Control of equilibrium and non-equilibrium Casimir forces

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Outline

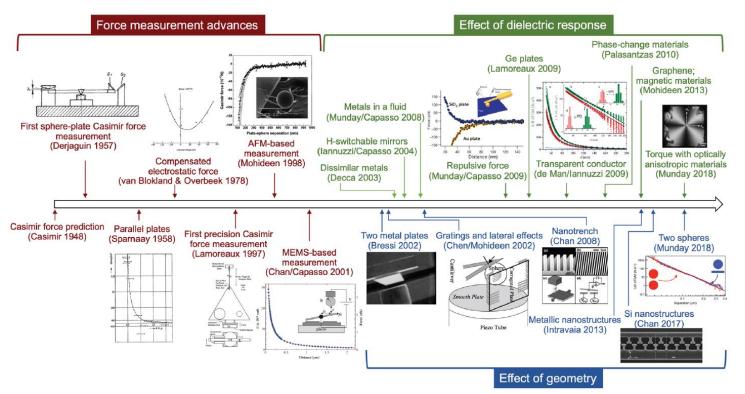
Brief overview of Casimir forces

Control theory for Casimir forces

Symmetry of Casimir forces in wavevector space

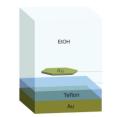
Conclusions

History of Casimir force research

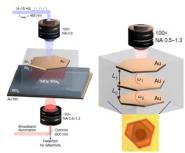


- T. Gong, M. R. Corrado, A. R. Mahbub, C. Shelden, and J. N. Munday, Nanophotonics 10, 523 (2021).
- J. N. Munday, KEK IPNS-IMSS-QUP joint workshop Feb. 8-10, 2022.

Past few years



Levitation in liquid [Science 364, 984 (2019)]



Active control, many-body in liquid [Nature, 597, 214 (2021)]

Literature of Casimir forces towards new force research^{4/26}

PRL 94, 240401 (2005)

PHYSICAL REVIEW LETTERS

week ending 24 JUNE 2005

Constraining New Forces in the Casimir Regime Using the Isoelectronic Technique

R. S. Decca, 1,* D. López, H. B. Chan, E. Fischbach, D. E. Krause, 5,4 and C. R. Jamell Department of Physics, Indiana University-Purdue University Indianapolis, Indianapolis, Indiana 46202, USA Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA ³Department of Physics, University of Florida, Gainesville, Florida 32611, USA ⁴Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA ⁵Physics Department, Wabash College, Crawfordsville, Indiana 47933, USA

(Received 1 February 2005; published 20 June 2005)

Hypothetical force difference



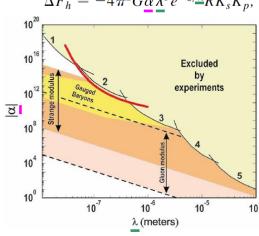


FIG. 4 (color). Values in the $\{\lambda, \alpha\}$ space excluded by experiments. The red curve represents limits obtained in this work. Curves 1 to 5 were obtained by Mohideen's group [7], our group [1], Lamoreaux [6], Kapitulnik's group [8], and Price's group [5], respectively. Also shown are theoretical predictions [21].

G: Gravitational constant

 K_s , K_p : Terms relating to densities of the sphere and the plate R: Radius of the sphere

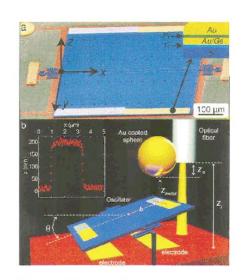
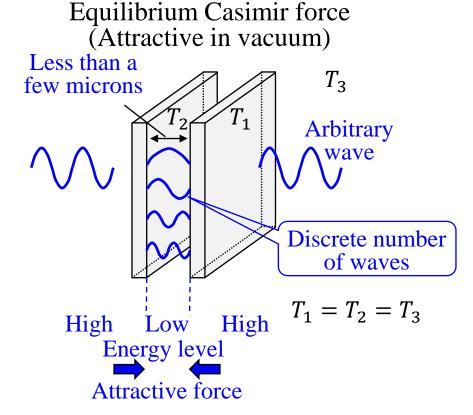
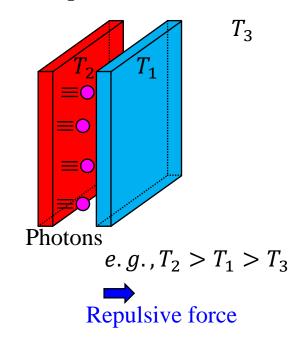


FIG. 1 (color). (a) Scanning electron microscope image of the MTO with the composite sample deposited on it. The coordinate system used in the Letter is indicated. Inset: schematic of the sample deposited on the MTO. The thickness of the different layers are (in order of deposition): $d_{Ti} = 1$ nm, $d_{Ge} = 200$ nm, $d_{\rm Pt} = 1$ nm, and $d_{\rm Au}^p = 150$ nm. The thickness of the layers deposited on the sphere (not shown) are: $d_{Cr} = 1$ nm and $d_{An}^s =$ 200 nm. (b) Experimental setup. The red dotted line indicates where AFM line cuts were taken. Inset: AFM profile of the sample interface.

Equilibrium and non-equilibrium Casimir forces



Non-equilibrium Casimir force (Can be repulsive in vacuum)



Note: The plates consist of reciprocal materials.

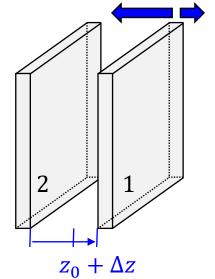
Stable and unstable Casimir force systems

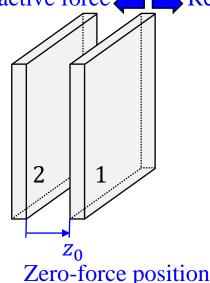
When attractive and repulsive force components acting on body 1 are balanced, zero-force position can exist.

Attractive force Repulsive force

Stable

Attractive force is enhanced.

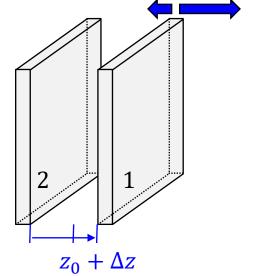




Zero-force position

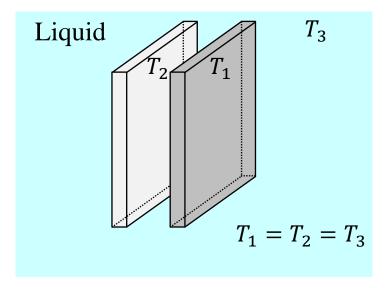
Unstable

Repulsive force is enhanced.



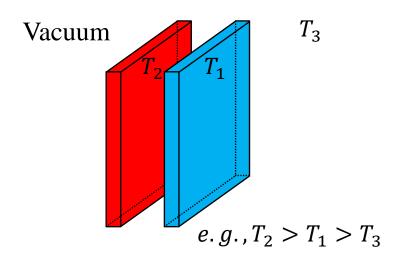
Examples of stable and unstable systems

Equilibrium Casimir force in liquid (Can be stable)



[See *Science* 364, 984 (2019)]

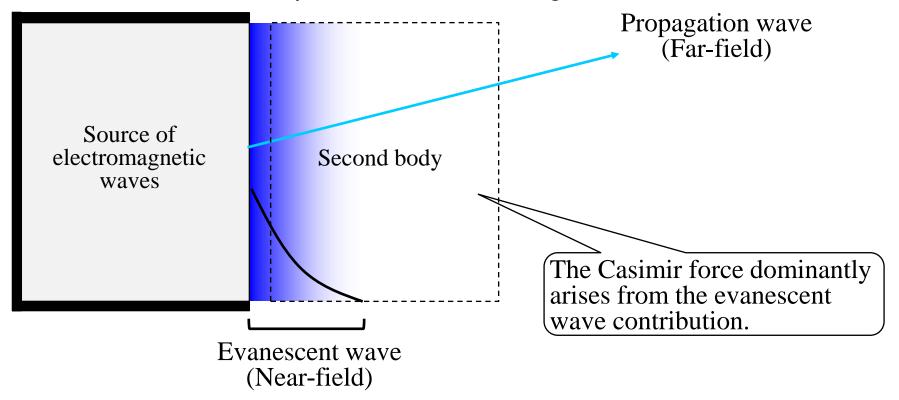
Non-equilibrium Casimir force in vacuum (Unstable)



Note: The plates consist of reciprocal materials.

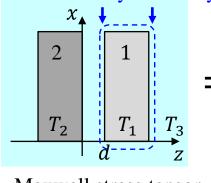
Propagation and evanescent waves

The propagation wave goes away from the electromagnetic source. The evanescent wave stays around the electromagnetic source.



Exterior boundary boundary

Casimir force calculation



Interior

Multiple

Maxwell stress tensor

$$T_{ij} = \epsilon_0 E_i E_j + \mu_0 H_i H_j - \delta_{ij} \frac{1}{2} (\epsilon_0 E^2 + \mu_0 H^2), (1)$$

Casimir force acting on body 1 in the two-body system (isotropic materials)

reflection

Casimir force calculation in equilibrium

Calculations of <u>equilibrium Casimir</u> <u>forces</u> can be simplified since photon exchange is balanced at the exterior boundary. (The interior boundary is only considered.)

 $\begin{array}{c|cccc}
2 & 1 \\
R_2 & \\
R_1 & \\
0 & d & z
\end{array}$ Interior

boundary

$$T \equiv T_1 = T_2 = T_3$$

$$i\xi$$

Casimir force formula, integration along the real frequency axis
$$P_{t} = -\sum_{j=p,s} \int_{0}^{\infty} \frac{k_{\parallel} dk_{\parallel}}{2\pi} \int_{0}^{\infty} \frac{d\omega}{2\pi} 4\hbar \left[n(\omega, T) + \frac{1}{2} \right] Re[k_{z}Z(\omega, \beta)], (11a)$$
$$Z(\omega, \beta) = \frac{R_{1}(\omega, \beta)R_{2}(\omega, \beta)e^{i2k_{z}d}}{1 - R_{1}(\omega, \beta)R_{2}(\omega, \beta)e^{i2k_{z}d}}, (11b)$$

- · Understanding the mechanism
- · Long calculation time

Wick rotation approach, integration along the imaginary frequency axis (E.M. Lifshitz, Sov. Phys. 1956)

$$P_{t} = \sum_{j=p,s} \int_{0}^{\infty} \frac{k_{\parallel} dk_{\parallel}}{2\pi} 2k_{B} T \sum_{n=0}^{\infty'} q_{0,n} Z(i\xi_{n},\beta), (12a)$$

$$Z(i\xi_{n},\beta) = \frac{R_{1}(i\xi_{n},\beta)R_{2}(i\xi_{n},\beta)e^{-2q_{0,n}d}}{1 - R_{1}(i\xi_{n},\beta)R_{2}(i\xi_{n},\beta)e^{-2q_{0,n}d}}, (12b),$$

- · Significantly reduced calculation time
- · Little observation of the mechanism

Reciprocal and non-reciprocal materials

$$\widehat{R}(\boldsymbol{k}_{\parallel}) = \begin{bmatrix} R^{s \to s}(\boldsymbol{k}_{\parallel}) & R^{p \to s}(\boldsymbol{k}_{\parallel}) \\ R^{s \to p}(\boldsymbol{k}_{\parallel}) & R^{p \to p}(\boldsymbol{k}_{\parallel}) \end{bmatrix}, (21)$$

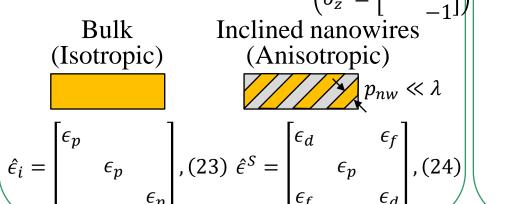
S-polarization \ $E \otimes H$ P-polarization $H \odot E$

Incident Vacuum Reciprocal or non-reciprocal material

Reciprocal materials
$$\hat{p}(-\mathbf{k}) = \hat{a} \hat{p}^T(\mathbf{k}) \hat{a}$$

Reciprocal materials
$$\hat{R}(-\boldsymbol{k}_{\parallel}) = \hat{\sigma}_{z}\hat{R}^{T}(\boldsymbol{k}_{\parallel})\hat{\sigma}_{z}, \quad (22)$$

$$\left(\hat{\sigma}_{z} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$
ulk Inclined nanowires ropic) (Anisotropic)





Magnetic Weyl InSb semimetals $\otimes B$

Non-reciprocal materials

Eq. (22) can be violated.

$$\hat{\epsilon}^{A} = \begin{bmatrix} \epsilon_{d} & i\epsilon_{f} \\ & \epsilon_{p} \\ -i\epsilon_{f} & \epsilon_{d} \end{bmatrix}, (25)$$

Our Casimir force research

Fundamental understanding Particle physics Toward industry Dynamic control of Casimir forces in vacuum [2] Symmetry argument in Casimir forces Control theory [5] Trajectory tracking[6] New force search via zero Casimir force[1] Non-reciprocal 1 Silicon T₁ Show $z, v, or T_1$ Sio 2 SiO₂ T₂ Weyl semimetal details Controller 3 Silicon T Dynamic control of Casimir [3]Casimir force [4]Exterior control Zero Casimir force New force forces in liquid[7] is insensitive to of non-equilibrium material loss Casimir force $-Max Z(\omega, \beta) Max$ Fthano Graphene nano-flake SiO2 [1] Y. Ema. M. Hazumi, H. Iizuka, K. graphene SiO, substrate Mukaida, and K. Nakayama, Phys. Rev. $\beta (\mu m^{-1})$ D 108, 016009 (2023).

- [2] H. Iizuka and S. Fan, Phys. Rev. B 108, 075429 (2023).
- [3] H. Iizuka and S. Fan, J. Optical Society of America B 36, 2981 (2019).
- [4] H. Iizuka and S. Fan, J. Optical Society America B 38, 151-158 (2021).
- [5] H. Iizuka and S. Fan, Applied Physics Letters 118, 144001 (2021).
- [6] H. Iizuka and S. Fan, J. Quantitative Spectroscopy Radiative Transfer 289, 108281 (2022).
- , [7] H. Toyama, T. Ikeda, and H. Iizuka, Phys. Rev. B 108, 245402 (2023).

Outline

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Control theory for Casimir forces

Symmetry of Casimir forces in wavevector space

Conclusions

Casimir force to Industries

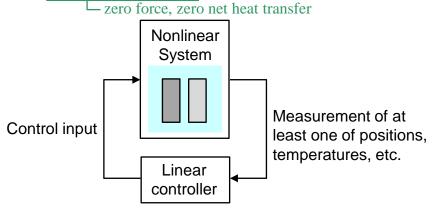
Control theory has been widely used in industries.

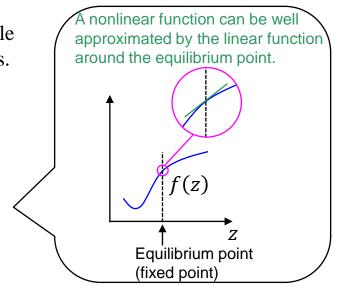




We introduce control theory to manipulate Casimir forces for the first time.

We show that a linear controller enables a nonlinear system interacting through non-equilibrium Casimir forces to be stable around the equilibrium point with a limited number of sensors.





System

 $\dot{x} = Ax$

Regulator

 $u = -K_r x$

Linear control system (1/2)

Dynamics of a linear system is given by

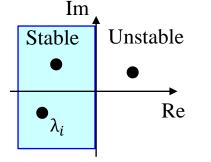
$$\frac{dx}{dt} = Ax, (51)$$

$$| \subseteq \text{State vector } x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$
System
$$\dot{x} = Ax$$

$$| \dot{x} = Ax$$

The system is stable if all eigenvalues meet

$$Re[\lambda_i] < 0, (52)$$



Eigenvalues (unstable)

The controller changes the dynamics.

of the dynamics.
$$\frac{dx}{dt} = Ax + Bu, (53)$$

$$Control input$$

$$u = -K_r x, (54)$$

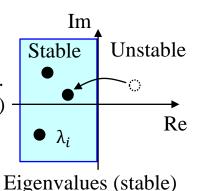
$$Gain matrix of the regulator$$

Eqs. (53),(54) give

$$\frac{dx}{dt} = (A - BK_r)x, (55)$$

If the controllability matrix is full ranked, the system is "controllable". (All λ_i can have negative real parts.)

$$Q = [A, AB, \dots AB^{n-1}], (56)$$



Steve Brunton, Control bootcamp, YouTube.

B. Friedland, Control System Design: An Introduction to State-Space Methods (McGraw-Hill, New York, 1986).

Linear control system (2/2)

The estimator (Kalman filter) enables the system to be stable with limited measurement results.

$$\frac{dx}{dt} = Ax + Bu, (61)$$

$$y = Cx, (62)$$

$$C = \begin{bmatrix} 0 & 1 & 0 & \dots \end{bmatrix}$$

$$y = Cx, (62)$$

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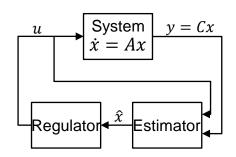
$$C = \begin{bmatrix} 0 & 1 & 0 & \dots \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & \dots \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 &$$

If the observability matrix is full ranked, the system is "observable". (The system is stable with limited measurement results.)

$$N = [C, CA, \dots CA^{n-1}], (65)$$



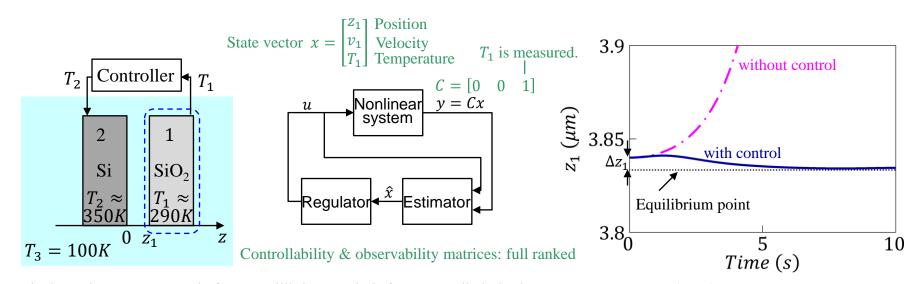
Steve Brunton, Control bootcamp, YouTube.

B. Friedland, Control System Design: An Introduction to State-Space Methods (McGraw-Hill, New York, 1986).

Control of non-equilibrium Casimir force

Consider a system consisting of a silicon plate and a SiO₂ plate in a low-temperature environment. The silicon plate is fixed and the temperature can be adjusted while the SiO₂ plate is moved and the temperature is determined through the thermal emission exchange.

We theoretically prove that the SiO_2 plate can stay at a distance away from the silicon plate indefinitely by monitoring the temperature of the SiO_2 plate and adjusting the temperature of the silicon plate.



H. Iizuka and S. Fan, "Control of non-equilibrium Casimir force," Applied Physics Letters 118, 144001 (2021).

Trajectory tracking through the control of Casimir force

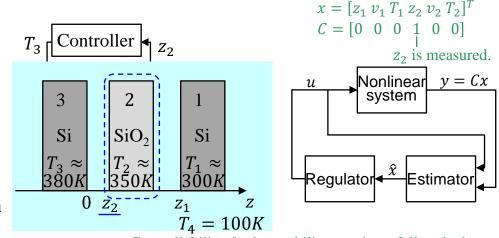
An autonomous vehicle can follow a path as we set by control theory, which is called trajectory tracking.



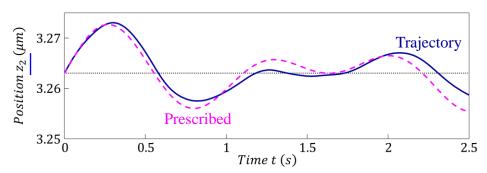


Trajectory tracking is applied to a three-body system interacting through non-equilibrium Casimir forces.

The position of the middle SiO₂ plate (blue line) follows a prescribed trajectory (pink dashed line) by adjusting the temperature of the left silicon plate (single parameter) while the right silicon plate is kept around the equilibrium position.



Controllability & observability matrices: full ranked



H. Iizuka and S. Fan, "Trajectory tracking through the control of non-equilibrium Casimir force," Journal of Quantitative Spectroscopy and Radiative Transfer 289, 108281 (2022).

Outline

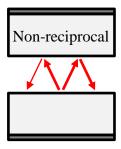
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[2] Symmetry argument in Casimir forces



[2] H. Iizuka and S. Fan, Phys. Rev. B 108, 075429 (2023).

Casimir lateral force

Environment $\epsilon_0 T_3$ Anisotropic reciprocal material $\epsilon_2^S = \begin{vmatrix} \epsilon_d & \epsilon_f \\ \epsilon_p & \epsilon_d \end{vmatrix}, (24) \quad \text{or} \quad \epsilon_2^A = \begin{vmatrix} \epsilon_d & i\epsilon_f \\ \epsilon_p & \epsilon_d \end{vmatrix}, (25)$ Body 2 $\hat{\epsilon}_2 T_2$

function

Anisotropic reciprocal material
$$\epsilon_d$$

non-reciprocal material

InSb/Weyl

Body 1 $\epsilon_{1p}T_1$ (Isotropic)

Casimir lateral force acting on body 2

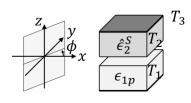
 $\boldsymbol{F}_{2}^{\parallel}(T_{1},T_{2}) = \int_{\Omega} d\omega \int_{\Omega} d\boldsymbol{k}_{\parallel} \underline{\boldsymbol{F}_{2}^{\parallel}(\omega,\boldsymbol{k}_{\parallel},T_{1},T_{2})},(41)$

 $F_2^{\parallel}(\omega, \mathbf{k}_{\parallel}, T_1, T_2) = -F_{1 \to 2}^{\parallel}(\omega, \mathbf{k}_{\parallel}, T_1) - F_{2 \to 1}^{\parallel}(\omega, \mathbf{k}_{\parallel}, T_2), (42)$

Perfect electric conductor No photon exchange between the two-body system and the environment. $T_{1} = T_{2} = T_{3} = T \text{ (Equilibrium)} \underbrace{\tilde{F}_{l \to m}^{\parallel}(\omega, \mathbf{k}_{\parallel})}_{\text{Exchange}} = \begin{cases} Tr \left[(-1)^{l} \left(\hat{I} - \hat{R}_{m}^{\dagger} \hat{R}_{m} \right) \widehat{D}_{lm} \left(\hat{I} - \hat{R}_{l} \hat{R}_{l}^{\dagger} \right) \widehat{D}_{lm}^{\dagger} \right], & (k_{\parallel} < k_{0}) \\ Tr \left[(-1)^{l} \left(\hat{R}_{m}^{\dagger} - \hat{R}_{m} \right) \widehat{D}_{lm} \left(\hat{R}_{l} - \hat{R}_{l}^{\dagger} \right) \widehat{D}_{lm}^{\dagger} e^{-2\kappa_{z0}d} \right], (k_{\parallel} > k_{0}) \end{cases}, (44)$

 $T_1 \neq T_2$ (Non-equilibrium)

 $\boldsymbol{F}_{1\to2}^{\parallel}(\omega,\boldsymbol{k}_{\parallel},T_{1}) = \left[n(\omega,T_{1}) + \frac{1}{2}\right] \frac{h\boldsymbol{k}_{\parallel}}{\Omega \pi^{3}} \tilde{F}_{1\to2}^{\parallel}(\omega,\boldsymbol{k}_{\parallel}), (43)$



Casimir lateral force in $(\omega, \mathbf{k}_{\parallel})$ space $F_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2, T_3)$

Exchange function $\tilde{F}_{l\to m}^{\parallel}(\omega, \boldsymbol{k}_{\parallel})$

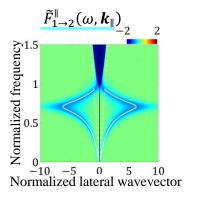
White lines: Dispersion curves

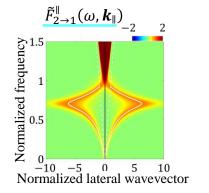
Casimir lateral force, reciprocal

Casimir lateral force is symmetric for reciprocal systems in equilibrium and non-equilibrium.

$$F_2^{\parallel}(\omega, \mathbf{k}_{\parallel}, T_1, T_2) = -F_2^{\parallel}(\omega, -\mathbf{k}_{\parallel}, T_1, T_2), (45)$$

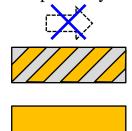
Symmetric for k_{\parallel}





Lateral force does not occur in equilibrium and nonequilibrium for reciprocal systems.

 $\tilde{F}_{1\rightarrow 2}^{\parallel} + \tilde{F}_{2\rightarrow 1}^{\parallel} \qquad a\tilde{F}_{1\rightarrow 2}^{\parallel} + b\tilde{F}_{2\rightarrow 1}^{\parallel}$





Casimir lateral force, non-reciprocal



Casimir lateral force in $(\omega, \mathbf{k}_{\parallel})$ space $F_2^{\parallel}(\omega, \boldsymbol{k}_{\parallel}, T_1, T_2, T_3)$

Exchange function $\tilde{F}_{l\to m}^{\parallel}(\omega, \boldsymbol{k}_{\parallel})$

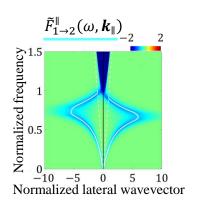
White lines: Dispersion curves

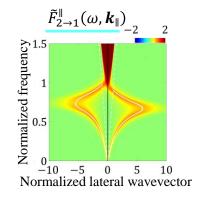
Symmetry of the Casimir lateral force is broken for non-reciprocal systems in equilibrium and non-equilibrium.

$$\tilde{F}_{1 \to 2}^{\parallel} + \tilde{F}_{2 \to 1}^{\parallel}$$
 Cancelled out

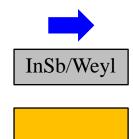
$$a\tilde{F}_{1\rightarrow 2}^{\parallel} + b\tilde{F}_{2\rightarrow 1}^{\parallel}$$

Asymmetric for k_{\parallel}

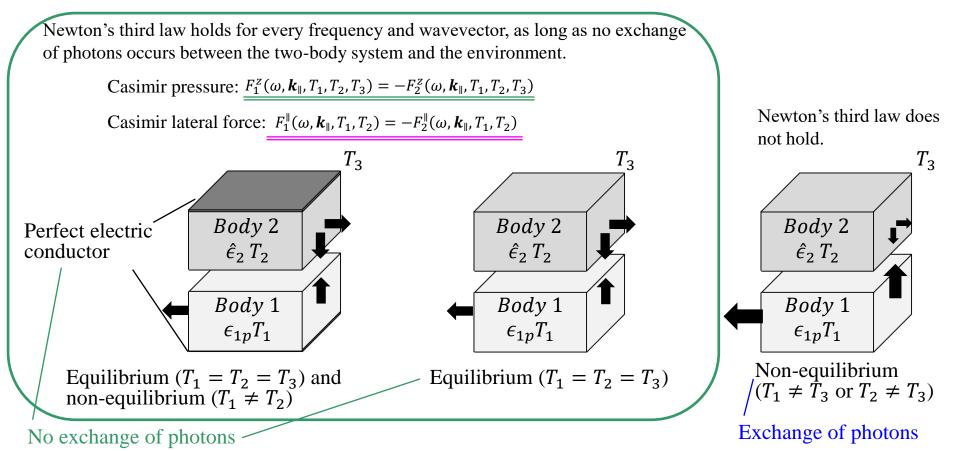




Lateral force can occur in non-equilibrium for non-reciprocal systems.



Newton's third law



The above is true for both reciprocal $\hat{\epsilon}_2 = \hat{\epsilon}_2^S$ and non-reciprocal $\hat{\epsilon}_2 = \hat{\epsilon}_2^A$ materials.

Conclusions

A brief overview of Casimir forces was presented.

Parallel-plate systems for non-equilibrium Casimir forces are controllable and observable by introducing a linear controller.

Symmetry of Casimir forces in wavevector space was discussed. This understanding is helpful for investigating Casimir forces using Weyl semimetals.

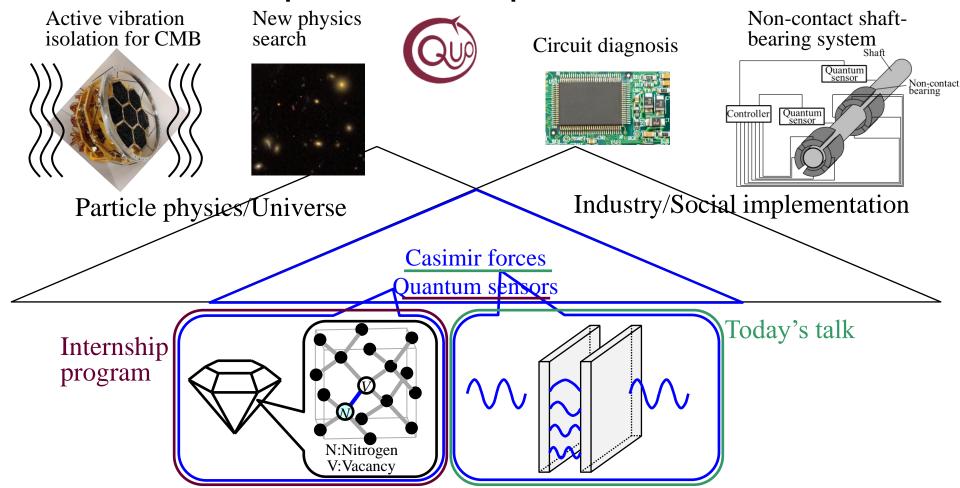
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Research scope of QUP quantum sensor cluster



QUP Internship program

https://www2.kek.jp/qup/en/jobs/qupip.html

Jobs QUPIP

Announcement

2023.10.18

QUP Internship Program (QUPIP)

The International Center for Quantum-field Measurement Systems for Studies of the Universe and Particles (WPI-QUP) invites applications for young researchers (post-doctoral fellows and graduate students) to stay and work with QUP researchers. QUP is developing a new measurement system using existing quantum fields to explore unknown quantum fields under "Bringing New Eyes to Humanity."

Hideo Iizuka (Research Location: KEK Tsukuba, Japan)

Our aim is to search for new quantum fields enabled by quantum effects in a single lab space. We have started setting up a measurement system using nitrogen-vacancy centers in diamond for light dark matter search.

Example study subjects for the young researchers

- \blacksquare Setup of the ODMR (optically detected magnetic resonance) measurement system.
- Evaluation of optical properties of diamond samples.

Quantum sensors

