On $K_L \to (\pi^0)\ell^+\ell^-$ decays

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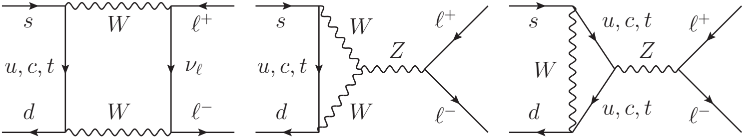
MH, Hoid, Ruiz de Elvira JHEP 04 (2024) 071, work in progress

Outline

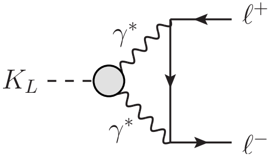
1. Improved Standard-Model prediction for $K_L \to \ell^+\ell^-$

2. Limits on modified Z couplings

$K_L \to \ell^+\ell^-$: anatomy



- **Dominant LD contribution** from $K_L \to \gamma^*\gamma^*$
- SD contribution from W boxes and Z exchange
- Need to control LD part to extract BSM constraints
- Consider normalized amplitude



$$R_L^\ell = \frac{\text{Br}[K_L \to \ell^+\ell^-]}{\text{Br}[K_L \to \gamma\gamma]} = 2\sigma_\ell(M_K^2)\left(\frac{\alpha}{\pi} r_\ell\right)^2 |\mathcal{A}_\ell(M_K^2)|^2$$

$$\sigma_\ell(M_K^2) = \sqrt{1 - \frac{4m_\ell^2}{M_K^2}} \qquad r_\ell = \frac{m_\ell}{M_K}$$

- Branching fractions for $K_L \to \mu^+\mu^-$ BNL E871 2000, KEK E137 1995, BNL E791 1995 and $K_L \to e^+e^-$ BNL E871 1998

$$\text{Br}[K_L \to \mu^+\mu^-] = 6.84(11) \times 10^{-9} \qquad \text{Br}[K_L \to e^+e^-] = 8.7^{+5.7}_{-4.1} \times 10^{-12}$$

- Normalization $\text{Br}[K_L \to \gamma\gamma] = 5.47(4) \times 10^{-4}$ known fairly well from PDG fit and $\frac{\text{Br}[K_L\to\gamma\gamma]}{\text{Br}[K_L\to 3\pi]} = 2.802(18) \times 10^{-3}$ KLOE 2003, NA48 2003
- To optimize uncertainties, use:
  - $\frac{\text{Br}[K_L\to\mu^+\mu^-]}{\text{Br}[K_L\to\pi^+\pi^-]} = 3.477(53) \times 10^{-6}$ BNL E871 2000, KEK E137 1995, BNL E791 1995
  - $\frac{\text{Br}[K_L\to\pi^+\pi^-]}{\text{Br}[K_L\to\gamma\gamma]} = 3.596(44)$ PDG global fit

$\hookrightarrow R_L^\mu = \frac{\text{Br}[K_L\to\mu^+\mu^-]}{\text{Br}[K_L\to\gamma\gamma]} = 1.250(24) \times 10^{-5}$

$K_L \to \ell^+\ell^-$: decomposition of the amplitude

- Imaginary part dominated by $\gamma\gamma$ cut

$$\mathrm{Im}_{\gamma\gamma}\mathcal{A}_\ell(M_K^2) = \frac{\pi}{2\sigma_\ell(M_K^2)} \log\big[y_\ell(M_K^2)\big] \qquad y_\ell(M_K^2) = \frac{1-\sigma_\ell(M_K^2)}{1+\sigma_\ell(M_K^2)}$$

- Real part in ChPT

$$\mathrm{Re}\,\mathcal{A}_\ell(M_K^2) = \frac{1}{\sigma_\ell(M_K^2)}\bigg[\mathrm{Li}_2\big[-y_\ell(M_K^2)\big] + \frac{1}{4}\log^2\big[y_\ell(M_K^2)\big] + \frac{\pi^2}{12}\bigg] + 3\log\frac{m_\ell}{\mu} - \frac{5}{2} + \chi(\mu)$$

- Low-energy constant $\chi(\mu)$ receives both LD and SD contributions

$$\chi(\mu) = \chi_{\mathrm{LD}}(\mu) + \chi_{\mathrm{SD}} \qquad \chi_{\mathrm{SD}}^{\mathrm{SM}} = -1.80(6)$$

↪ uncertainty from CKM, more on the sign later

- $\chi_{\mathrm{LD}}(\mu)$ regulates the UV divergence in ChPT for point-like $K_L \to \gamma^*\gamma^*$

↪ determined by **$K_L \to \gamma^*\gamma^*$ transition form factor** (TFF)

$K_L \to \gamma^*\gamma^*$: constraints

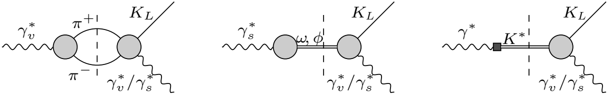
$K_L \to \gamma^*\gamma^*$ amplitude

$$\mathcal{A}^{\mu\nu}[K_L \to \gamma^*(q_1,\mu)\gamma^*(q_2,\nu)] = i\epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta} c(q_1^2, q_2^2)$$

- **Experimental constraints** on TFF $c(q_1^2, q_2^2)$
  - Normalization: $|c(0,0)| = 3.389(14) \times 10^{-9}\,\text{GeV}^{-1}$ from $K_L \to \gamma\gamma$
  - Slope b_{K_L}:

    $$\tilde{c}(q^2, 0) = \frac{c(q^2,0)}{c(0,0)} = 1 + b_{K_L} q^2 + \mathcal{O}(q^4)$$

    $\hookrightarrow$ from $K_L \to \ell^+\ell^-\gamma$
  - **Two-pion cuts** from $K_L \to \pi^+\pi^-\gamma$
- **Asymptotic behavior** from partonic calculation Isidori, Unterdorfer 2004
- Previously: model ansatz for TFF BMS (Bergström, Massó, Singer), DIP (D'Ambrosio, Isidori, and Portolés)
- Here: combine all constraints in **dispersive approach** MH, Hoid, Ruiz de Elvira 2024



- Dominant cuts from $\pi^+\pi^-$ (isovector) and ω,ϕ (isoscalar)
- K^* contribution relevant for the slope (on-shell $K^* \simeq K\pi \to \gamma$ forbidden)

$$b_{K_L} = 2.72(11)\,\text{GeV}^{-2} \gg M_\rho^{-2} \simeq 1.66\,\text{GeV}^{-2}$$

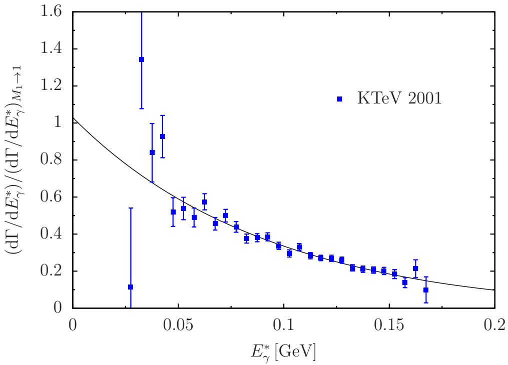
- Isospin decomposition

$$c^{\text{disp}}(q_1^2,q_2^2) = c_{vv}(q_1^2,q_2^2) + c_{vs}(q_1^2,q_2^2) + c_{sv}(q_1^2,q_2^2) + c_{ss}(q_1^2,q_2^2) + c_{K^*}^{\text{disp}}(q_1^2,q_2^2)$$
$$c_{K^*}^{\text{disp}}(q_1^2,q_2^2) = c_{vK^*}(q_1^2,q_2^2) + c_{K^*v}(q_1^2,q_2^2) + c_{sK^*}(q_1^2,q_2^2) + c_{K^*s}(q_1^2,q_2^2)$$

$\hookrightarrow$ singly-virtual $c_{vv}(q^2,0) + c_{vs}(q^2,0)$ from $K_L \to \pi^+\pi^-\gamma$ spectrum

- $SU(3)$ symmetry for doubly-virtual weights (and K^*)

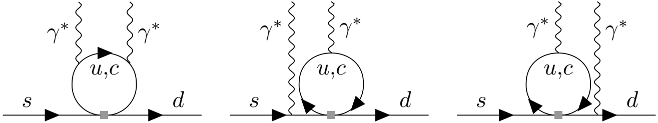
$K_L \to \pi^+\pi^-\gamma$ and $K_L \to \ell^+\ell^-\gamma$ spectra



- Data from KTeV 2001 by digitizing plot, KTeV 2006, FNAL E731 1993 not accessible at all
- Same problem for $K_L \to e^+e^-\gamma$ KTeV 2007, $K_L \to \mu^+\mu^-\gamma$ KTeV 2001

  $\hookrightarrow$ only fits of BMS and DIP model, original data not available
- Have to match to model parameters, **data preservation efforts crucial**

Asymptotic contribution



- Partonic calculation Isidori, Unterdorfer 2004, Simma, Wyler 1990, Herrlich, Kalinowski 1992

$$c^{\text{asym}}(q_1^2, q_2^2) = \frac{16\alpha G_F V_{us}^* V_{ud} F_K}{9\pi\sqrt{2}} \big[C_2(\mu) + 3C_1(\mu)\big] \Big[I(q_1^2, q_2^2) + T(q_1^2) + T(q_2^2)\Big]$$

- Write loop functions as dispersion integral ($q = u, c$), e.g.

$$T_q(q^2) = \frac{q^2}{\pi} \int_{s_q}^{\infty} ds \frac{\text{Im}\, T_q(s)}{s(s - q^2)}$$

$\hookrightarrow$ choose threshold s_q to separate the low-energy physics

- Wilson coefficients $C_2(M_W) = 1$, $C_1(M_W) = 0$, but large cancellation at low scales

$\hookrightarrow$ RG corrections important Buchalla et al. 1996

Sign of LD–SD interference

- Arguments for a relative minus between LD and SD Isidori, Unterdorfer 2004
  1. Pion pole almost saturates normalization $|c(0,0)| = 3.389(14) \times 10^{-9}\,\text{GeV}^{-1}$

$$c(0,0)\big|_{\pi^0} = \frac{2G_8F_\pi\alpha}{\pi}\frac{M_K^2}{M_K^2 - M_\pi^2} \qquad \big|c(0,0)\big|_{\pi^0} \simeq 4.2 \times 10^{-9}\,\text{GeV}^{-1}$$

  2. η, η' interfere destructively in any realistic mixing scheme Gómez Dumm, Pich 1998
  3. From 1 and 2, conclude that signs of $\mathcal{A}[K_L \to \gamma\gamma]$ and $\mathcal{A}[K_L \to \pi^0 \to \gamma\gamma]$ coincide
  4. Sign of G_8 not observable, but $G_8 < 0$ can be inferred from matrix elements of four-quark operators $\langle\pi^0|\mathcal{H}_W|K_L\rangle$ in a factorization assumption Pich, de Rafael 1996
  5. From 3 and 4, conclude $c(0,0) < 0$
- Positive interference of direct and indirect CP violation in $K_L \to \pi^0\ell^+\ell^-$ M. Knecht, Kaons@CERN 2023 also corresponds to $G_8 < 0$ Buchalla et al. 2003
- Can we get additional evidence from lattice QCD? talk by E.-H. Chao

Detour: η–η' mixing

- $K_L \to \gamma^*\gamma^*$ in $U(3)$ ChPT Ecker, Neufeld, Pich 1992, Gómez Dumm, Pich 1998

$$c(0,0) = \frac{2G_8 F_\pi \alpha}{\pi} \frac{M_K^2}{M_K^2 - M_\pi^2} \hat{c}(0,0) \qquad |\hat{c}_{\text{exp}}(0,0)| \simeq 0.8_{\mathcal{O}(p^2)} \ldots 1.2_{\mathcal{O}(p^4)}$$

$$\hat{c}(0,0) = 1 - \xi_\eta \frac{(c_\theta - 2\sqrt{2}s_\theta)(c_\theta + 2\sqrt{2}\rho s_\theta)}{3} + \xi_{\eta'} \frac{(2\sqrt{2}c_\theta + s_\theta)(2\sqrt{2}\rho c_\theta - s_\theta)}{3} \qquad \xi_P = \frac{M_K^2 - M_\pi^2}{M_P^2 - M_K^2}$$

  with η–η' mixing angle θ ($c_\theta = \cos\theta$, $s_\theta = \sin\theta$) and $\rho = 1 \Leftrightarrow$ nonet symmetry

- Limiting cases:
  - $SU(3)$: $\hat{c}(0,0) \propto 4M_K^2 - 3M_\eta^2 - M_\pi^2 = 0$
  - $\theta = -\arcsin\frac{1}{3} \simeq -19.5^\circ$: $\hat{c}(0,0) = 1 - \frac{16}{27}\xi_\eta(1-\rho) + \frac{7}{27}\xi_{\eta'}(1+8\rho)$

    $\hookrightarrow$ η contribution vanishes in nonet limit $\Rightarrow$ $\hat{c}(0,0) \simeq 1.8$ for $\rho = 1$

- Phenomenology: $\theta \simeq -20^\circ$, nonet symmetry works reasonably well for $P \to \gamma\gamma$

- Important role of η' well known for HLbL

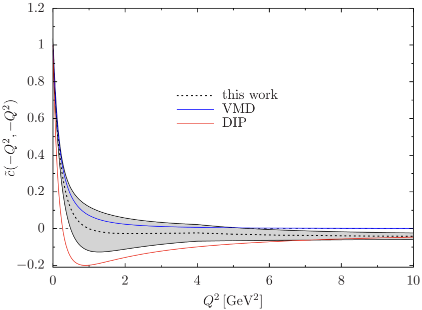
- Various higher-order effects make $\hat{c}(0,0) \simeq 1.8 \to 1.2$ well plausible, but

  $\hat{c}(0,0) \simeq 1.8 \to -1.2$ would imply almost 200% correction

Uncertainty estimates

- **Dispersive error**
  - Variation in cutoff parameters and VMD assumptions for doubly-virtual weights
  - Sum rule for $c(0,0)$ fulfilled at 115%, then imposed as constraint
  - Errors propagated from $K_L \to \pi^+\pi^-\gamma$ fit
- **Asymptotic error**
  - Use NLL RG in 't Hooft–Veltman scheme, variation to LL RG and NDR as error estimate
  - In loop integral: set $\mu^2 = |q_1^2 + q_2^2|/2$, $C_i(\mu)$ constant below $\mu_{\text{cut}} = 2\,\text{GeV}$
  - Lattice input to improve transition to asymptotic region?
- **Experimental error**
  - Uncertainty in slope parameter b_{K_L}
  - Could be improved with new data on $K_L \to \ell^+\ell^-\gamma$

Diagonal form factor and low-energy constant



	Re $\mathcal{A}_\mu^{\text{LD}}$	Re $\mathcal{A}_e^{\text{LD}}$	χ_{LD}^μ	χ_{LD}^e
VMD	−0.68(11)	31.36(26)	4.44(11)	7.6(3)
DIP	0.57(28)	32.46(71)	5.70(28)	8.7(7)
This work	−0.16(38)	31.68(98)	4.96(38)	8.0(1.0)

- Find less pronounced minimum than DIP, closer to VMD
- LEC at $\mu = 0.77\,\text{GeV}$ not lepton-flavor universal

  $\hookrightarrow$ higher chiral orders
- DIP and VMD errors propagated from input slope

Summary of $K_L \to \ell^+\ell^-$ predictions

Key results

$$\text{Im}\,\mathcal{A}_\mu(M_K^2) = -5.20(0) \qquad \text{Im}\,\mathcal{A}_e(M_K^2) = -21.59(1)$$

$$\text{Re}\,\mathcal{A}_\mu^{\text{LD}}(M_K^2) = -0.50_{\text{disp}} + 0.34_{\text{asym}} = -0.16(21)_{\text{disp}}(27)_{\text{asym}}(17)_{\text{exp}}[38]_{\text{total}}$$

$$\text{Re}\,\mathcal{A}_e^{\text{LD}}(M_K^2) = 31.99_{\text{disp}} - 0.31_{\text{asym}} = 31.68(59)_{\text{disp}}(73)_{\text{asym}}(27)_{\text{exp}}[98]_{\text{total}}$$

$$\text{Re}\,\mathcal{A}_\mu^{\text{LD+SD}}(M_K^2) = -1.96(39) \qquad \text{Re}\,\mathcal{A}_e^{\text{LD+SD}}(M_K^2) = 29.9(1.0)$$

- Avenues for further improvements:
  - Data for $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \ell^+\ell^-\gamma$ spectra
    $\hookrightarrow$ reduction of dispersive and experimental (slope) uncertainty
  - Interplay with lattice QCD
    $\hookrightarrow$ sign between LD and SD, matching to asymptotic contribution

BSM constraints: modified Z couplings

- Compare different channels regarding their SD sensitivity

  $\hookrightarrow$ **modified Z couplings** Buras, Silvestrini 1999

$$\mathcal{L}_{\text{EFT}} = C_A^\ell \bar{s}\gamma^\mu\gamma_5 d\, \bar{\ell}\gamma_\mu\gamma_5\ell + \text{h.c.} = -\frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} C_{7A}^\ell\, \bar{s}\gamma^\mu(1-\gamma_5)d\, \bar{\ell}\gamma_\mu\gamma_5\ell + \text{h.c.}$$

$$\mathcal{L}_Z = \frac{G_F}{\sqrt{2}}\frac{e}{2\pi^2}M_Z^2\frac{\cos\theta_W}{\sin\theta_W}Z_{ds}\,\bar{s}\gamma^\mu(1-\gamma_5)d\, Z_\mu + \text{h.c.}$$

$$C_A^\ell = \frac{G_F}{\sqrt{2}}V_{us}^* V_{ud} C_{7A}^\ell = \frac{\alpha(M_Z)G_F}{2\pi\sqrt{2}\sin^2\theta_W}Z_{ds}$$

- For $K_L \to \mu^+\mu^-$ we find

$$\text{Re}\,\mathcal{A}_\ell^{\text{BSM}} = -\frac{4m_\ell F_K}{\mathcal{N}}\text{Re}\,C_A^\ell \qquad \mathcal{N} = -\frac{\alpha}{\pi}\frac{m_\ell}{M_K}\sqrt{\frac{16\pi\Gamma[K_L\to\gamma\gamma]}{M_K}}$$

$$\text{Re}\,Z_{ds}^{\text{BSM}} = 1.6(5)_{\text{exp}}(8)_{\text{SM}}\times 10^{-4} \qquad |\text{Re}\,Z_{ds}^{\text{BSM}}| < 2.8\times 10^{-4} \quad \text{at 90\% C.L.}$$

- In the following: study sensitivity/complementarity of other channels

  $\hookrightarrow K_S \to \ell^+\ell^-$, $K \to \pi\nu\bar{\nu}$, $K \to \pi\ell^+\ell^-$

$K_S \to \mu^+\mu^-$

- Master formula

$$\mathrm{Br}[K_S \to \ell^+\ell^-]\big|_{\mathrm{SD}} = \frac{\tau_S M_K}{8\pi}\sigma_\ell(M_K^2)\big|C_{\mathrm{SD}}\big|^2$$

$$C_{\mathrm{SD}} = -\frac{\sqrt{2}\,G_F\alpha(M_Z)m_\ell F_K}{\pi\sin^2\theta_W}\Big(\mathrm{Im}\,\lambda_t Y(x_t) + \mathrm{Im}\,Z_{ds}^{\mathrm{BSM}}\Big)$$

- SM prediction Ecker, Pich 1991 and experimental limit LHCb 2020

$$\mathrm{Br}[K_S \to \mu^+\mu^-]\big|_{\mathrm{SM}} \simeq 5.0(3)\times 10^{-12} \qquad \mathrm{Br}[K_S \to \mu^+\mu^-] < 2.1\times 10^{-10}$$

- Resulting limit

$$|\mathrm{Im}\,Z_{ds}^{\mathrm{BSM}}| < 45\times 10^{-4} \quad \text{at 90\% C.L.}$$

- Possible improvements
  - SM prediction carries $\simeq 30\%$ uncertainty from higher chiral orders

    $\hookrightarrow$ **dispersion relations** Colangelo, Stucki, Tunstall 2016
  - How far can the limit improve at LHCb? talk by D. Martínez Santos

$K \to \pi\nu\bar{\nu}$

- Master formulae

$$\text{Br}[K^+ \to \pi^+\nu\bar{\nu}] = \kappa_+(1+\Delta_{\text{EM}})\left[\left(\frac{\text{Im}\,\lambda_t X_t + \text{Im}\,Z_{ds}^{\text{BSM}}}{\lambda^5}\right)^2 + \left(\frac{\text{Re}\,\lambda_c}{\lambda}(P_c+\delta P_{c,u}) + \frac{\text{Re}\,\lambda_t X_t + \text{Re}\,Z_{ds}^{\text{BSM}}}{\lambda^5}\right)^2\right]$$

$$\text{Br}[K_L \to \pi^0\nu\bar{\nu}] = \kappa_L r_{\epsilon_K}\left(\frac{\text{Im}\,\lambda_t X_t + \text{Im}\,Z_{ds}^{\text{BSM}}}{\lambda^5}\right)^2$$

- $K^+ \to \pi^+\nu\bar{\nu}$ has *CP*-even and *CP*-odd contributions, current limits NA62 2021

$$\text{Re}\,Z_{ds}^{\text{BSM}} = -1.1(1.5)_{\text{exp}}(0.2)_{\text{SM}} \times 10^{-4} \qquad |\text{Re}\,Z_{ds}^{\text{BSM}}| < 3.1 \times 10^{-4} \quad \text{at 90\% C.L.}$$

$$\text{Im}\,Z_{ds}^{\text{BSM}} = 2.4(2.3)_{\text{exp}}(0.4)_{\text{SM}} \times 10^{-4} \qquad |\text{Im}\,Z_{ds}^{\text{BSM}}| < 5.4 \times 10^{-4} \quad \text{at 90\% C.L.}$$

assuming $\text{Im}\,Z_{ds}^{\text{BSM}} = 0$ or $\text{Re}\,Z_{ds}^{\text{BSM}} = 0$, respectively

- Similar limit for $\text{Re}\,Z_{ds}^{\text{BSM}}$ as from $K_L \to \mu^+\mu^-$, but need to disentangle $\text{Im}\,Z_{ds}^{\text{BSM}}$
- Limit from $K_L \to \pi^0\nu\bar{\nu}$ KOTO, Kaons@CERN 2023

$$\text{Im}\,Z_{ds}^{\text{BSM}} = -2(11)_{\text{exp}} \times 10^{-4} \qquad |\text{Im}\,Z_{ds}^{\text{BSM}}| < 18 \times 10^{-4} \quad \text{at 90\% C.L.}$$

$\hookrightarrow$ direct probe of $\text{Im}\,Z_{ds}^{\text{BSM}}$

$K_L \to \pi^0 \ell^+ \ell^-$: SM prediction

- Master formula Buchalla et al. 2003, Isidori et al. 2003

$$\text{Br}[K_L \to \pi^0 \ell^+ \ell^-] = 10^{-12} \times \left[C^\ell_{\text{mix}} + C^\ell_{\text{int}} \left(\frac{\text{Im}\,\lambda_t}{10^{-4}} \right) + C^\ell_{\text{dir}} \left(\frac{\text{Im}\,\lambda_t}{10^{-4}} \right)^2 + C^\ell_{\text{CPC}} \right]$$

$$C^e_{\text{mix}} = 15.7(3) a_S^2 \qquad C^e_{\text{int}} = 8.91 \bar{y}_{7V} a_S \qquad C^e_{\text{dir}} = 2.67 (\bar{y}^2_{7V} + \bar{y}^2_{7A}) \qquad C^e_{\text{CPC}} \approx 0$$

$$C^\mu_{\text{mix}} = 3.7(1) a_S^2 \qquad C^\mu_{\text{int}} = 2.12 \bar{y}_{7V} a_S \qquad C^\mu_{\text{dir}} = 0.63 (\bar{y}^2_{7V} + \bar{y}^2_{7A}) + 0.85 \bar{y}^2_{7A} \qquad C^\mu_{\text{CPC}} = 5.2(1.6)$$

- Ingredients:
  - $K_S \to \pi^0 \gamma^*$ form factor for indirect *CP* violation
    $\hookrightarrow$ $\text{Br}[K_S \to \pi^0 \ell^+ \ell^-]$ NA48 2003, 2004 + VMD assumptions: $a_S = 1.2(2)$
  - Sign of interference positive M. Knecht, Kaons@CERN 2023
  - For muon channel: *CP*-conserving two-photon contribution Isidori et al. 2003
  - Wilson coefficients: $\bar{y}^{\text{SM}}_{7V} = 0.73(4)$, $\bar{y}^{\text{SM}}_{7A} = -0.68(3)$ Buchalla et al. 2003
- SM predictions and experimental limits KTeV 2000, 2004

$$\text{Br}[K_L \to \pi^0 e^+ e^-]\big|_{\text{SM}} = 3.89(95) \times 10^{-11} \qquad \text{Br}[K_L \to \pi^0 e^+ e^-] < 2.8 \times 10^{-10}$$

$$\text{Br}[K_L \to \pi^0 \mu^+ \mu^-]\big|_{\text{SM}} = 1.52(22)(16)[28] \times 10^{-11} \qquad \text{Br}[K_L \to \pi^0 \mu^+ \mu^-] < 3.8 \times 10^{-10}$$

$K_L \to \pi^0\ell^+\ell^-$: modified Z couplings

- Wilson coefficients modified according to

$$\bar{y}_{7V} = \bar{y}_{7V}^{\text{SM}} + \frac{1-4\sin^2\theta_W}{2\pi\sin^2\theta_W}\frac{\text{Im}\,Z_{ds}^{\text{BSM}}}{\text{Im}\,\lambda_t} \qquad \bar{y}_{7A} = \bar{y}_{7A}^{\text{SM}} - \frac{1}{2\pi\sin^2\theta_W}\frac{\text{Im}\,Z_{ds}^{\text{BSM}}}{\text{Im}\,\lambda_t}$$

- Resulting limits

$$\text{Im}\,Z_{ds}^{\text{BSM}} = -1.7(8.4)_{\text{exp}}(0.1)_{\text{SM}} \times 10^{-4} \qquad |\text{Im}\,Z_{ds}^{\text{BSM}}| < 14 \times 10^{-4} \quad \text{at 90\% C.L.}$$

$$\text{Im}\,Z_{ds}^{\text{BSM}} = -1.5(13.8)_{\text{exp}}(0.1)_{\text{SM}} \times 10^{-4} \qquad |\text{Im}\,Z_{ds}^{\text{BSM}}| < 23 \times 10^{-4} \quad \text{at 90\% C.L.}$$

$\hookrightarrow$ for now, some room before theory uncertainties kick in, but need to improve

- **$K_S \to \pi^0\gamma^*$ form factor**:
  - Improved measurement of $K_S \to \pi^0\ell^+\ell^-$ at LHCb, ideally including spectrum
    talk by D. Martínez Santos
  - Lattice QCD talk by A. Portelli
- **Two-photon corrections**:
  - **Dispersive methods**: need to reconstruct $K_L \to \pi^0\gamma^*\gamma^*$ instead of $K_L \to \gamma^*\gamma^*$ as for $K_L \to \ell^+\ell^-$, some similarities to $\eta^{(\prime)} \to \pi^0\ell^+\ell^-$ Schäfer et al. 2023
  - Lattice QCD talk by E.-H. Chao

Modified Z couplings: summary

Channel	Standard Model	Experiment	$\lvert\mathrm{Re}\, Z_{ds}^{\mathrm{BSM}}\rvert \times 10^4$	$\lvert\mathrm{Im}\, Z_{ds}^{\mathrm{BSM}}\rvert \times 10^4$
$K_L \to \mu^+\mu^-$	$7.44^{+0.41}_{-0.34} \times 10^{-9}$	$6.84(11) \times 10^{-9}$	< 2.8	–
$K_S \to \mu^+\mu^-$	$5.0(1.6) \times 10^{-12}$	$< 2.1 \times 10^{-10}$	–	< 45
$K^+ \to \pi^+\nu\bar{\nu}$	$8.2(5) \times 10^{-11}$	$(10.6^{+4.0}_{-3.4} \pm 0.9) \times 10^{-11}$	< 3.1	< 5.4
$K_L \to \pi^0\nu\bar{\nu}$	$2.79(23) \times 10^{-11}$	$< 2.0 \times 10^{-9}$	–	< 18
$K_L \to \pi^0 e^+e^-$	$3.61(94) \times 10^{-11}$	$< 2.8 \times 10^{-10}$	–	< 14
$K_L \to \pi^0\mu^+\mu^-$	$1.52(28) \times 10^{-11}$	$< 3.8 \times 10^{-10}$	–	< 23

Conclusions

- **Improved SM prediction for $K_L \to \ell^+\ell^-$**
  - New calculation of the LD part using dispersion relations
  - Uncertainty still theory dominated, but getting close to experiment
  - Further improvements possible, including interplay with lattice QCD and data

    $\hookrightarrow$ new measurements of $K_L \to \ell^+\ell^-$, $K_L \to \ell^+\ell^-\gamma$, $K_L \to \pi^+\pi^-\gamma$ well motivated!
- **Limits on modified Z couplings**
  - Example to compare reach of different channels
  - $K_L \to \ell^+\ell^-$ (CP even), by definition, complementary to $K_L \to \pi^0\nu\bar{\nu}$ (CP odd)
  - Possible way to "save" BSM sensitivity of CP-even part in $K^+ \to \pi^+\nu\bar{\nu}$?
  - $K_L \to \pi^0\ell^+\ell^-$ could reach similar level as $K_L \to \pi^0\nu\bar{\nu}$
  - Eventually need better input for $K_S \to \pi^0\gamma^*$ form factor

    $\hookrightarrow$ $K_S \to \pi^0\ell^+\ell^-$ at LHCb, lattice QCD
  - Two-photon contribution for $K_L \to \pi^0\mu^+\mu^-$

    $\hookrightarrow$ similar methods as for $K_L \to \mu^+\mu^-$