) on the right
- Lots of new input
  - Dispersive study of VVA correlator Lüdtke, Procura, Stoffer, to appear
  - Lattice calculation of γZ two-point function Mainz 2022, ...
  - α_s corrections to VVA for heavy quarks Melnikov 2006 (never used!)
- Irrelevant for bottom line, but should still be updated

Electroweak contribution: update



	WP20	Update
1-loop	194.79(1)	194.79(1)
2-loop, bosonic	−19.96(1)	−19.96(1)
2-loop, Higgs	−1.51(1)	−1.51(1)
2-loop, VVA, (u, d, e)	−2.28(20)	
2-loop, VVA, (c, s, μ)	−4.63(30)	
2-loop, VVA, (t, b, τ)	−8.21(10)	
2-loop, fermionic (rest)	−4.64(10)	
3-loop, NLL	0.0(2)	
total	153.6(1.0)	

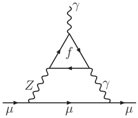
- "Fermionic (rest)" $\Rightarrow$ γZ two-point function
- Commonly used
  - Marciano 1993: $8\pi^2\bar{\Pi}^{\gamma Z}(-M_Z^2) = 6.88(50)$
  - Jegerlehner 1986: $8\pi^2\bar{\Pi}^{\gamma Z}(-M_Z^2) = 5.87(4)$
- Can test $SU(3)$ assumptions with lattice

  $\hookrightarrow$ gives value in between
- Use $8\pi^2\bar{\Pi}^{\gamma Z}(-M_Z^2) = 6.3(5)$

  $\hookrightarrow -4.58(2)$
- Main error 0.10 from higher orders in $\frac{M_Z^2}{m_t^2}$ and $1 - 4s_W^2$

Electroweak contribution: update

	WP20	Update
1-loop	194.79(1)	194.79(1)
2-loop, bosonic	−19.96(1)	−19.96(1)
2-loop, Higgs	−1.51(1)	−1.51(1)
2-loop, VVA, (u, d, e)	−2.28(20)	
2-loop, VVA, (c, s, μ)	−4.63(30)	
2-loop, VVA, (t, b, τ)	−8.21(10)	
2-loop, fermionic (rest)	−4.64(10)	−4.58(10)
3-loop, NLL	0.0(2)	
total	153.6(1.0)	

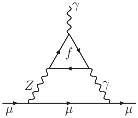


(b)

- VVA determined by $w_{L,T}(Q^2)$
  $\hookrightarrow$ single scale
- For third generation use pQCD
  $\hookrightarrow -8.21(10)$ CMV 2003
- Error estimates from leading log
- With Melnikov 2006, can actually calculate α_s corrections
  $\hookrightarrow -8.16(1)$

Electroweak contribution: update

	WP20	Update
1-loop	194.79(1)	194.79(1)
2-loop, bosonic	−19.96(1)	−19.96(1)
2-loop, Higgs	−1.51(1)	−1.51(1)
2-loop, VVA, (u, d, e)	−2.28(20)	
2-loop, VVA, (c, s, μ)	−4.63(30)	
2-loop, VVA, (t, b, τ)	−8.21(10)	−8.16(1)
2-loop, fermionic (rest)	−4.64(10)	−4.58(10)
3-loop, NLL	0.0(2)	
total	153.6(1.0)	

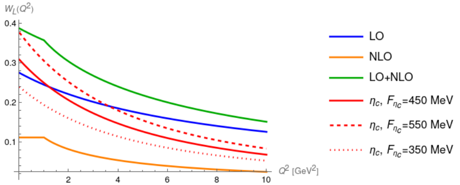


(b)

- VVA for $\{u, d\}$ studied extensively for HLbL
- Constraints
  - Normalization from ChPT Knecht 2020
  - OPE ($1/Q^2$ and $1/Q^4$) CMV 2003
  - Residues and cuts from dispersive approach Lüdtke, Procura, Stoffer
- Simplified analysis yields $-2.1(1)$ preliminary, to be self consistent with VVA analysis

Electroweak contribution: update

	WP20	Update
1-loop	194.79(1)	194.79(1)
2-loop, bosonic	−19.96(1)	−19.96(1)
2-loop, Higgs	−1.51(1)	−1.51(1)
2-loop, VVA, (u, d, e)	−2.28(20)	−2.1(1)
2-loop, VVA, (c, s, μ)	−4.63(30)	
2-loop, VVA, (t, b, τ)	−8.21(10)	−8.16(1)
2-loop, fermionic (rest)	−4.64(10)	−4.58(10)
3-loop, NLL	0.0(2)	
total	153.6(1.0)	



- Same strategy for s as for $\{u, d\}$
  $\hookrightarrow$ small changes
- α_s corrections to charm large, $+0.46$
  $\hookrightarrow$ larger than leading-log estimate
- Below a matching scale $Q_0^2 = (1.5^2–2.5^2)\,\text{GeV}^2$ replace pQCD by η_c pole
- Error from scale variation and F_{η_c}
  $\hookrightarrow -4.0(2)$ error can likely be improved a little

Electroweak contribution: update

	WP20	Update
1-loop	194.79(1)	194.79(1)
2-loop, bosonic	−19.96(1)	−19.96(1)
2-loop, Higgs	−1.51(1)	−1.51(1)
2-loop, VVA, (u, d, e)	−2.28(20)	−2.1(1)
2-loop, VVA, (c, s, μ)	−4.63(30)	−4.0(2)
2-loop, VVA, (t, b, τ)	−8.21(10)	−8.16(1)
2-loop, fermionic (rest)	−4.64(10)	−4.58(10)
3-loop, NLL	0.0(2)	0.0(2)
total	153.6(1.0)	154.5(3)

- **Preliminary** numerics indicate upward shift of 0.9 units
- Main reasons for difference:
  - α_s corrections for charm
  - CMV 2003 used $m_c = 1.5\,\text{GeV}$
    $\hookrightarrow$ their result is $\simeq 0.3$ smaller
  - Small shifts for first- and third-generation VVA
- Currently looking into NLL
  $\hookrightarrow$ might be possible with modern EFT technology Naterop, Stoffer, work in progress