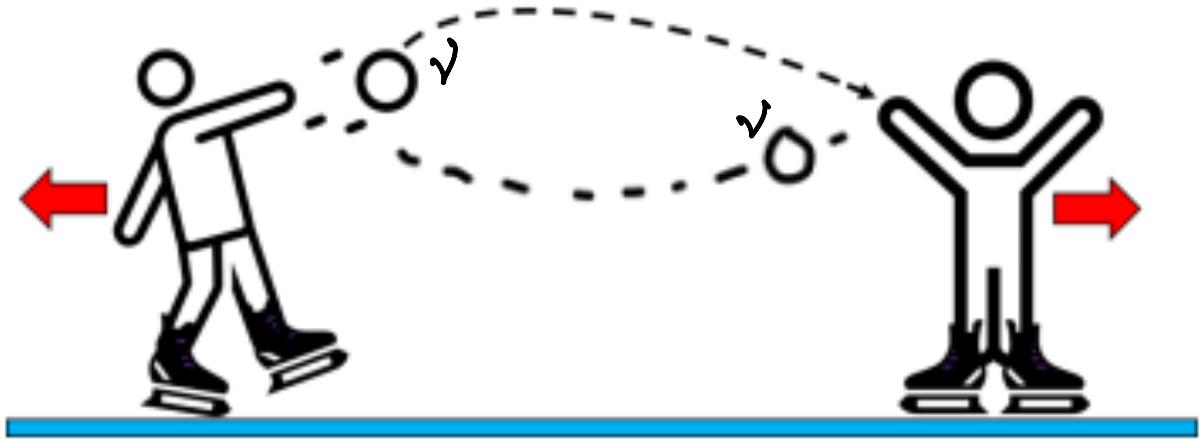


Two neutrino forces

\- *And where to find them*

Mitrajyoti Ghosh
Florida State University
JUST Particle Physics Workshop,
National Taiwan University, June 2024

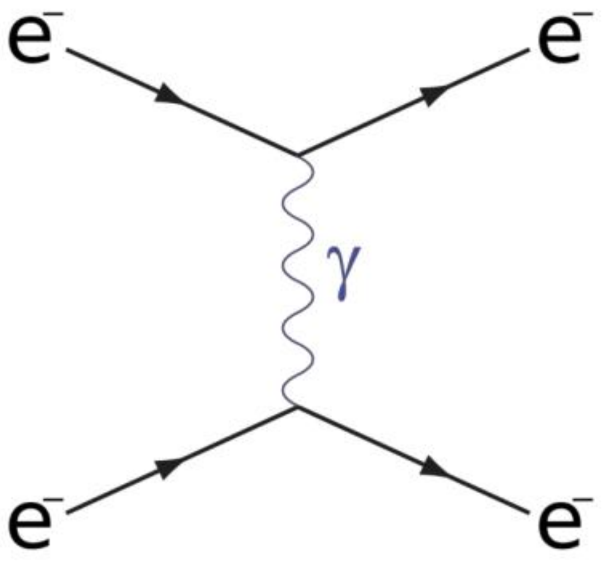
Forces from virtual particle exchange

In the non-relativistic limit, you can compute a scattering amplitude using a static potential.

Conversely, you can find the "force law" from the scattering amplitude.

$$i\mathcal{M} \sim \int \mathrm{d}^3\mathbf{r} \;\; V(\mathbf{r}) \; e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$V(\mathbf{r}) \sim i \int \mathrm{d}^3\mathbf{q} \;\; \mathcal{M} \; e^{-i\mathbf{q}\cdot\mathbf{r}}$$



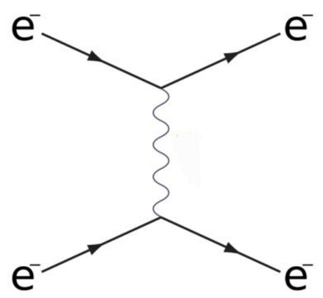
The Coulomb Force for example:

$$\mathcal{M} \sim \frac{1}{\mathbf{q}^2}$$

$$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \;\; \frac{1}{\mathbf{q}^2} \; e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{1}{r}$$

Gaining weight

With massive propagators, the force falls off exponentially with the mass of the propagator.



$$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \left(\frac{1}{\mathbf{q}^2 + m^2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-mr}}{r}$$

Heavier the mediator, less likely to exchange it at larger distances, smaller the force. Define RANGE $\sim 1/m$.

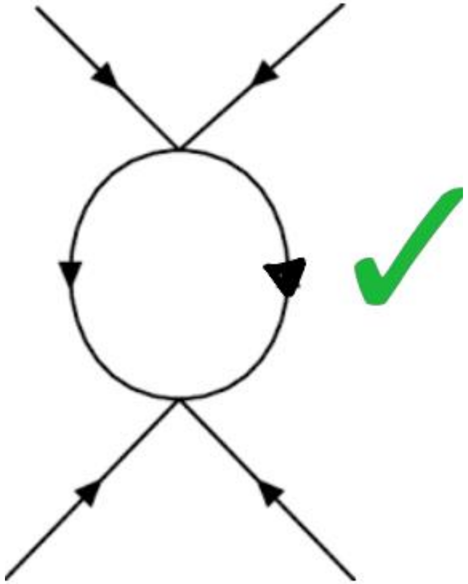
This is why the Weak force is "weak", not because the couplings are small, but because the mediators are ***HEAVY***.

Two Fermion Quantum Forces

Single fermion exchange cannot give rise to a classical force at tree level

But you can have one-loop 'quantum' effects with a fermion loop.

TWO-Fermion exchange force



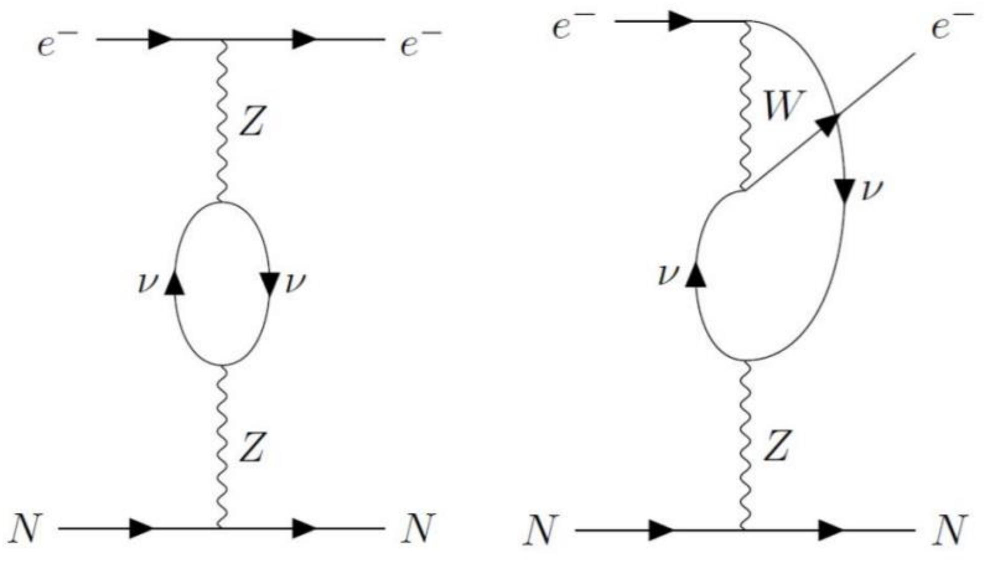
The 'special' case of two-neutrino exchange

Why special?

The neutrino force is longest-ranged 2-fermion force since neutrinos are the lightest fermions in the SM.

Why try to see it?

- *ALREADY exists, not New Physics!*
- *Sensitive to neutrino masses and the nature of neutrinos (Dirac or Majorana) and neutrino mixing parameters (PMNS matrix elements etc)*



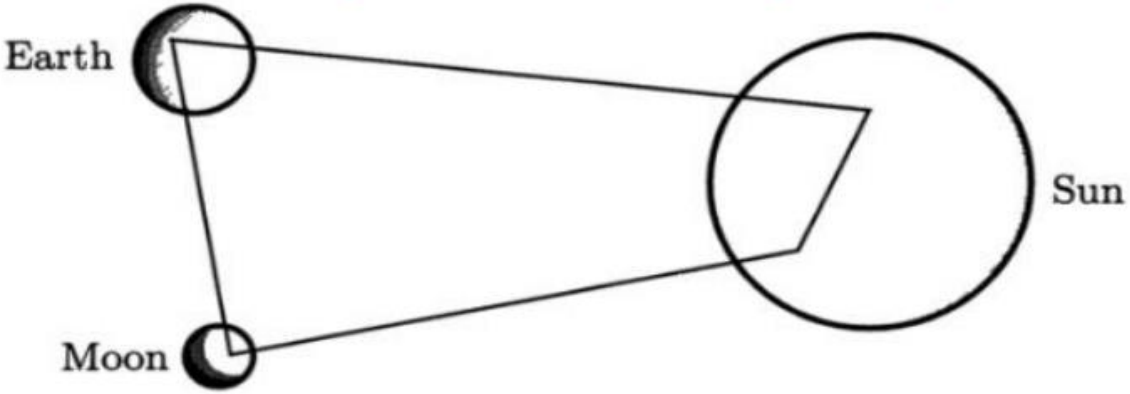
Note: These diagrams contribute to forces that are both short-ranged and long-ranged forces. Short ranged forces have a range $\sim 1/m_Z$, while long ranged forces have range $\sim 1/m_\nu$.

Some history

The neutrino-force as an explanation for gravity??

FEYNMAN LECTURES ON GRAVITATION

RICHARD P. FEYNMAN



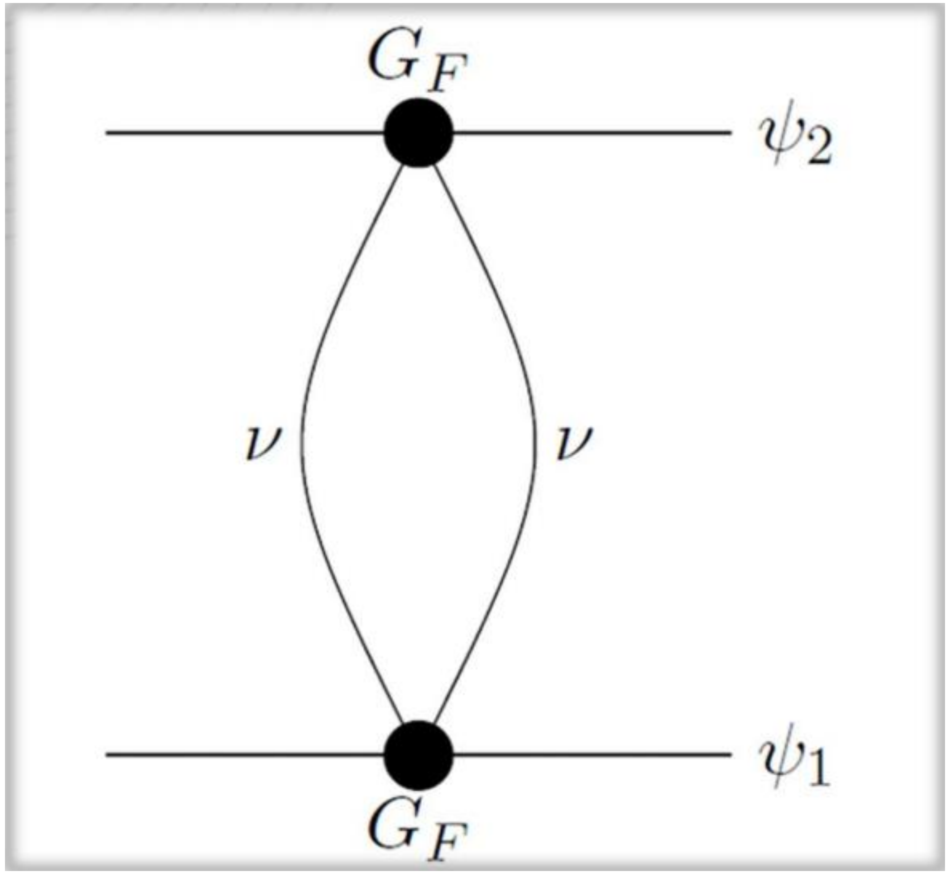
$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \qquad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi\rho(R) R^2 dR}{2R^3}, \qquad (2.4.5)$$

2-Neutrino Force v1.0 (1964 edition)



The force between two weakly charged fermions due to neutrino exchange was computed in the 60's in the 4-Fermi approximation (See Feinberg, Sucher, 1964):

$$V(r) = \frac{G_F^2}{4\pi^3 r^5}$$

Neutrino mass was incorporated later in the 90's (Masso, Grifols, Toldra, 1996)

$$V_{\nu\nu}^{\text{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3} \frac{K_3(2m_\nu r)}{r^2}, \qquad V_{\nu\nu}^{\text{Majorana}}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3} \frac{K_2(2m_\nu r)}{r^3}$$

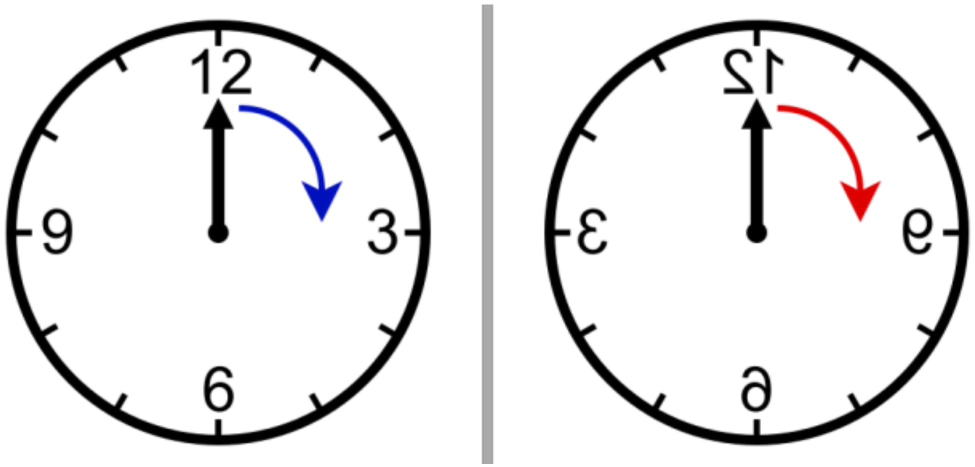
Suppressed by two powers of Fermi's constant
TOO SMALL TO SEE! HOPELESS!

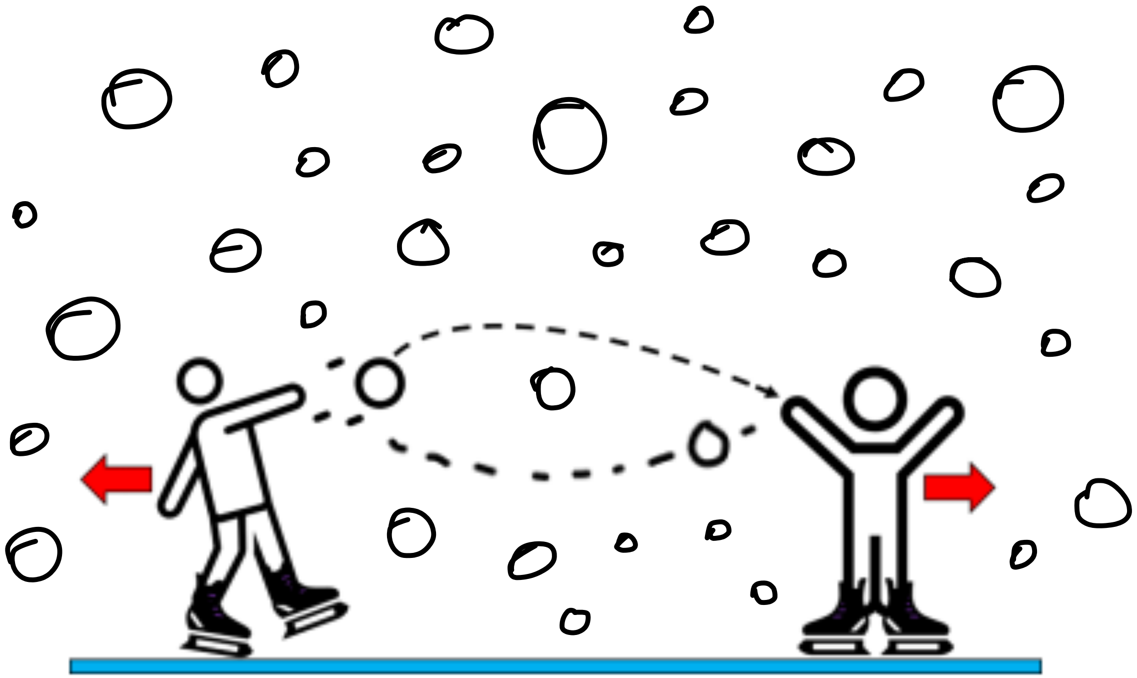
BUT THIS IS NOT A COMPLETE CALCULATION. More to the story!

BUT...

Do neutrino forces have any other properties that are not captured in the radial dependence alone?

Yes! They violate parity. Can use this to our advantage!





Can they be enhanced somehow?

Yes! Can try to use a neutrino background.

Plan for today

Atomic Parity Violation

- *Parity violation in atoms via neutrino forces*
- *How to see APV*
- *Results in hydrogen (because anything else too hard for me!)*

Based on MG, Grossman, Tangarife, 1912.09444

Enhancement via Backgrounds

- *What backgrounds do?*
- *What kind of backgrounds work best?*
- *Realistic? Feasible?*

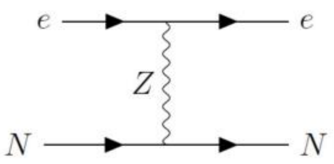
Based on MG, Grossman, Tangarife, Xu, Yu 2209.07082. See 2405.16801 for more discussion.

Atomic Parity Violation

Why APV?

Q: Why do we need atomic systems? Can't we do scattering instead?

A: Tree level parity violation will dominate. We really want to exploit the long-ranged nature of the neutrino force



The How

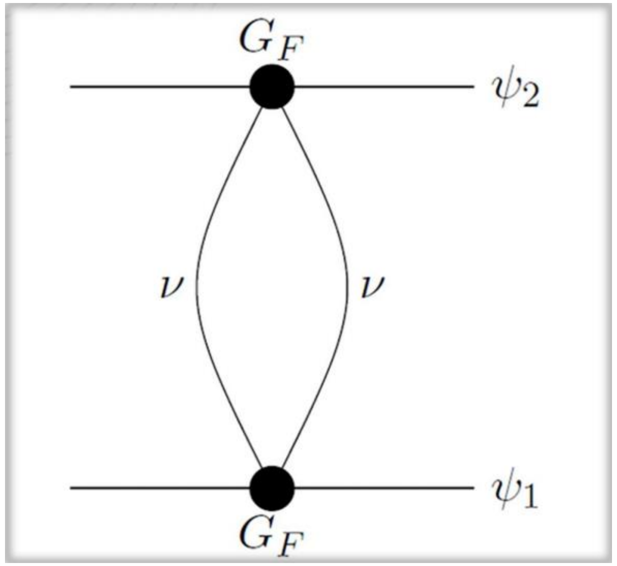
Q: So, what do we want to calculate/measure?

A: Essentially, perturbations to atomic wavefunctions due to the parity violating neutrino force.

(Well yes! But first complete the old calculation from the 60's)

2-Neutrino Force v2.0 (2018 edition)

$$V_{PNC}^{\text{loop}} \approx \sum_i \frac{G_A}{m_e}\left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)\left[(2\vec{\sigma}_p \cdot \vec{p}_e)V_{\nu_i\nu_i}(r) + (\vec{\sigma}_e \times \vec{\sigma}_p)\cdot\vec{\nabla}V_{\nu_i\nu_i}(r)\right]$$



$$V_{\nu\nu}^{\text{Dirac}}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3}\frac{K_3(2m_\nu r)}{r^2}$$

Notes:

1. *This expression is valid for hydrogen only. Other systems will have different terms, with the general expression for the 2-fermion force being*

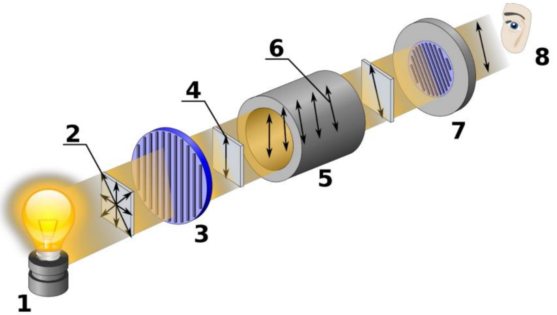
   $$V_{PNC}(r) = H_1 F(r)\vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r)\vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N)\cdot\vec{\nabla}\left[F(r)\right]$$

2. *We only consider Dirac neutrinos for now.*
3. *Parity Violation is in the spin-dependence. You can spin dependent forces that conserve parity, but you cannot have parity violation without spin-dependence.*

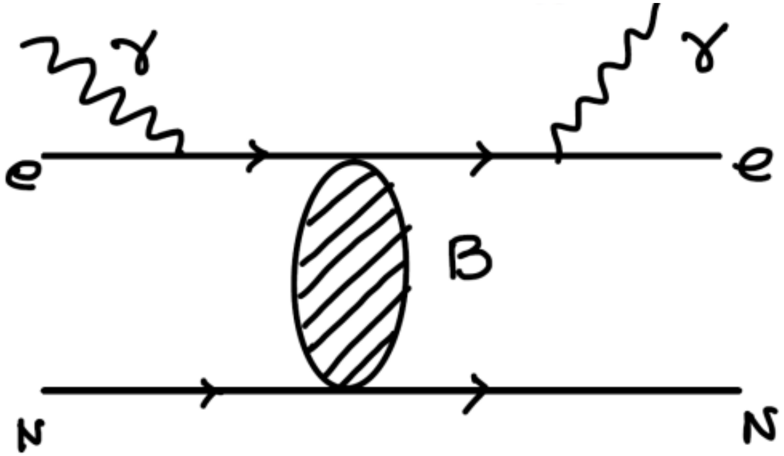
Parity Violation and Atomic birefringence

The idea

Spin-dependent parity violating forces will affect dipole transition rates in atoms. L- and R-polarized light will be treated differently by these forces, leading to birefringence.



Diagrammatically:



If B = QED stuff,

$$f_L = f_R$$

No optical rotation.

But if B = electroweak stuff,

$$f_L \neq f_R$$

Optical Activity!

Using Selection Rule Violation: an easier(?) detour

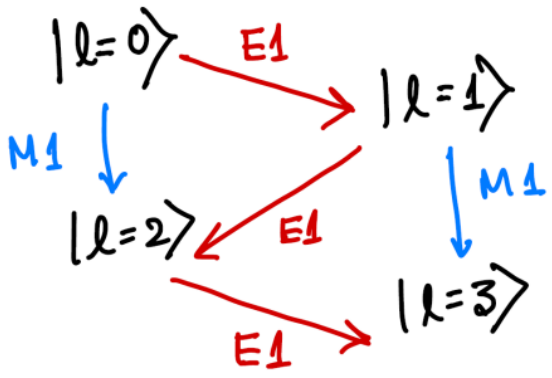
Personal taste: using Griffith's QM quicker than performing the calculation in Field Theory

$$\hat{H} = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{r} + V_{PC} + V_{PNC}$$

Neutrino stuff go here!

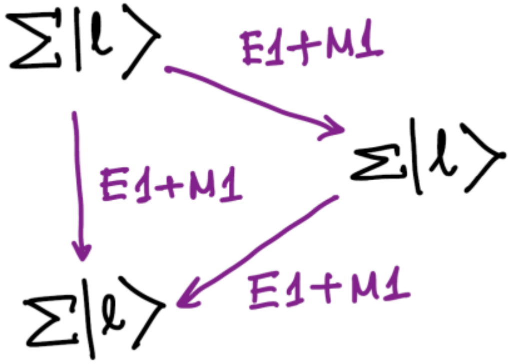
Parity Conserved

Note: Parity $= (-1)^\ell$



Selection Rules Hold!

Parity Violated



No selection rules anymore!

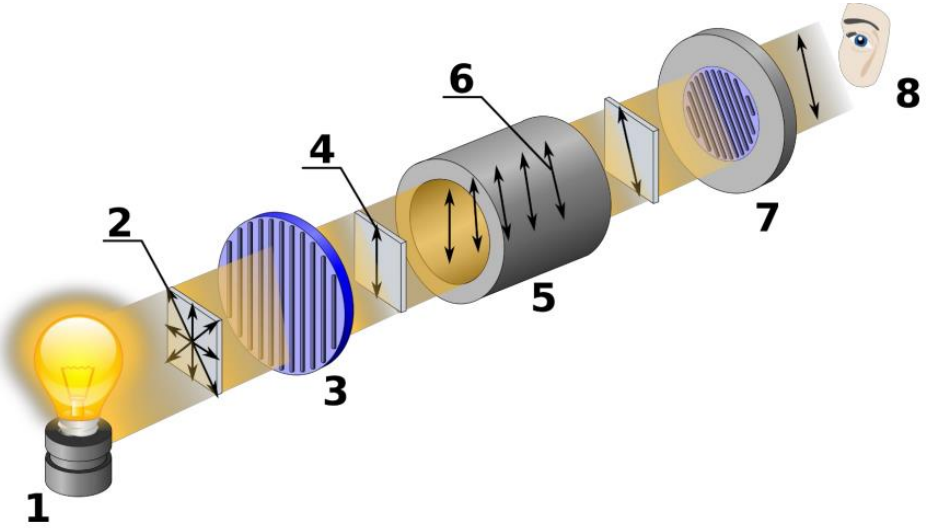
The Claim:

I can calculate birefringence effects using E1 and M1 transition rates.

Neutrino induced Optical Rotation

Rotation of plane of polarization per unit length of sample due to the neutrino force in hydrogen gas is given by:

$$\left(-\frac{1}{4}+s_W^2+\frac{1}{2}|U_{ei}|^2\right)\left(-7.7\times10^{-33}+3.7\times10^{-32}\nu_i^2\right)$$



$$\nu_i \equiv \frac{1}{\alpha}\frac{m_{\nu_i}}{m_e}$$

This is: a_0/λ_{ν_i}

a_0 = Bohr Radius of hydrogen

λ_{ν_i} = Compton wavelength of neutrino mass eigenstate i

NOTE

Cannot use states with low angular momentum (YET!), because the tree level effects dominate. We used l=2 states.

To conclude:

ATOMIC PARITY VIOLATION:

NEUTRINO INDIRECT DETECTION

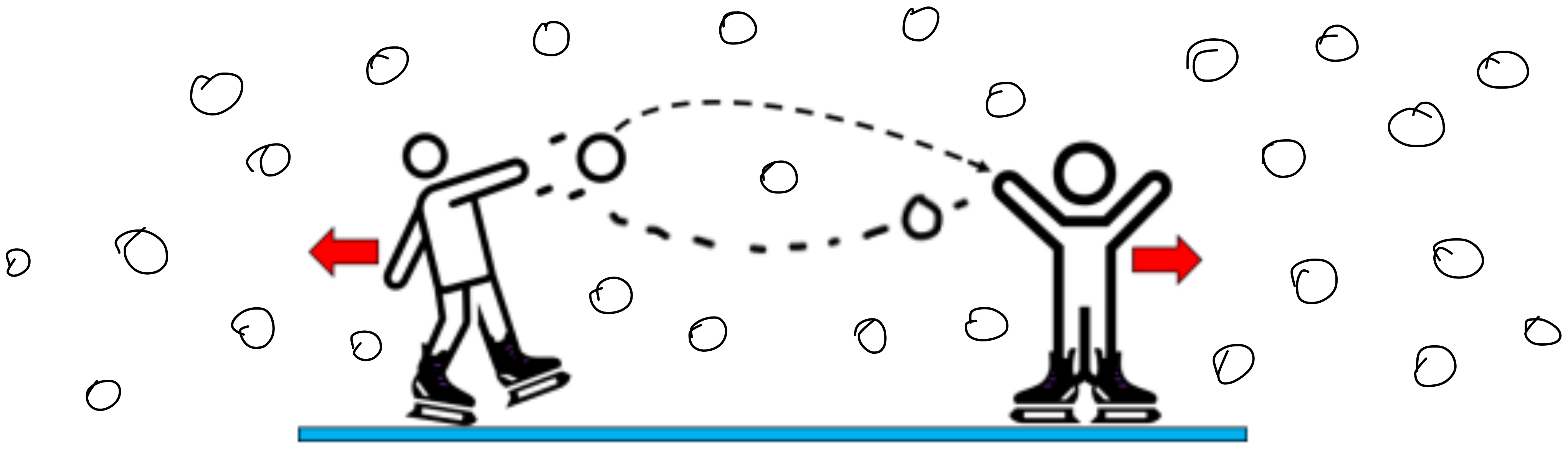
Positives

An indirect method. Does not require a neutrino beam. It is intrinsically the biggest long-range parity violating effect in the Standard Model.

Sensitive to many neutrino properties. Hydrogen samples do not probe all of these properties, but other situations are sensitive to other neutrino stats.

***Larger atoms have bigger effects**. Hard to compute by hand, but not hard on a computer.*

Negatives mostly on the experimental side. Need more brainstorming to come up with an experiment that can do this!



Enhancing the force via a neutrino background

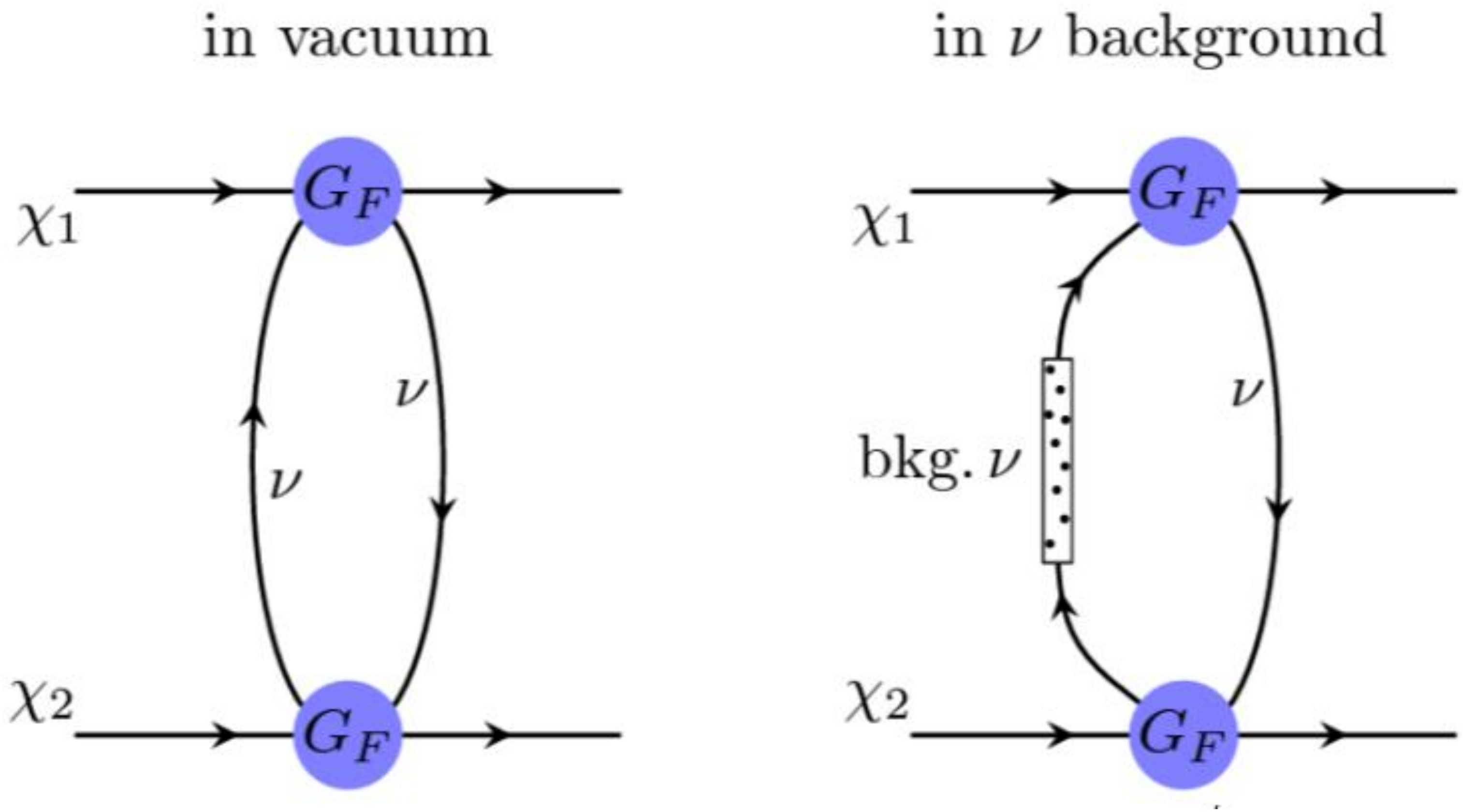
The fermion propagator in a background

$$(\not{p} + m)\left\{\frac{i}{p^2 - m^2 + i\epsilon} + (2\pi)\delta\left(p^2 - m^2\right)\left[\Theta\left(p^0\right) n_+(\mathbf{p}) + \Theta\left(-p^0\right) n_-(\mathbf{p})\right]\right\}$$

Vacuum propagator — Background correction

Additional term in the Fermion that comes with a delta function that puts the fermion on shell. Proportional to the background density of fermions.

Neutrino force in a background



The cosmic neutrino background

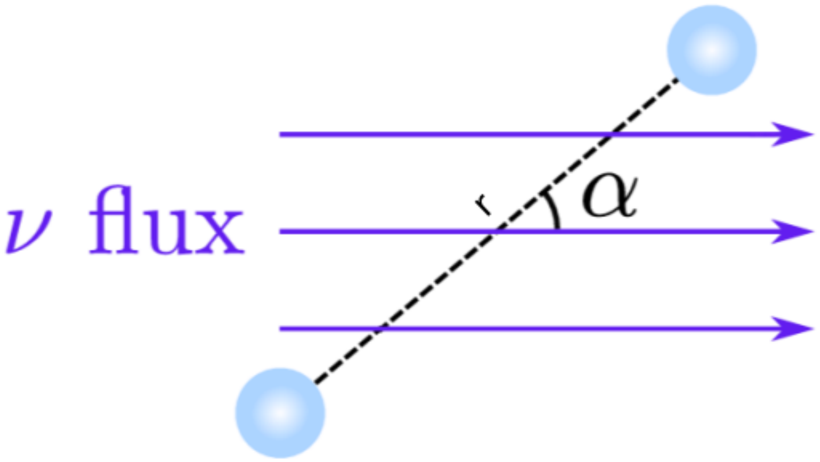
$$n_{\pm}(\mathbf{k}, T) = \exp\left[\left(\pm\mu - \kappa\right)/T\right]$$

	$r \ll T^{-1}$	$r \gg T^{-1}$
Dirac:	$-\frac{14.4}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{m_\nu T^3}{r}$	$-\frac{1}{32\pi^3} G_F^2 g_V^1 g_V^2 \frac{m_\nu}{T} \frac{1}{r^5}$
Majorana:	$-\frac{248.9}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{T^5}{m_\nu r}$	$-\frac{1}{8\pi^3} G_F^2 g_V^1 g_V^2 \frac{1}{m_\nu T} \frac{1}{r^7}$

These results are obtained for a single family neutrino when the distance between test particles $r \gg 1/m_\nu$.

They do not provide any significant enhancement to the force. The background is too diffuse.

Monochromatic directional backgrounds

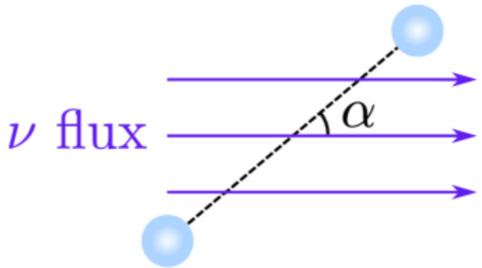


Test objects are placed in the path of a beam of neutrinos, making an angle with the beam direction.

For simplicity, we assume massless neutrinos with beam energy E_ν and flux Φ_0.

The neutrino flux from a reactor at large enough distance from the source can be approximated by this.

The leading behavior



Vectorial Couplings

Beam parameters

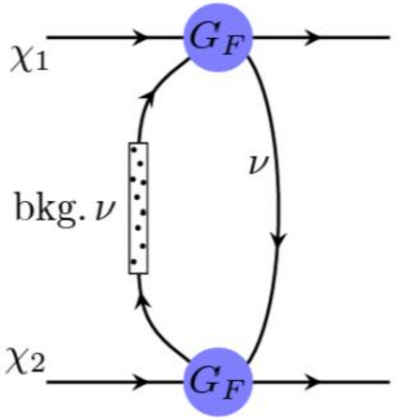
Radial dependence

Oscillations

$$V_{\rm bkg}\left(r \gg E_\nu^{-1}, \alpha \ll 1\right) = -\frac{g_V^1 g_V^2}{\pi} G_F^2 \times \Phi_0 E_\nu \times \frac{1}{r} \times \cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$

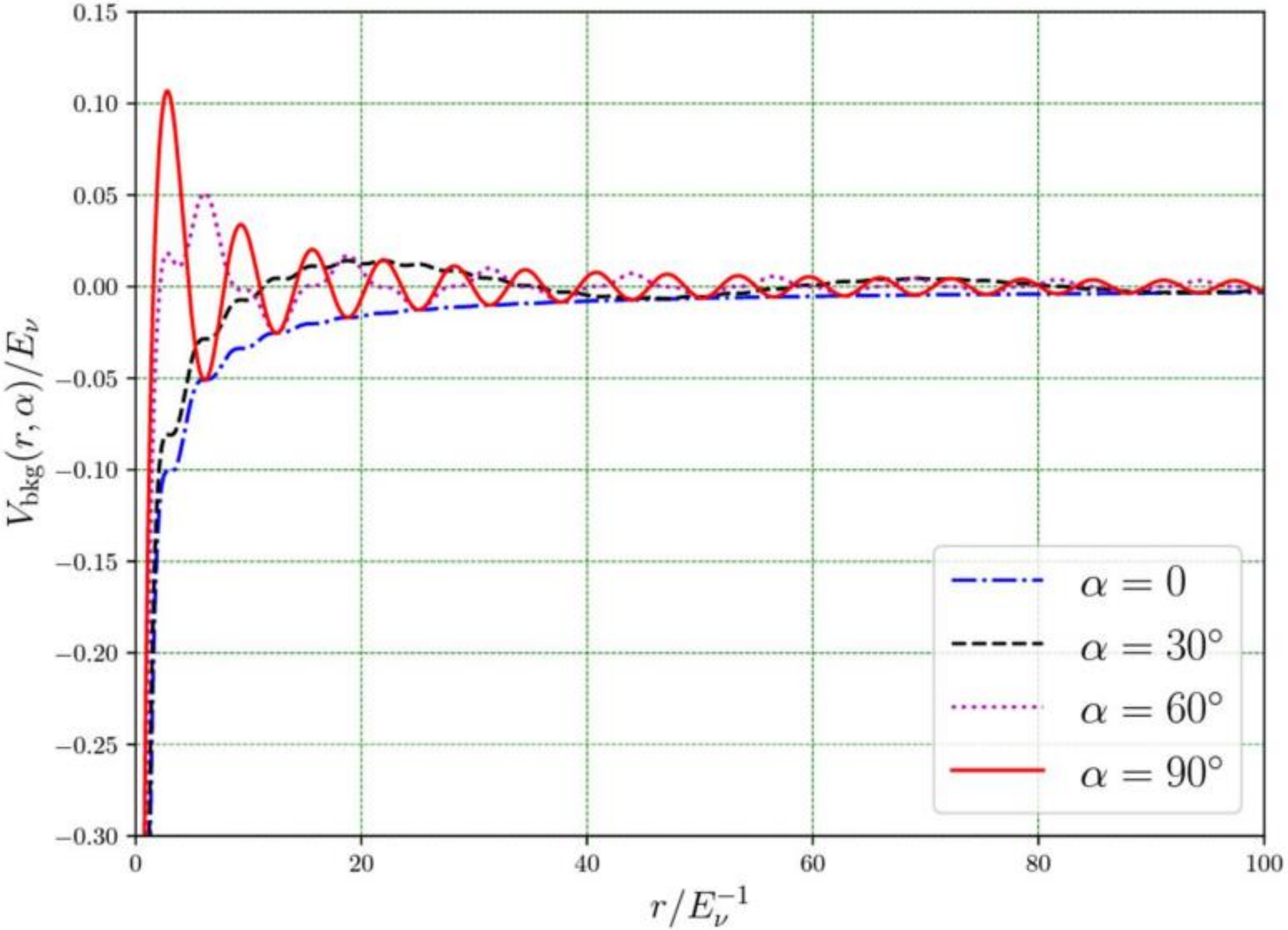
Intuitively:

Radial dependence: Background puts one of the neutrinos on shell. The force comes from a "tree" essentially.



Oscillations: effect should be maximised when the background neutrinos are in the same direction as exchanged neutrinos.

In a picture...



0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
$V_{\rm bkg}(r,\alpha)/E_\nu$
0
20
40
60
80
100
r/E_ν^{-1}
$\alpha = 0$
$\alpha = 30°$
$\alpha = 60°$
$\alpha = 90°$

The optimal case $\alpha = 0$

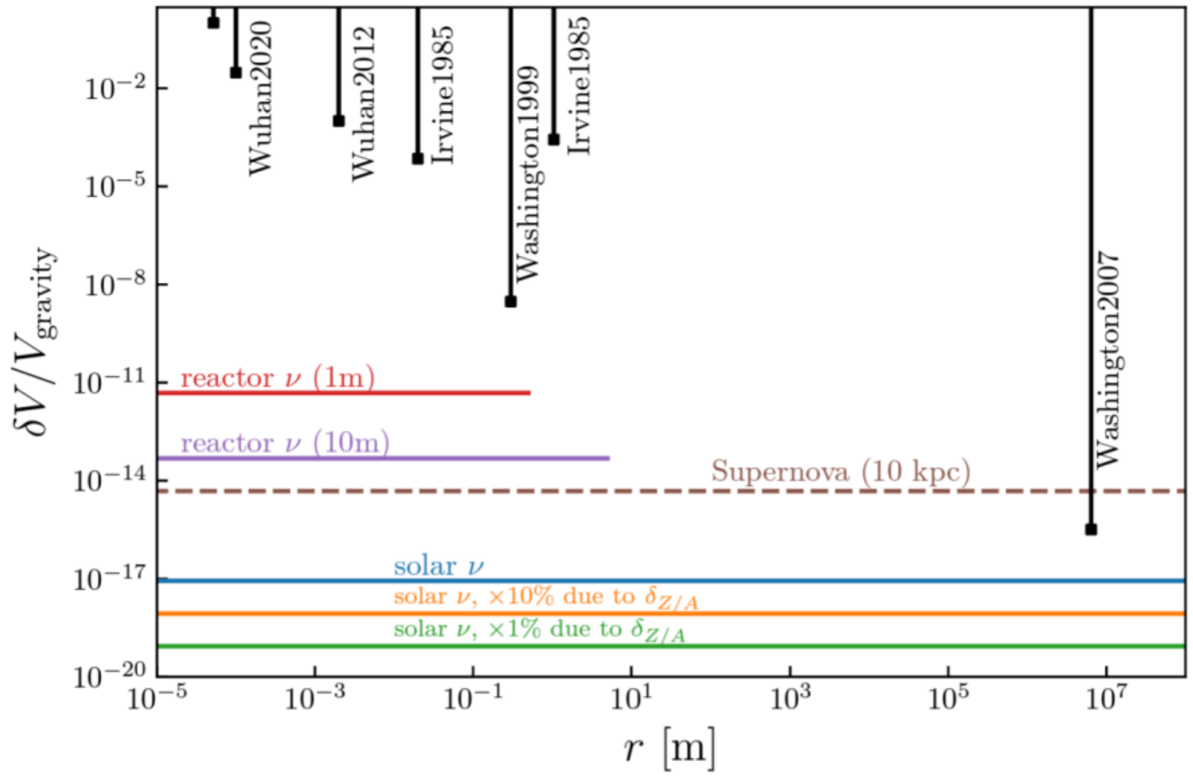
Number density of electron neutrinos in the beam

$$V_{\text{bkg}}(r) = -\frac{G_F^2 \Phi E_\nu}{\pi r} \frac{m_1 m_2}{m_p^2} \times f(A_1, A_2, Z_1, Z_2, n_e)$$

$$f(A_1, A_2, Z_1, Z_2, n_e) = \frac{1}{4}\left[n_e\left(\frac{3Z_1}{A_1}-1\right)\left(\frac{3Z_2}{A_2}-1\right) + (1-n_e)\left(1-\frac{Z_1}{A_1}\right)\left(1-\frac{Z_2}{A_2}\right)\right]$$

$$\frac{V_{\text{bkg}}(r)}{V_{\text{grav}}(r)} \sim 10^{-13}$$

Currently we can detect forces that are about 9 orders of magnitude smaller than gravity. Just need to go a leeetle further!

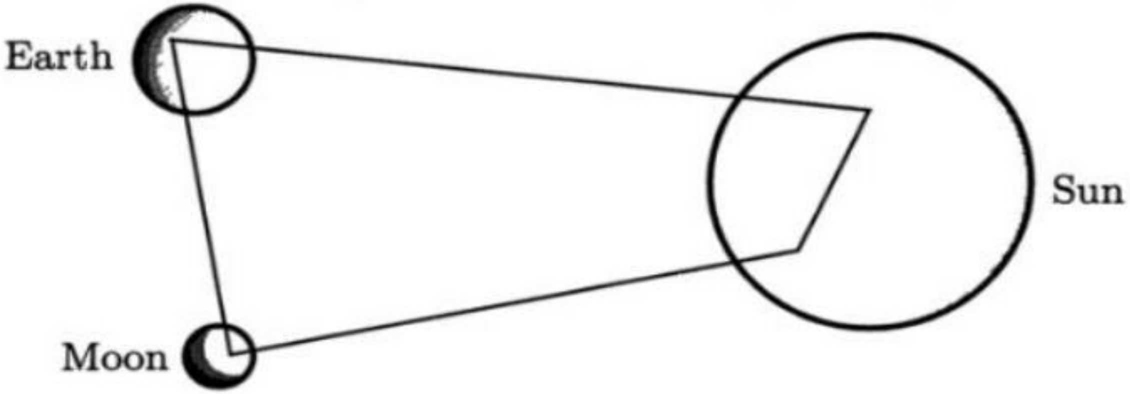


Reminds me of:

The neutrino-force as an explanation for gravity??

FEYNMAN LECTURES ON GRAVITATION

RICHARD P. FEYNMAN



$$E = -G'^3 m_1 m_2 m_3 \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \qquad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

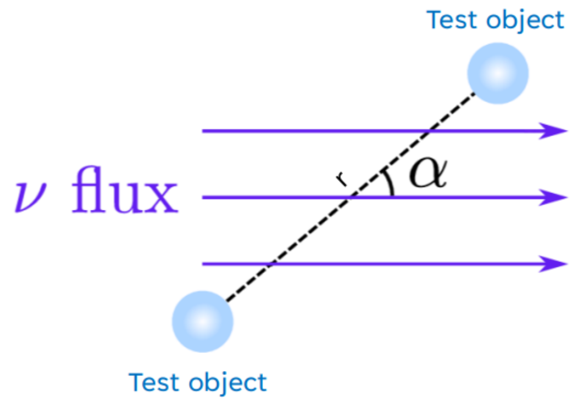
$$E = -\frac{G'^3 m_1 m_2 \pi^2}{r_{12}} \int \frac{4\pi\rho(R) R^2 dR}{2R^3}, \qquad (2.4.5)$$

Background

Us: $\frac{1}{r^5} \to \frac{1}{r}$, Feynman: $\frac{1}{r^4} \to \frac{1}{r}$

There are, as always, caveats...

$$\cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$



Finite size problems

Force is too wiggly! Objects need to be very small to not have the force between them cancel.

$$\Delta(\alpha^2) \lesssim (E_\nu r)^{-1}$$

Assuming that the beam is truly monochromatic, we find, for test objects of size R:

$$\alpha \lesssim (E_\nu R)^{-1}$$

If hydrogen atoms are used as test objects, we need $\alpha \ll 10^{-2}$.

NEED LOWER ENERGY BACKGROUNDS!

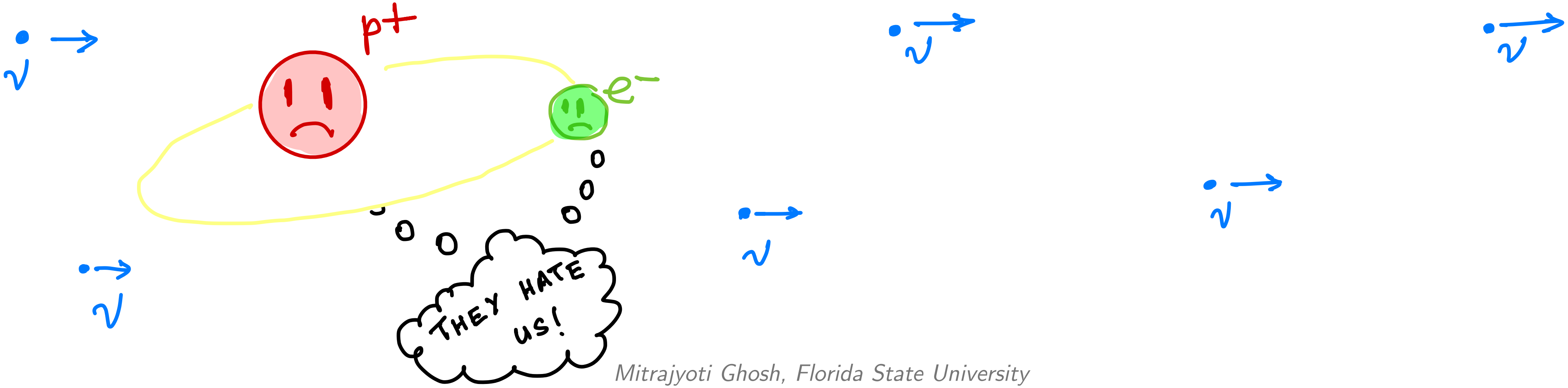
Monochromaticity problems

A real beam is not completely monochromatic, a spread in energy can also kill the enhancement.

What about APV in a neutrino background?

Realistic backgrounds do not sufficiently enhance APV in hydrogen since the distance between two neutrinos in a background is often much larger than the size of the atom.

Need to look for it in bigger atoms, maybe.



All in all

Takeaway 1: *The neutrino force is sensitive to many unknowns about neutrinos, but so far, our sensitivity to it is low. Its long range remains its biggest strength.*

Takeaway 2: *We can also apply all of the "forces" machinery here to New Physics searches. For example, new pseudoscalars will also contribute to APV and may even have bigger effects!*

Thank You for your attention!