Dark matter semi-annihilation for inert scalar multiplets

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Inert scalar multiplets

Inert scalar multiplets:

- $SU(2)_L$ scalar multiplet that does not acquire a vacuum expectation value and does not mix with the Higgs
- I will not consider singlets, since they have no gauge interactions.

Properties:

- They can be charged under some symmetry.
- The neutral component can be a dark matter candidate.

Previous work:

- n-plets with $n \in \{2, 3, 4, 5, 6, 7, 8\}$ have been studied.
- Combinations of different multiplets have been considered.
- Increasing experimental tension
- Generally assumes a $\mathbb{Z}_2$ symmetry

Semi-annihilation

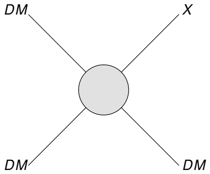
First proposed in:

- F. D'Eramo and J. Thaler, "Semi-annihilation of Dark Matter," JHEP 06 (2010) 109, arXiv:1003.5912 [hep-ph].

The process that controls DM abundance is:

$$DM\ DM \to DM\ X,$$

with X not being dark matter.



Advantages:

- Not strongly constrained by direct detection experiments
- Not strongly constrained by collider experiments

Goal

Questions:

- What models of inert multiplets include semi-annihilation?
- Can this impact the dark matter abundance?
- Can this relax some constraints?

Goal:

- Determine which models of inert scalar multiplets can:
  - Lead to efficient semi-annihilation
  - Avoid experimental constraints
  - Avoid stable charged particles in a technically natural way

Plan:

- One multiplet case . . . and why it fails
- Two multiplets case
- Analysis of the most promising model
- Some preliminary indirect detection results

One multiplet case: Potential I

Consider a complex scalar multiplet ϕ:

- Dimension n
- Hypercharge 0

The relevant renormalizable potential is

$$V = V_0 + V_A + V_B + V_C,$$

where

$$V_0 = -\mu^2 |H|^2 + \lambda_0 |H|^4 + m^2 |\phi|^2, \qquad V_A = \lambda_1 A_{abc} \phi^a \phi^b \phi^c + \text{h.c.},$$

$$V_B = \sum_{r=1}^{\alpha} \lambda_2^r B^r_{abcd} (H^a)^\dagger H^b (\phi^c)^\dagger \phi^d, \qquad V_C = \sum_{r=1}^{\beta} \lambda_3^r C^r_{abcd} (\phi^a)^\dagger (\phi^b)^\dagger \phi^c \phi^d.$$

The potential contains three types of terms:

- V_A: Responsible for semi-annihilation
- V_B: Can split masses and affect the abundance
- V_C: Dark matter self-interactions

One multiplet case: Potential II

- The A_{abc} term is non-zero for $n \in \{1, 5, 9, ...\}$.
- The potential respects a $\mathbb{Z}_3$ symmetry under which only ϕ transforms in the following way:

$$\phi \to e^{2\pi i/3}\phi.$$

- Unless $\mathbb{Z}_3$ is spontaneously broken, the lightest component is stable.
- No tree-level interactions with the Z boson for neutral component.
  - Good for direct detection constraints
- Suffers from two problems...

One multiplet case: Problem I

The potential includes a term that can be written as

$$\lambda_3^2 H^\dagger \tau^a H \phi^\dagger T^a \phi \;\Rightarrow\; -\frac{\lambda_3^2 v^2}{4}\phi^\dagger T^3 \phi,$$

where τ^a and T^a are the appropriate $SU(2)$ generators.

Example with $n = 5$:

$$V \supset -\frac{\lambda_3^2 v^2}{2}\phi_{++}^\dagger \phi_{++} - \frac{\lambda_3^2 v^2}{4}\phi_{+}^\dagger \phi_{+} + \frac{\lambda_3^2 v^2}{4}\phi_{-}^\dagger \phi_{-} + \frac{\lambda_3^2 v^2}{2}\phi_{--}^\dagger \phi_{--}.$$

The problem:

- At tree-level, the lightest component is charged.
- Setting $\lambda_3^2 = 0$ is not technically natural.

Ultimately:

- Can be solved by radiative corrections
- Requires fine-tuning, but not sufficient to exclude the model

One multiplet case: Problem II a

However, the model suffers from a serious problem:

- Semi-annihilation is not efficient.

There are two types of semi-annihilation processes:

1. $\phi^a \phi^b \to (\phi^c)^\dagger \;\; h$,
2. $\phi^a \phi^b \to (\phi^c)^\dagger \;\; B$,

where h is the Higgs boson and B a gauge boson A, Z, W^+ or W^-.

One multiplet case: Problem II b

Let's consider:

1. $\phi^a \phi^b \to (\phi^c)^\dagger \ \ h$,

Note:

- In the limit that EW symmetries are restored, this is forbiden by $SU(2)$ and $U(1)$.
- CS must be suppressed by some power of v/m.
- m must be in the TeV range to explain the current abundance.
- Therefore, this process is inefficient.

One multiplet case: Problem II c

Let's consider:

2. $\phi^a \phi^b \to (\phi^c)^\dagger \;\; B$,

with $v = 0$. The corresponding cross section is

$$\sigma = \frac{1}{64\pi s p_A^2} \int_{t_1}^{t_0} |M|^2 dt,$$

with

$$\int_{t_1}^{t_0} |M|^2 dt \propto \frac{-3\sqrt{s(s-4m^2)} + 2(s+2m^2)\tanh^{-1}\left(\sqrt{\frac{s-4m^2}{s}}\right)}{s-m^2} \approx \frac{\Delta s^{5/2}}{90 m^5},$$

with $\Delta s = s - 4m^2$. This leads to:

- $\sigma\beta \propto \beta^4$ (d-wave)
- $\langle \sigma\beta \rangle \propto T^2$

One multiplet case: Problem II d

- When the Higgs vev is taken into account, some processes are not really *d*-wave but are still suppressed by powers of v/m.
- If the potential includes $\lambda\phi_1^2\phi_2 + \text{h.c.}$:
  - *p*-wave if the incoming particles are distinct
  - *d*-wave otherwise
- Consequence of:
  - Gauge invariance
  - Conservation of angular momentum

One multiplet case: Final verdict

For that model:

- All semi-annihilation processes are either *p*-wave, *d*-wave or suppressed by v/m to some power.
- Semi-annihilation is inefficient.

Only other way to have semi-annihilation is:

- Include a term of the form $\lambda\phi^3 H +$ h.c..
- This would lead to efficient semi-annihilation.
- However, there are no neutral particles.

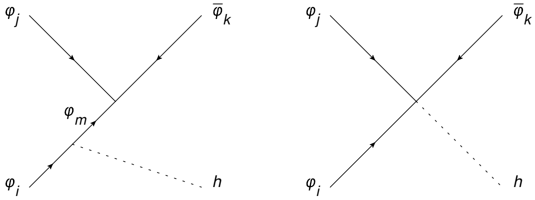
Conclusion

- There does not exist any renormalizable models of a single inert multiplet for which semi-annihilation is efficient.

One multiplet case: What did we learn?

Three potential solutions:

1. Include fields beyond inert multiplets
   - A possibility, but beyond the scope of this work.
2. Include $\lambda_1 \phi_1^3 + \text{h.c.}$ and a term in $\lambda_2 \phi_1 \phi_2^\dagger H^\dagger + \text{h.c.}$
   - Strongly constrained by direct detection experiments
3. Include $\lambda \phi_i \phi_j \phi_k H + \text{h.c.}$ with i, j and k not all equal



Two multiplets case: Potential I

The potential must include:

$$V \supset \lambda \phi_1^2 \phi_2 H + \text{h.c.}$$

Comments:

- ϕ_2 has to be even-dimensional and therefore complex.
- For both ϕ_1 and ϕ_2 to be non-trivially charged under a stabilizing symmetry and thus have semi-annihilation, ϕ_1 must also be complex.
- We will refer to the weak hypercharges of ϕ_1 and ϕ_2 as Y_1 and Y_2 and their sizes as n_1 and n_2, respectively.

Two multiplets case: Potential II

Only a finite number of B terms compatible with:

- $\lambda\phi_1^2\phi_2 H + \text{h.c.}$
- ϕ_1 and ϕ_2 non-trivially charged under a stabilizing symmetry
- Possible existence of stable neutral particles

Besides $\lambda H^\dagger H \phi_i^\dagger \phi_i$ and $\lambda H^\dagger \tau^a H \phi_i^\dagger T^a \phi_i$ which are always allowed, the possibilities are:

- $V_{B_1} = \lambda\phi_2^2 (H^\dagger)^2 + \text{h.c.}$, with $Y_1 = -1/2$, $Y_2 = 1/2$ and $\mathbb{Z}_4$.
- $V_{B_2} = \lambda\phi_1\phi_2^\dagger H^2 + \text{h.c.}$, with $Y_1 = -1/2$, $Y_2 = 1/2$ and $\mathbb{Z}_3$.
- $V_{B_3} = \lambda\phi_2^2 H^2 + \text{h.c.}$, with $Y_1 = 0$, $Y_2 = -1/2$ and $\mathbb{Z}_4$.
- $V_{B_4} = \lambda\phi_1\phi_2^\dagger H^\dagger + \text{h.c.}$, with $Y_1 = 0$, $Y_2 = -1/2$ and $\mathbb{Z}_3$.
- No B term, with $U(1)$.

Two multiplets case: Potential III

Where the symmetries are given by:

- $\mathbb{Z}_3$: $\phi_1 \to e^{2\pi i/3}\phi_1$ and $\phi_2 \to e^{2\pi i/3}\phi_2$.
- $\mathbb{Z}_4$: $\phi_1 \to i\phi_1$ and $\phi_2 \to -\phi_2$.
- $U(1)$: $\phi_1 \to e^{\pi\alpha i}\phi_1$ and $\phi_2 \to e^{-2\pi\alpha i}\phi_2$, with $\alpha \in \mathbb{R}$.

Two multiplets case: Direct detection + no stable charged particles

The only ways to suppress the direct detection signal of a stable scalar are:

- Split the masses of the CP-even and CP-odd parts of the neutral component
- Have dark matter be mostly a neutral component from an odd-dimensional multiplet of zero weak hypercharge

We will require one of these conditions to be satisfied for all dark matter components.

We would also like to avoid stable charged particles at tree level.

Two multiplets case: Conclusion

The only possibility is:

- $V_{B_4} = \lambda \phi_1 \phi_2^\dagger H^\dagger + \text{h.c.}$, with $Y_1 = 0$, $Y_2 = -1/2$ and $\mathbb{Z}_3$.

Comments:

- The cases of V_{B_1}, V_{B_3} and $U(1)$ are also not strictly excluded. They must however rely on loop corrections to avoid stable charged particles, satisfy special conditions to meet the direct detection constraints, or both.
- The scenario of V_{B_2} would be very difficult to salvage.

Potential I

The relevant potential is:

$$V = V_0 + V_A + V_B + V_C,$$

where

$$\begin{aligned}
V_0 =& -\mu^2|H|^2 + \lambda_0|H|^4 + m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2,\\
V_A =& \lambda_1 A_{abcd}\phi_1^a\phi_1^b\phi_2^c H^d + \lambda_2 B_{abc}\phi_1^a\phi_1^b\phi_1^c + \text{h.c.},\\
V_B =& \sum_{r=1}^{\alpha}\lambda_3^r C^r_{abcd}(H^a)^\dagger H^b(\phi_1^c)^\dagger\phi_1^d + \sum_{r=1}^{\beta}\lambda_4^r D^r_{abcd}(H^a)^\dagger H^b(\phi_2^c)^\dagger\phi_2^d\\
&+ [\lambda_5 E_{abc}\phi_1^a(\phi_2^b)^\dagger(H^c)^\dagger + \text{h.c.}],\\
V_C =& \sum_{r=1}^{\gamma}\lambda_6^r F^r_{abcd}(\phi_1^a)^\dagger(\phi_1^b)^\dagger\phi_1^c\phi_1^d + \sum_{r=1}^{\delta}\lambda_7^r G^r_{abcd}(\phi_2^a)^\dagger(\phi_2^b)^\dagger\phi_2^c\phi_2^d\\
&+ \sum_{r=1}^{\epsilon}\lambda_8^r H^r_{abcd}(\phi_1^a)^\dagger(\phi_2^b)^\dagger\phi_1^c\phi_2^d.
\end{aligned}$$

Potential II

The conditions for the $SU(2)$ contractions to be non-zero are

$$\begin{aligned} A: &\quad n_2 \in \{2n_1,\ 2n_1-2\ , 2n_1-4,\ \ldots\}, \\ B: &\quad n_1 \in \{1, 5, 9,\ \ldots\}, \\ E: &\quad n_1 - n_2 \in \{-1, 1\}. \end{aligned}$$

All other contractions are always non-vanishing.

The potential respects a $\mathbb{Z}_3$ symmetry under which

$$\phi_1 \to e^{2\pi i/3}\phi_1, \quad \phi_2 \to e^{2\pi i/3}\phi_2.$$

Unless $\mathbb{Z}_3$ is spontaneously broken, the lightest mass eigenstate is stable.

Abundance computation: Overview

To simplify computations, we will:

- Take the limit of unbroken EW symmetries,
- Work with isospin eigenstates.

All $2 \to 2$ cross sections can be decomposed as

$$\begin{aligned}\sigma([P_A P_B]_J^M \to [P_C P_D]_J^{M'}) &= \sigma_0([P_A P_B]_J \to [P_C P_D]_J)\delta^{MM'} \\ &= \frac{S_{CD}}{64\pi s p_A^2}\hat{M}([P_A P_B]_J \to [P_C P_D]_J)\delta^{MM'},\end{aligned}$$

where S_{CD} is $1/2$ if P_C and P_D are identical and 1 otherwise.

Abundance computation: CS I

For annihilation to Higgs pairs, the $\hat{M}$ coefficients are given by

$$\hat{M}([\phi_1^\dagger\phi_1]_0 \to [H^\dagger H]_0) = f_1\left(s, m_1, m_2, \sqrt{2n_1}\lambda_3^1, \sqrt{\frac{n_1}{2}}|\lambda_5|^2\right),$$

$$\hat{M}([\phi_2^\dagger\phi_2]_0 \to [H^\dagger H]_0) = f_1\left(s, m_2, m_1, \sqrt{2n_2}\lambda_4^1, \frac{n_1}{\sqrt{2n_2}}|\lambda_5|^2\right),$$

$$\hat{M}([\phi_1^\dagger\phi_1]_1 \to [H^\dagger H]_1) = f_1\left(s, m_1, m_2, \sqrt{\frac{(n_1^2-1)n_1}{24}}\lambda_3^2, \frac{(n_1-n_2)}{\sqrt{6}}\frac{\sqrt{n_1(n_1^2-1)}}{n_2}|\lambda_5|^2\right),$$

$$\hat{M}([\phi_2^\dagger\phi_2]_1 \to [H^\dagger H]_1) = f_1\left(s, m_2, m_1, \sqrt{\frac{(n_2^2-1)n_2}{24}}\lambda_4^2, \frac{(n_1-n_2)}{\sqrt{6}}\sqrt{\frac{n_2^2-1}{n_2}}|\lambda_5|^2\right),$$

where

$$f_1(s, m_A, m_B, C_A, C_B) =$$

$$\sqrt{s(s-4m_A^2)}\left(C_A^2 + \frac{C_B^2}{(m_A-m_B)^2+m_B^2 s}\right) - 4C_AC_B\tanh^{-1}\left(\frac{\sqrt{s(s-4m_A^2)}}{s-2m_A^2+2m_B^2}\right).$$

Abundance computation: CS II

For annihilation to pairs of gauge bosons, the $\hat{M}$ coefficients are given by

$$\hat{M}([\phi_1^\dagger \phi_1]_0 \to [WW]_0) = f_2\left(s, m_1, \frac{(n_1^2-1)\sqrt{n_1}}{4\sqrt{3}} g^2\right),$$

$$\hat{M}([\phi_2^\dagger \phi_2]_0 \to [WW]_0) = f_2\left(s, m_2, \frac{(n_2^2-1)\sqrt{n_2}}{4\sqrt{3}} g^2\right),$$

$$\hat{M}([\phi_1^\dagger \phi_1]_2 \to [WW]_2) = f_2\left(s, m_1, \frac{1}{2}\sqrt{\frac{n_1(n_1^2-1)(n_1^2-4)}{30}} g^2\right),$$

$$\hat{M}([\phi_2^\dagger \phi_2]_2 \to [WW]_2) = f_2\left(s, m_2, \frac{1}{2}\sqrt{\frac{n_2(n_2^2-1)(n_2^2-4)}{30}} g^2\right),$$

$$\hat{M}([\phi_2^\dagger \phi_2]_1 \to [WB]_1) = f_2\left(s, m_2, \sqrt{\frac{n_2(n_2^2-1)}{12}} Y_2 g g'\right),$$

$$\hat{M}([\phi_2^\dagger \phi_2]_0 \to [BB]_0) = f_2\left(s, m_2, (Y_2 g')^2\right),$$

where

$$f_2(s, m, C) = \frac{8C^2}{s}\left[(s+4m^2)\sqrt{s(s-4m^2)} - 8m^2(s-2m^2)\tanh^{-1}\left(\sqrt{\frac{s-4m^2}{s}}\right)\right].$$

Abundance computation: CS III

For semi-annihilation involving a Higgs boson, we will assume that the B tensor is zero, λ_2 is small or λ_5 is small. If at least one of these conditions is satisfied, then the $\hat{M}$ coefficients are given by

$$\hat{M}\left([\phi_1\phi_1]_{J_0} \to [\phi_2^\dagger H^\dagger]_{J_0}\right) = \frac{4|\lambda_1|^2}{R}\frac{\sqrt{s-4m_1^2}(s-m_2^2)}{\sqrt{s}},$$

$$\hat{M}\left([\phi_1\phi_2]_{J_1} \to [\phi_1^\dagger H^\dagger]_{J_1}\right) = 4|\lambda_1|^2\left|C_{n_1 n_2}^{R_1'}\right|^2\frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}(s-m_1^2)}{s},$$

$$\hat{M}\left([\phi_1\phi_2]_{J_2} \to [\phi_1^\dagger H^\dagger]_{J_2}\right) = 4|\lambda_1|^2\left|C_{n_1 n_2}^{R_2'}\right|^2\frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}(s-m_1^2)}{s},$$

where

$$R = 4\left\lfloor\frac{n_2}{4}\right\rfloor + 1, \qquad R_1' = n_1 + 1, \qquad R_2' = n_1 - 1,$$

$$J_0 = \frac{R-1}{2}, \qquad J_1 = \frac{R_1'-1}{2}, \qquad J_2 = \frac{R_2'-1}{2},$$

and

$$\left|C_{n_1 n_2}^{R'}\right|^2 = \frac{1}{R'}\left(\frac{1}{2} - \frac{n_2}{4n_1}(R'-n_1)(n_1-n_2)(-1)^{\frac{n_1-1}{2}}\right).$$

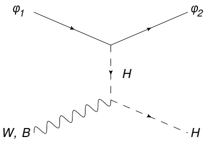
Abundance computation: CS IV

Define the effective cross sections

$$\begin{aligned}
\sigma(\phi_1\phi_1 \to \mathrm{SM}\ \mathrm{SM}) = {} & \frac{1}{2n_1^2}\sigma_0\left([\phi_1^\dagger\phi_1]_0 \to [H^\dagger H]_0\right) + \frac{3}{2n_1^2}\sigma_0\left([\phi_1^\dagger\phi_1]_1 \to [H^\dagger H]_1\right) \\
& + \frac{1}{2n_1^2}\sigma_0\left([\phi_1^\dagger\phi_1]_0 \to [WW]_0\right) + \frac{5}{2n_1^2}\sigma_0\left([\phi_1^\dagger\phi_1]_2 \to [WW]_2\right), \\
\sigma(\phi_2\phi_2 \to \mathrm{SM}\ \mathrm{SM}) = {} & \frac{1}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_0 \to [H^\dagger H]_0\right) + \frac{3}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_1 \to [H^\dagger H]_1\right) \\
& + \frac{1}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_0 \to [WW]_0\right) + \frac{5}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_2 \to [WW]_2\right) \\
& + \frac{1}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_0 \to [BB]_0\right) + \frac{3}{2n_2^2}\sigma_0\left([\phi_2^\dagger\phi_2]_1 \to [WB]_1\right), \\
\sigma(\phi_1\phi_1 \to \phi_2\ \mathrm{SM}) = {} & \frac{R}{4n_1^2}\sigma_0\left([\phi_1\phi_1]_{J_0} \to [\phi_2^\dagger H^\dagger]_{J_0}\right), \\
\sigma(\phi_1\phi_2 \to \phi_1\ \mathrm{SM}) = {} & \frac{R_1'}{2n_1 n_2}\sigma_0\left([\phi_1\phi_2]_{J_1} \to [\phi_1^\dagger H^\dagger]_{J_1}\right) + \frac{R_2'}{2n_1 n_2}\sigma_0\left([\phi_1\phi_2]_{J_2} \to [\phi_1^\dagger H^\dagger]_{J_2}\right).
\end{aligned}$$

Abundance computation: CS V

As long as λ_5 is not vanishingly small, ϕ_1 and ϕ_2 will remain in relative chemical equilibrium during freeze-out via interactions like:



We can then further define effective thermally averaged cross sections as

$$\langle\sigma\beta(\phi\phi \to \text{SM SM})\rangle = \left(\frac{Y_1^{\text{eq}}}{Y^{\text{eq}}}\right)^2 \langle\sigma\beta(\phi_1\phi_1 \to \text{SM SM})\rangle + \left(\frac{Y_2^{\text{eq}}}{Y^{\text{eq}}}\right)^2 \langle\sigma\beta(\phi_2\phi_2 \to \text{SM SM})\rangle,$$

$$\langle\sigma\beta(\phi\phi \to \phi \text{ SM})\rangle = \left(\frac{Y_1^{\text{eq}}}{Y^{\text{eq}}}\right)^2 \langle\sigma\beta(\phi_1\phi_1 \to \phi_2 \text{ SM})\rangle + \frac{Y_1^{\text{eq}} Y_2^{\text{eq}}}{(Y^{\text{eq}})^2} \langle\sigma\beta(\phi_1\phi_2 \to \phi_1 \text{ SM})\rangle.$$

Abundance computation: CS VI

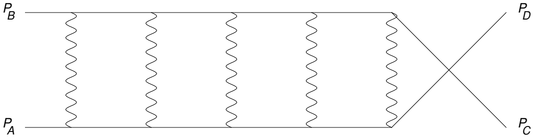
Define $x = m_1/T$ and use

$$\frac{dt}{dx} = \sqrt{\frac{45}{4\pi^3}}\frac{g_*^{1/2}}{h_{\mathrm{eff}}}\frac{M_{\mathrm{Pl}}}{m_1 T}, \quad g_*^{1/2} = \frac{h_{\mathrm{eff}}}{g_{\mathrm{eff}}^{1/2}}\left(1 + \frac{T}{3h_{\mathrm{eff}}}\frac{dh_{\mathrm{eff}}}{dT}\right).$$

We are then left with a single equation to solve:

$$\frac{dY}{dx} = -s_E\frac{dt}{dx}\left[\langle\sigma\beta(\phi\phi \to \mathrm{SM\ SM})\rangle(Y^2 - (Y^{\mathrm{eq}})^2) + \langle\sigma\beta(\phi\phi \to \phi\ \mathrm{SM})\rangle Y(Y - Y^{\mathrm{eq}})\right].$$

Abundance computation: Sommerfeld enhancement



Consider two colliding particles P_A and P_B of weak isospins J_A and J_B and weak hypercharge Y_A and Y_B, respectively. Assume they form a state of weak isospin J. Define

$$\alpha = \frac{g^2}{8\pi}\left[J(J+1) - J_A(J_A+1) - J_B(J_B+1)\right] + \frac{(g')^2}{4\pi} Y_A Y_B \eta,$$

where $\eta = -1$ for the collision of a particle and an antiparticle and $\eta = 1$ otherwise. The cross section is then enhanced by a multiplicative factor of

$$S_{\mathrm{SE}} = -\frac{2\pi\alpha}{\beta}\frac{1}{1 - e^{\frac{2\pi\alpha}{\beta}}}.$$

Abundance computation: Broken EW

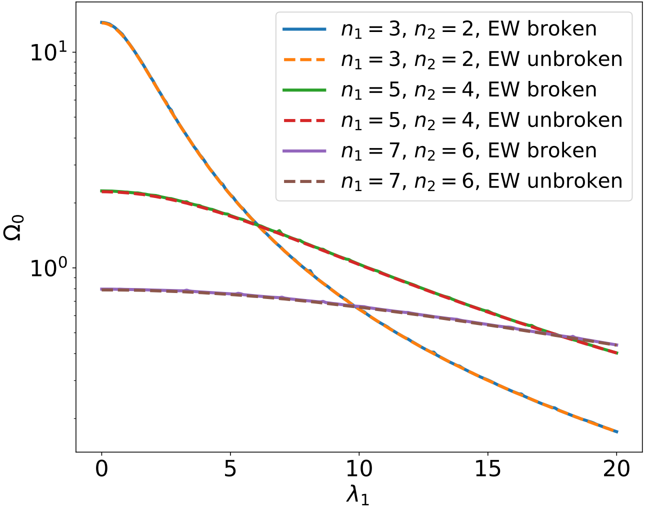
As a validation test, we also computed the abundance with v set to its current value.

Procedure:

- Diagonalize mass matrices
- Compute interactions in terms of mass eigenstates
- Compute all amplitudes
- Compute all thermally averaged cross sections
- Write and solve the Boltzmann equation

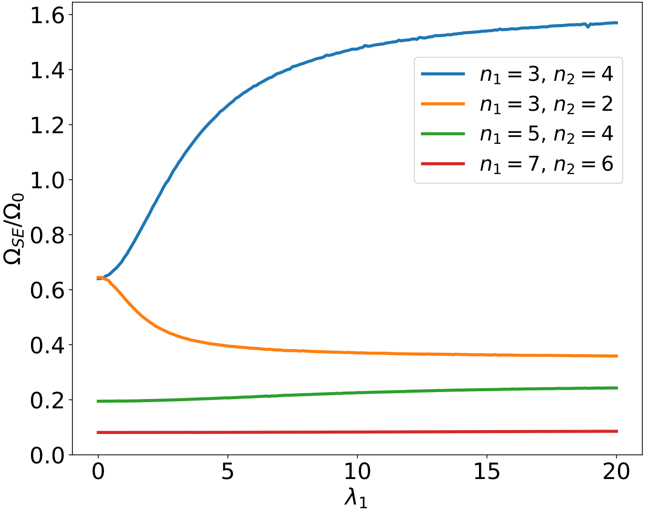
Abundance computation: Comparison

Without Sommerfeld enhancement



$m_1 = 10$ TeV, $m_2 = 11$ TeV, all other parameters set to 0

Abundance computation: Sommerfeld factor

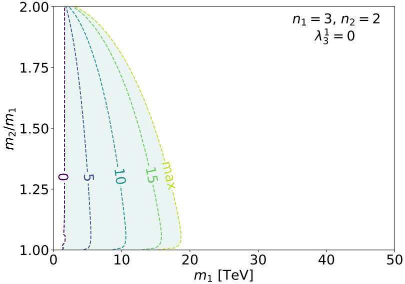


$m_1 = 10$ TeV, $m_2 = 11$ TeV, all other parameters set to 0

Results I

Plots:

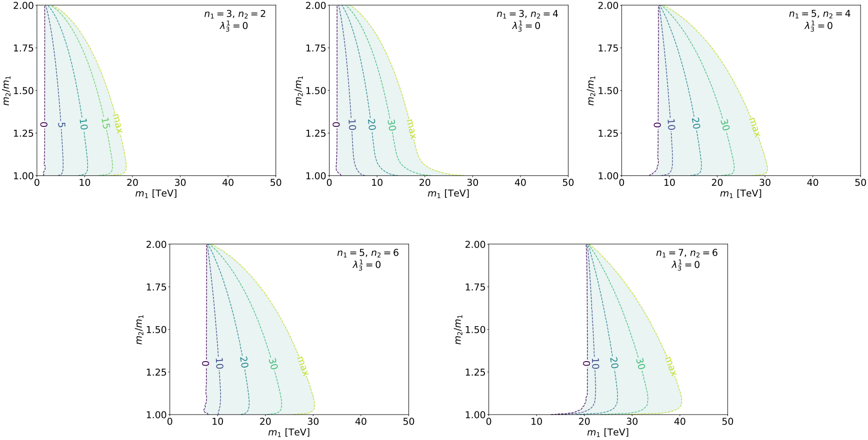
- Adjust λ_1 to get correct Ω
- Contours of λ_1
- All other parameters set to 0.
- Allowed region in green.



Comments:

- Allows for a wide range of m_1
- m_2/m_1 can differ considerably from 1, unlike coannihilation.
- Less relative range of m_1 for larger multiplets

Results II


$n_1 = 3$, $n_2 = 2$
$\lambda_3^1 = 0$
$n_1 = 3$, $n_2 = 4$
$\lambda_3^1 = 0$
$n_1 = 5$, $n_2 = 4$
$\lambda_3^1 = 0$
$n_1 = 5$, $n_2 = 6$
$\lambda_3^1 = 0$
$n_1 = 7$, $n_2 = 6$
$\lambda_3^1 = 0$
m_2/m_1
m_1 [TeV]
0
5
10
15
20
30
40
50
max
1.00
1.25
1.50
1.75
2.00

Sommerfeld enhancement at low velocity I

Parametrize the interactions as:

$$\mathcal{L}^{\text{light}} = \quad - i\hat{A}_{ii} A_\mu \left(\hat{\phi}_i^\dagger \partial^\mu \hat{\phi}_i - \partial^\mu \hat{\phi}_i^\dagger \hat{\phi}_i\right) - i\hat{C}_{ij} Z_\mu \left(\hat{\phi}_i^\dagger \partial^\mu \hat{\phi}_j - \partial^\mu \hat{\phi}_j^\dagger \hat{\phi}_i\right) \\ - \left[i\hat{F}_{ij} W_\mu^+ \left(\hat{\phi}_i^\dagger \partial^\mu \hat{\phi}_j - \partial^\mu \hat{\phi}_i^\dagger \hat{\phi}_j\right) + \text{h.c.}\right] - \hat{\Omega}_{ij} h \hat{\phi}_i^\dagger \hat{\phi}_j.$$

Consider the scattering $\hat{\phi}_i^\dagger \hat{\phi}_i \to \hat{\phi}_j^\dagger \hat{\phi}_j$. The associated potential is:

$$V_{ij} = -\frac{\hat{A}_{ii}^2}{4\pi r}\delta_{ij} - \frac{\hat{C}_{ii}^2}{4\pi r}\delta_{ij} e^{-m_Z r} - \frac{\hat{F}_{ij}^2}{4\pi r} e^{-m_W r} - \frac{\hat{F}_{ji}^2}{4\pi r} e^{-m_W r} - \frac{\hat{\Omega}_{ii}^2}{16\pi \hat{m}_0^2 r}\delta_{ij} e^{-m_h r} + \delta \hat{m}_{ij}.$$

Consider the scattering $\hat{\phi}_i \hat{\phi}_{i'} \to \hat{\phi}_j \hat{\phi}_{j'}$, where $\hat{\phi}_{i'}$ $(\hat{\phi}_{j'})$ is the mass eigenstate with opposite charge to $\hat{\phi}_i$ $(\hat{\phi}_j)$. The associated potential is:

$$\tilde{V}_{ij} = \left[\frac{\hat{A}_{ii}\hat{A}_{i'i'}}{4\pi r}\delta_{ij} + \frac{\hat{C}_{ii}\hat{C}_{i'i'}}{4\pi r}\delta_{ij} e^{-m_Z r} + \frac{\hat{F}_{ij}\hat{F}_{j'i'}}{4\pi r} e^{-m_W r} \right. \\ \left. + \frac{\hat{F}_{ji}\hat{F}_{i'j'}}{4\pi r} e^{-m_W r} - \frac{\hat{\Omega}_{ii}\hat{\Omega}_{i'i'}}{16\pi \hat{m}_0^2 r}\delta_{ij} e^{-m_h r} + \delta \tilde{m}_{ij}\right] s_i s_j,$$

with $s_a = \sqrt{1 + \delta_{aa'}}$.

Sommerfeld enhancement at low velocity II

One must solve the following radial Schrödinger equations:

$$-\frac{1}{\hat{m}_0}\frac{d^2}{dr^2}u_{ia}(r)+V_{ij}u_{ja}(r)=Eu_{ia}(r),$$
$$-\frac{1}{\hat{m}_0}\frac{d^2}{dr^2}\tilde{u}_{ia}(r)+\tilde{V}_{ij}\tilde{u}_{ja}(r)=E\tilde{u}_{ia}(r).$$

with boundary conditions

$$u_{ia}(0)=\delta_{ia},\quad \frac{d}{dr}u_{ia}(\infty)=ik_i u_{ia},$$
$$\tilde{u}_{ia}(0)=\delta_{ia},\quad \frac{d}{dr}\tilde{u}_{ia}(\infty)=i\tilde{k}_i\tilde{u}_{ia},$$

where $k_i=\sqrt{\hat{m}_0(E-\delta\hat{m}_{ii})}$ and $\tilde{k}_i=\sqrt{\hat{m}_0(E-\delta\tilde{m}_{ii})}$. The matrices R and $\tilde{R}$ are defined through

$$u_{ia}(r\to\infty)=R_{ia}e^{ik_ir},\quad \tilde{u}_{ia}(r\to\infty)=\tilde{R}_{ia}e^{i\tilde{k}_ir}.$$

The matrices of Sommerfeld factors are then simply given by

$$s=R^T,\quad \tilde{s}=\tilde{R}^T.$$

Sommerfeld enhancement at low velocity III

Consider an annihilation process between a $\hat{\phi}_0^\dagger$ and a $\hat{\phi}_0$ to a final state f. The Sommerfeld enhanced cross section is given by

$$(\sigma v)_f = (s^\dagger \Gamma_f s)_{nn},$$

with n corresponding to the neutral one. The annihilation rates are:

$$\Gamma_{ZZ} = \frac{g^4 c_W^4}{8\pi m_1^2} \hat{T}_N \hat{T}_N^T + \frac{\hat{\lambda}_+ \hat{\lambda}_+^T}{64\pi m_1^2}, \qquad \Gamma_{ZA} = \frac{g^4 c_W^2 s_W^2}{4\pi m_1^2} \hat{T}_N \hat{T}_N^T, \qquad \Gamma_{AA} = \frac{g^4 s_W^4}{8\pi m_1^2} \hat{T}_N \hat{T}_N^T,$$

$$\Gamma_{W^+W^-} = \frac{g^4}{64\pi m_1^2} \hat{T}_C \hat{T}_C^T + \frac{\hat{\lambda}_- \hat{\lambda}_-^T}{32\pi m_1^2}, \qquad \Gamma_{hh} = \frac{\hat{\lambda}_+ \hat{\lambda}_+^T}{64\pi m_1^2},$$

where

$$(\hat{T}_N)_i = (T_3)_{ii}, \quad (\hat{T}_C)_i = \left(\{T^+, T^-\}\right)_{ii}, \quad \hat{\lambda}_\pm \equiv -\lambda_3^1 \pm \lambda_3^2 \hat{T}_3/2, \quad (\hat{T}_3)_i = (T_3)_{ii}.$$

Sommerfeld enhancement at low velocity IV

Consider an annihilation process of two $\hat{\phi}_0$ to a final state f. The Sommerfeld enhanced cross section is given by

$$(\sigma v)_f = 2(\tilde{s}^\dagger \Gamma_f \tilde{s})_{nn},$$

The rate for semi-annihilation is

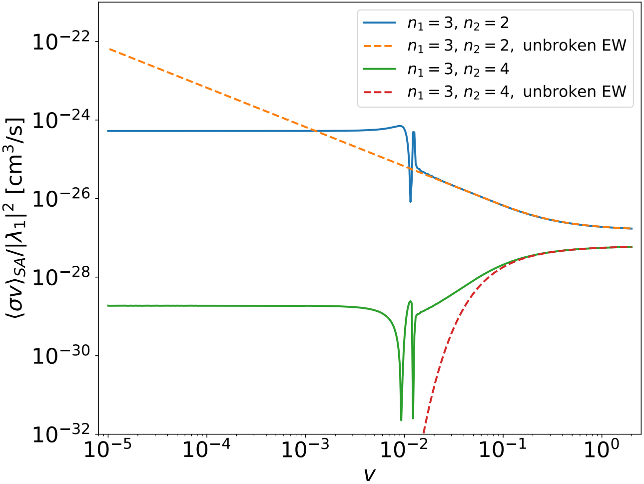
$$\Gamma_{\tilde{\phi}^\dagger W^-} = 2\Gamma_{\tilde{\phi}^\dagger Z} = 2\Gamma_{\tilde{\phi}^\dagger h} = \frac{|\lambda_1|^2}{32\pi m_1^2}\left(1 - \frac{m_2^2}{4m_1^2}\right)\tilde{M}_{n_1 n_2},$$

where

$$\tilde{M}_{32} = \frac{1}{3}\begin{pmatrix} 2 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix},\ \tilde{M}_{34} = \frac{1}{15}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix},$$

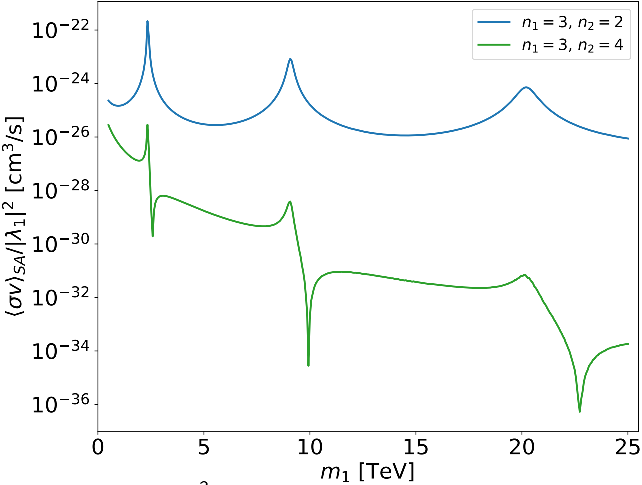
$$\tilde{M}_{54} = \tilde{M}_{56} = \frac{1}{35}\begin{pmatrix} 4 & 2 & -2\sqrt{2} \\ 2 & 1 & -\sqrt{2} \\ -2\sqrt{2} & -\sqrt{2} & 2 \end{pmatrix},\ \tilde{M}_{76} = \frac{1}{210}\begin{pmatrix} 25 & 0 & -15 & 10\sqrt{2} \\ 0 & 0 & 0 & 0 \\ -15 & 0 & 9 & -6\sqrt{2} \\ 10\sqrt{2} & 0 & -6\sqrt{2} & 8 \end{pmatrix}.$$

Sommerfeld enhancement at low velocity V



$m_1 = 8$ TeV, $m_2/m_1 = 1.2$, all other parameters set to 0

Sommerfeld enhancement at low velocity VI



$m_2/m_1 = 1.2$, $v = 10^{-3}$, all other parameters set to 0

Particle spectra

The spectra of particles produced in annihilations must also be known.

Regular annihilation:
PPPC 4 DM ID (Cirelli, 1012.4515)

Semi-annihilation:
Three annihilations channels:

$$\phi_1^i \phi_1^{i'} \to (\phi_2^0)^\dagger h, \quad \phi_1^i \phi_1^{i'} \to (\phi_2^0)^\dagger Z, \quad \phi_1^i \phi_1^{i'} \to (\phi_2^-)^\dagger W^-,$$

Three decays channels:

$$\begin{array}{lll} (\phi_2^0)^\dagger \to (\phi_1^0)^\dagger h, & (\phi_2^0)^\dagger \to (\phi_1^0)^\dagger Z, & (\phi_2^0)^\dagger \to (\phi_1^+)^\dagger W^+, \\ (\phi_2^-)^\dagger \to (\phi_1^-)^\dagger h, & (\phi_2^-)^\dagger \to (\phi_1^-)^\dagger Z, & (\phi_2^-)^\dagger \to (\phi_1^0)^\dagger W^+. \end{array}$$

So, nine possible spectras. Spectra generated with `Pythia`.

HESS

Limits on dark matter annihilation in the galactic core

We recast their limits on continuum searches.

Highly dependent on the dark matter density profile in the galactic core...
but it is not very well known.

Einasto with core radius r_c:

$$\rho_{\mathrm{E}}(r) = \rho_s \exp\left[-\frac{2}{\alpha_s}\left(\left(\frac{\max(r, r_c)}{r_s}\right)^{\alpha_s} - 1\right)\right].$$

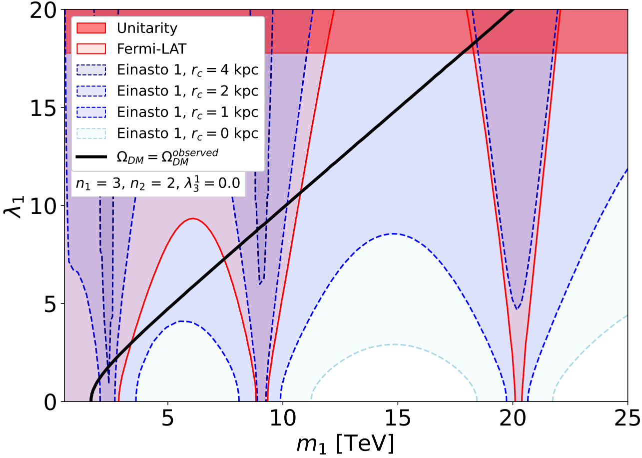
$\rho_s = 0.079\ \mathrm{GeV/cm}^3$, $r_s = 20.0$ kpc and $\alpha_s = 0.17$.

Fermi-LAT

Limits on dark matter annihilation in dwarf galaxies

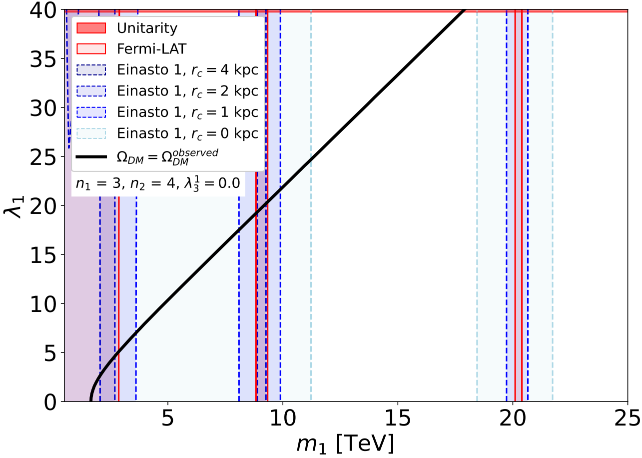
Very reliable, but not very strong limits.

Results I



$m_2/m_1 = 1.2$

Results II



$m_2/m_1 = 1.2$

Conclusion

Goal:

- Determine which models of inert scalar multiplets can:
  - Lead to efficient semi-annihilation
  - Avoid experimental constraints
  - Avoid stable charged particles in a technically natural way

Take home points:

- There are no efficient and viable model of one multiplet semi-annihilation.
- There is one category of models of two multiplets that satisfies all our requirements.
- Other models are not excluded, but must satisfy other constraints.
- All multiplet sizes are allowed if the galactic core is large enough.
- $n_1 = 3$ and $n_2 = 4$ is allowed even for very cuspy profiles.

Thanks!