Majorana phases beyond neutrinoless double beta decay

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KEK Theory Meeting on Particle Physics Phenomenology

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Details: A. Dery, S. Gori, Y. Grossman, ZL, JHEP 10 (2024) 100, 2406.18647

The standard model + neutrino mass

- Clearest empirical evidence that the minimal SM is incomplete:
  - Dark matter
  - Baryon asymmetry of the Universe
  - Neutrino mass
  - Inflation in the early universe [have a plausible theoretical picture]
  - Dark energy [cosmological constant? need to know more?]
- We do not even know the Lagrangian that describes the particles we have observed

$$\mathcal{L} = -Y_e^{ij}\, \overline{L_{Li}^I}\, \phi\, e_{Rj}^I - \begin{cases} \frac{Y_\nu^{ij}}{\Lambda}\, L_{Li}^I L_{Lj}^I\, \phi\, \phi & \text{violates lepton number} \\ Y_\nu^{ij}\, \overline{L_{Li}^I}\, \tilde{\phi}\, \nu_{Rj}^I & \text{requires } \nu_R \text{ fields} \end{cases}$$

Do neutrino mass terms violate lepton number? 10 or 12 parameters in lepton sector?

The mixing of neutrinos

- Fermions with same quantum numbers mix, Yukawas define mass eigenstates:

$$M = \begin{pmatrix} M_{a1} & M_{a2} & M_{a3} \\ M_{b1} & M_{b2} & M_{b3} \\ M_{c1} & M_{c2} & M_{c3} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \underbrace{\begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}}_{\text{if Majorana, 1 if Dirac}}$$

- The additional phases $\eta_{1,2}$ do not affect oscillation experiments, only lepton $\#$ violation
- It is often said that we should measure all parameters in the Lagrangian... although...

  (Wolfenstein: *'I do not care what the values of the Wolfenstein parameters are, so you should not either; the only question is if their independent determinations give consistent results'*)
- However, it is $\eta_{1,2}$ and not δ, which is the least known parameter of the PMNS matrix

  Can we ever hope to measure a different linear combination than what enters $0\nu\beta\beta$?

Neutrino oscillation measurements

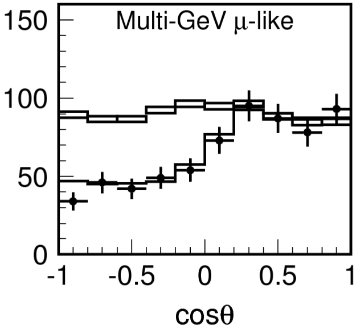
- Three mixing angles have been measured
- Oscillation between two flavors $(\delta m^2 = m_1^2 - m_2^2)$

$$P_{\rm osc} = \sin^2(2\theta)\ \sin^2\left(1.27\,\frac{\delta m^2}{\rm eV^2}\,\frac{L}{\rm km}\,\frac{\rm GeV}{E}\right)$$

- Atmospheric neutrinos:

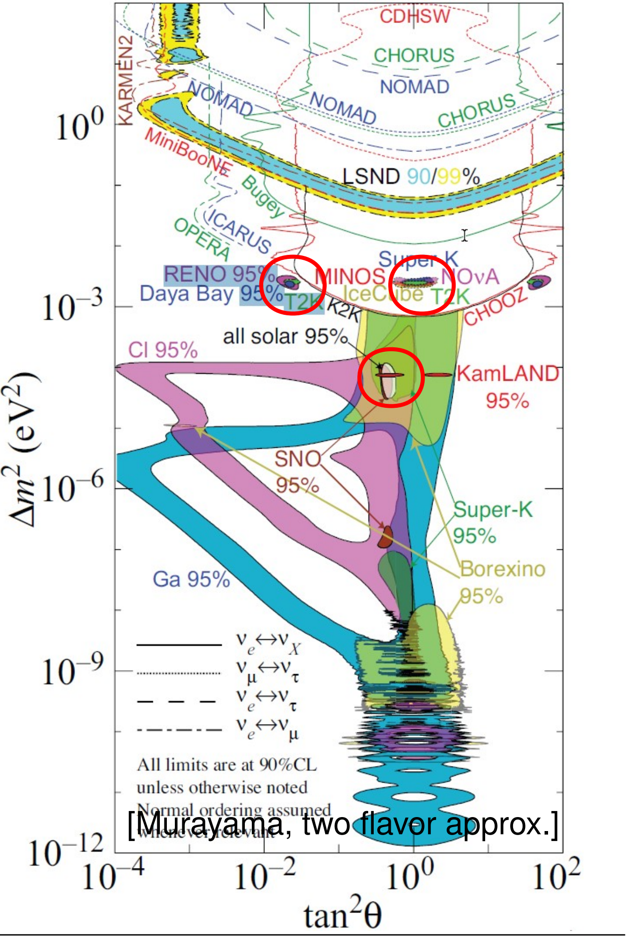
$$1 \sim (10^{-3}) \times (10^{1\ldots4})\ /\ (10^{0\pm1})$$

half of up-going ν_μ get lost



- Solar neutrinos: $\delta m^2\, L/E \gg 1$
- Two mass-squared differences have been measured, but not the absolute mass scale

(Short baseline anomalies not easy to fit, even with 4 flavors)



[Murayama, two flavor approx.]

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Neutrinos — many unknowns

- Are neutrinos $=$ their own antiparticles?

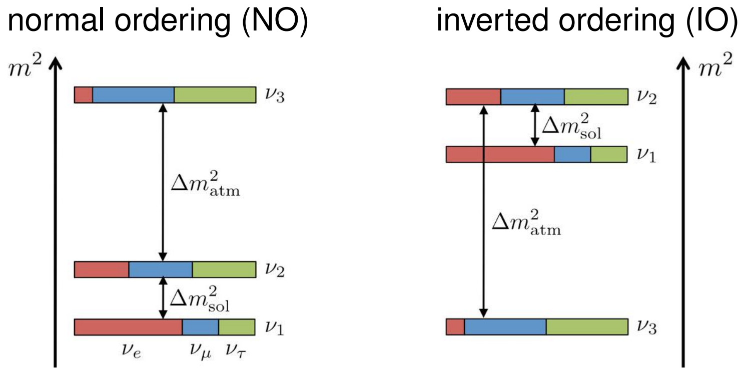
  (Different than all other known particles? Theoretically favored, most leptogenesis models)

- What is the absolute mass scale?

  Two mass-squared differences measured
  At least one state $m_{\nu_i} \gtrsim 50\,\mathrm{meV}$

  Cosmology: $\sum m_i < 0.072\,\mathrm{eV}$ [DESI 2024]

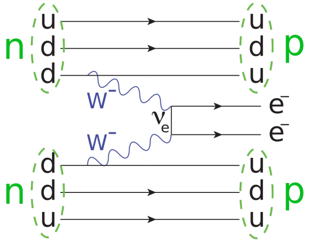
  (n.b.: preference for $\sum m_i < 0$ [e.g., 2407.07878])



- Value of CP violating phase δ ?

- Is the mass ordering “normal” (NO) or “inverted” (IO)?

  If IO, neutrinoless double beta decay ($0\nu\beta\beta$) experiments will decide
  If NO, may or may not see $0\nu\beta\beta$, even in Majorana case



Neutrinos — a history of surprises

- Most theorists’ expectations around early 1990’s:
  - Solar neutrino problem will go away, we do not understand the Sun Wrong
  - If it does not, solution must be small angle MSW, since it’s cute Wrong
  - Expect $\Delta m^2_{23} \sim 10 - 100\mathrm{eV}^2$, since it’s cosmologically interesting (DM) Wrong
  - Expect $\theta_{23} \sim V_{cb} \simeq 0.04$, motivated by simple GUT models Wrong
  - Atmospheric neutrino anomaly will go away, because it requires large mixing angle — the first that became compelling ($\Rightarrow$ Nobel, 2002) Wrong
  - 2000s: tribimaximal mixing ansatz, predicted θ_{13} near zero Wrong
    $\theta_{13} \sim 9^\circ$, not too small — helps CP violation searches

  [inspired by H. Murayama]

- Experiments crucial, independent of prevailing theoretical “guidance”

Quark vs. lepton mixing

- Mixing matrix parameters, assuming 3-generation unitarity:

$$U_{\rm PMNS}: \quad \sin\theta_{12} = 0.550 \pm 0.011 \qquad \sin\theta_{13} = 0.148 \pm 0.002$$

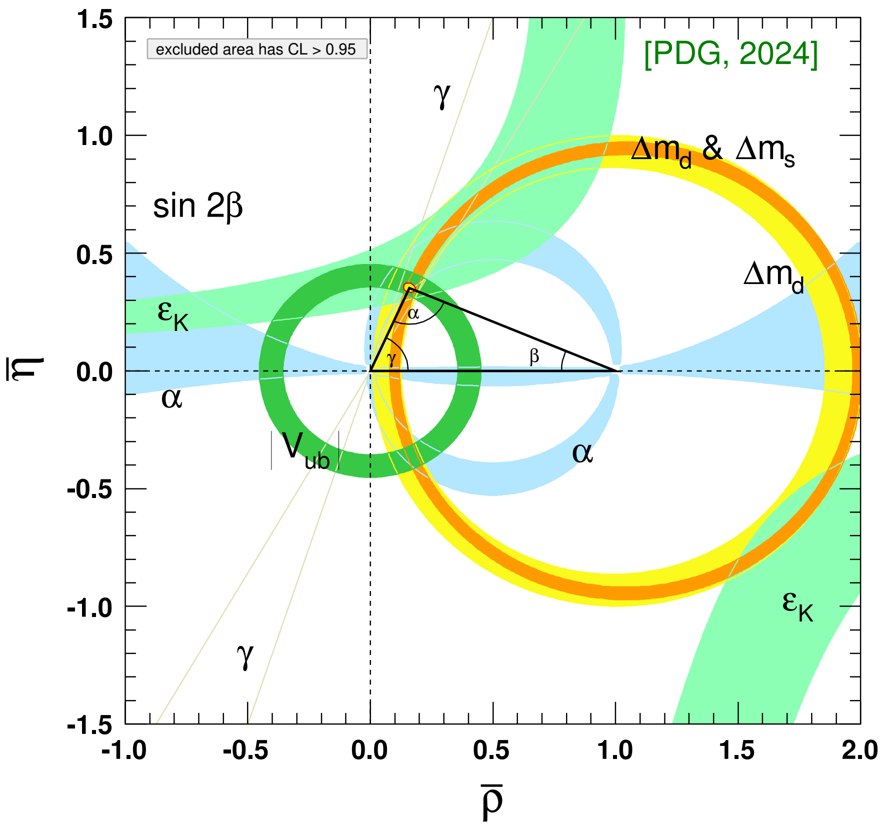
$$\sin\theta_{23} = 0.749 \pm 0.010 \qquad \delta = (177^{+19}_{-20})^\circ$$ [unconstrained at 3σ] [νfit 2024, NO, converted]

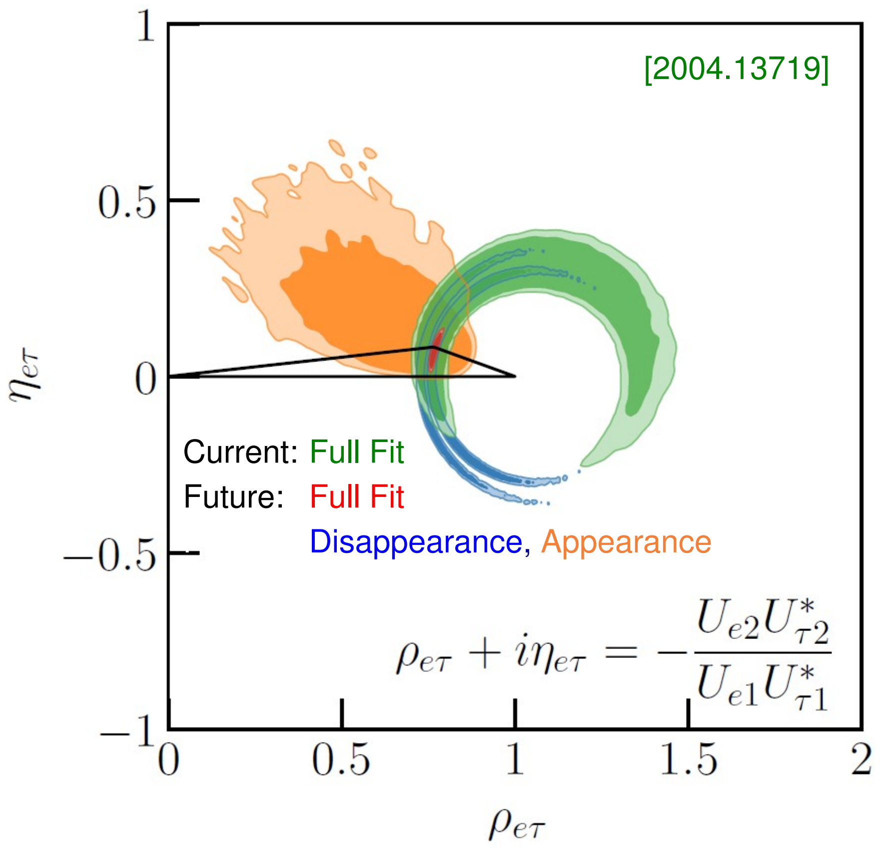
$$V_{\rm CKM}: \quad \sin\theta_{12} = 0.2250 \pm 0.0007 \qquad \sin\theta_{13} = 0.0037 \pm 0.0001$$

$$\sin\theta_{23} = 0.0418 \pm 0.0008 \qquad \delta = (65.7 \pm 1.5)^\circ$$ [PDG 2024]

- Are the origin of quark and lepton masses and mixings related?
- Some lepton processes are especially clean; quark sector much more rich
- Neutrino FCNCs seem impossible to search for; e.g., $\nu_i \to \nu_j\,\gamma$, $X \to \nu_i\bar\nu_j(Y)$
- SM flavor puzzle extended: why lepton and quark masses and mixings so different?

Quark vs. lepton mixing (2)





Why don’t we know yet if Majorana or Dirac?

- In the $m_\nu \to 0$ limit, the distinction between a Dirac and Majorana fermions disappears

  Except for oscillation experiments, no consequence of $m_\nu \neq 0$ has ever been observed

- Any experiment to probe the nature of neutrino mass involve suppression by $\propto m_\nu / E$

  FCNC neutrino decays allowed, e.g., $\nu_3 \to \nu_1 \gamma$, $\nu_3 \to \nu_2 \nu_1 \bar{\nu}_1$ — rates extremely small

- The smoking gun signature would be the observation of lepton number violation

  (Majorana neutrinos are their own antiparticles, thus cannot carry any quantum number)

- Neutrinos we can study are always ultrarelativistic

  Exception: cosmic neutrino background $(T_\nu \sim 2\,\mathrm{K} \sim 2 \times 10^{-4}\,\mathrm{eV})$

  (In Majorana case, both ν and $\bar{\nu}$ interact with detector)

Experimental challenges detecting LNV

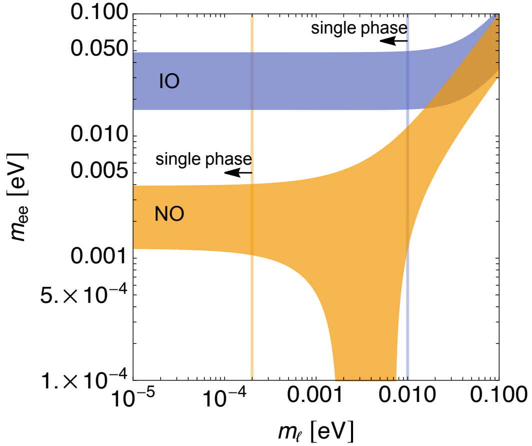
- The rates are suppressed by m_ν, need huge statistics

  Best chance is $0\nu\beta\beta$, rate $\propto m_{ee}^2$

  $$m_{ee} = \left| \sum_{i=1}^{3} m_i\, U_{ei}^2 \right|$$

  Planned experiments will reach $m_{ee} \sim 0.01\,\mathrm{eV}$, decisive for IO

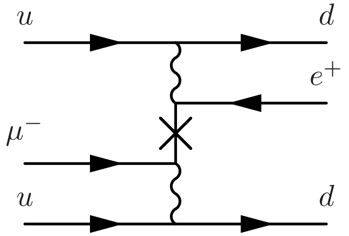
  For NO, m_{ee} can vanish even if neutrinos are Majorana



- Second best: $\mu^- \to e^+$ conversion, search for $pp\mu^- \to nne^+$

  Proportional to $m_{\mu e} = \left| \sum_{i=1}^{3} m_i\, U_{ei} U_{\mu i} \right|$

  Mu2e and COMET will improve current bound to $\sim 10^{-16}$



- Expectation from PMNS much smaller; patterns would give powerful constraints

Invariants and conventions

- Mixing matrix: $U = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}$

  These quantities are not physical / phase convention independent

  E.g., the Majorana phases η_1 and η_2 can be shifted to 2nd at 3rd (or 1st and 3rd) entries

- Define: Dirac phase $=$ CP violating phase measurable in LN conserving processes

  Majorana phases $=$ only accessible through LNV processes

- For CKM, it is well known that 4 elements are needed to define a physical CPV quantity:

  $t_{\alpha i\beta j} = U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}$

  For Majorana fermions, fewer phases can be absorbed in field redefinitions, hence $s_{\alpha ij} = U_{\alpha i}U^*_{\alpha j}$ are physical

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Our choice of parameters

- For Dirac fermions, 3 mixing angles and a phase:

$$\{|t_{e2e3}|,\ |t_{e3e3}|,\ |t_{\mu 2e3}|,\ \Psi_D\}, \qquad \Psi_D = \arg(t_{\mu 2e3}) = \arg\left(c_{12}c_{23}e^{-i\delta} - s_{12}s_{23}s_{13}\right)$$

($-|t_{\mu 2e3}| \sin\Psi_D = J$, the leptonic Jarlskog invariant, the same way as for CKM)

- For Majorana fermions, 2 additional phases:

$$\{\Phi_{12}, \Phi_{23}\} \equiv \{\Phi^e_{12}, \Phi^e_{23}\} \qquad \Phi^\alpha_{ij} = \arg\left(s_{\alpha ij}\right)$$

E.g., $\Phi_{12} = \eta_1 - \eta_2$, etc.

Generalizing m_{ee}

- If interested in other LNV processes besides $0\nu\beta\beta$, it is natural to generalize:

$$m_{\alpha\beta} = \left| \sum_{i=1}^{3} m_i U_{\alpha i} U_{\beta i} \right| \qquad \alpha, \beta \in \{e, \mu, \tau\}$$

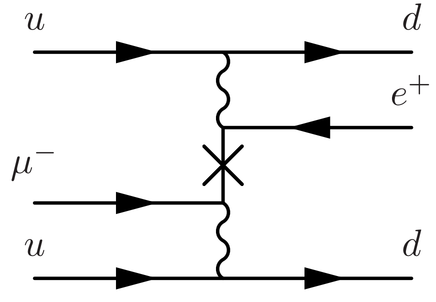
Explicit expressions are not too illuminating, simplest is:

$$\begin{aligned} m_{ee}^2 \; = \; & (m_1^2 c_{12}^4 + m_2^2 s_{12}^4) c_{13}^4 + m_3^2 s_{13}^4 + 2 m_1 m_2 s_{12}^2 c_{12}^2 c_{13}^4 \cos\left[2(\eta_1 - \eta_2)\right] \\ & + 2 m_1 m_3 c_{12}^2 s_{13}^2 c_{13}^2 \cos\left[2(\eta_1 + \delta)\right] + 2 m_2 m_3 s_{12}^2 s_{13}^2 c_{13}^2 \cos\left[2(\eta_2 + \delta)\right] \end{aligned}$$

- The rate of a lepton number violating process is proportional to the corresponding $m_{\alpha\beta}^2$

For example, the $\mu^- \to e^+$ conversion rate $\propto m_{\mu e}^2$

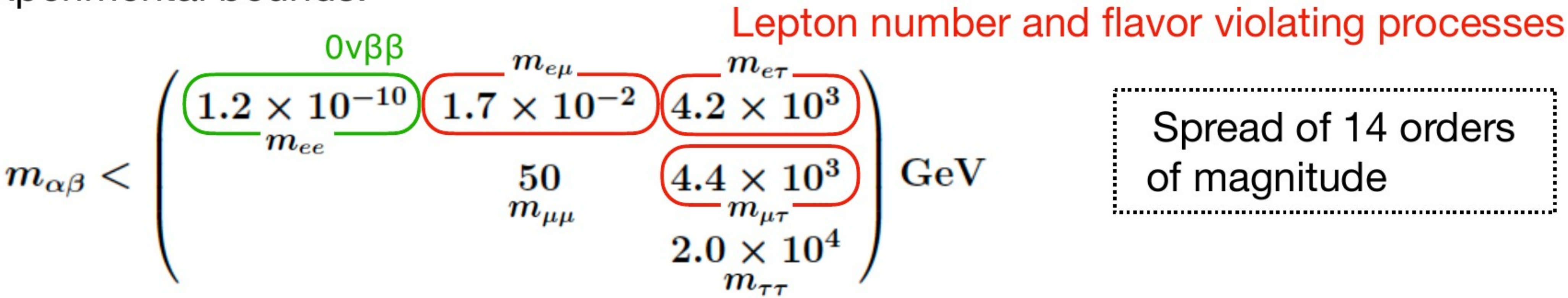
Sensitivity to each Majorana phase scales as the product of corresponding masses, $m_i\, m_j \;\Rightarrow\;$ large suppression



$\mu^-(N, Z) \to e^+(N, Z-2)$
(in suitable nuclei)

Experimental bounds, in a nutshell

Experimental bounds:



Lepton number and flavor violating processes

0νββ

$$m_{\alpha\beta} < \begin{pmatrix} \underset{m_{ee}}{\boxed{1.2 \times 10^{-10}}} & \overset{m_{e\mu}}{\boxed{1.7 \times 10^{-2}}} & \overset{m_{e\tau}}{\boxed{4.2 \times 10^{3}}} \\ & \underset{m_{\mu\mu}}{50} & \underset{m_{\mu\tau}}{\boxed{4.4 \times 10^{3}}} \\ & & \underset{m_{\tau\tau}}{2.0 \times 10^{4}} \end{pmatrix} \text{GeV}$$

updated from Rodejohann, Zuber, 0011050 [from S. Gori]

Spread of 14 orders of magnitude

- Experimental sensitivity to $0\nu\beta\beta$ searches are by far the best, since macroscopic amounts of nuclei are used, rather than particle beams
- Mu2e & COMET will improve the bound on $m_{\mu e}$ by ~ 4 orders of magnitude

  Tiny rates: $\sim 3 \times 10^{-22} (m_{\mu e}^2 / m_e^2) |M|^2$

The single phase limit

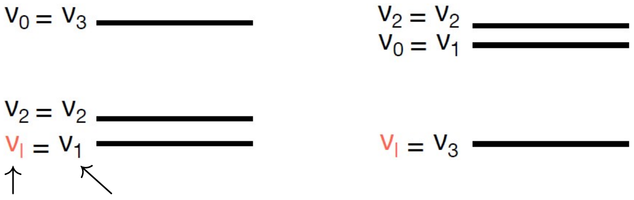
- We define $\{\nu_\ell,\ \nu_2,\ \nu_o\}$ so that ν_ℓ is always the lightest state

- In the $m_{\nu_\ell} \to 0$ limit, the Majorana phase dependence of all $m_{\alpha\beta}$ is the same (instead of $\eta_{1,2}$ only one η)

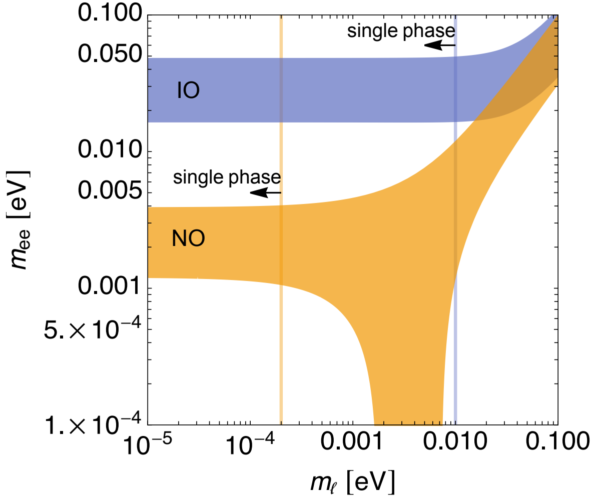
- **Proof:** a symmetric rank-2 neutrino Yukawa matrix has 5 real and 5 imaginary parameters (for rank-3: 6 real & 6 imag.)

  The charged lepton Yukawas contain $9+9$ parameters

  The global $U(3)_L \times U(3)_E$ symmetry is completely broken, allowing to remove $6+12$ parameters, leaving 5 masses, 3 mixing angles, and 2 phases (one Dirac and one Majorana) as physical parameters

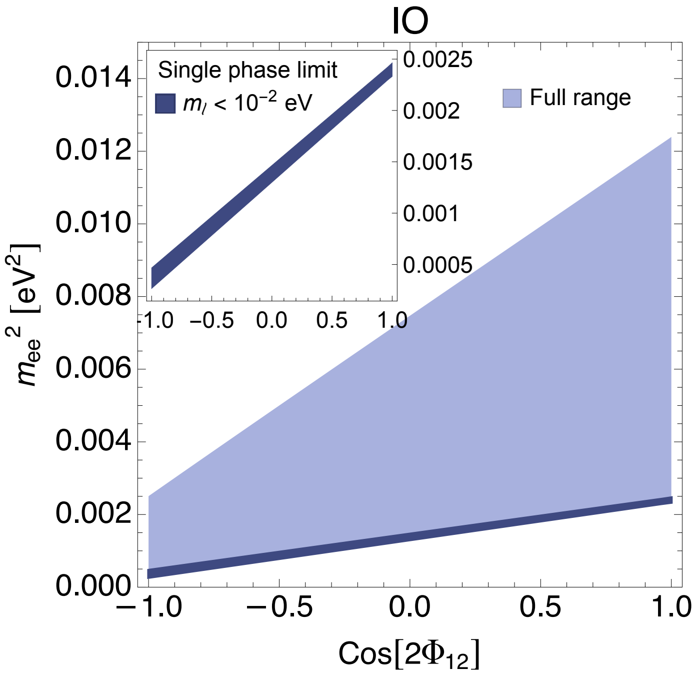


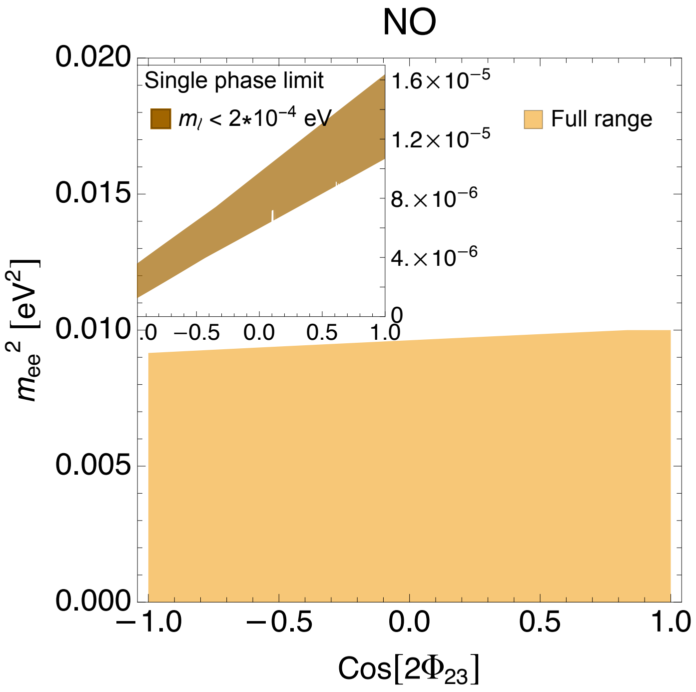
our / PDG convention



vertical lines show where single phase limit becomes a good approximation

Majorana phase from m_{ee} in single phase limit





Insets show that as m_l gets small, measurement of m_{ee} determine Φ_{2o}

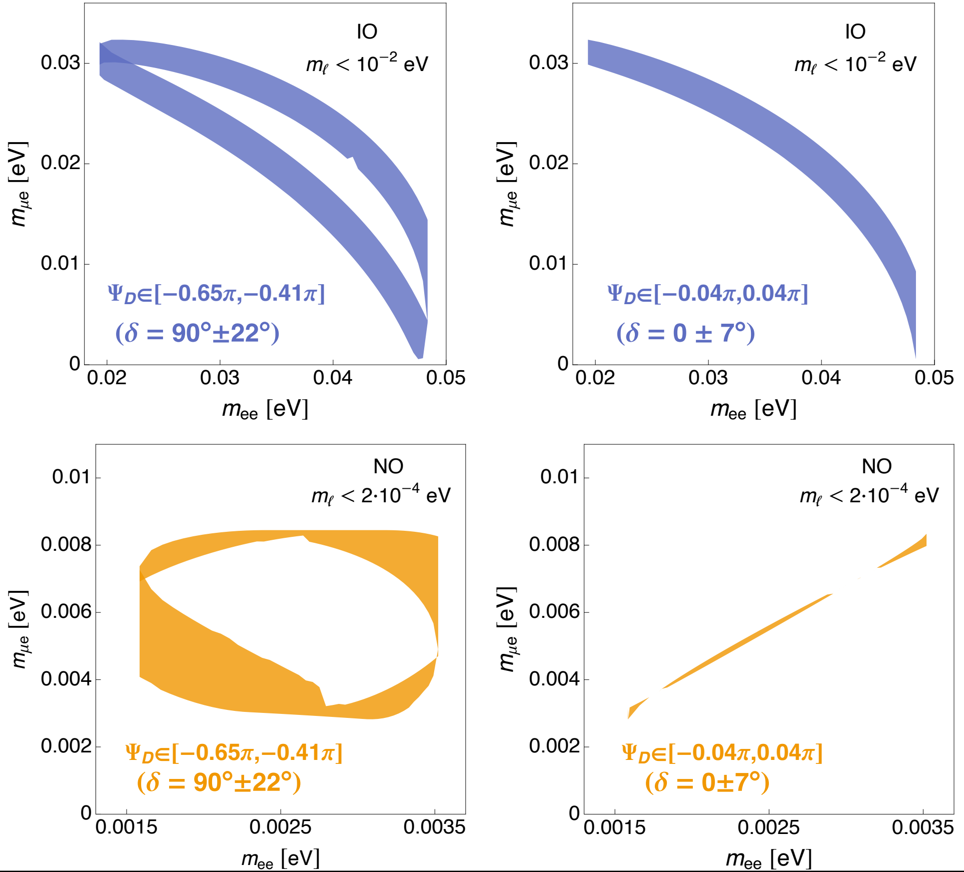
- If the lightest m_ν gets small, the correlation between m_{ee} and all other $m_{\alpha\beta}$ get stronger

Predicting $m_{\mu e}$ from m_{ee}

Correlation of m_{ee} and $m_{\mu e}$, assuming two possible measured ranges of δ:

$\delta = (0 \pm 7)^\circ$ and $\delta = (90 \pm 22)^\circ$

(HyperK expectations with 10 years of data)

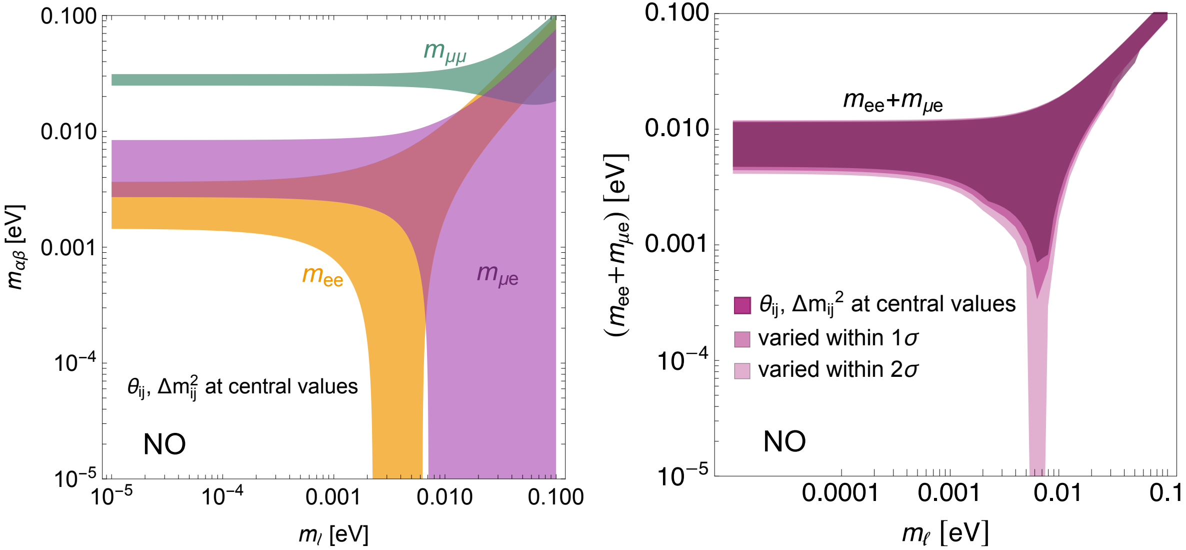


A no-loose “theorem”

- With current PMNS and Δm^2 central values, m_{ee} and $m_{\mu e}$ cannot simultaneously vanish (even in case of NO)

  $\Rightarrow$ In principle, LNV is detectable with better $m_{\mu e}$ sensitivity, even for NO

- Tantalizing PMNS values: $m_{ee} + m_{\mu e}$ cannot vanish (barely, at the 2σ level)

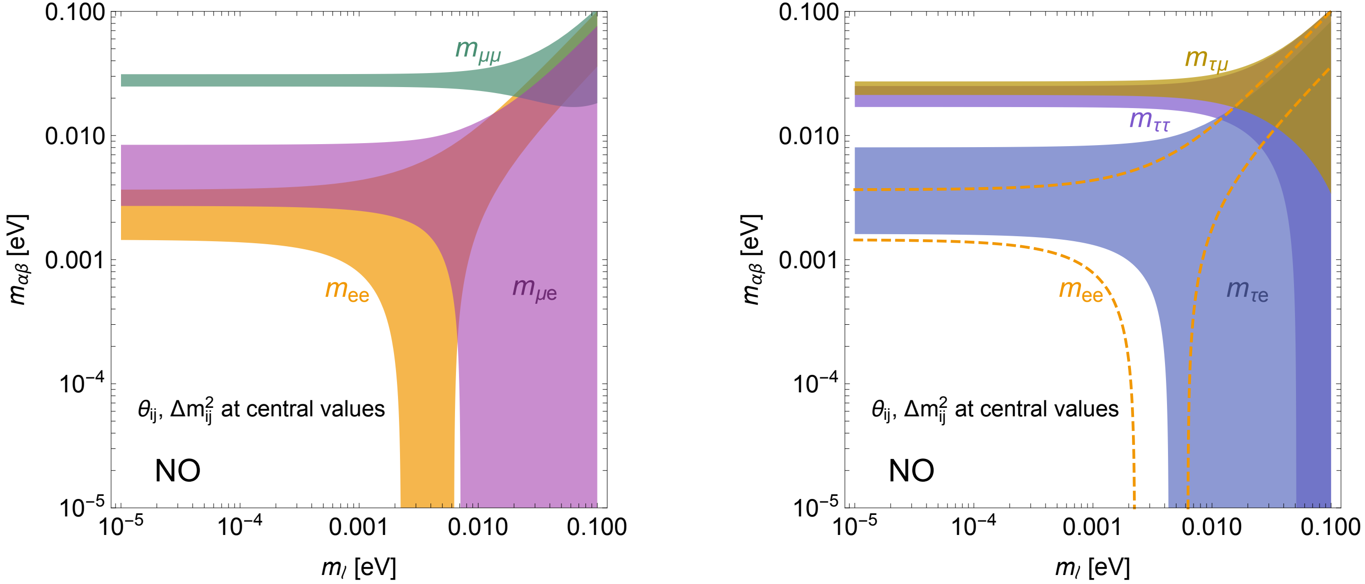


[Dery, Gori, Grossman, ZL, 2406.18647]

- Very challenging (impossible?) to reach such sensitivity

  Neutron stars contain $\sim 10^{55}\,\mu^-$, can it be used somehow? [Dery, Gori, Grossman, ZL, in progress]

Similar story for $m_{\tau i}$



- Experimental prospects to probe these may be even more remote

Summary

- Whether neutrino mass terms violate lepton number is the most significant open question about particles we have seen — experimental discovery would be transformational
- $0\nu\beta\beta$ experiments are sensitive to one linear combination of the two Majorana phases
  (i) may discover LN violation; (ii) may rule it out (IO); (iii) may not find out (NO)
- Can we ever determine both Majorana phases? Seems very challenging
- Single phase limit: as the lightest neutrino mass gets smaller, all m_{α_β} depend on a single combination of the two Majorana phases
- How far can sensitivity to $m_{\mu e}$ be pushed?
- Nonrelativistic neutrinos? Room for new ideas!