Low-energy QCD theory from a theoretical perspective

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Outline

- pions
- new amplitude approaches
- why relevant at low-energy QCD?

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Main motivation

- Kaon experiment: laboratory of a broad physical program
- e.g. when we have kaons – we have inevitably also pions
- kaon factory $\rightarrow$ pion factory
- leads e.g. to the study of $\pi^0 \rightarrow e^+e^-$
- another example: the core decay $\pi^0 \rightarrow \gamma\gamma$: next page

$\pi^0 \to \gamma\gamma$: short comment

[kk, Moussallam '09]

$$\text{theory: } \Gamma = (8.09 \pm 0.11)\ \text{eV} \qquad \text{or } \tau = 8.04 \pm 0.11 \times 10^{-17}\ \text{s}$$

$$\text{PrimEx I+II: } \Gamma = (7.80 \pm 0.12)\ \text{eV} \qquad \text{or } \tau = 8.34 \pm 0.13 \times 10^{-17}\ \text{s}$$

$$\longrightarrow 1.8\ \sigma \text{ discrepancy}$$

F_π is a crucial ingredient

- F_π vs $\hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]
- using $\pi^0 \to \gamma\gamma$:

$$F_\pi = 93.85 \pm 1.4\,\text{MeV}$$

$$\text{cf with } \hat{F}_\pi = 92.22(7)$$

$$(1.2\sigma \text{ difference})$$

- our F_π from PDG is based on π_{l2} and SM using [Marciano, Sirlin'93]
- important input V_{ud}: new update by [Hardy,Towner '20]

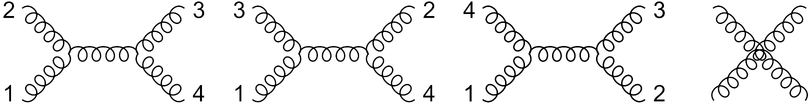
$$0.97418(26) \to 0.97373(31)$$

Amplitudes

- important in particle physics: Lagrangian $\to$ Feynman rules $\to$ amplitudes $\to$ cross-section
- new initiative to study these objects more deeply
- annual conferences: . . . , Prague 22, CERN 23, IAS 24, Seoul 25
- amplitudes as key object of theoretical studies
- example $\to$ next page

QCD: gluon amplitudes

- important in high-energy collider experiments (LHC)
- using conventional methods: complicated already at the tree-level



- intermediate steps are complicated but the final result "nice"
- standard methods hard/impossible for higher multiplicity
- surprisingly some results super simple and closed for all multiplicities

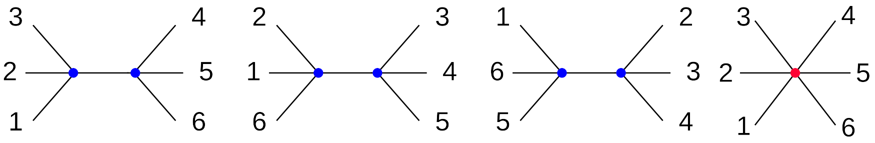
$$A_n(- - + \ldots +) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

(so called MHV, [Parke, Taylor '86])

pion amplitudes

[KK, Novotny, Trnka '13]

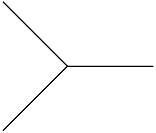
- We want to study low-energy QCD
- focus on dynamics of pions, kaons, . . .
- very complicated already at the tree-level for large n
- simplify the problem: massless, large N_c (one trace $\rightarrow$ cyclic ordering)
- 4pt: $A = s_{13}$
- 6pt:



pion amplitudes: new surprising way to calculate

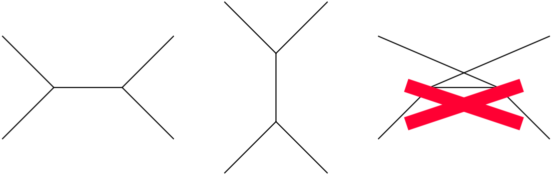
[Arkani-Hamed et al '23-'24]

The simplest model: $\mathrm{Tr}(\phi^3)$
only one vertex:



$= 1$

e.g. the 4pt amplitude:



$$A = \frac{1}{s_{12}} + \frac{1}{s_{14}}$$

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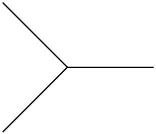

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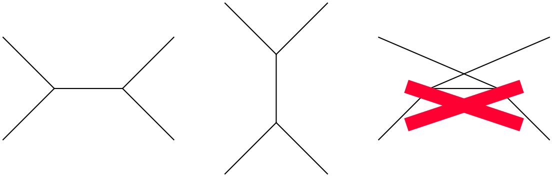
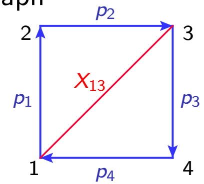


dual graph



$X_{ij} = (p_i + \ldots + p_{j-1})^2$

$$A = \frac{1}{s_{12}} + \frac{1}{s_{14}} = \frac{1}{X_{13}} + \frac{1}{X_{24}}$$

pion amplitudes: new surprising way to calculate

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The magic:

$$A = \frac{1}{X_{13}} + \frac{1}{X_{24}}$$

odd/even shifts:

$$X_{ee} \to X_{ee} + \delta, \qquad X_{oo} \to X_{oo} - \delta$$

$$X_{eo} \to X_{eo}$$

Do it in $Tr(\phi^3)$ amplitude and expand in small momenta for large δ:

$$A \to \frac{1}{X_{13} - \delta} + \frac{1}{X_{24} + \delta} \sim -X_{13} - X_{24} = s_{13}$$

which is the 4pt NLSM!

Novel way to calculate pion amplitudes

- True up to all multiplicity!
- can be extended to the loop level
- masses can be added naturally (under investigation)
- we hope we can also include higher orders (under investigation)
- More interestingly - scaffolding for gluons, and via double copy also gravity (under investigation)
- natural explanation from strings
- It aims to common geometric structure for all these theories!

Conclusion: NLSM still full of surprises

- amplitudes methods are important to uncover hidden structures
- true also for the low energy QCD
- It would be very surprising if the above miracles have no footprint in the low-energy data
- the key place to look is the $O(p^4)$ low-energy constants
- last ChPT $O(p^4)$ LECs estimate: [Bijnens, Ecker '14] $\leftarrow K_{l4}$ NA48
- There are many other, both old and new theoretical methods (dispersive techniques, BCJ, positivity bounds [Alvarez, Bijnens, Sjö '22] ...)

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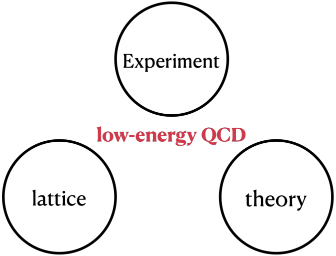

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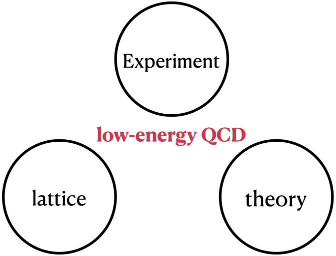


ありがとう

Backup slides

Summary of Classification of EFTs: “soft-bootstrap”

Non-trivial cases

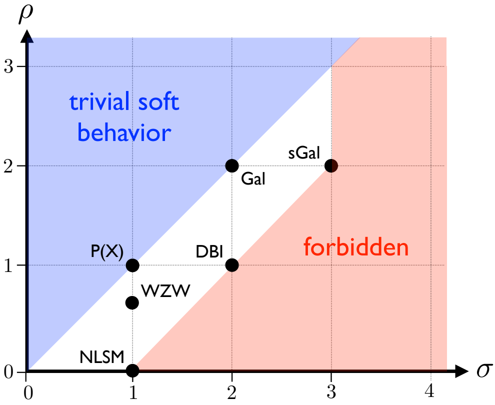
$$\text{for: } \mathcal{L} = \partial^m \phi^n : \qquad m < \sigma n \;\Leftrightarrow\; \sigma > \frac{(n-2)\rho + 2}{n}$$

i.e.

ρ	σ at least
0	1
1	2
2	2
3	3

non-trivial regime for $\rho \leq \sigma$

[C. Cheung, K. Kampf, J. Novotny, C. H. Shen and J. Trnka '17]



String theory considerations

string monodromy relations: [Plahte '70]

- open string amplitudes are calculated as disk integr.
- n vertex operators insertions on the boundary
- different orderings correspond to different choices of contours in the integrals over the insertion points
- linear relations among amplitudes from contour deformations

e.g. at 4pt:

$$A_4(1324) + \mathrm{e}^{i\pi\alpha' u} A_4(1234) + \mathrm{e}^{-i\pi\alpha' t} A_4(1342) = 0$$

in α' expansion leads to KK and BCJ relations.

String theory considerations

Z theory [Carrasco, Mafra, Schlotterer '17]
iterated integrals over the boundary of a disk worldsheet and naturally incorporate two notions of ordering

we can motivate it via the 4pt example, do the simple game:
Veneziano open string amplitude

$$A_4 = \frac{\Gamma(-1-\alpha' s)\Gamma(-1-\alpha' t)}{\Gamma(-2-\alpha' u)}$$

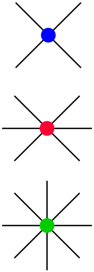
assume you want the correct Regge behaviour and expansion in α' starts with $O(\alpha'^1)$
We will get

$$Z_\times = B(-\alpha' u, -\alpha' s) - B(-\alpha' s, 1-\alpha' t) - B(-\alpha' u, 1-\alpha' t)$$

expansion in α' corresponds to NLSM and higher orders!

Higher-orders NLSM

40 years of ChPT: up to NNNLO $O(p^8)$
from the amplitude perspective?
yes!: [Dai, Low, Mehen, Mohapatra '20], [KK '21]



	#mesons	#terms
p^2	4	1
p^4	4	2
p^6	4	2
	6	5
p^8	4	3
	6	22
	8	17

Higher-orders NLSM

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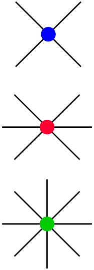

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p^2	4	1
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	8	17

Higher-orders NLSM: scalar BCJ bootstrap

[Brown,KK,Oktem,Paranjape, Trnka '23]

BCJ

$$\sum_{i=2}^{n-1}(s_{12}+\ldots+s_{1i})A_n(2,\ldots,i,1,i+1,\ldots,n)=0\,,$$

We focused on the statement [Gonzalez, Penco, Trodden'19]:

$$\text{BCJ} \;\Rightarrow\; \text{Adler.}$$

For recent studies of the KLT bootstrap see also [Chi, Elvang, Herderschee, Jones, Paranjape '21], [Chen, Elvang, Herderschee '23]

Higher-orders NLSM: scalar BCJ bootstrap

[Brown,kk,Oktem,Paranjape, Trnka '23]

- 4pt

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14	16	18
Soft amplitudes	1	2	2	3	3	4	4	5	5
BCJ amplitudes	1	0	1	1	1	1	2	1	2

not the final answer!

Higher-orders NLSM: scalar BCJ bootstrap

[Brown,kk,Oktem,Paranjape, Trnka '23]

- 4pt

$\mathcal{O}(p^{\#})$	2	4	6	8	10	12	14	16	18
Soft amplitudes	1	2	2	3	3	4	4	5	5
BCJ amplitudes	1	0	1	1	~~1~~ **0**	1	~~2~~ **1**	1	~~2~~ **1**

not the final answer!

- analysis of 6pt (up to $O(p^{18})$ and 8pt (up to $O(p^{10})$): many surprised relations among coefficients of different orders, e.g.

$$\alpha^{(10)} \sim \left(\alpha^{(6)}\right)^2$$

- what are "BCJ Lagrangians"?
  - NLSM
  - Z-theory

Geometrical picture

very active and quickly developing field

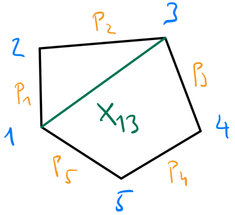
Arkani-Hamed et al. 1711.09102, 2311.09284, 2312.16282, 2401.00041, 2401.05483, 2402.06719, 2403.04826, . . .

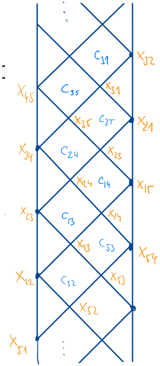
"trace ϕ^3" theory: ABHY associahedron

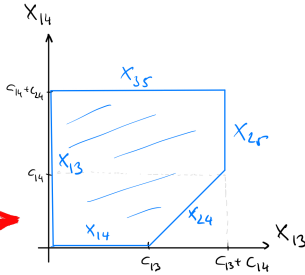
Important choice of basis:

$$X_{ij} = (p_i + p_{i+1} + \dots p_{j-1})^2$$

$$c_{ij} \equiv -2 p_i \cdot p_j$$
$$= X_{ij} + X_{i+1,j+1} - X_{ij+1} - X_{i+1}$$







Geometrical picture

It implies the existence of zeros of relevant amplitudes!

Are there some implications for other theories?

Bartsch, Brown, kk, Oktem, Paranjape, Trnka'24: Yes! via double copy

what is double copy? – first discovered as a relation between closed and open string amplitudes (KLT)

$$\text{Gravity} \sim YM * YM$$

more generic than that! – e.g. at 4pt (always at tree level)

$$M_4(1234) = -is_{12}A_4(1234)\tilde{A}_4(1243)$$

i.e.

$$\text{sGal} = \text{NLSM*NLSM}$$

true at all multiplicity!

We used hidden zero to prove the Galileon zeros

Geometric origin of permutation-invariant theories? unknown

EFT: simplest case

- focus on two derivatives: $\partial_\mu \phi \partial^\mu \phi \phi^n$
- Single field is a trivial case $\rightarrow$ have to consider multi-flavours $\phi_1, \phi_2 \dots$
- case by case studies: of two, three, ... flavours

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^i + \lambda_{ijkl}\partial_\mu \phi^i \partial^\mu \phi^j \phi^k \phi^l + \lambda_{i_1 \dots l_6}\partial_\mu \phi^{i_1}\partial^\mu \phi^{i_2}\phi^{i_3}\dots \phi^{i_6} + \dots$$

- Very complicated generally
- Assume some simplification using the group structure

$$\phi = \phi^a T^a$$

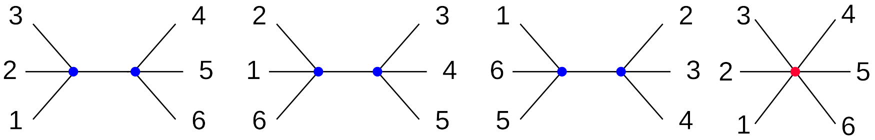
- similar to the 'gluon case': flavour ordering

$$A^{a_1 \dots a_n} = \sum_{perm} \mathrm{Tr}(T^{a_1}\dots T^{a_n})A(p_1, \dots p_n)$$

First example: NLSM

[KK, Novotny, Trnka '13]

bottom-up analysis, first non-trivial case, the 6pt amplitude:



power-counting:

$$\lambda_4^2 \; p^2 \frac{1}{p^2} p^2 + \lambda_6 \; p^2$$

in order to combine the pole and contact terms we need to consider some limit. The most natural candidate: we will demand soft limit, i.e.

$$A \to 0, \qquad \text{for} \quad p \to 0$$

$$\Rightarrow \quad \lambda_4^2 \sim \lambda_6 \qquad \text{corresponds to NLSM}$$

How to extend it to all orders (n-pt)? $\rightarrow$ new recursion relations