Strategies to test beyond the Standard Model with rare Kaon decays

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Based on 2206.14748, 2311.04878, 2404.03643 & work in progress

In collaboration with G. D'Ambrosio, A. Iyer and S. Neshatpour

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Introduction

Standard Model : A success story!

3 gauge couplings

2 Higgs parameters

6 quark masses

3 quark mixing angles + 1 phase

3 (+3) lepton masses

(3 lepton mixing angles + 1 phase)

() = with neutrino masses

$\left.\right\}$ Flavour parameters

Flavour is at the heart of the Standard Model!

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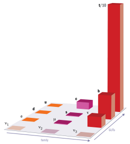
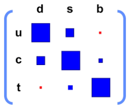
Still, a **deep understanding** of the inner structure of the SM **remains to be achieved**:

- hierarchy of the masses



- peculiar pattern of the CKM matrix elements



- Why 3 generations of quarks and leptons
- What fixes the size of CP violation
- ...

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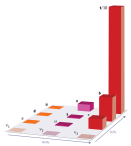

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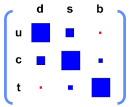

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Flavour parameters

Flavour is at the heart of the Standard Model!

Need for a more complete and predictive theory!

Flavour physics provides a **unique pathway** to understanding the fundamental organizing principle of the SM

Kaon Physics

Kaon decays are highly effective in probing new physics

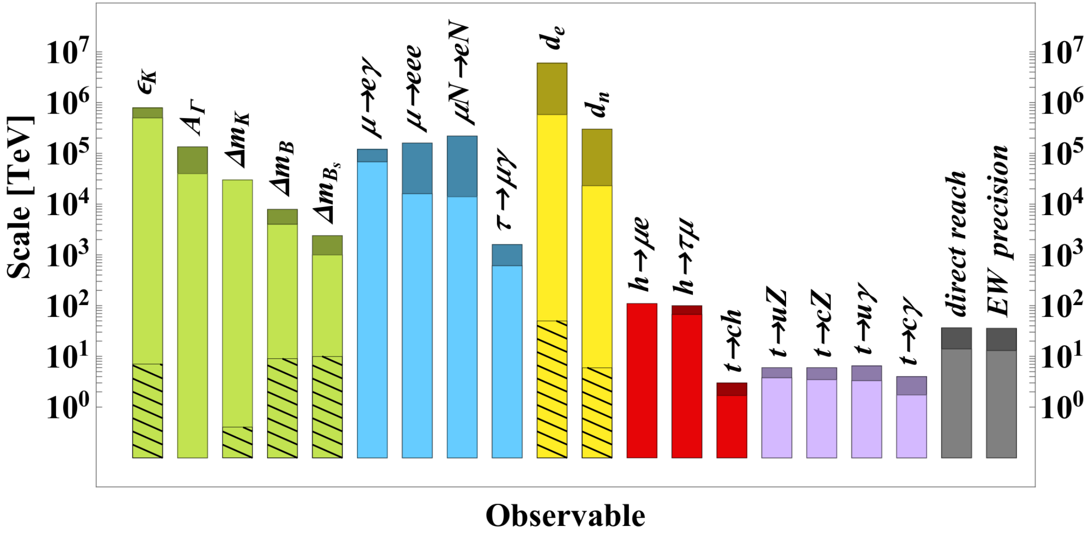
Even more suppressed than B-decays

→ **CKM** and **GIM** suppression of the SM contributions

→ kaon observables generally exhibit greater sensitivity to new physics than B-meson decays

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \frac{C_5}{\Lambda_M}\mathscr{O}^{(5)} + \sum_a \frac{C_6^a}{\Lambda^2}\mathscr{O}_a^{(6)} + \cdots$$

Assuming new physics in $\mathscr{O}_a^{(6)}$
and $C_6^a \sim \mathscr{O}(1)$



From : Physics Briefing Book : Input for the European Strategy for Particle Physics Update 2020

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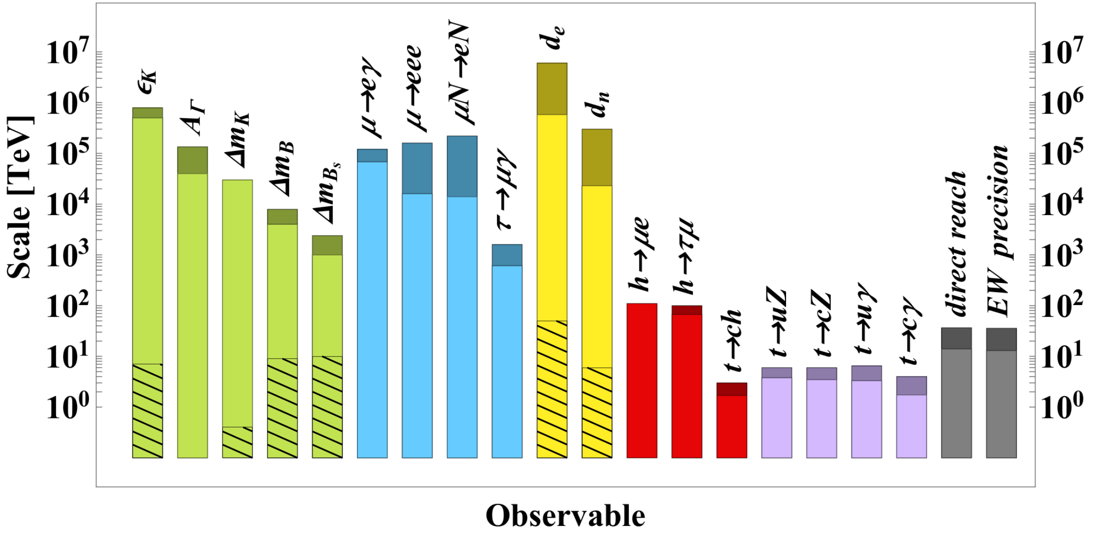

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The first indications of new physics could emerge through flavour measurements!

Rare kaon decays

- The rare decays of a charged or neutral kaon into a pion plus a pair of charged or neutral leptons are strongly suppressed in the SM

  → historical tools to study Flavor Changing Neutral Currents (FCNC)

- The "gold-plated" rare kaon decays $K^+ \to \pi^+ \nu\nu$ and $K_L \to \pi^0 \nu\nu$ do not suffer from large hadronic uncertainties

  → rates very precisely predicted in SM
  → complementary to B physics

- Mostly experimentally clean due to the limited number of possible decay channels

  → complementary probes of New Physics

Rare kaon decays

Weak effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell$$

Semi-leptonic local operators:

$$O_L^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\nu}_\ell\,\gamma^\mu (1-\gamma_5)\,\nu_\ell)\,,$$

$$O_9^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu \ell)\,, \qquad O_{10}^\ell = (\bar{s}\gamma_\mu P_L d)\,(\bar{\ell}\gamma^\mu \gamma_5 \ell)\,,$$

Scalar and pseudoscalar operators:

$$O_S = (\bar{s}P_R d)(\bar{\ell}\ell)\,, \qquad O_P = (\bar{s}P_R d)(\bar{\ell}\gamma_5 \ell)\,,$$

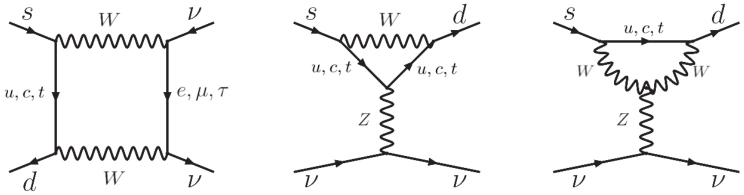
+ the chirality-flipped counterparts

NP contributions: $C_k \to C_k^{\rm SM} + \delta C_k$

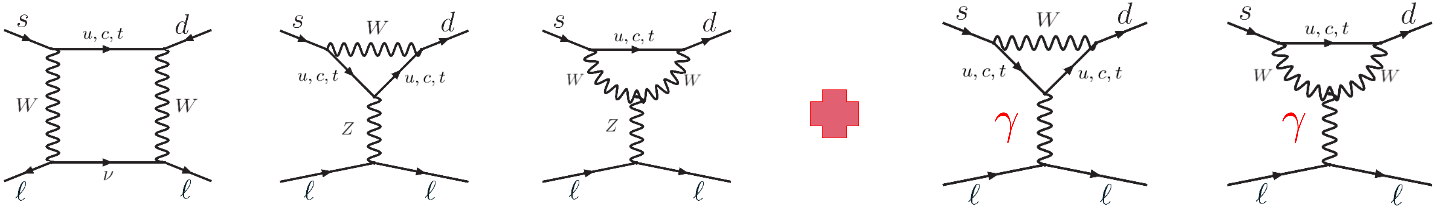
Rare kaon decays

- SD dominated
  - $K^+ \to \pi^+ \nu\nu$ and $K_L \to \pi^0 \nu\nu$ (golden channels)

Excellent theoretical precision!



- LD dominated
  - $K_L \to \mu\mu$, $K_S \to \mu\mu$, $K^+ \to \pi^+ \ell\ell$ and $K_L \to \pi^0 \ell\ell$, ...



$K^+ \to \pi^+ \nu\nu$

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu}) = \frac{\kappa_+(1+\Delta_{\rm EM})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_\ell \left[\mathrm{Im}^2 \left(\lambda_t C_L^\ell\right) + \mathrm{Re}^2 \left(-\frac{\lambda_c X_c}{s_w^2} + \lambda_t C_L^\ell \right) \right]$$

$\lambda_i = V^*_{is} V_{id}$

- Sum over the 3 neutrino flavours
- Electromagnetic radiative correction: $\Delta_{EM} \approx -0.003$ [Mescia, Smith '07]
- In the SM (top loop): $C^\ell_{L,\rm SM} = -X_{\rm SM}(x_t)/s_W^2$ NNLO QCD and 2-loop EW [Buchalla, Buras,'99; Misiak, Urban '99, Broad et al. '10]
- charm contribution: $X_c = \lambda^4 [P_c^{\rm SD} + \delta P_{c,u}^{\rm LD}]$ SD: NNLO QCD and NLO EW; LD: ChPT SD:[Buras et al. '05; Brod et al. '08] LD:[Isidori et al.'05]
- O_L matrix elements known from $K_{3\ell}$ branching ratios → included in κ_+ [Mescia, Smith '07]
- $\Gamma_{\rm SD}/\Gamma$ >90%

$K^+ \to \pi^+ \nu\nu$

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$\lambda_i = V_{is}^* V_{id}$

SM prediction:

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = (7.86 \pm 0.61) \times 10^{-11}$$

[D'Ambrosio, Iyer, FM, Neshatpour '22]

Sources of uncertainty:

SD ~ 2%, LD ~ 3%, Parametric ~ 7%

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = (7.73 \pm 0.61) \times 10^{-11}$$

[Brod, Gorbahn, Stamou '21]

$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = (8.60 \pm 0.42) \times 10^{-11}$$

[Buras, Venturini '22]

$K^+ \to \pi^+ \nu\nu$

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$$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{NA62}} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}$$

[NA62 Coll., Cortina Gil et al. ‘21]

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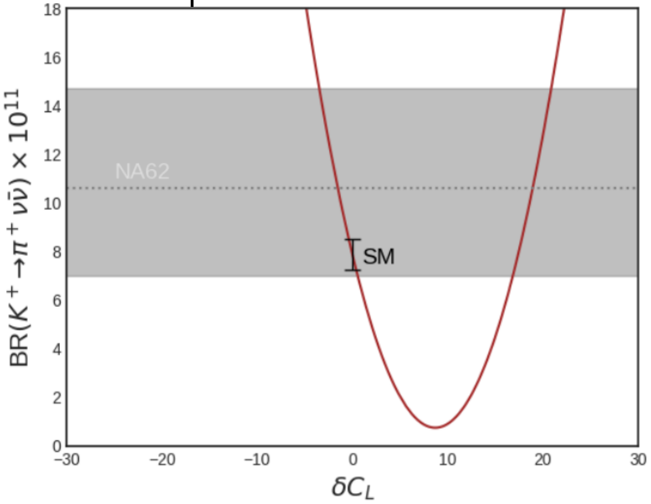
New Physics effects:

Lepton flavour universal



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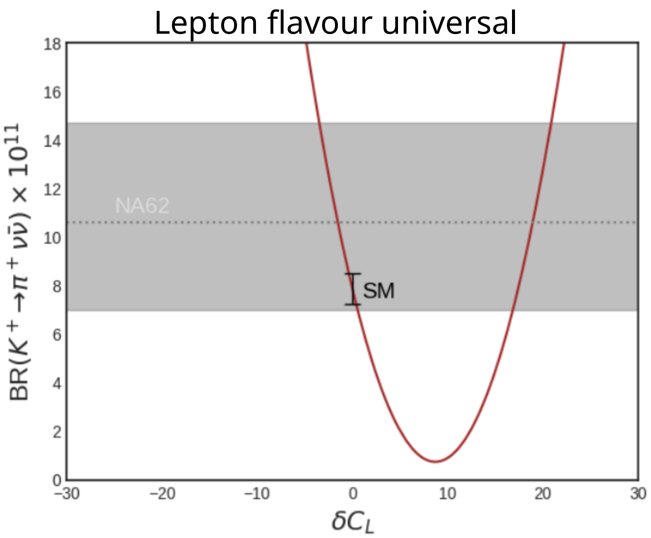

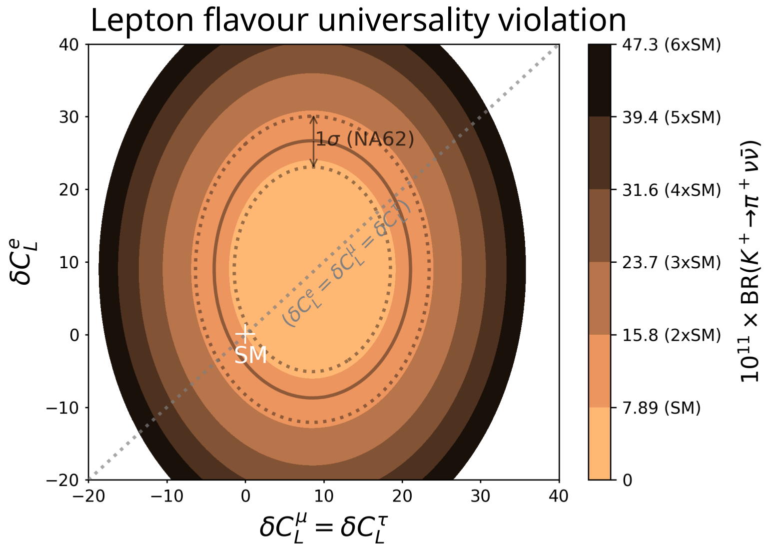

$K_L \to \pi^0 \nu\nu$

Theoretically very clean!

Sensitive to the CP-violating phase of the CKM matrix

Branching ratio:

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_w^4 \sum_\ell \mathrm{Im}^2 \left(\lambda_t C_L^\ell\right)$$

- Sum over the 3 neutrino flavours
- κ_L encodes the hadronic matrix element
- $C_{L,SM}$ same as for $K^+ \to \pi^+ \nu\nu$
- charm contributions below 1%
- 99% SD

$K_L \to \pi^0 \nu\nu$

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_w^4 \sum_\ell \mathrm{Im}^2\left(\lambda_t C_L^\ell\right)$$

SM prediction:

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} = (2.68 \pm 0.30) \times 10^{-11}$$

[D'Ambrosio, Iyer, FM, Neshatpour '22]

Sources of uncertainty:

SD ~ 2%, LD ~ 1%, Parametric ~ 11%

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} = (2.59 \pm 0.29) \times 10^{-11}$$

[Brod, Gorbahn, Stamou '21]

$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{SM}} = (2.94 \pm 0.15) \times 10^{-11}$$

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$K_L \to \pi^0 \nu\nu$

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$$\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu})_{\mathrm{KOTO}} < 3.0 \times 10^{-9} \text{ at } 90\% \text{ CL}$$ [KOTO Coll., Ahn et al. '18]

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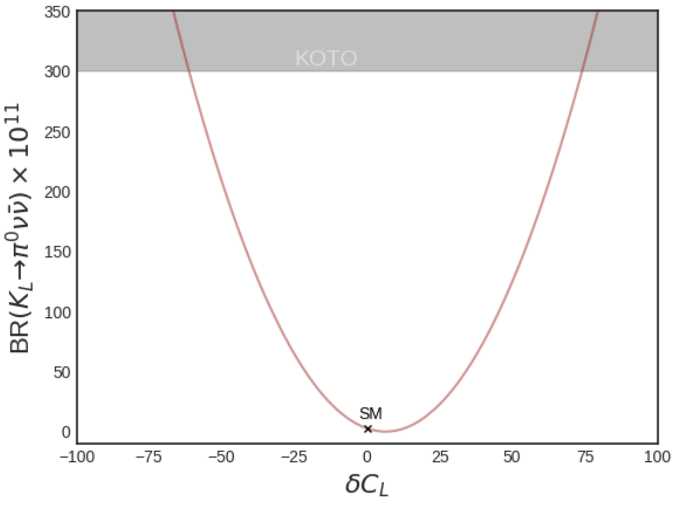
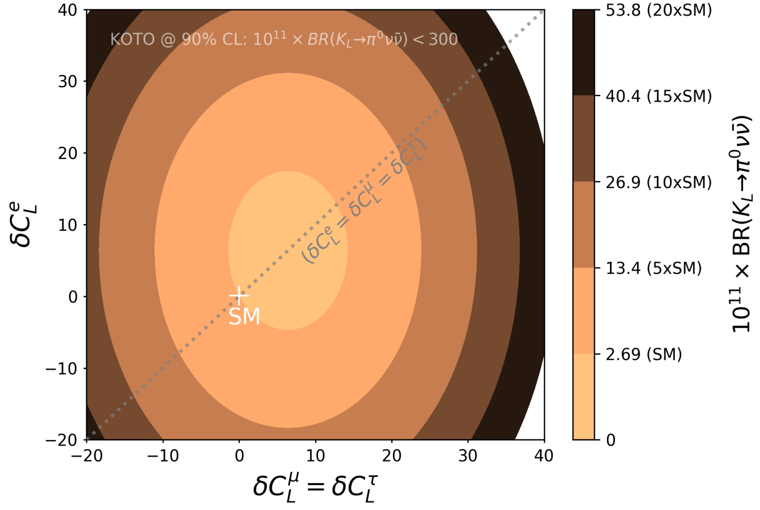


Lepton flavour universality violation



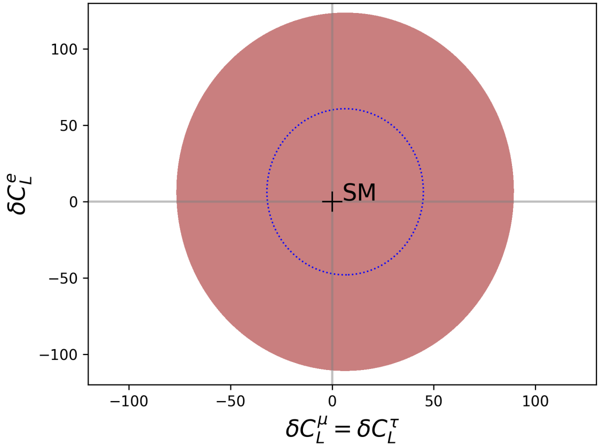
$K_L \to \pi^0 \nu\nu$ - Grossman–Nir bound

The matrix elements for $K_L \to \pi^0 \nu\nu$ and $K^+ \to \pi^+ \nu\nu$ transitions are related through isospin resulting in the Grossman–Nir bound :

$$BR(K_L \to \pi^0 \nu\nu) \leq 4.3 \times BR(K^+ \to \pi^+ \nu\nu)$$

Valid in the presence of most NP models

Considering the NA62 measurement of the charged mode:



S. Neshatpour, Symmetry 2024, 16(8), 946

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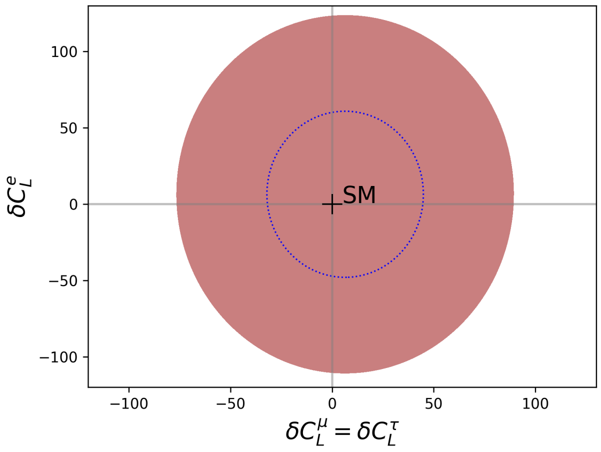

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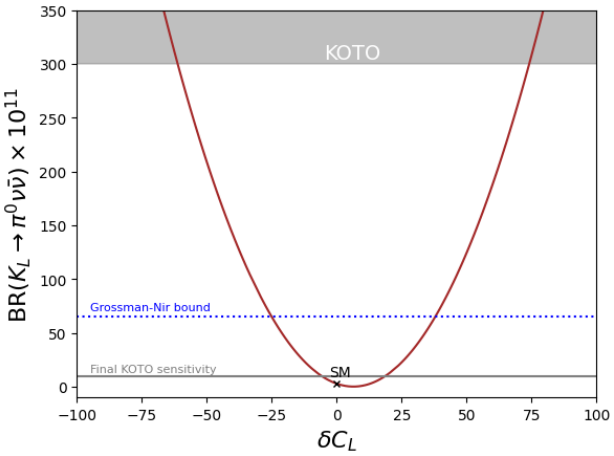
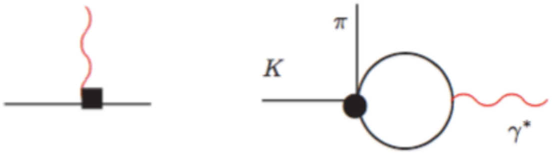


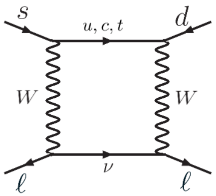
$K^+ \to \pi^+ \ell\ell$

Analogous mode to $B \to K \ell\ell$
CP- conserving, different from $K_L \to \pi^0 \ell\ell$
Can also be sensitive to scalar contribution

Long-distance dominated, mediated by single photon exchange $K^+ \to \pi^+ \gamma^*$



LD >> SD



$\Longrightarrow$ precise SM prediction not yet possible

The one–photon exchange has been studied at $O(p^6)$ in the chiral expansion
Gilman, Wise, '80; Ecker et al. '87; D'Ambrosio et al '98

→ includes an unknown combination of chiral couplings
→ described as a linear expansion: $(a_+ + b_+ z)$ $\qquad z = m^2(\ell\ell) / M_K$

LFUV in $K^+ \to \pi^+\ell\ell$

$$\mathcal{A}(z) \propto G_F M_K^2 (a + bz) + W^{\pi\pi}(z)$$ $z = m^2(\ell\ell) / M_K^2$

form factor parameters loop term

a_+ and b_+ are not theoretically determined precise enough to probe short distance physics

LFU predicts the same form factors a and b, for ℓ = e,μ

$a^{ee} \neq a^{\mu\mu}$ indicates LFUV NP: $$a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2}\,\mathrm{Re}\,[V_{td}V_{ts}^*(\boxed{C_9^\mu - C_9^e})]$$

[Crivellin, D'Ambrosio, Hoferichter, Tunstall '16]

Channel	a_+	b_+	Reference
ee	-0.561 ± 0.009	-0.694 ± 0.040	E865 '99 and NA48/2 '09 comb. [D'Ambrosio, Greynat, Knecht '18]
$\mu\mu$	-0.575 ± 0.013	-0.722 ± 0.043	NA62 Coll. '22

Lattice: determination at the 10% uncertainty can be expected

LFUV in $K^+ \to \pi^+\ell\ell$

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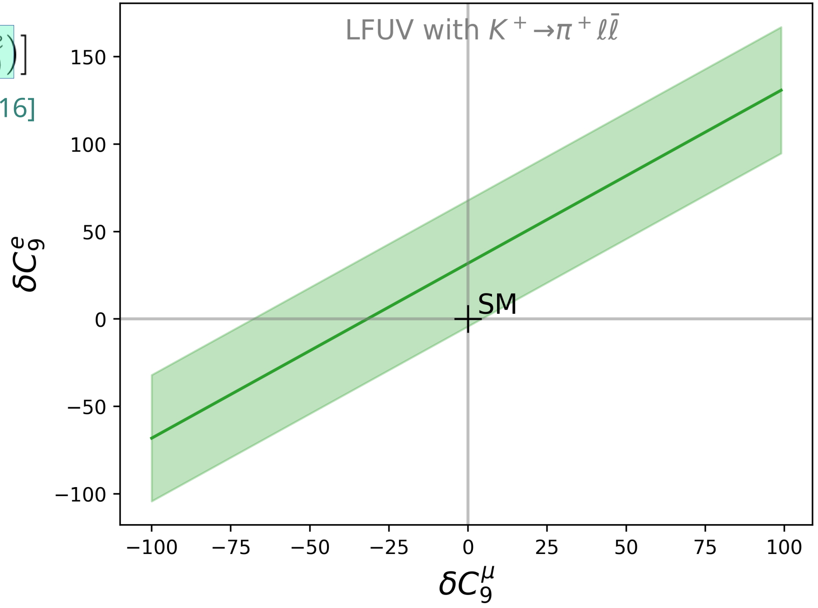
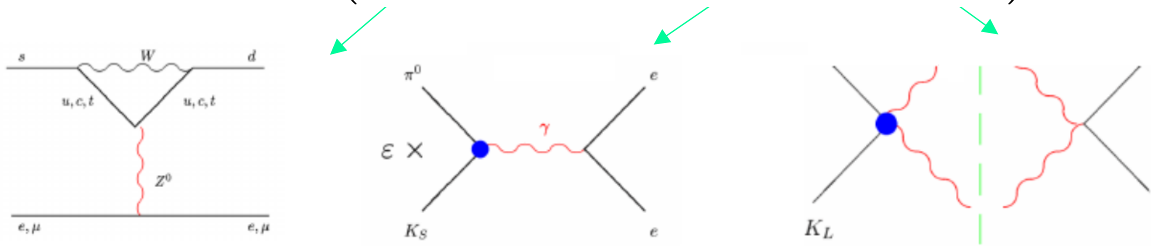


Lattice: determination at the 10% uncertainty can be expected

$K_L \to \pi^0 \ell \ell$

Smoking gun for the detection of direct CP violation!

$$\mathrm{BR}(K_L \to \pi^0 \ell \bar{\ell}) = \left(C^\ell_{\mathrm{dir}} \pm C^\ell_{\mathrm{int}} |a_S| + C^\ell_{\mathrm{mix}} |a_S|^2 + C^\ell_{\gamma\gamma} \right) \cdot 10^{-12}$$



[Dambrosio et al. '98; Isidori et al. '04; Mescia, Smith, Trine '06]

- C_{dir}: direct CP-violating term: purely short-distance effect contributing via the vector and axial Wilson coefficients C_9 and C_{10}
  It is proportional to the imaginary part of λ_t
- C_{mix}: indirect CP-violating term: long-distance dominated contribution of the single photon exchange via the $K_S \to \pi^0 \gamma^*$
  It is proportional to ε
- C_{int}: interference term of the above two contributions
- $C_{\gamma\gamma}$: CP-conserving term: long-distance-dominated contribution via two virtual photon exchanges.

$K_L \to \pi^0 \ell \ell$

	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)\boxed{(w_{7V}^2 + w_{7A}^2)}$	$(11.3 \pm 0.3)\boxed{w_{7V}}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)\boxed{(w_{7V}^2 + 2.32 w_{7A}^2)}$	$(2.63 \pm 0.06)\boxed{w_{7V}}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine '06]

$$\boxed{w_{7V}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_9\right] \,, \quad \boxed{w_{7A}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407 \times 10^{-4}} C_{10}\right]$$

SM prediction:

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\left(1.55^{+0.60}_{-0.48}\right) \times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\left(0.94^{+0.21}_{-0.20}\right) \times 10^{-11}$$

[D'Ambrosio, Iyer, FM, Neshatpour '22]

Experimental results:

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11} \qquad \text{at } 90\% \text{ CL}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11} \qquad \text{at } 90\% \text{ CL}$$

[KTeV '00 and '03]

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	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)\boxed{(w_{7V}^2 + w_{7A}^2)}$	$(11.3 \pm 0.3)\boxed{w_{7V}}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)\boxed{(w_{7V}^2 + 2.32 w_{7A}^2)}$	$(2.63 \pm 0.06)\boxed{w_{7V}}$	3.36 ± 0.20	5.2 ± 1.6

[Mescia, Smith, Trine '06]

$$\boxed{w_{7V}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407\times 10^{-4}}\boxed{C_9}\right], \quad \boxed{w_{7A}} = \frac{1}{2\pi}\mathrm{Im}\left[\frac{\lambda_t^{sd}}{1.407\times 10^{-4}}\boxed{C_{10}}\right]$$

SM prediction:

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\,\left(1.55^{+0.60}_{-0.48}\right) \times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\,\left(0.94^{+0.21}_{-0.20}\right) \times 10^{-11}$$

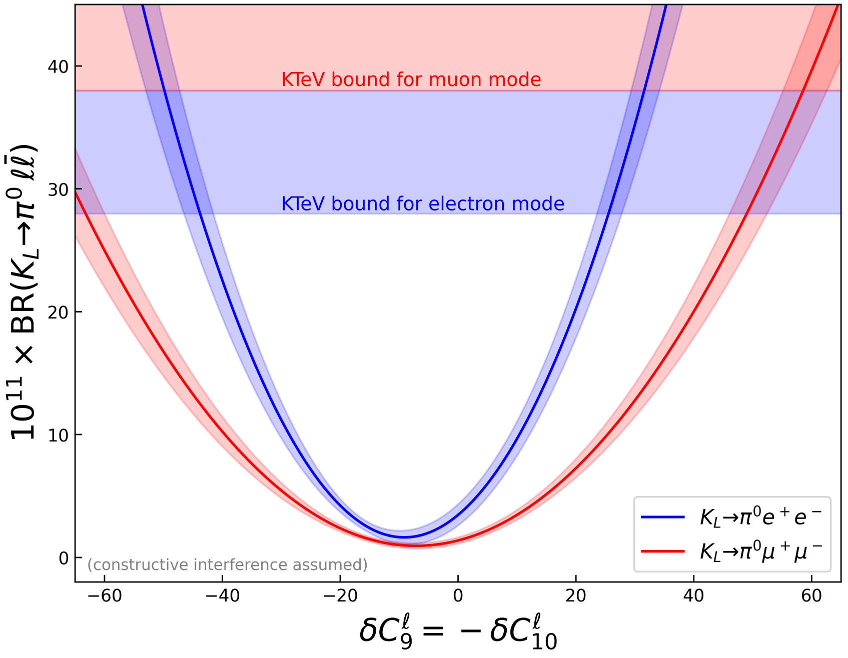
[D'Ambrosio, Iyer, FM, Neshatpour '22]

Expreimental results:

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11} \qquad \text{at 90\% CL}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11} \qquad \text{at 90\% CL}$$

[KTeV '00 and '03]



The electron channel is more sensitive to NP contributions than the muon channel!

$K_S \to \pi^0 \ell\ell$

The branching ratio of the neutral mode $K_S \to \pi^0 \ell\ell$ is about two orders of magnitude smaller than that of the charged mode

→ even more challenging to extract information on short-distance physics

The experimental determination of a_S is crucial for the SM prediction of the branching ratio of $K_L \to \pi^0 \ell\ell$ which is sensitive to NP contributions.

The branching ratio has been measured by NA48/1 experiment:

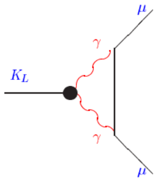
$$|a_S^{ee}| = 1.06^{+0.25}_{-0.21} \text{ and } |a_S^{\mu\mu}| = 1.54^{+0.40}_{-0.32}$$

The LHCb upgrade will be able to reduce the uncertainty in the determination of a_S

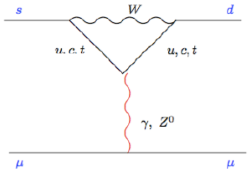
$K_L \to \mu\mu$

$K_L \to \mu\mu$ is long distance dominated, mediated by two photons via $K_L \to \gamma^*\gamma^*$

$$\mathrm{BR}(K_L \to \mu\bar{\mu}) = \tau_L \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left| N_L^{\mathrm{LD}} - \left(\frac{2m_\mu}{m_K}\frac{G_F \alpha_e}{\sqrt{2}\pi}\right) \mathrm{Re}\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^{\ell}\right]\right|^2 \qquad \beta_\mu = \sqrt{1 - 4m_\mu^2/M_K^2}$$



LD >> SD



Short distance: Y_c (charm contribution) and $C_{10}^{SM} = -Y(x_t)/s_W^2$ (top contribution)

Long distance: 2-photon exchange:

- absorptive part: calculable with good precision, almost saturates the experimental measurement
- dispersive part: smaller, but introduces a large theoretical uncertainty

$$N_L^{\mathrm{LD}} \propto (\chi_{\mathrm{disp}} + i\chi_{\mathrm{abs}}) \longrightarrow \quad N_L^{\mathrm{LD}} = \pm\,[0.54(77) - 3.95i] \times 10^{-11}\,(\mathrm{GeV})^{-2}$$

[D'Ambrosio et al. '86 '97;
Gomez Dumm, Pich '98;
Knecht et al. '99;
Isidori, Unterdorfer '03]

[D'Ambrosio et al. '17]
[Hoferichter et al. '23]

Prediction depends on the sign of $A(K_L \to \gamma\gamma)$ contribution determining the effect of the SD-LD interference

See talk by Martin Hoferichter on Sunday

$K_L \to \mu\mu$

$K_L \to \mu\mu$ is long distance dominated, mediated by two photons via $K_L \to \gamma^*\gamma^*$

$$\mathrm{BR}(K_L \to \mu\bar{\mu}) = \tau_L \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left| N_L^{\mathrm{LD}} - \left(\frac{2m_\mu}{m_K}\frac{G_F \alpha_e}{\sqrt{2}\pi}\right) \mathrm{Re}\left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell\right]\right|^2$$

SM prediction:

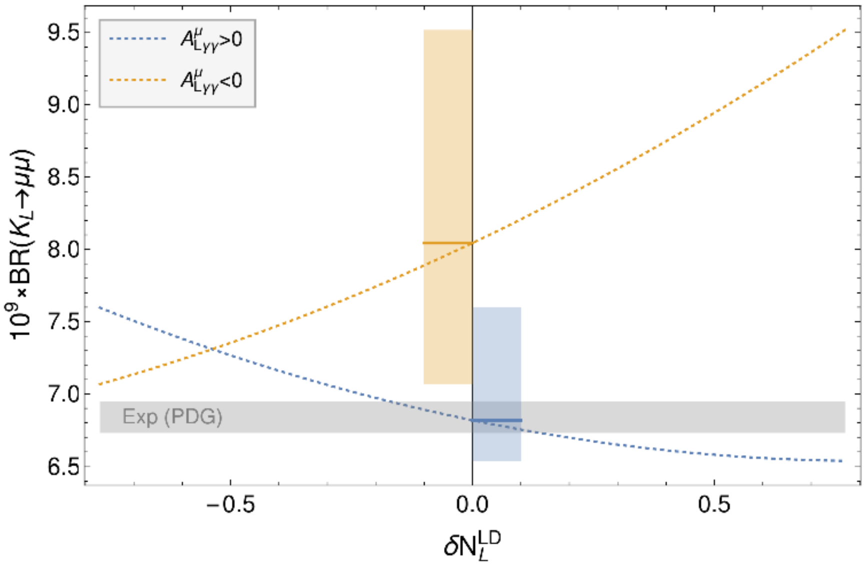
$$\mathrm{BR}(K_L \to \mu\bar{\mu})_{\mathrm{SM}} = \begin{cases} \mathrm{LD}(+): & (6.82^{+0.77}_{-0.24} \pm 0.04) \times 10^{-9} \\ \mathrm{LD}(-): & (8.04^{+1.46}_{-0.97} \pm 0.09) \times 10^{-9} \end{cases}$$

[D'Ambrosio, Iyer, FM, Neshatpour '22]

Experimental results:

$$\mathrm{BR}(K_L \to \mu\bar{\mu})_{\mathrm{exp}} = (6.84 \pm 0.11) \times 10^{-9}$$

[PDG]



Precise experimental measurement (less than 2% uncertainty)!
Large and asymmetric theoretical uncertainties!

$K_L \to \mu\mu$

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[D'Ambrosio, Iyer, FM, Neshatpour '22]

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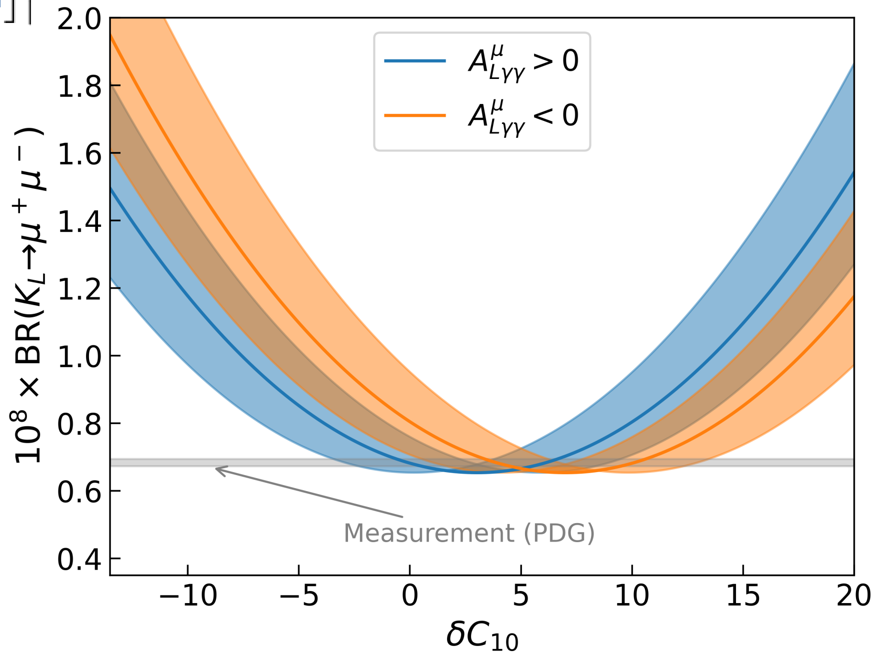

Precise experimental measurement (less than 2% uncertainty)!
Large and asymmetric theoretical uncertainties!

$K_S \to \mu\mu$

LD contribution for $K_S \to \mu\mu$ is cleaner: The leading $O(p^4)$ chiral contribution of $K_S \to \pi^+ \pi^- \to \gamma\gamma \to \mu^+ \mu^-$ is theoretically under better control

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \mathrm{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

$$\beta_\mu = \sqrt{1 - 4m_\mu^2 / M_K^2}$$

$$N_S^{\mathrm{LD}} = (-2.65 + 1.14i) \times 10^{-11}\,(\mathrm{GeV})^{-2}$$

[D'Ambrosio et al. '86 '97;
Gomez Dumm, Pich '98;
Knecht et al. '99;
Isidori, Unterdorfer '03]

The theoretical prediction is not affected by sign ambiguity

$K_S \to \mu\mu$

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \mathrm{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

SM prediction:

$$\mathrm{BR}(K_S \to \mu\bar{\mu})^{\mathrm{SM}} = (5.15 \pm 1.50) \times 10^{-12}$$

[D'Ambrosio, Iyer, FM, Neshatpour '22]

Experimental results:

$$\mathrm{BR}(K_S \to \mu\mu) < 2.1(2.4) \times 10^{-10} \ @90(95)\% \ \mathrm{CL}$$

[LHCb, '20]

$K_S \to \mu\mu$

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \mathrm{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t \boxed{C_{10}^\ell} \right] \right\}$$

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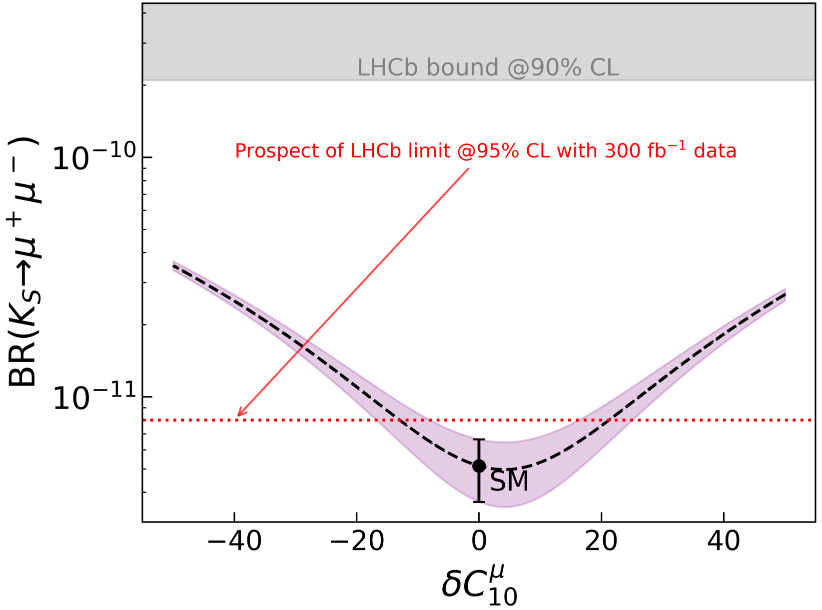

→ Future measurements of BR($K_S \to \mu\mu$) at LHCb will be crucial for exploring new physics scenarios involving scalar and pseudoscalar contributions!

→ Interference effects between $K_L \to \mu\mu$ and $K_S \to \mu\mu$ could provide valuable insights into short-distance physics

D'Ambrosio, Kitahara '17; Dery et al. '21

$K_S \to \mu\mu$ – scalar contribution

Add the scalar operator:

$$H_{eff}^{scalar} = C_s \mathcal{O} + \tilde{C}_s \tilde{\mathcal{O}}$$

$$\mathrm{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| B_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \mathrm{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

$$B_S = N_S^{\mathrm{LD}} - \frac{m_s M_K}{m_s + m_d} \mathrm{Re}(C_S - \tilde{C}_S)$$

[Chobanova et al '17]

How to get a handle on the scalar operator?

$K^+ \to \pi^+ \ell\ell$

Let's go back to $K^+ \to \pi^+ \ell\ell$

[Gao. '03,
Chen et al. '03]

$$\frac{d^2\Gamma}{dz\, d\cos\theta} = \frac{G_F^2 M_K^5}{2^8 \pi^3} \beta_\ell \, \lambda^{1/2}(z) \times \Big\{ |f_V|^2 \frac{\alpha^2}{16\pi^2} \lambda(z)(1 - \beta_\ell^2 \cos^2\theta) + |f_S|^2 \, z \beta_\ell^2$$

$$+ \mathrm{Re}(f_V^* f_S) \frac{\alpha\, r_\ell}{\pi} \beta_\ell \lambda^{1/2}(z) \cos\theta \Big\} \,, \qquad r_\ell = m_\ell / M_K$$

$$A_{\mathrm{FB}}(z) = \frac{\alpha G_F^2 M_K^5}{2^8 \pi^4} r_\ell \, \beta_\ell^2(z) \lambda(z) \mathrm{Re}\,(f_V^* f_S) \Big/ \left(\frac{d\Gamma(z)}{dz} \right)$$

Difficult to do this for the electron mode

AFB is non-zero only in case there are simultaneously vector and scalar contributions!

Bounds on f_S

$(K^+ \to \pi^+\mu^+\mu^-)$		
NA48/2	exp	$\|f_S\| <$
A_{FB}	$(-2.4 \pm 1.8) \times 10^{-2}$	4.2×10^{-5}
BR	$(9.62 \pm 0.21) \times 10^{-8}$	1.0×10^{-4}
NA62	exp	$\|f_S\| <$
A_{FB}	$(0.0 \pm 0.7) \times 10^{-2}$	7.7×10^{-6}
BR	$(9.16 \pm 0.06) \times 10^{-8}$	5.6×10^{-5}

$(K^+ \to \pi^+e^+e^-)$		
E865	exp	$\|f_S\| <$
A_{FB}	–	–
BR	$(2.988 \pm 0.040) \times 10^{-7}$	6.8×10^{-5}
NA48/2	exp	$\|f_S\| <$
A_{FB}	–	–
BR	$(3.14 \pm 0.04) \times 10^{-7}$	6.8×10^{-5}

[D'Ambrosio, Iyer, FM, Neshatpour '24]

- we constrain the scalar interactions by examining both the BR and the AFB
- The upper bounds on f_S from both observables demonstrate the sensitivity of current experimental measurements
- The most stringent limit on f_S arises from the NA62 measurement of AFB, highlighting its potential to probe new physics scenarios involving scalar interactions
- NA62 will have further results for $K^+ \to \pi^+\ell\ell$ (for muons and electrons)

Global analysis

All observables

Rare kaon observables

Observable	SM prediction	Experimental results
$\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}$
$\mathrm{BR}(K_L^0 \to \pi^0 \nu\nu)$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL
$\mathrm{LFUV}(a_+^{\mu\mu} - a_+^{ee})$	0	-0.014 ± 0.016
$\mathrm{BR}(K_L \to \mu\mu)\ (+)$	$(6.82^{+0.77}_{-0.29}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$
$\mathrm{BR}(K_L \to \mu\mu)\ (-)$	$(8.04^{+1.47}_{-0.98}) \times 10^{-9}$	
$\mathrm{BR}(K_S \to \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL
$\mathrm{BR}(K_L \to \pi^0 ee)(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL
~~$\mathrm{BR}(K_L \to \pi^0 ee)(-)$~~	~~$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$~~	
$\mathrm{BR}(K_L \to \pi^0 \mu\mu)(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL
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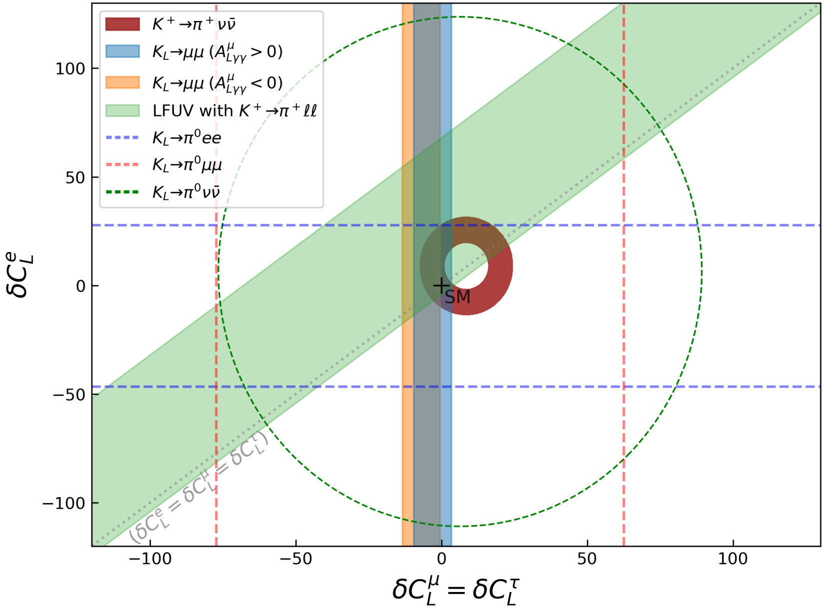
$$\delta C_L^e \neq \delta C_L^\mu = \delta C_L^\tau$$



We assume NP contributions of the charged and neutral leptons related to each other by the $SU(2)_L$ gauge symmetry and we work in the chiral basis

$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$

Bounds from individual observables:

Coloured regions: 68% CL measurements
Dashed lines: 90% upper limits

All observables

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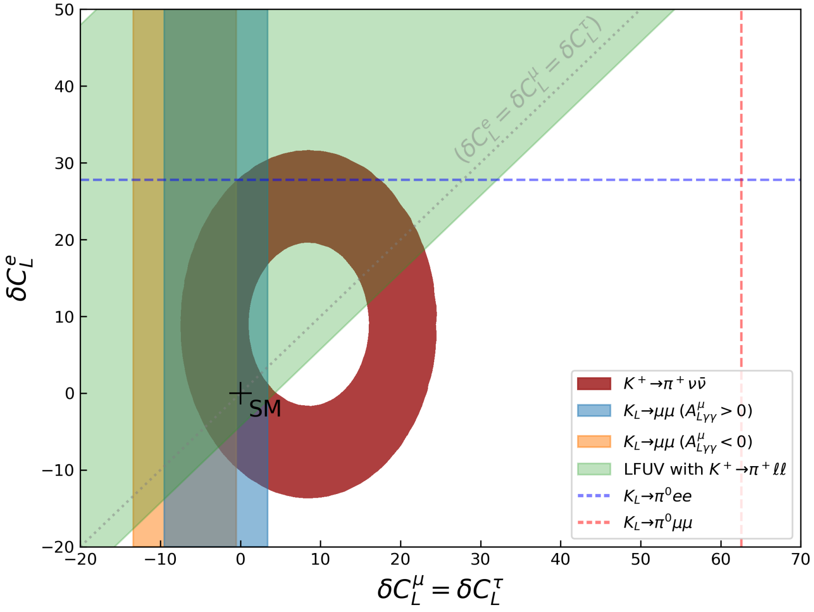

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Bounds from individual observables:

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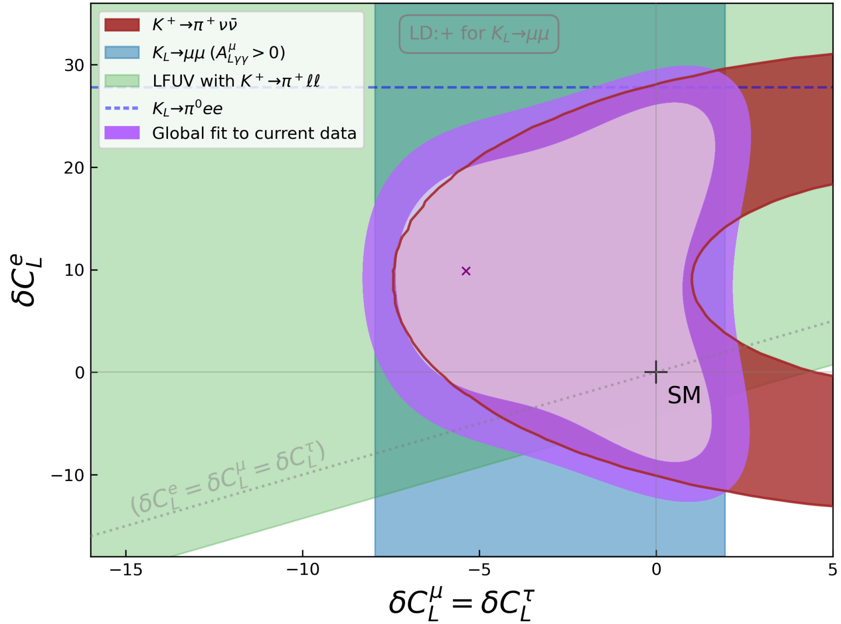
Dashed lines: 90% upper limits

All observables / Global fit

Fit (with SuperIso public program) for positive LD contributions to $K_L \to \mu\mu$



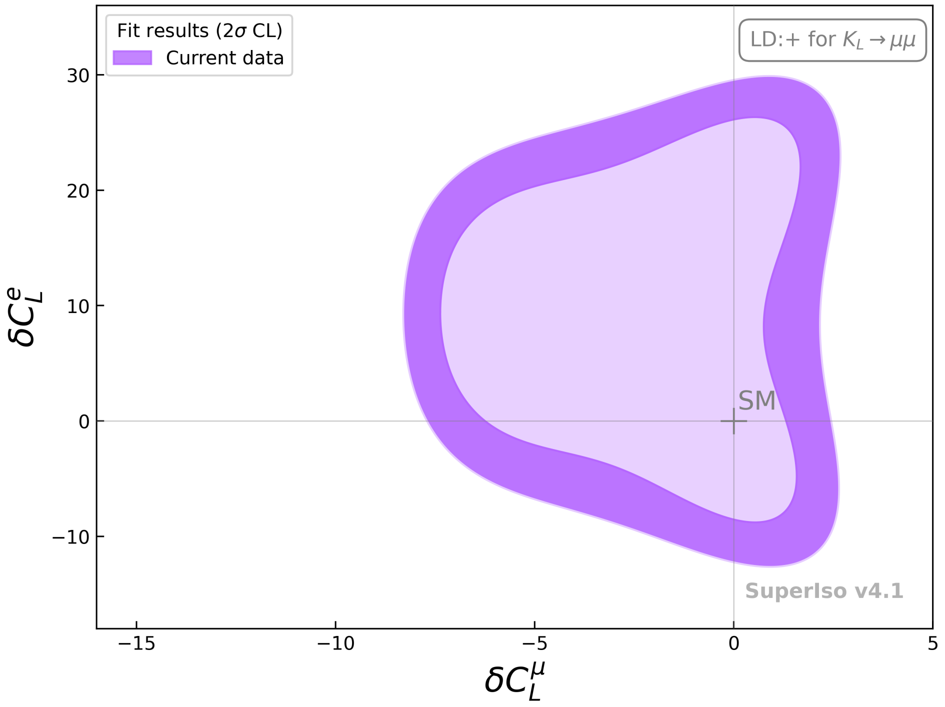
Lighter / darker purple region: 68% / 95% CL of global fit

Main constraining observables BR($K^+ \to \pi^+ \nu\nu$) followed by BR($K_L \to \mu\mu$)

Prospects

Prospects for future measurements

Prospects for KOTO-II

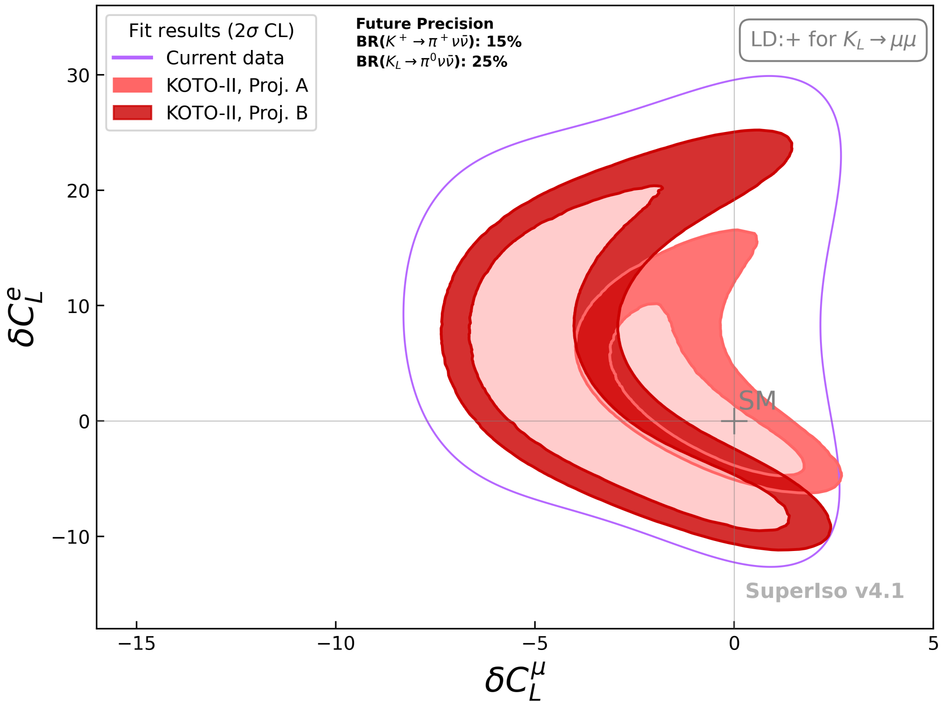


current sutuation

Projection A
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

Projection B
All measurements give current best-fit point with target precision of KOTO-II

Prospects for KOTO-II



current sutuation

Scenario 1

- Final NA62 precision for $K^+ \to \pi^+ \nu\nu$
- Final KOTO-II precision for $K_L \to \pi^0 \nu\nu$

Projection A
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

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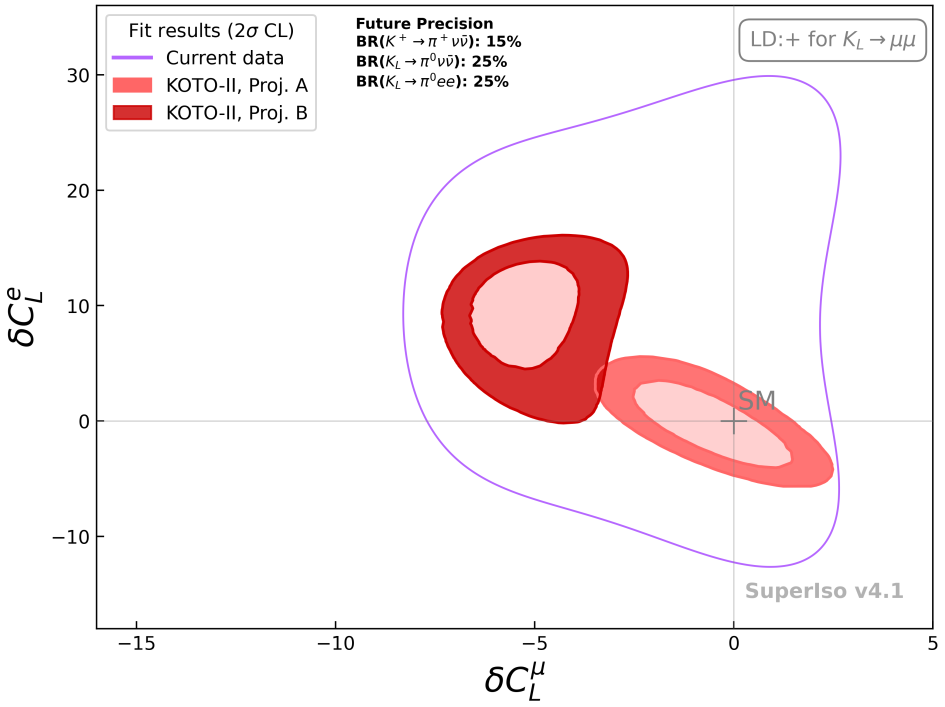
Prospects for KOTO-II



current sutuation

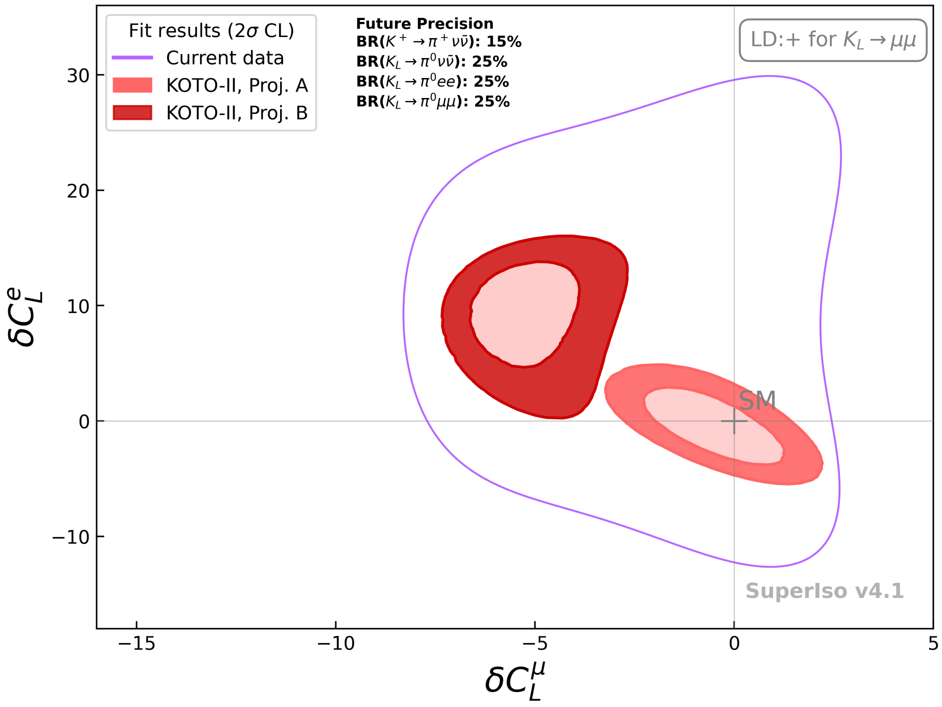
Scenario 2

- Final NA62 precision for $K^+ \to \pi^+ \nu\nu$
- Final KOTO-II precision for $K_L \to \pi^0 \nu\nu$
- KOTO-II $K_L \to \pi^0 ee$

Projection A
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

Projection B
All measurements give current best-fit point with target precision of KOTO-II

Prospects for KOTO-II



current sutuation

Scenario 3

- Final NA62 precision for $K^+ \to \pi^+ \nu\nu$
- Final KOTO-II precision for $K_L \to \pi^0 \nu\nu$
- KOTO-II $K_L \to \pi^0 ee$
- KOTO-II $K_L \to \pi^0 \mu\mu$

Projection A

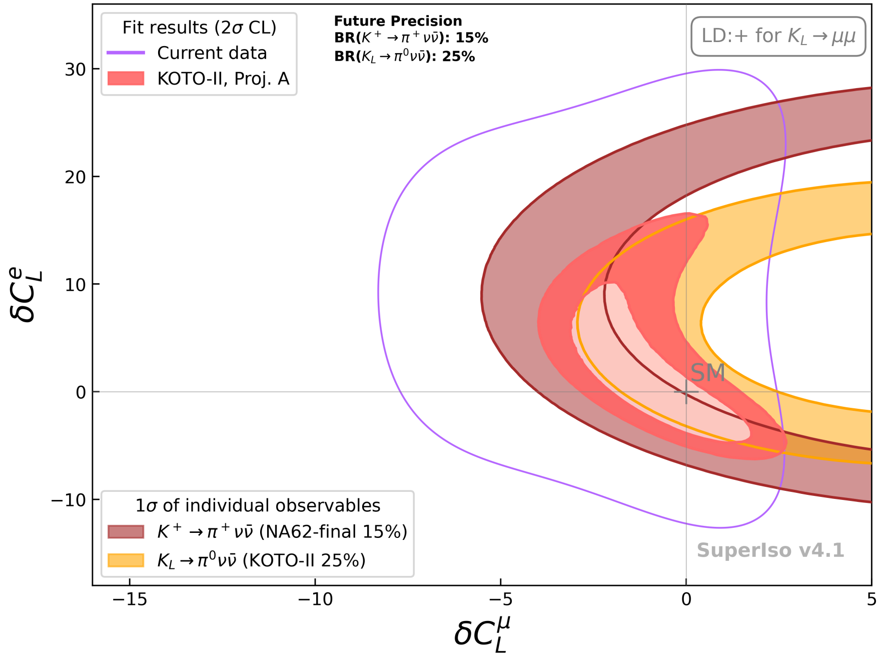
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

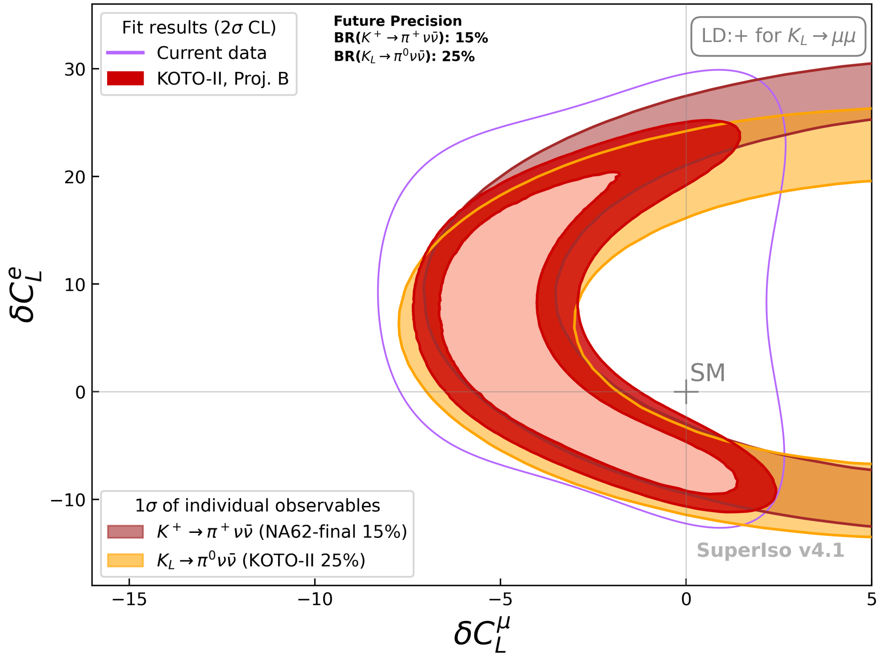
Projection B

All measurements give current best-fit point with target precision of KOTO-II

Impact of the main decays

Scenario 1



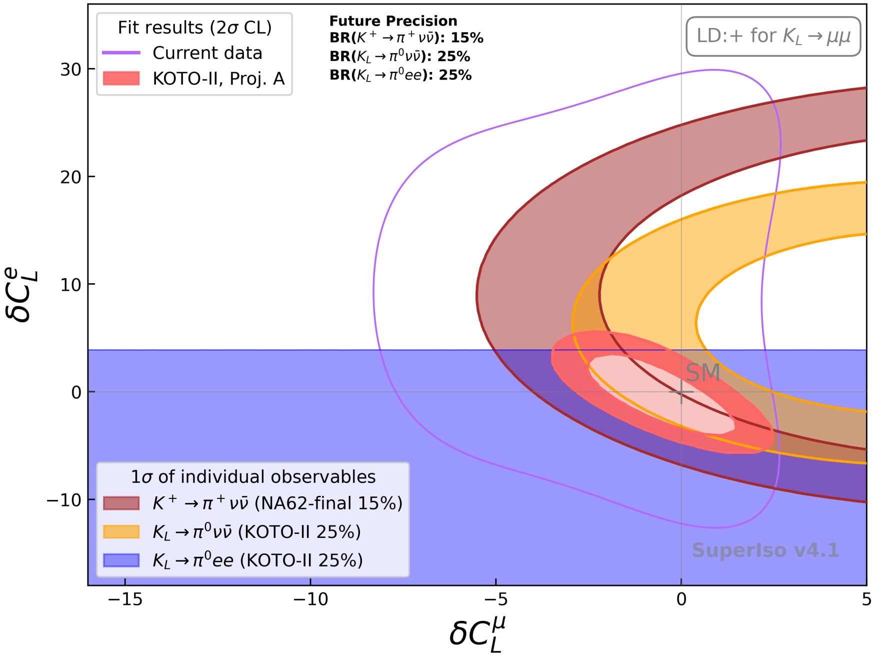


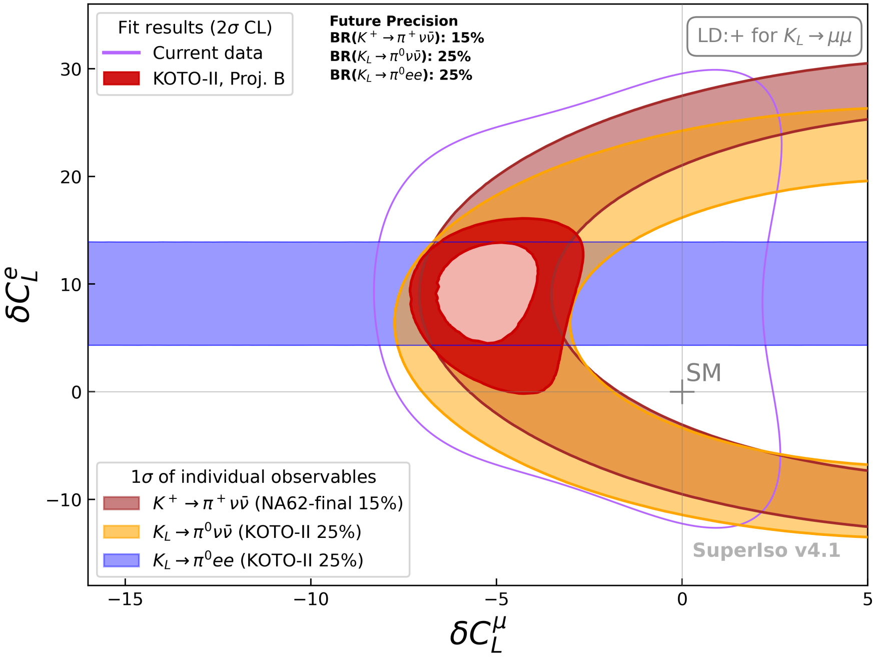
Projection A
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

Projection B
All measurements give current best-fit point with target precision of KOTO-II

Impact of the main decays

Scenario 2





Projection A

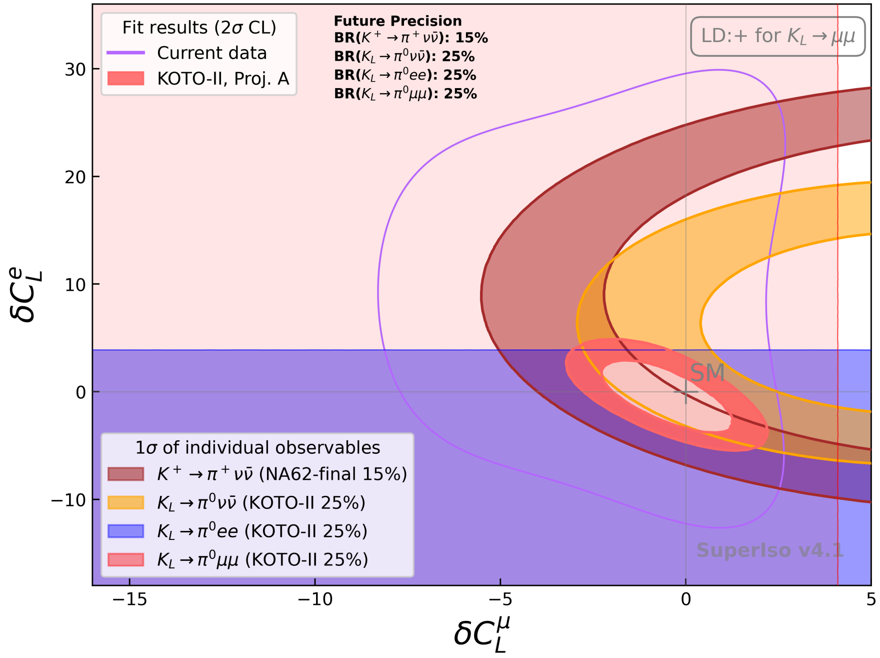
Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

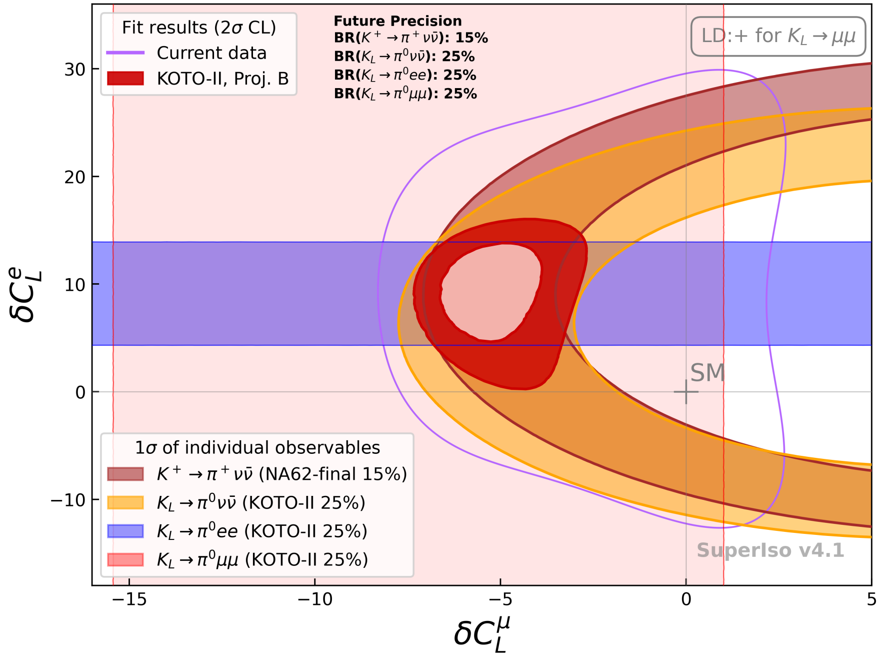
Projection B

All measurements give current best-fit point with target precision of KOTO-II

Impact of the main decays

Scenario 3





Projection A

Observables already measured are kept, others assumed at their SM values, all with target precision of KOTO-II

Projection B

All measurements give current best-fit point with target precision of KOTO-II

Conclusions

- Rare kaon decays offer valuable insights into short-distance physics

  ➪ providing indirect portal to new physics

- $K \to \pi \nu\nu$ decays are predicted in the SM with very high precision

  ➪ An experimental measurement of $K_L \to \pi\nu\nu$ will be of utmost important

  ➪ Together with $K_L \to \pi^0 ee$ and $K_L \to \pi^0 \mu\mu$ provides a great potential for probing and distinguishing new physics scenarios

  ➪ will be further enhanced via advancements in theoretical precision using continuum, data-driven approaches and lattice calculations

Improvement in the theoretical and experimental determination of rare kaon decays offers promising avenue for uncovering signs of new physics

Rosemary Fowler discovered the **kaon** particle during her doctoral research in **1948**

She received an honorary doctorate from Sir Paul Nurse, chancellor of the University of Bristol on 22 July **2024**

… at the age of **98**!


ありがとう