

Cogenesis by a sliding pNGB with global symmetry non-restoration

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Contents

1. Introduction

- ✓ Spontaneous baryogenesis/leptogenesis
- ✓ Conventional misalignment mechanism

2. pNGB with global symmetry non-restoration

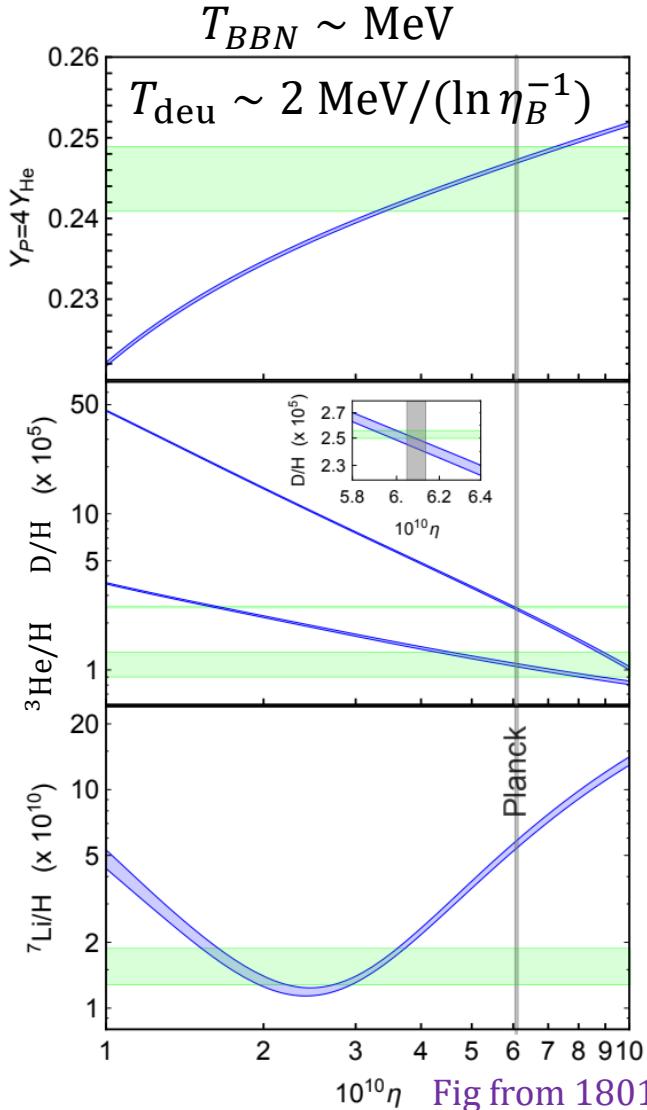
- ✓ Setup & properties
- ✓ pNGB dynamics

3. Cogenesis scenario

4. Summary

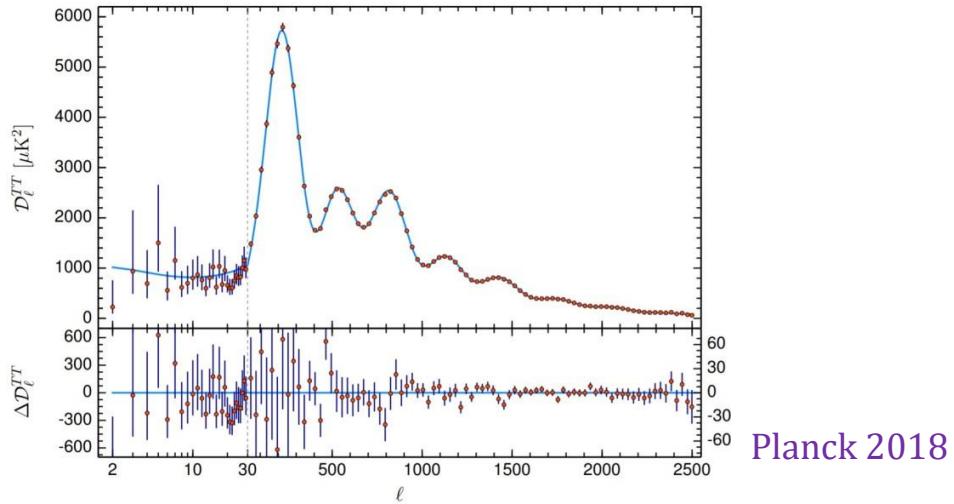
Baryon asymmetry of the Universe

Observation: Our universe is made of matter, not antimatter.



$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6 \times 10^{-10}$$
$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{S} = 9 \times 10^{-11}$$

→ Consistent with CMB fitting



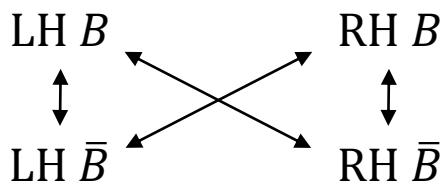
$Y_B \sim 10^{-10}$ should be produced before BBN, somehow.

(main topic of this talk)

To generate baryon asymmetry...

Sakharov conditions

- **B number changing process**
obvious
- **C and CP violation**
no net effect if any of them is conserved



- **Departure from thermal equilibrium**
$$\left(n_B = \sum_q \int \frac{d^3 p}{(2\pi)^3} f_q(p) \right) = \left(n_{\bar{B}} = \sum_{\bar{q}} \int \frac{d^3 p}{(2\pi)^3} f_{\bar{q}}(p) \right)$$

In SM

Weak sphaleron

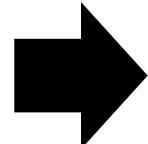
SU(2) Chern-Simons number transition
 $\rightarrow B + L$ transition

C violation: electroweak interaction

CP violation: CKM matrix (**not enough**)

EWPT (cross-over... **not enough**)

Some freeze-out process (... **not enough**)

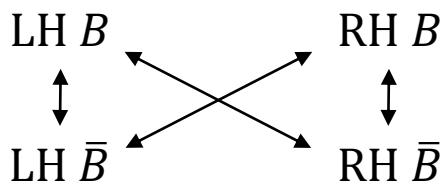


We need new physics.
(there are many scenarios)

To generate baryon asymmetry...

Sakharov conditions

- **B number changing process**
obvious
- **C and CP violation**
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- ~~Departure from thermal equilibrium~~
$$\left(n_B = \sum_q \int \frac{d^3 p}{(2\pi)^3} f_q(p) \right) \neq \left(n_{\bar{B}} = \sum_{\bar{q}} \int \frac{d^3 p}{(2\pi)^3} f_{\bar{q}}(p) \right)$$



Spontaneous baryogenesis/leptogenesis (Cohen, Kaplan, 87, 88)
 $f_q \neq f_{\bar{q}}$ in the presence of CPT violation (due to background field dynamics)

In SM

Weak sphaleron

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C violation: electroweak interaction

CP violation: CKM matrix (**not enough**)

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Spontaneous baryogenesis/leptogenesis

- Consider a pseudo-scalar field a with a current interaction

$$L = \dots + \frac{\partial_\mu a}{f_a} \psi^\dagger \bar{\sigma}^\mu \psi \quad (\psi^\dagger \bar{\sigma}^0 \psi = n_\psi - n_{\bar{\psi}})$$

and its homogenous motion:

$$\theta \equiv \frac{a}{f_a}, \quad \dot{\theta} \neq 0 \Rightarrow H = \dots - \int d^3x \dot{\theta} (n_\psi - n_{\bar{\psi}}) = \dots - \dot{\theta} Q_\psi$$

$$\Rightarrow \langle n_\psi - n_{\bar{\psi}} \rangle \sim \int dp p^2 \left(e^{-(E-\dot{\theta})/T} - e^{-(E+\dot{\theta})/T} \right) \sim c_\psi \dot{\theta} T^2$$

- Chemical equilibration (Charge transportation)

Depending on what interactions are in the thermal bath, there are a series of chemical equilibrations, i.e. asymmetry re-distribution.

e.g. Top quark Yukawa $\rightarrow \mu_{q_3} + \mu_{t^c} + \mu_H = 0$ $\frac{\mu_i}{T} \sim \frac{n_i - n_{\bar{i}}}{n_i + n_{\bar{i}}}$
EW sphaleron $\rightarrow \sum_i (3\mu_{q_i} + \mu_{l_i}) = 0$

...

- Scenario is specified by

✓ How its motion ($\dot{\theta} \neq 0$) is generated

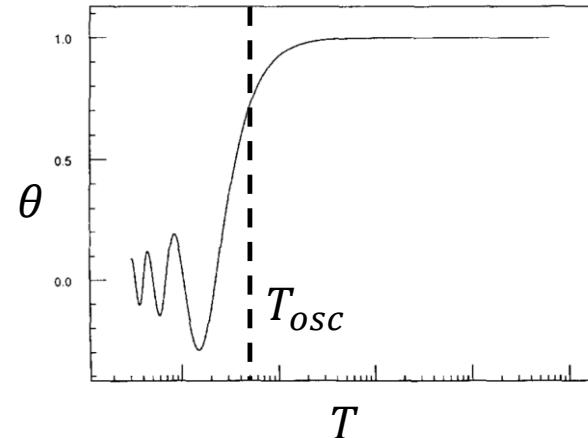
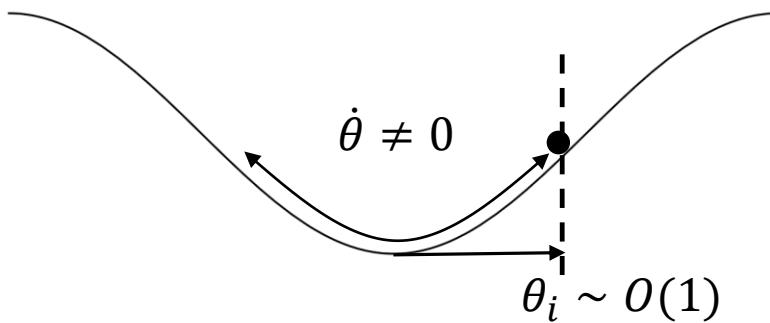
e.g. coherent oscillation, kinetic misalignment, first-order phase transition, etc.

An example

Misalignment mechanism

Cohen, Kaplan, 87

- θ was stuck at $O(1)$ initial misalignment angle, and starts oscillation when $H(T_{osc}) = m_a$.

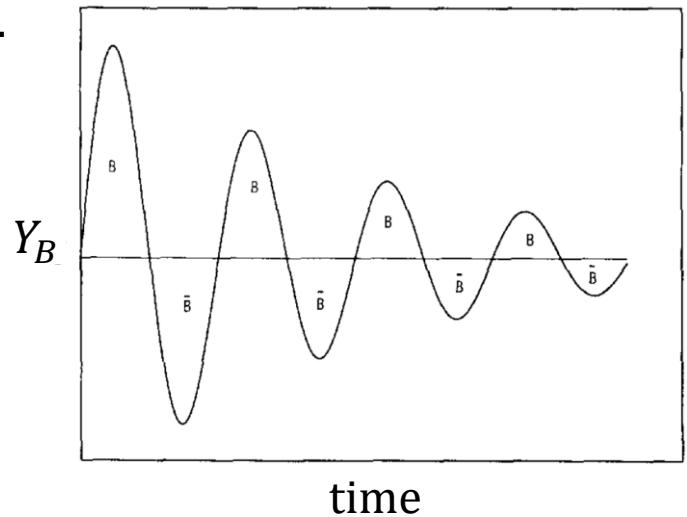


- $\dot{\theta}$ acts as a source term in the Boltzmann equation.

$$\dot{n}_B + 3H n_B = -\Gamma_B (n_B - c_B \dot{\theta} T^2)$$



T_{B*} : decoupling
temperature of Γ_B

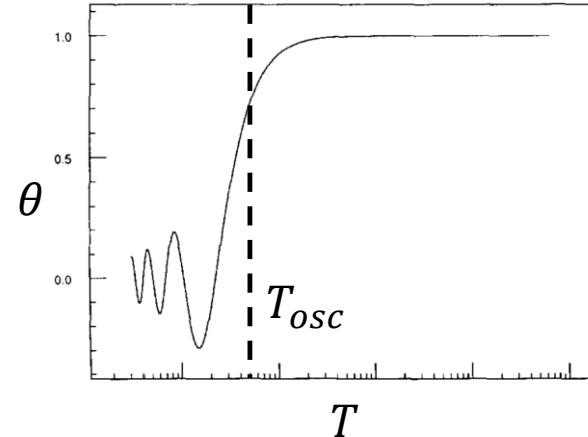
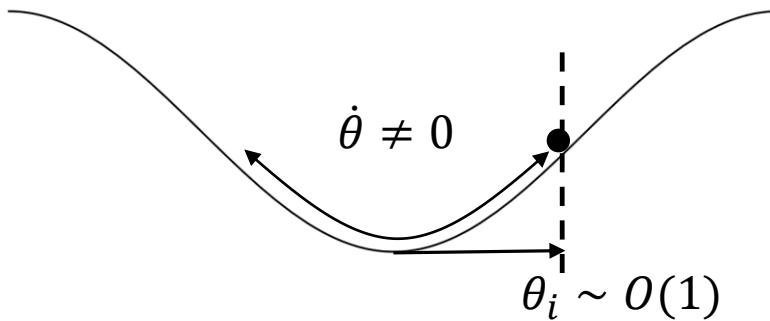


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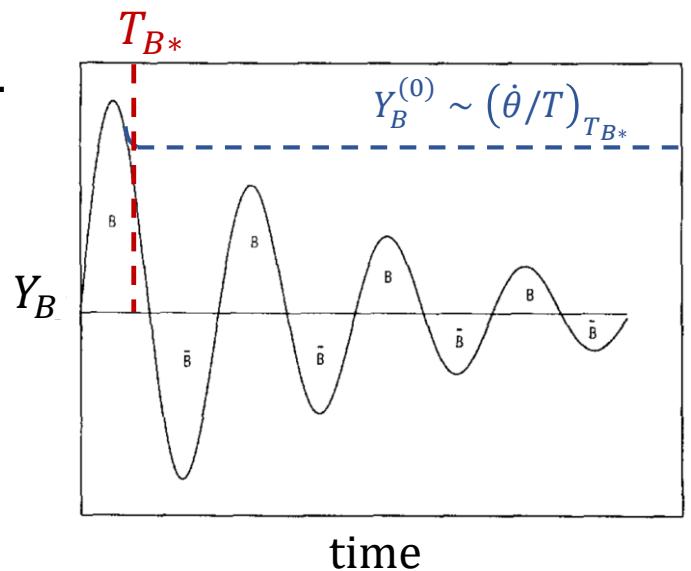


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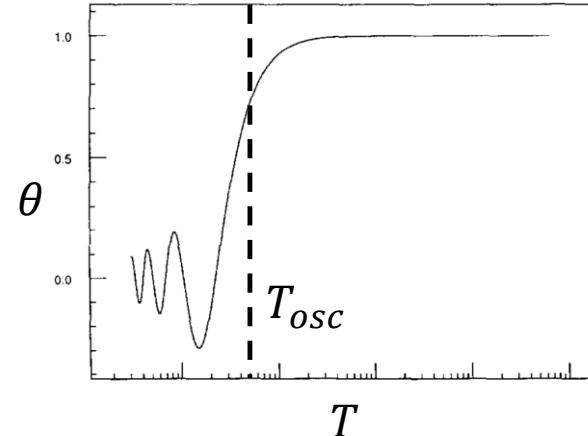
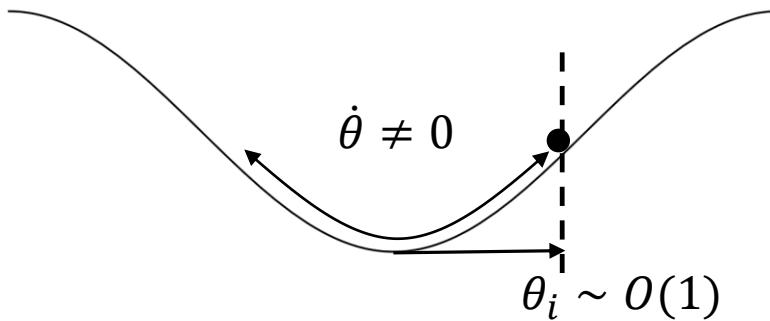


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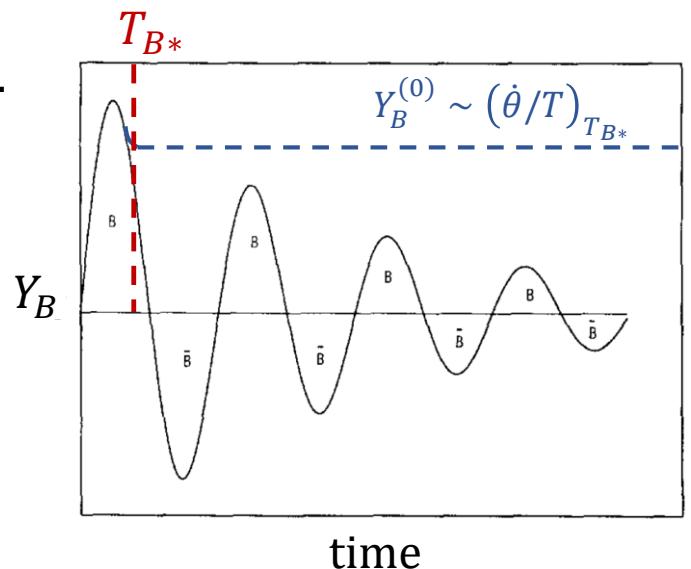
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T_{B*} : decoupling
temperature of Γ_B

- Q: can θ -oscillation also be DM?



Is “co”genesis possible for standard misalignment?

Quick estimation

$$Y_B^{(0)} = \frac{n_B}{s} \Big|_{T_{B^*}} \sim \frac{\dot{\theta}(T_{B^*})}{g_* T_{B^*}} \lesssim \frac{m_a}{g_* T_{osc}} \sim \frac{1}{g_*} \sqrt{\frac{m_a}{M_{pl}}} \Rightarrow \begin{aligned} m_a &\gtrsim 100 \text{ GeV} \\ T_{osc} &\gtrsim 10^{10} \text{ GeV} \\ f_a &\gg T_{osc} \end{aligned}$$

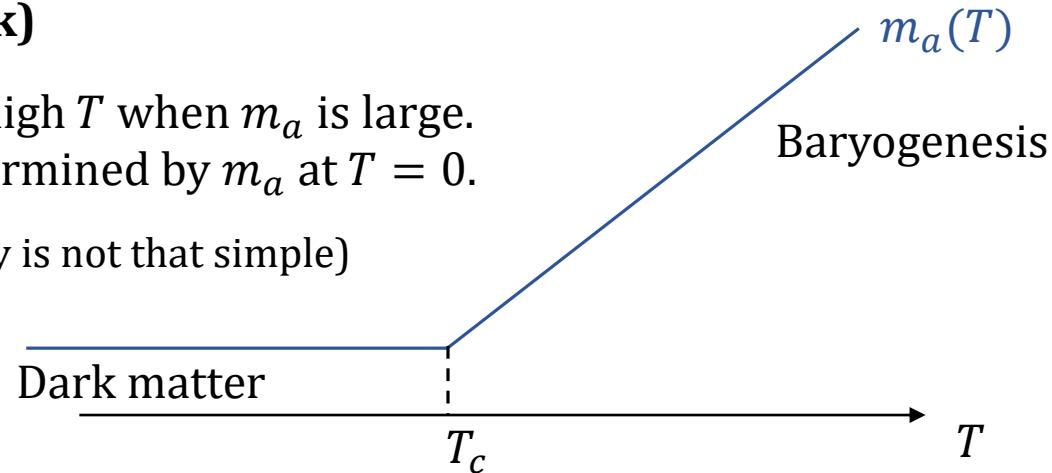
↑
Tuning of $T_{osc} \sim T_{B^*}$ is needed
to have the maximal value

⇒ **θ oscillation cannot be DM**
(its energy density is too large)
(when m_a is constant)

What if $m_a(T) \propto T^\#$? (this work)

Baryogenesis is completed at high T when m_a is large.
Dark matter abundance is determined by m_a at $T = 0$.

(original motivation... but the story is not that simple)



Realization of $m_a(T) \propto T^\#$ by symmetry non-restoration

A complex scalar field $\Phi = \frac{\phi}{\sqrt{2}} \exp[i\theta]$ whose axial mode is the pNGB for cogenesis.

$$V(\Phi) = \lambda_\phi |\Phi|^4 - m_0^2 |\Phi|^2 \quad \Rightarrow f_a^{(0)} = \langle \phi \rangle = \frac{m_0}{\sqrt{\lambda_\phi}}$$

Thermal corrections

$$V_T(\phi) = \frac{1}{4} \lambda_\phi \phi^4 - \frac{1}{2} m_0^2 \phi^2 + \underbrace{(D T^2 \phi^2 - E T \phi^3 + \dots)}_{\text{thermal corrections (high-}T\text{ expansion)}}$$

$$= \frac{1}{4} \lambda_\phi \phi^4 - \frac{1}{2} (m_0^2 - 2D T^2) \phi^2 + \dots \quad D \sim g^2 + y^2 + \lambda_{\text{mix}}$$

- $D > 0$: Usual scenario with symmetry restoration at high T
- $D < 0$: Symmetry non-restoration at high T

$$\langle \phi \rangle_T = f_a(T) \simeq \sqrt{\left(f_a^{(0)}\right)^2 + c_\lambda T^2} \simeq \begin{cases} f_a^{(0)} & , \quad T < T_c \equiv f_a^{(0)} / \sqrt{c_\lambda} \\ \boxed{\sqrt{c_\lambda} T} & , \quad T > T_c \end{cases}$$

with $c_\lambda = |D|/\lambda_\phi$

\uparrow
coupling with SM Higgs
or additional scalars

Realization of $m_a(T) \propto T^\#$ by symmetry non-restoration

- VEV of radial mode increases as T above T_c

$$D < 0 \quad \boxed{f_a(T) \simeq \sqrt{c_\lambda} T} \quad , \quad T > T_c \equiv f_a^{(0)} / \sqrt{c_\lambda}$$

- An explicit $U(1)$ breaking operator to generate pNGB potential

$$V_{U(1)} = \frac{1}{\Lambda} \Phi^5 + \text{h. c.} \quad \Rightarrow V_{\text{pNGB}}(\theta) \sim \frac{\phi^5}{\Lambda} (1 - \cos(5\theta))$$

$$\Rightarrow m_a(T) \sim \frac{f_a(T)^{3/2}}{\Lambda^{1/2}} \simeq \begin{cases} m_a^{(0)} & , \quad T < T_c \equiv f_a^{(0)} / \sqrt{c_\lambda} \\ \boxed{m_a^{(0)} (T/T_c)^{3/2}} & , \quad T > T_c \end{cases}$$

- ✓ In the end, we need $\Lambda \gg M_{\text{Pl}}$.

This can be achieved by considering $V = \frac{1}{M_{\text{Pl}}^2} X \Phi^5 \rightarrow \left(\Lambda = \frac{M_{\text{Pl}}^2}{\langle X \rangle} \right)$

with a proper discrete symmetry to prevent higher dimensional operators from dominating.

Realization of $m_a(T) \propto T^\#$ by symmetry non-restoration

An explicit example: Φ with an additional complex scalar S

$$V(\Phi, S) = \lambda_\phi |\Phi|^4 - 2\lambda_{\phi s} |\Phi|^2 |S|^2 + \lambda_s |S|^4 - m_0^2 |\Phi|^2 + m_s^2 |S|^2$$

Stability condition: $\lambda_\phi \lambda_s > \lambda_{\phi s}^2$

Consistency at one-loop: $\lambda > \frac{\lambda' \lambda''}{16\pi^2}$ for $\lambda, \lambda', \lambda'' = \lambda_\phi, \lambda_s, \lambda_{\phi s}$

$$V_T(\phi, s) = V + V_{CW} + \frac{T^4}{2\pi^2} \sum J_B \left(\frac{m_i^2(\phi)}{T^2} \right)$$

$$\simeq \frac{1}{4} \lambda_\phi \phi^4 - \frac{1}{2} \left(m_0^2 + \frac{1}{6} \lambda_{\phi s} T^2 \right) \phi^2 + \dots$$

Large $c_\lambda = \frac{\lambda_{\phi s}}{6\lambda_\phi}$: $\lambda_s \sim O(1)$, $\lambda_{\phi s} \sim \lambda_{\phi s}^2$, $\lambda_{\phi s} \ll 1$

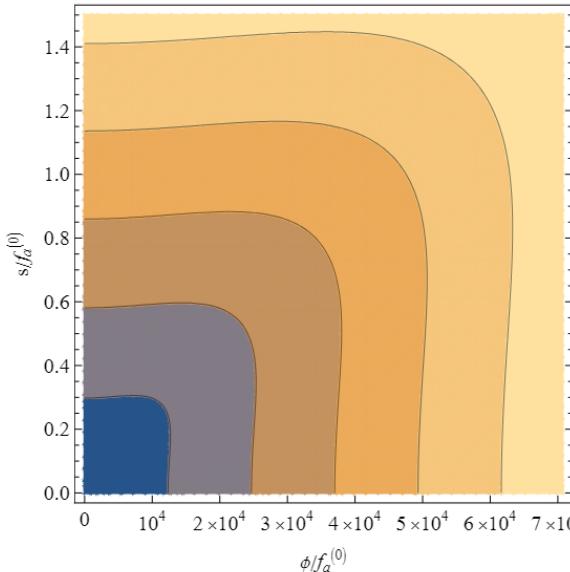
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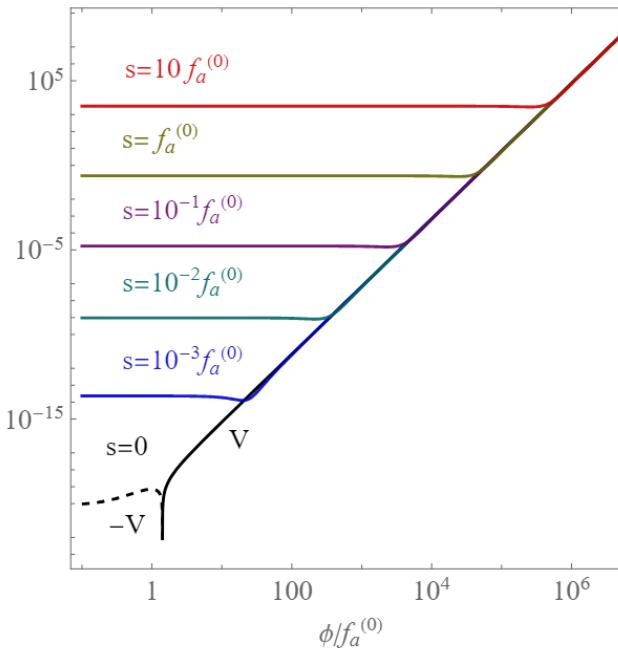
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$$V^{1/4}/f_a^{(0)}$$



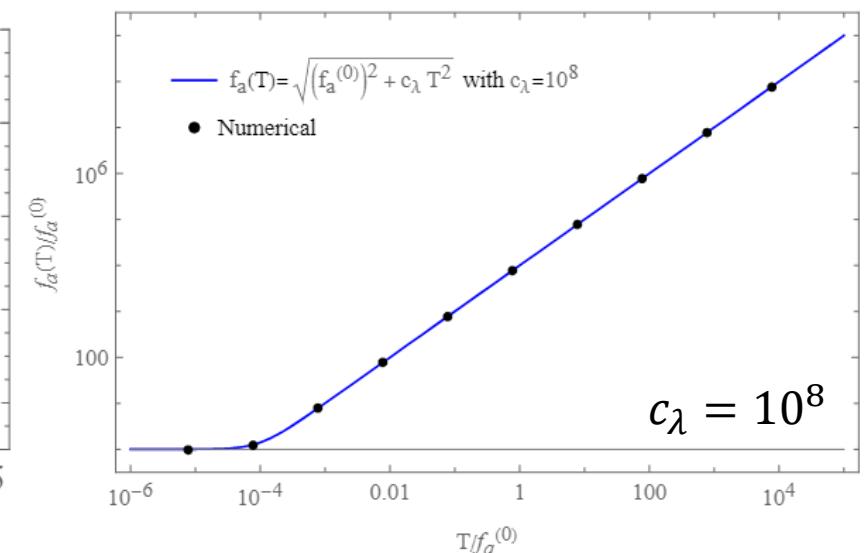
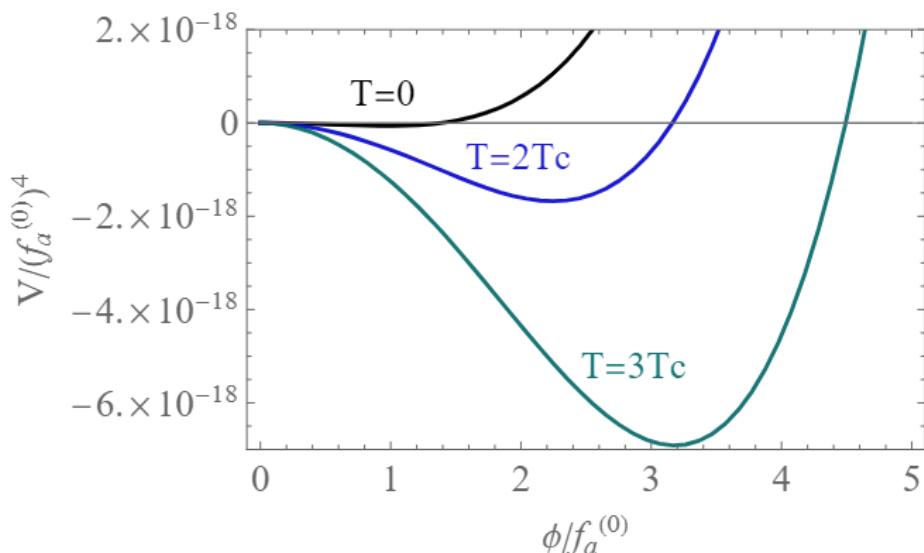
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pNGB dynamics with symmetry non-restoration

- Assuming that ϕ follows its potential minimum (which must be justified later),

$$\ddot{\theta} + \left(3H + 2\frac{\dot{f}_a}{f_a} \right) \dot{\theta} = -\frac{1}{5} m_a^2(T) \sin 5\theta$$

$\simeq H$ for $T > T_c$

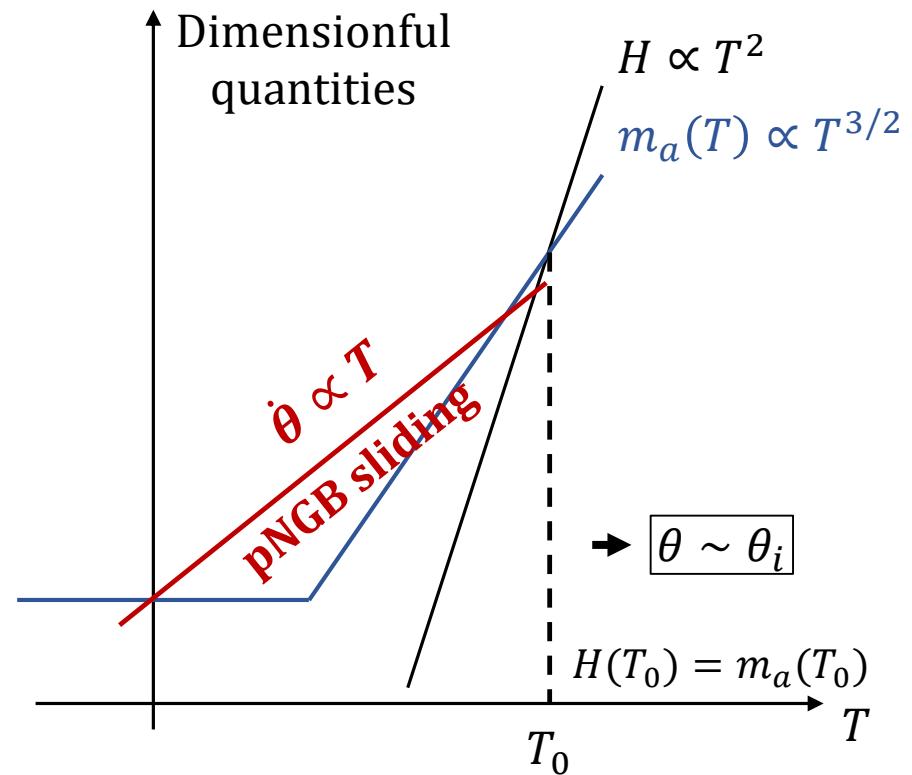
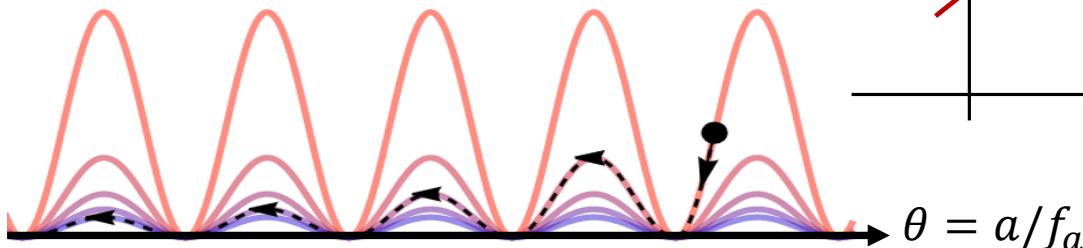
- $H \propto T^2$ (radiation-dominated)

- $m_a \propto T^{3/2}$ for $T > T_c$

- At high T , $H(T) > m_a(T) \Rightarrow \theta \simeq \theta_i$.

- At T_0 ($H(T_0) = m_a(T_0)$), $\dot{\theta} \simeq m_a(T_0)$

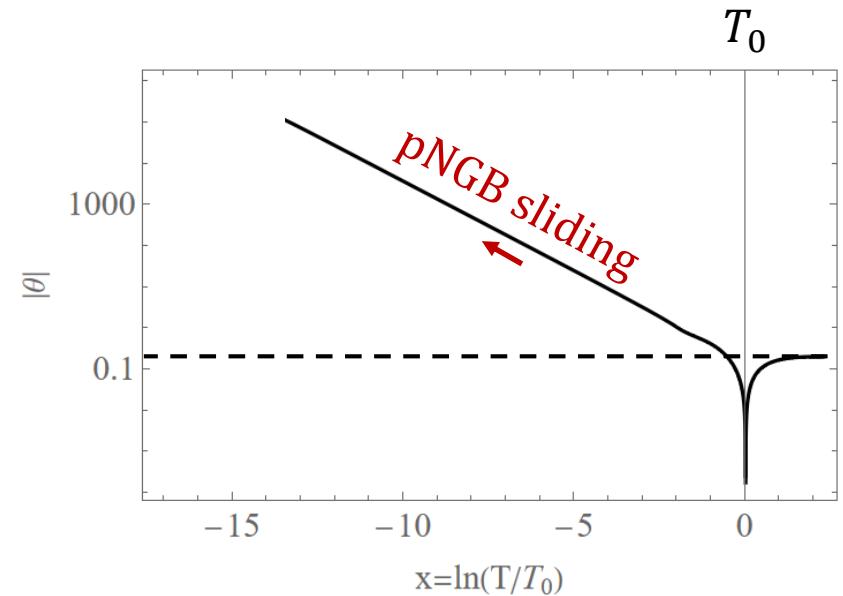
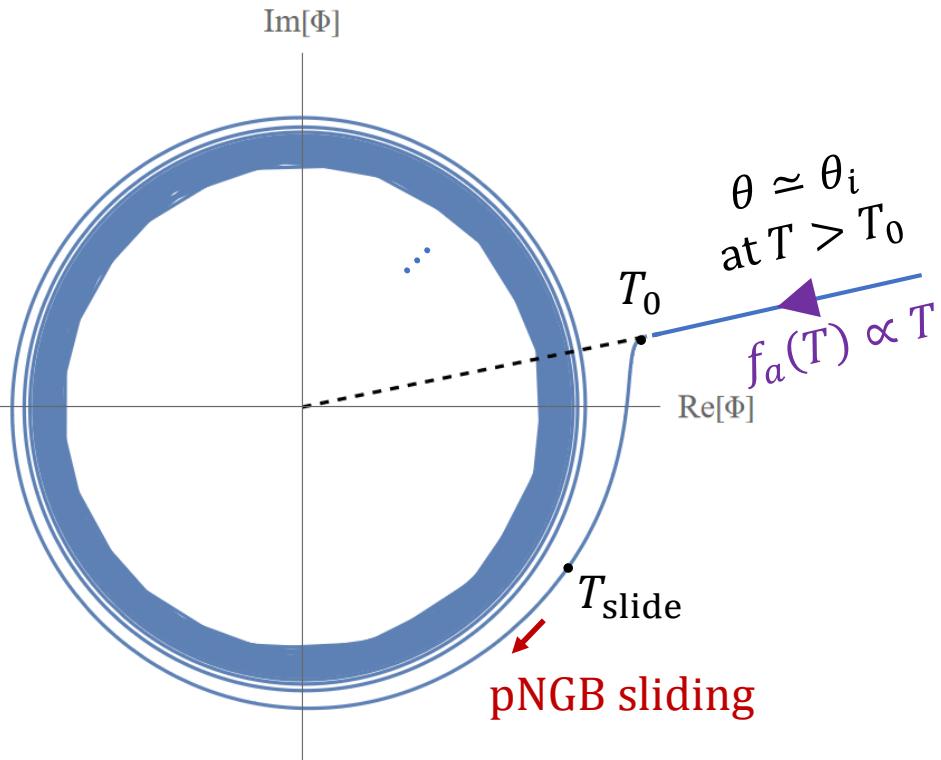
- During the first dropping, potential barrier decreases faster than the redshift of K.E.



pNGB dynamics with symmetry non-restoration

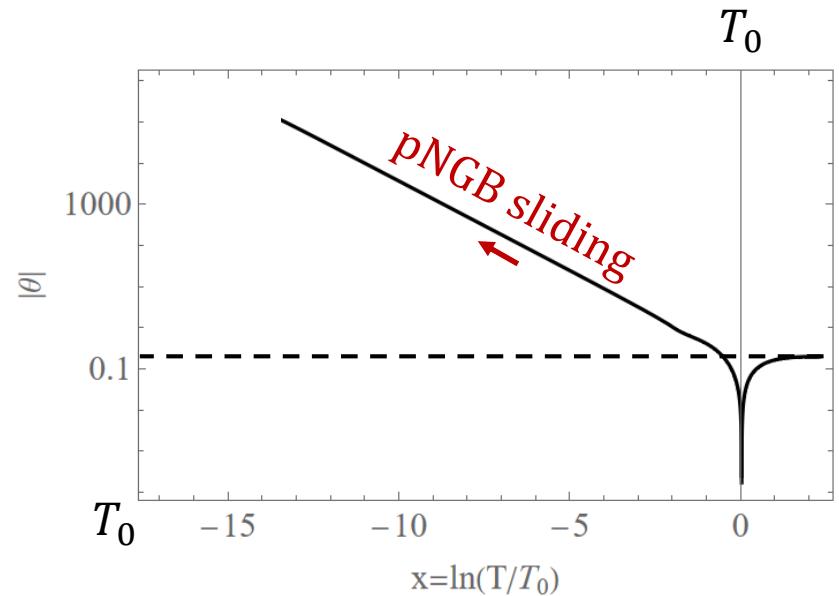
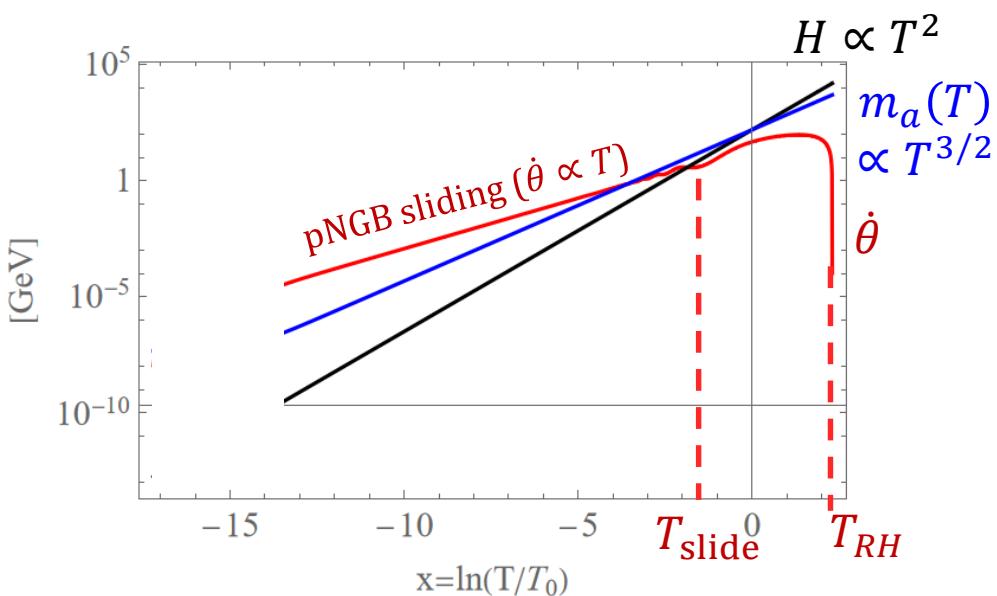
$$\ddot{\theta} + \left(3H + 2 \frac{\dot{f}_a}{f_a} \right) \dot{\theta} = -\frac{1}{5} m_a^2(T) \sin 5\theta$$

$\simeq H$ for $T > T_c$



pNGB dynamics with symmetry non-restoration

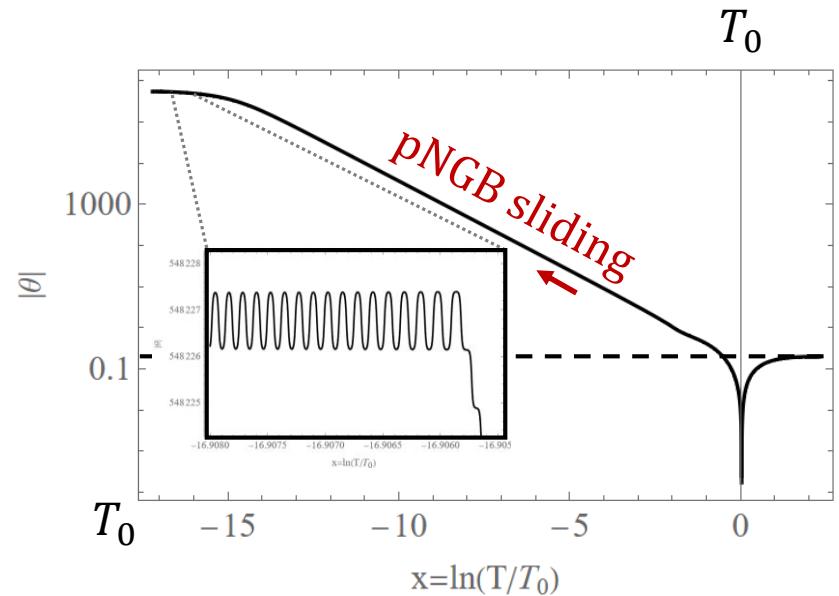
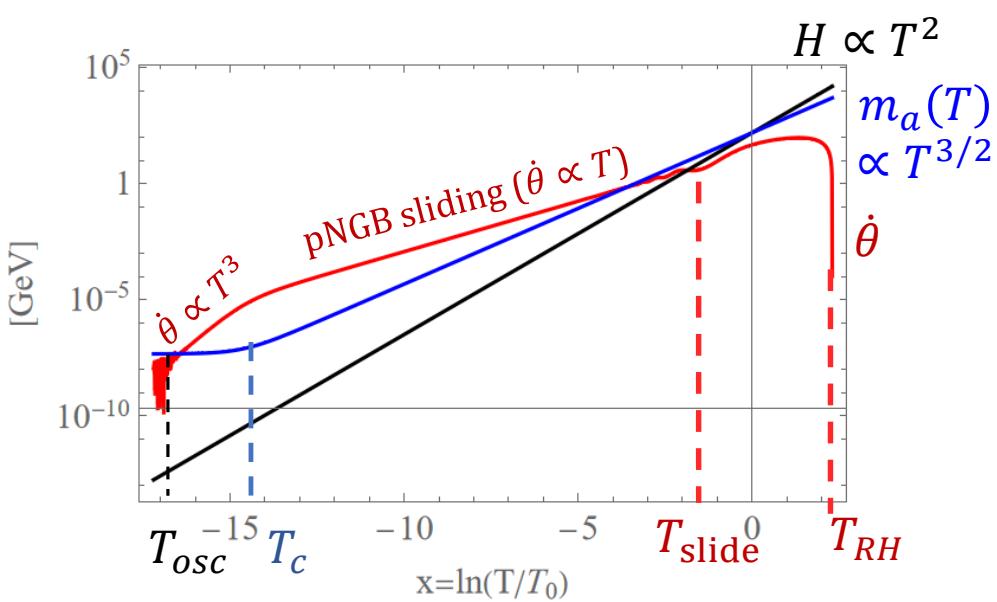
$$\ddot{\theta} + \underbrace{\left(3H + 2\frac{\dot{f}_a}{f_a}\right)\dot{\theta}}_{\simeq H \text{ for } T > T_c} = -\frac{1}{5}m_a^2(T) \sin 5\theta$$



pNGB dynamics with symmetry non-restoration

$$\ddot{\theta} + \left(3H + 2\frac{\dot{f}_a}{f_a} \right) \dot{\theta} = -\frac{1}{5} m_a^2(T) \sin 5\theta$$

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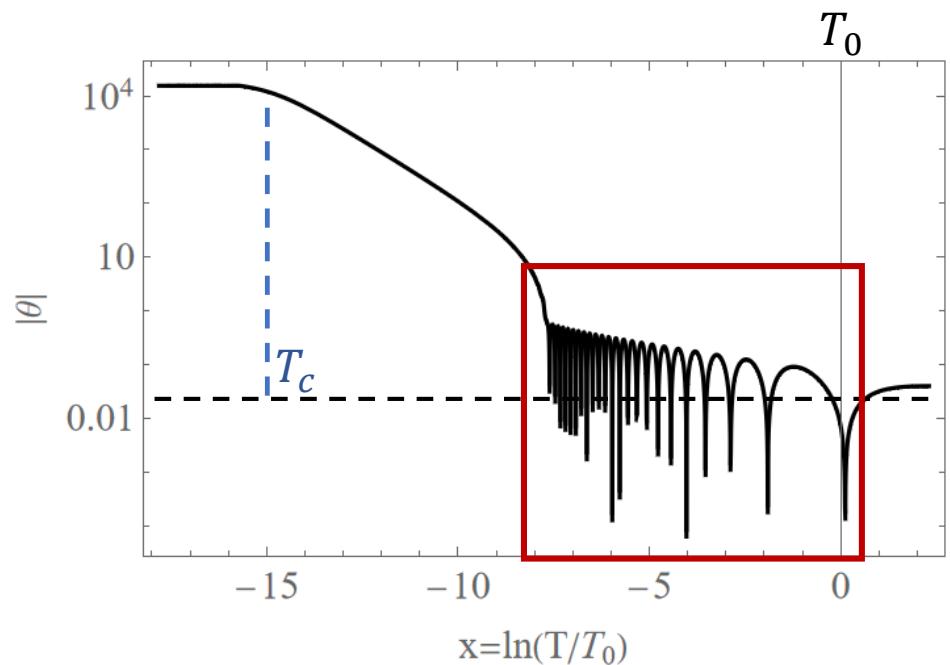
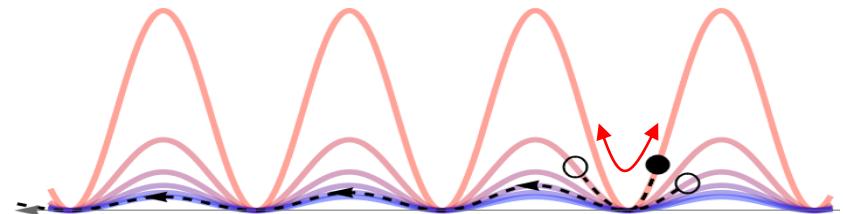
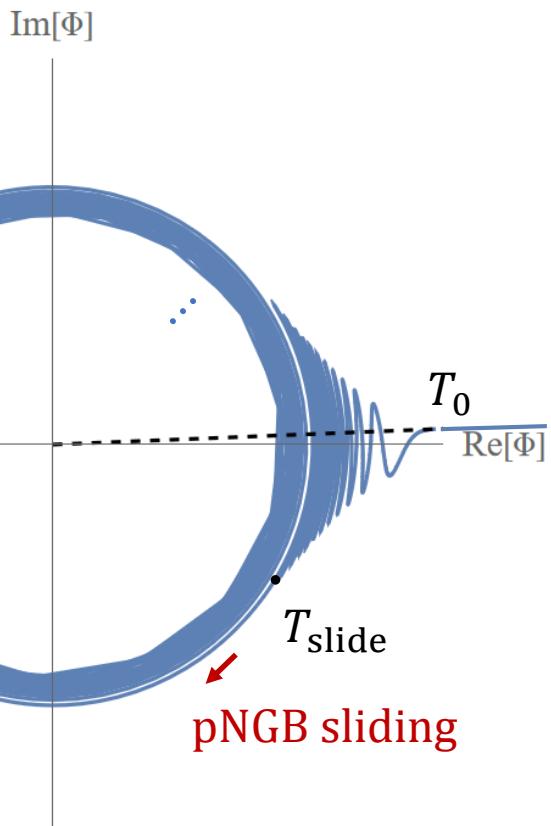
- At $T < T_c$, $\dot{f}_a \simeq 0$, so $\dot{\theta} \propto T^3$ until K. E < barrier height (i.e. the pNGB gets trapped).
- Then, pNGB starts oscillation ($T < T_{osc}$), and becomes dark matter.

pNGB dynamics

When $5\theta_i \sim 0.2$

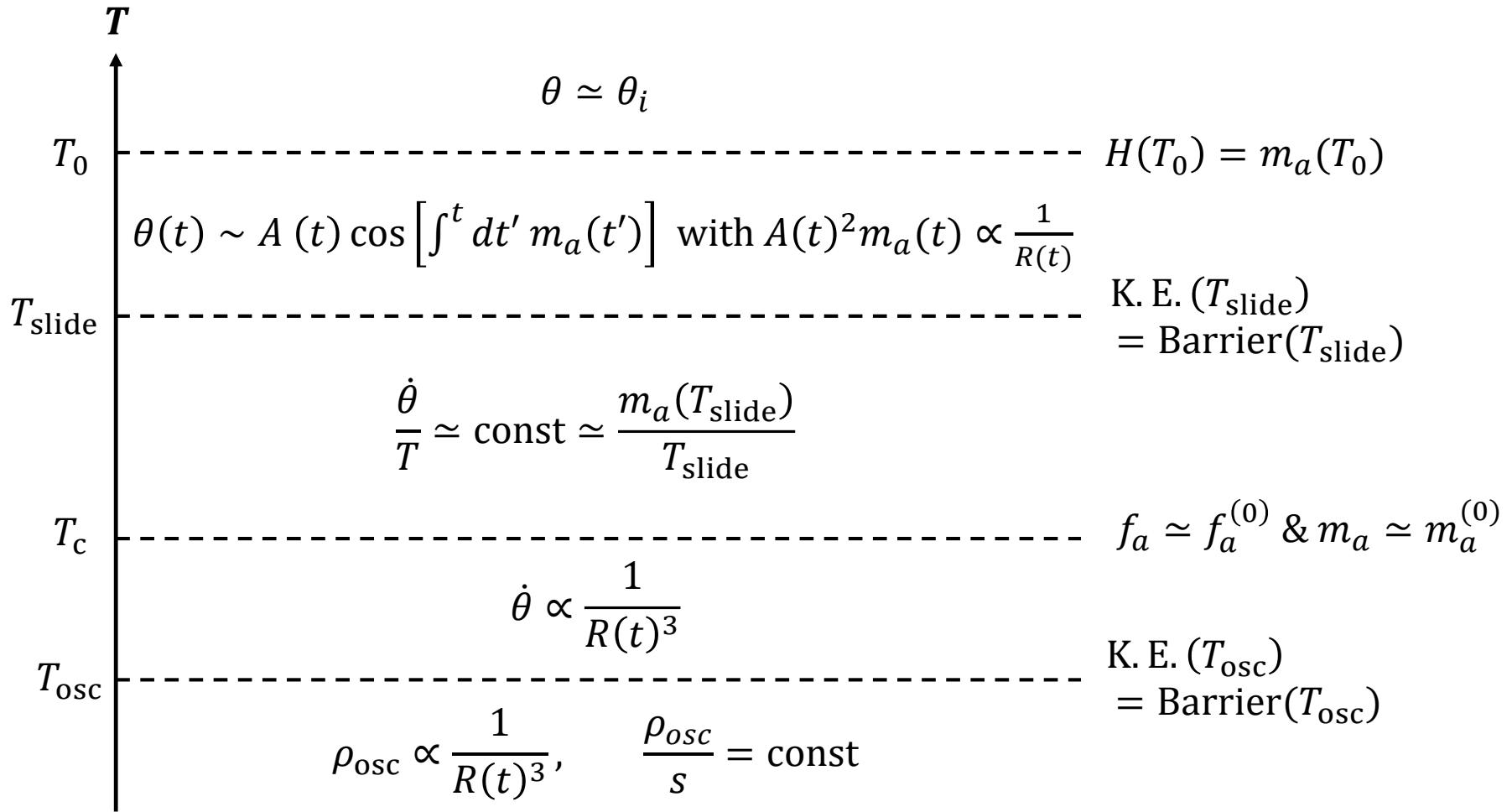
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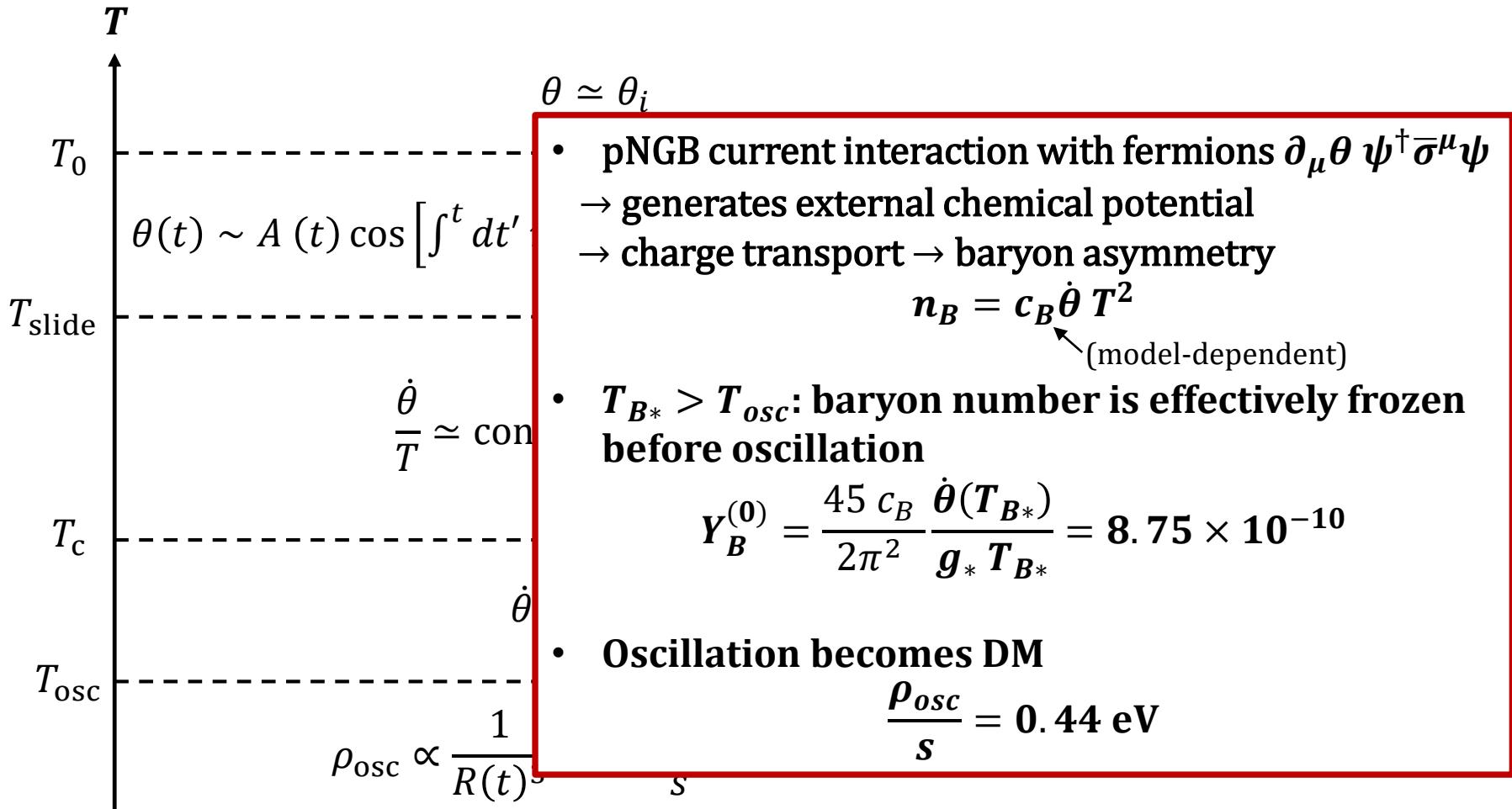
Summary of pNGB dynamics

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Summary of pNGB dynamics

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Required conditions in the scalar sector

- **Conditions for ϕ**
 - ✓ Scalar field that gives $-|D| T^2 \phi^2$ correction must be in the thermal bath.
(Higgs or singlet scalars)
 - ✓ ϕ should be in the thermal bath \rightarrow its relic abundance must decay before BBN.
 \rightarrow mixing with the Higgs boson with $m_\phi^{(0)} > 2m_e \sim \text{MeV}$ and $\sin \theta_{h\phi} > 10^{-5}$
 - ✓ ϕ must follow the potential minimum
 - Time scale of ϕ dynamics is much shorter than others: $m_\phi(T) \gg m_a(T), H$
 - Thermal friction \rightarrow sufficiently large damping can be provided.
- **Conditions for a**
 - ✓ “Particle” pNGB should not be produced (hot DM component).
Estimation of freeze-in process of $\phi\phi \rightarrow a a \Rightarrow c_\lambda > 10^7$.
 - ✓ Lifetime of pNGB “DM” must be large enough. Depending on decay channel, there are several constraints.

Type-I seesaw with Majoron

- Interaction lagrangian in the lepton sector

$$\begin{aligned}
 -\Delta L &= \left(\frac{1}{2} y \Phi \nu^c \nu^c + Y_D H l \nu^c + \text{h. c.} \right) \\
 &\rightarrow \left(\frac{1}{2} M_N \nu^c \nu^c + Y_D H l \nu^c + \text{h. c.} \right) + \frac{1}{2} \partial_\mu \theta J_{B-L}^\mu \\
 \psi &\rightarrow e^{\frac{Q_{B-L}}{2} i \theta} \psi
 \end{aligned}$$

\downarrow
 generates external
 chemical potential $\propto \dot{\theta}$

$$y \langle |\Phi| \rangle = M_N,$$

$a = p\text{NGB}$
 from spontaneous
 $U(1)_{B-L}$ breaking
=Majoron

- $B - L$ changing process is required: any process involving M_N ,**
 e.g. inverse decay of ν^c , ...

$$\begin{aligned}
 \dot{n}_B + 3H n_B &= -\Gamma_B (n_B - c_B \dot{\theta} T^2) \\
 \Gamma_B &= \min(\Gamma_{EW}, \Gamma_{M_N})
 \end{aligned}$$

- Inverse decay of ν^c is active for**

$$\frac{M_N}{7} \lesssim T \lesssim 10M_N$$

$$\text{in our case } M_N = \frac{1}{\sqrt{2}} y f_a(T) = \frac{1}{\sqrt{2}} y c_\lambda^{1/2} T \text{ for } T > T_c$$

$$\Rightarrow \text{active at } T > T_c \text{ if } \frac{\sqrt{2}}{10 \sqrt{c_\lambda}} \lesssim y \lesssim \frac{7\sqrt{2}}{\sqrt{c_\lambda}}$$

Type-I seesaw with Majoron

$$0.1 \frac{1}{\sqrt{c_\lambda}} \lesssim y \lesssim 10 \frac{1}{\sqrt{c_\lambda}}$$

- Potential terms in the scalar sector

$$\Delta V = -2 \lambda_{h\phi} |H|^2 |\Phi|^2 - (2 \lambda_{\phi s_i} |\Phi|^2 |s_i|^2) + \dots$$

$$\rightarrow c_\lambda = \frac{\frac{4}{12} \lambda_{h\phi} + \left(\frac{N_s}{12} \lambda_{\phi s_i} \right) - \frac{1}{12} y^2}{\lambda_\phi} \equiv \frac{\lambda_{\text{mix}}}{3\lambda_\phi}$$

if needed

- We ensure that **y does not spoil symmetry non-restoration**,
i.e. no tuning of (large number) – (large number) is required.

$$\lambda_\phi \sim \lambda_{\text{mix}}^2, \quad \lambda_{h\phi} \sim \lambda_{\phi s_i} \sim \lambda_{\text{mix}}, \quad y \lesssim \sqrt{\lambda_{\text{mix}}}$$

- Baryon number is frozen around $T_{EW} \simeq 130 \text{ GeV}$**
when electroweak sphaleron is decoupled.

Type-I seesaw with Majoron

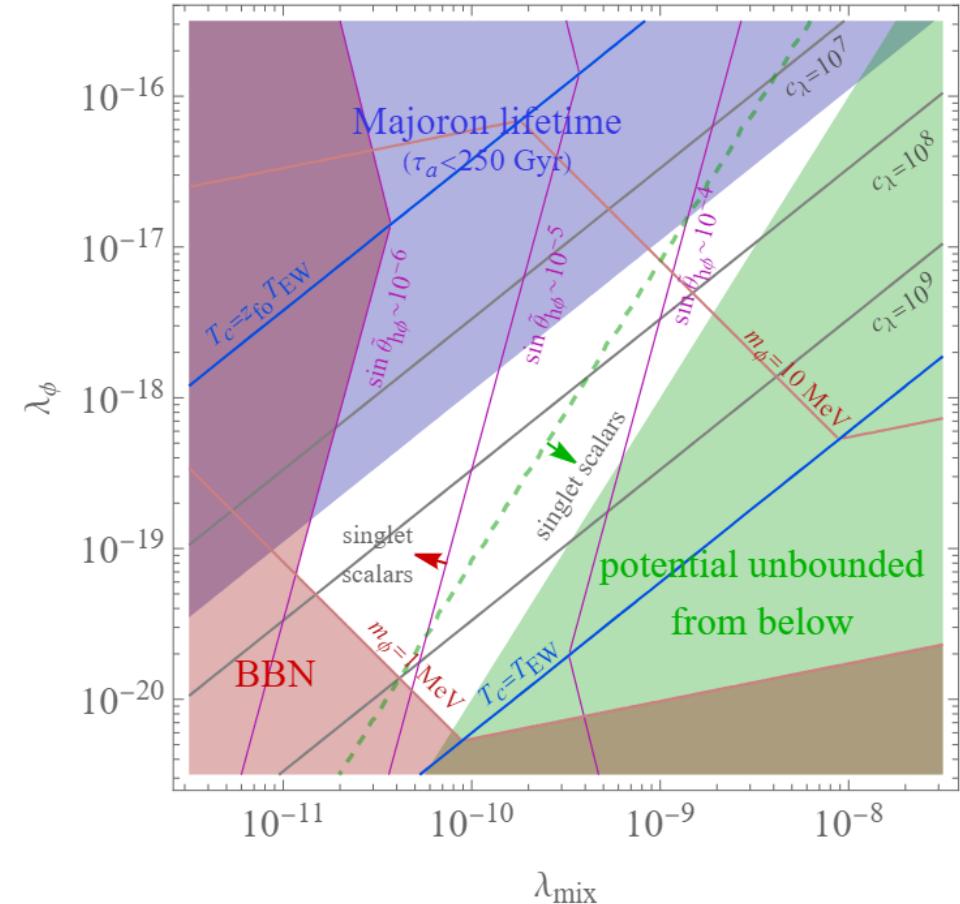
- Free parameters: $\lambda_{\text{mix}}, \lambda_\phi, m_a^{(0)}, f_a^{(0)}, (g_*, y, \theta_i), (T_{RH})$ $\left\{ \begin{array}{l} g_* \sim 100 \\ 0.1 \frac{1}{\sqrt{c_\lambda}} \lesssim y \lesssim 7 \frac{1}{\sqrt{c_\lambda}} \\ 5\theta_i \sim O(1) \\ T_{RH} > T_0 \end{array} \right.$
- Two conditions for cogensis: DM abundance and Y_B

$$\boxed{\begin{aligned} m_a^{(0)} &\simeq 5 \text{ eV} \left(\frac{10^8}{c_\lambda}\right)^{5/9} \\ f_a^{(0)} &\simeq 3 \times 10^6 \text{ GeV} \left(\frac{10^8}{c_\lambda}\right)^{5/18} \end{aligned}}$$

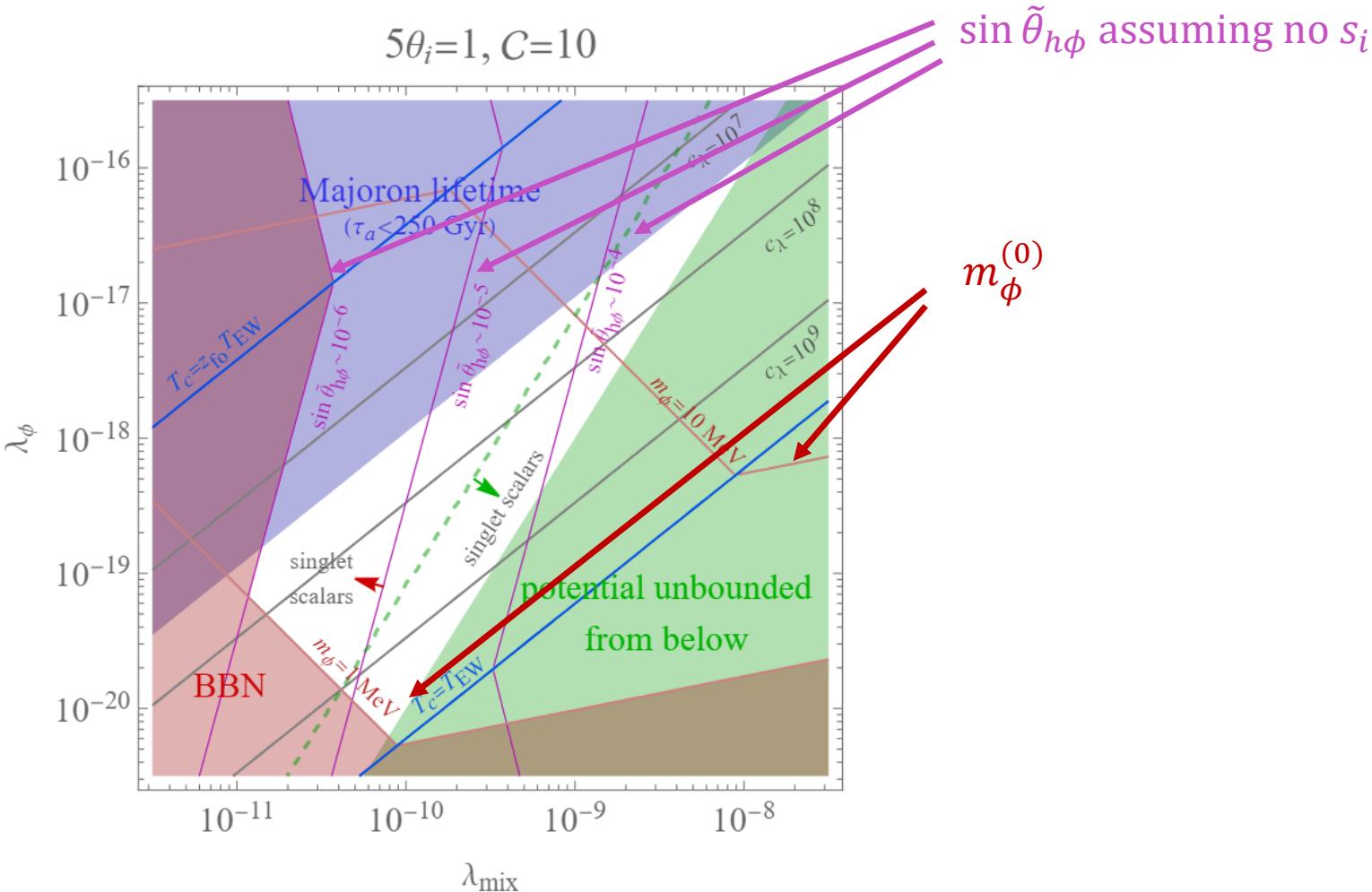
$$c_\lambda = \frac{\lambda_{\text{mix}}}{3\lambda_\phi}$$

Viable parameter space

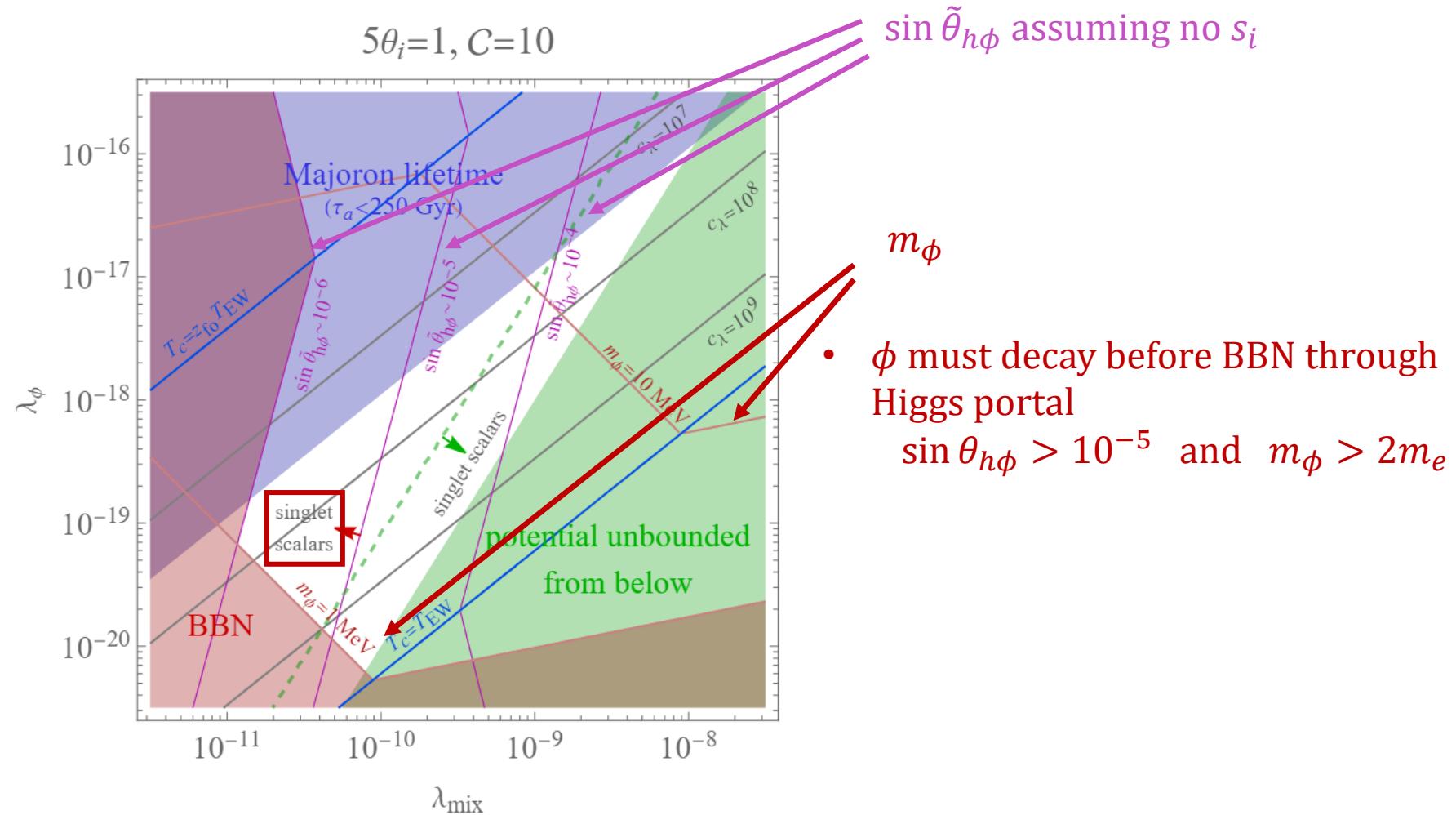
$$5\theta_i=1, C=10$$



Viable parameter space

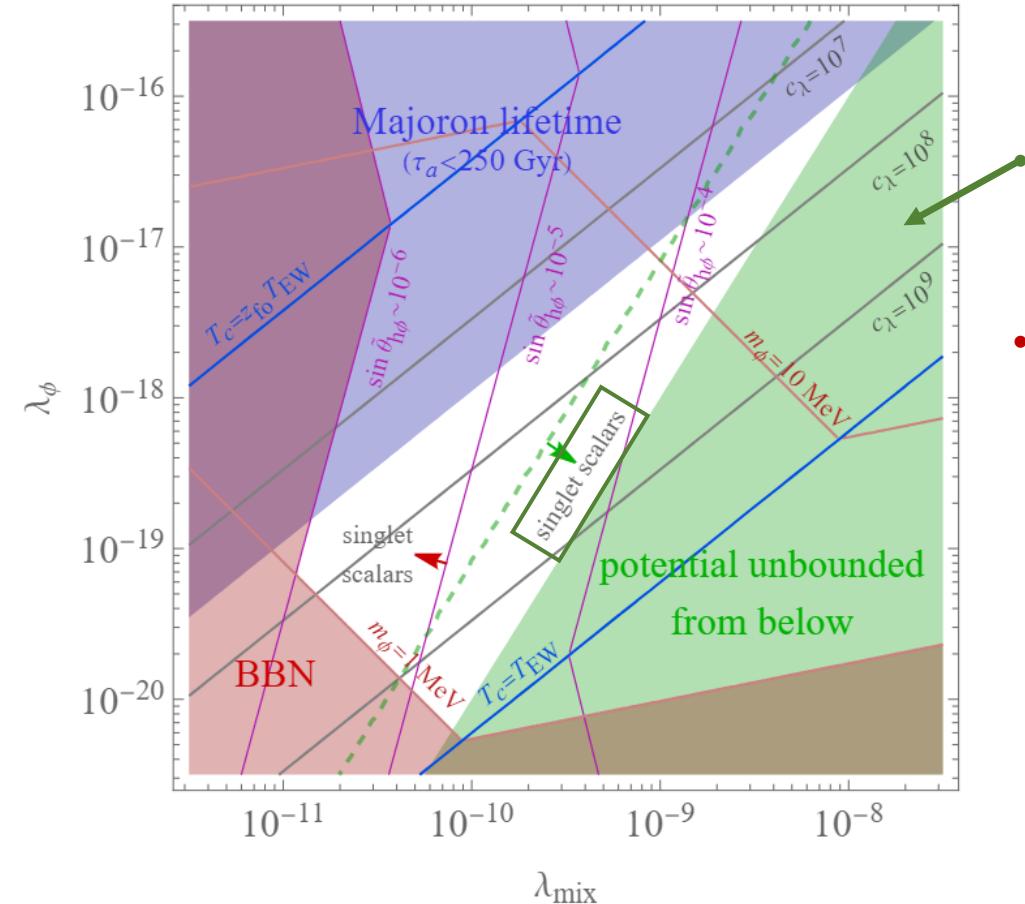


Viable parameter space



Viable parameter space

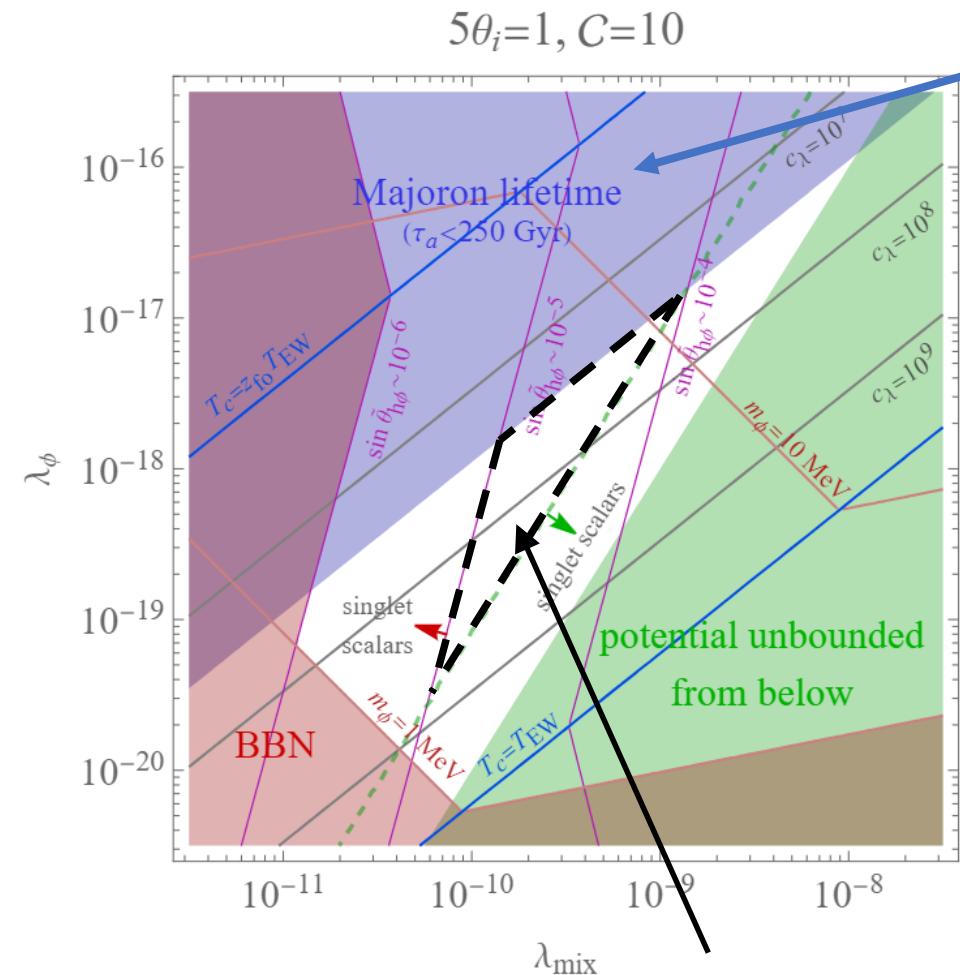
$$5\theta_i=1, C=10$$



Potential unbounded from below
 $\lambda_\phi \lambda_h < \lambda_{\text{mix}}^2$ or $\lambda_\phi \lambda_s < \lambda_{\text{mix}}^2$

- ϕ must decay before BBN through Higgs portal
 $\sin \theta_{h\phi} > 10^{-5}$ and $m_\phi > 2m_e$

Viable parameter space

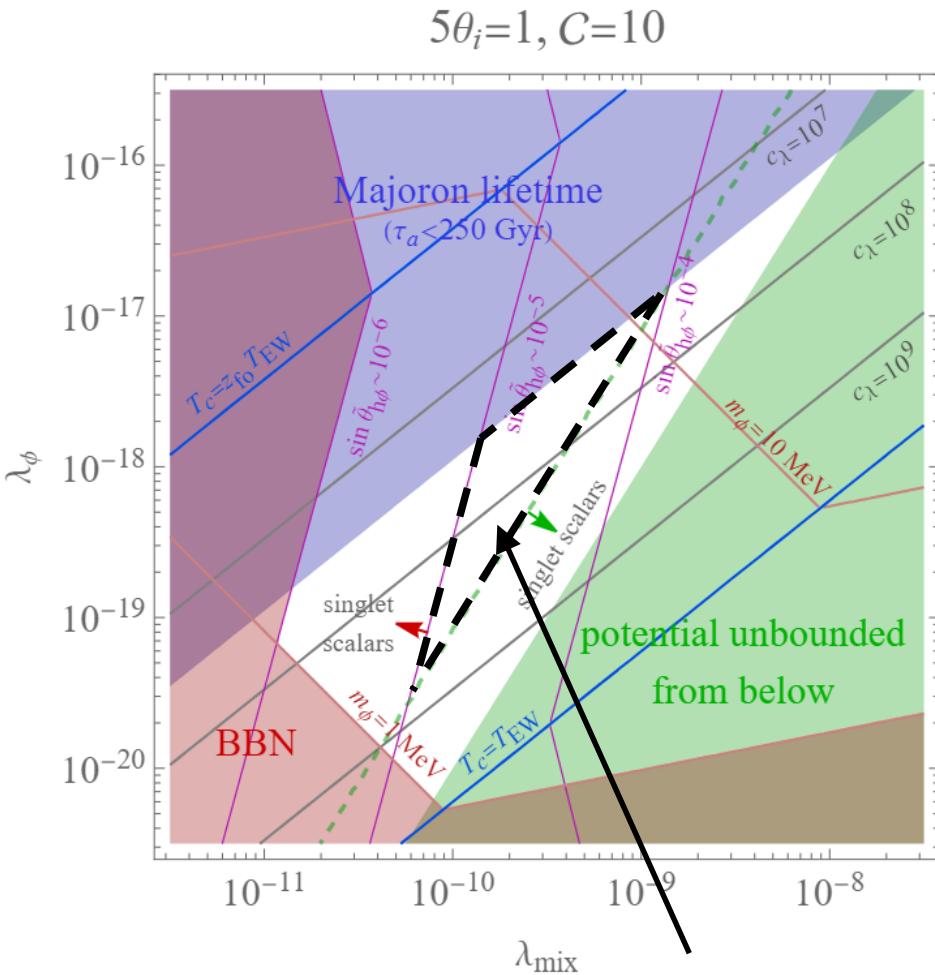


a Lifetime constraint from CMB & BAO
 $\tau_a > 250$ Gyr

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Minimal Higgs portal works
without singlet scalars

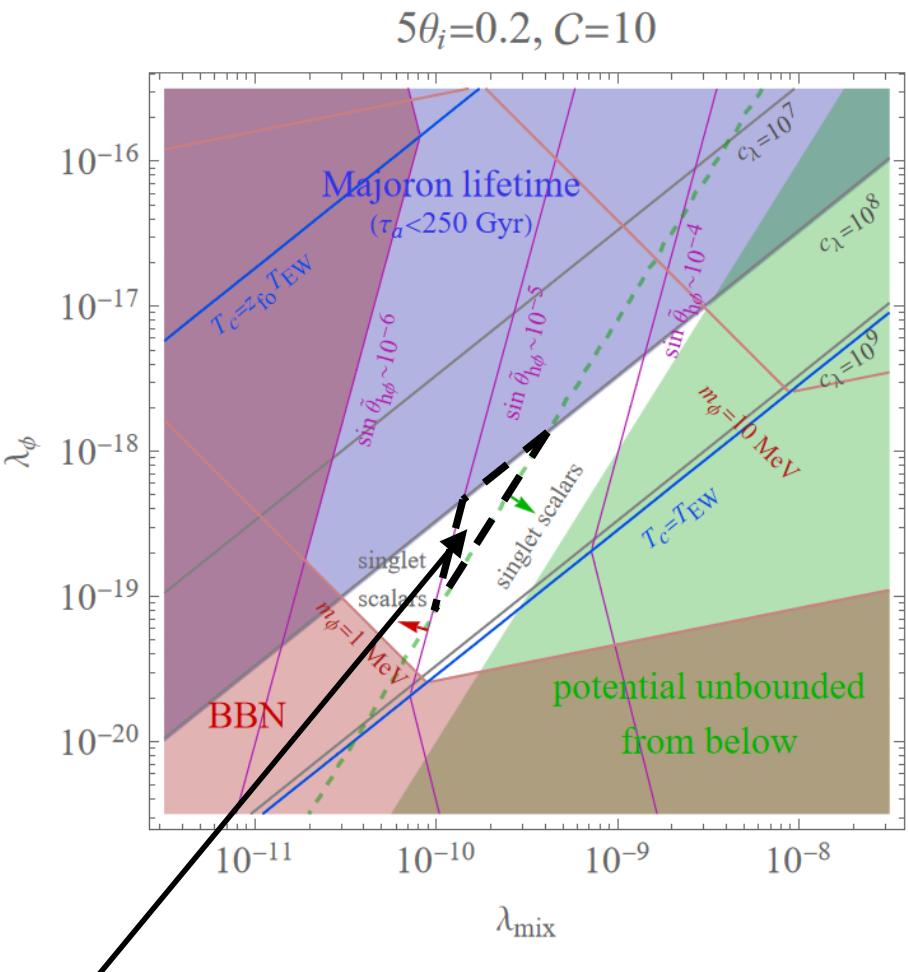
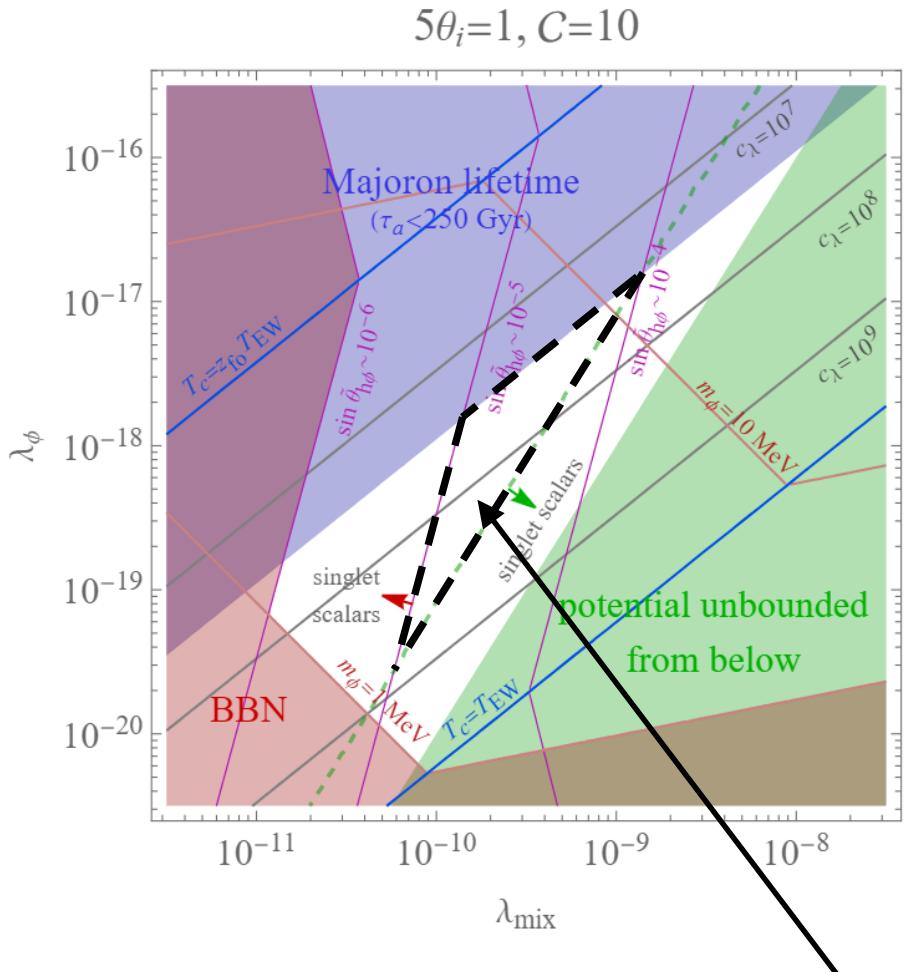
Viable parameter space



Minimal Higgs portal works
without singlet scalars

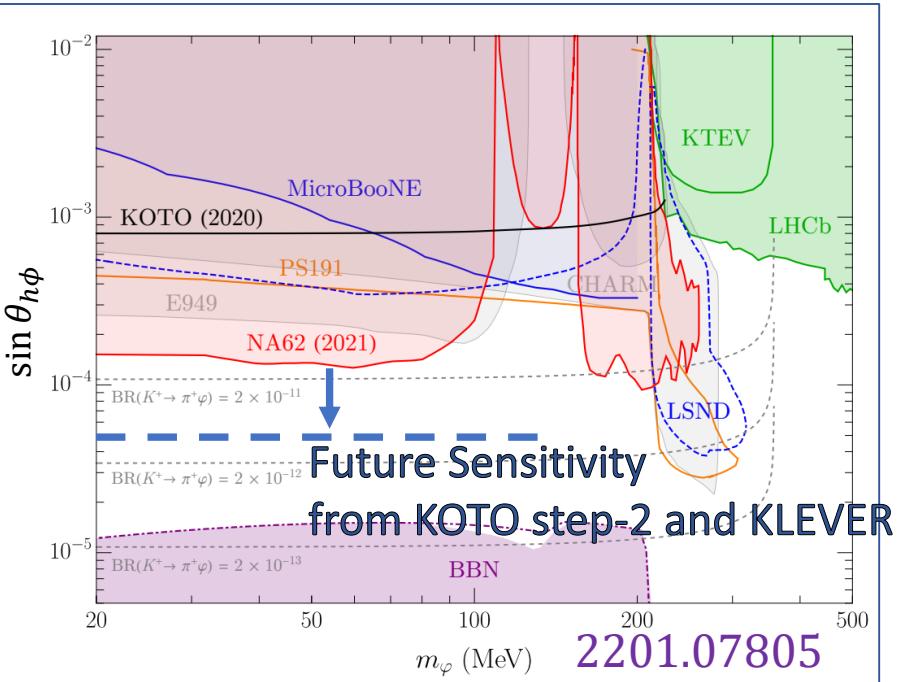
- *a* Lifetime constraint from CMB & BAO
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- Potential unbounded from below
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- ϕ must decay before BBN through Higgs portal
 $\sin \theta_{h\phi} > 10^{-5}$ and $m_\phi > 2m_e$
- Singlet scalars are expected to be around EW scale to avoid tuning.
- $T_c \sim T_{EW}$ (accidentally!)
 $\rightarrow M_N^{(0)} \sim 100$ GeV

Viable parameter space

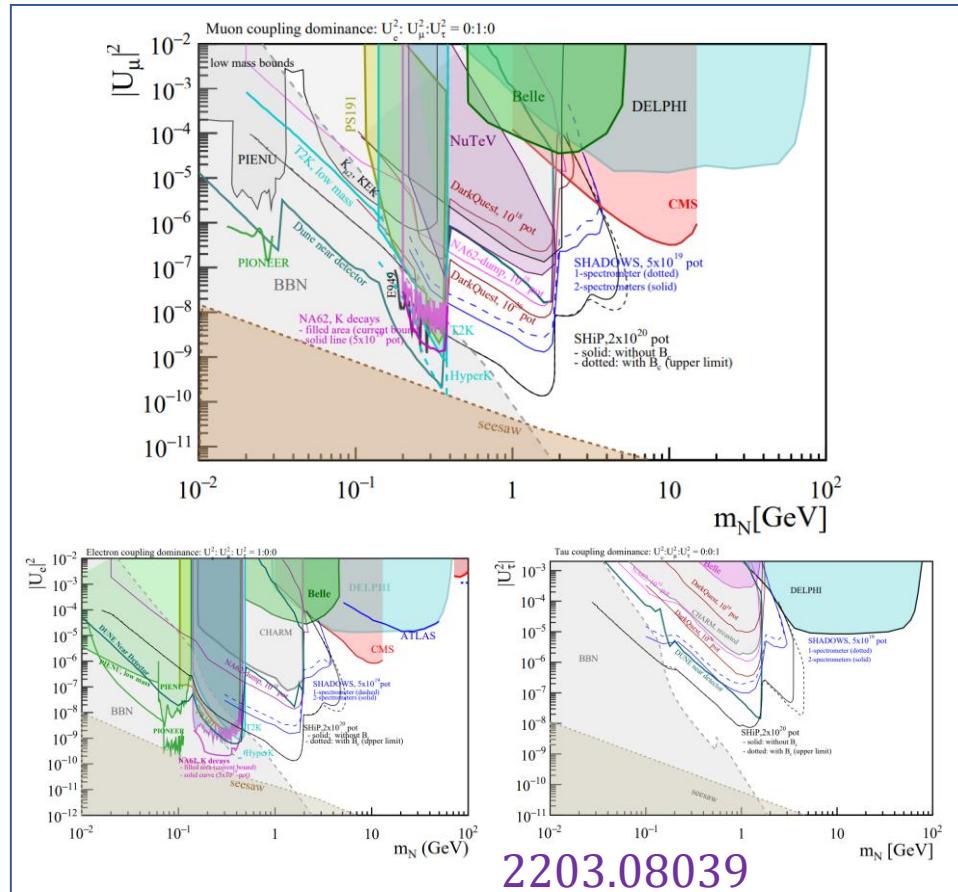


Future sensitivity

Radial mode ϕ search ($\text{MeV} < m_\phi < 20\text{MeV}$)

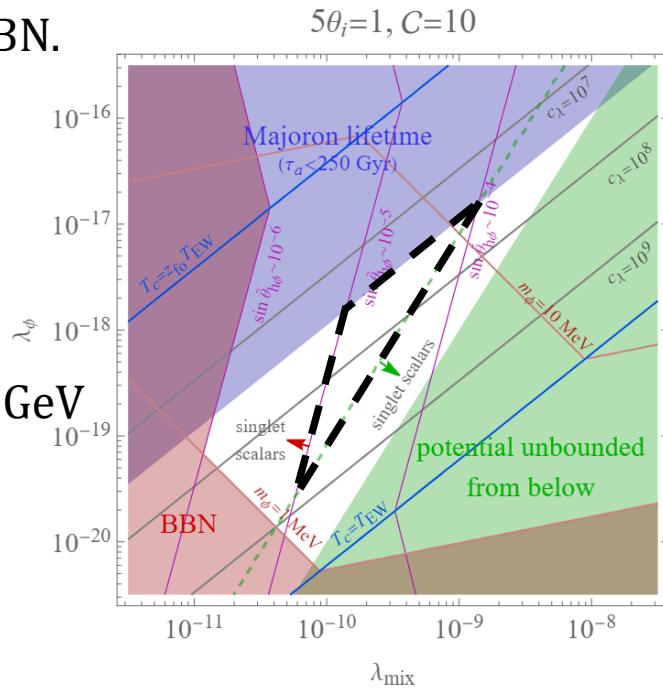


Heavy Neutral Lepton search heaviest $M_N^{(0)} \sim T_c \sim 100 \text{ GeV}$



Summary

- (Global symmetry non-restoration) + (dim-5 explicit br operator) \rightarrow pNGB sliding
- Cogenesis is possible when $m_a^{(0)} \sim 5$ eV and $f_a^{(0)} \sim 3 \times 10^6$ GeV.
- For the Majoron case, we find a viable parameter space.
 - ✓ $\sin \theta_{h\phi} > 10^{-5}$ & $m_\phi > 2m_e$ is needed to avoid BBN.
 → Kaon experiments may test it.
 - ✓ $M_N^{(0)} \lesssim 100$ GeV for the leptogenesis
 → HNL searches (beamdump, LHC, ...)
 - ✓ Singlet scalars in non-minimal model around 100 GeV
 → LHC and future colliders may test it.



Thank you!

Thank you!

Lepton asymmetry generation

$$\begin{aligned}
 \dot{n}_{l_\alpha} + 3Hn_{l_\alpha} &= \frac{n_N}{n_N^{(eq)}} \Gamma^{(eq)}(N \rightarrow l_\alpha H) - \frac{n_{l_\alpha} n_H}{n_{l_\alpha}^{(eq)} n_H^{(eq)}} \Gamma^{(eq)}(l_\alpha H \rightarrow N) \\
 &\quad \text{||} \\
 - \left. \right) \dot{n}_{\bar{l}_\alpha} + 3Hn_{\bar{l}_\alpha} &= \frac{n_N}{n_N^{(eq)}} \Gamma^{(eq)}(N \rightarrow \bar{l}_\alpha \bar{H}) - \frac{n_{\bar{l}_\alpha} n_{\bar{H}}}{n_{\bar{l}_\alpha}^{(eq)} n_{\bar{H}}^{(eq)}} \Gamma^{(eq)}(\bar{l}_\alpha \bar{H} \rightarrow N) \\
 \hline
 \dot{n}_{\Delta l_\alpha} + 3Hn_{\Delta l_\alpha} &= \begin{array}{c} O(\epsilon_{CP}) \\ \text{Source term} \\ \text{in conventional} \\ \text{thermal leptogenesis} \end{array} - \left(\frac{n_{\Delta l_\alpha}}{n_{l_\alpha}^{(eq)}} + \frac{n_{\Delta H}}{n_H^{(eq)}} \right) \Gamma^{(eq)}(l_\alpha H \rightarrow N) \\
 &\quad \text{Wash-out term} \\
 &\quad \downarrow \dot{\theta} \neq 0: \mu_i \rightarrow \mu_i - \frac{(B-L)_i}{2} \dot{\theta} \\
 \dot{n}_{\Delta l_\alpha} + 3Hn_{\Delta l_\alpha} &= - \left(\left(\frac{n_{\Delta l_\alpha}}{n_{l_\alpha}^{(eq)}} - \frac{\dot{\theta}}{T} \right) + \frac{n_{\Delta H}}{n_H^{(eq)}} \right) \Gamma^{(eq)}(l_\alpha H \rightarrow N) \\
 &\quad \text{Source term: wash-in mechanism}
 \end{aligned}$$

pNGB dynamics

When $5\theta_i \sim 0.2$

$$\ddot{\theta} + \left(3H + 2 \frac{\dot{f}_a}{f_a} \right) \dot{\theta} = -\frac{1}{5} m_a^2(T) \sin 5\theta$$

$\simeq H$ for $T > T_c$

