The possible solution to V_{cb} puzzle from nontrivial analyticity of quark propagator

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Outline

- Introduction
- Nontrivial analytic structure of Quark propagator
- Numerical result
- Summary

- V_{cb} puzzle

$$V_{CKM}=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

B meson decays

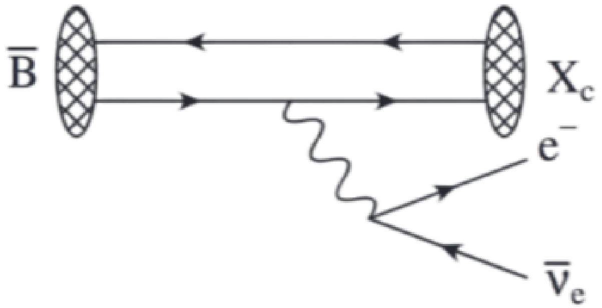
Two access to determine $|V_{cb}|$:

1. inclusive B decay

Operator Product Expansion (OPE)

$$|V_{cb}^{\rm OPE}| = (42.16 \pm 0.50) \times 10^{-3}$$

[Gambino, et al., 2107.00604]



(semileptonic decay $B \to X_c \ell \bar{\nu}$)

2. exclusive B decay (such as $B \to D^{(*)} \ell \bar{\nu}$)

$|V_{cb}| = (39.7 \pm 0.6) \times 10^{-3}$ (SM(2/1/0) scheme),

$|V_{cb}| = (39.3 \pm 0.6) \times 10^{-3}$ (SM(3/2/1) scheme)

[Iguro, Watanabe, 2004.10208]

→ 3.0 σ deviation between the both determinations.

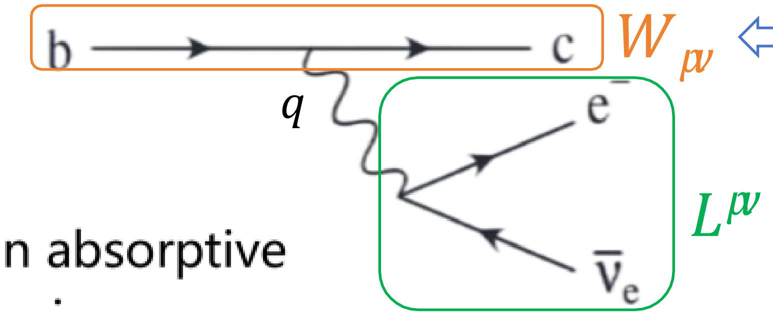
[PDG2025]

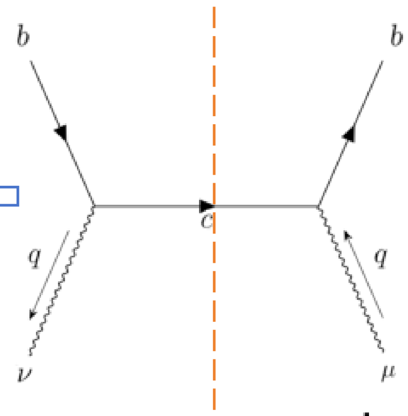
Possibility: violation of the quark-hadron duality

• Inclusive semileptonic B decay [Manohar, Wise, 9308246]

Decay width:

$$d\Gamma \propto |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu}$$





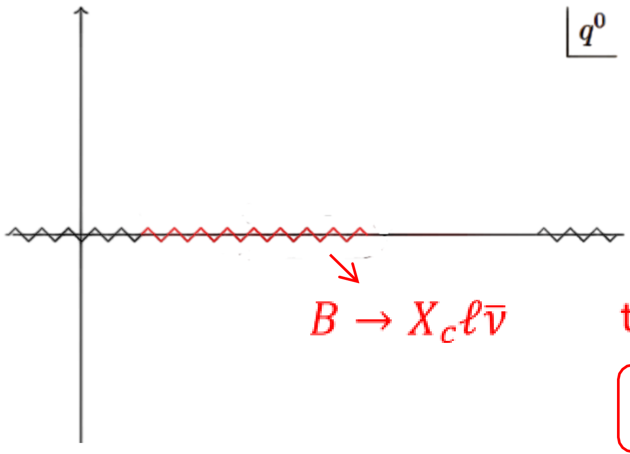
$T_{\mu\nu}\big|_{B\to X_c}$

The hadronic tensor $W_{\mu\nu}$ is related to an absorptive part of the forward scattering tensor $T_{\mu\nu}$:

$$T^{\mu\nu} = -i\int d^4x e^{-iq\cdot x} \langle H_v| T[J^{\dagger\mu}(x)J^\nu(0)]|H_v\rangle .$$

$$W^{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}T^{\mu\nu}\Big|_{B\to X_c}$$

The analyticity structure of $T_{\mu\nu}$ in the complex plane of $q\cdot v = q^0$ with q^2 fixed:



$$T^{\mu\nu} \propto \frac{1}{p^2 - M^2 + i\varepsilon}$$

Solve $(p_B - q)^2 = M_{X_c}^2$:

$$-\infty < q^0 < \frac{M_B^2 + q^2 - M^2_{X_c^{\min}}}{2M_B}$$

the kinematic region for $B\to X_c\ell\bar{\nu}$:

$$\sqrt{q^2} < q^0 < (M_B^2 + q^2 - M^2_{X_c^{\min}})/2M_B$$

The $W^{\mu\nu}$ is given by the discontinuities across a cut of the amplitudes $T^{\mu\nu}$.

- $B \to X_c \ell \bar{\nu}$

[Manohar, Wise, 9308246]

Using the mentioned optical theorem identity, the double differential rate ($\alpha = q^2/[2m_b(q^0 - E_\ell)]$):

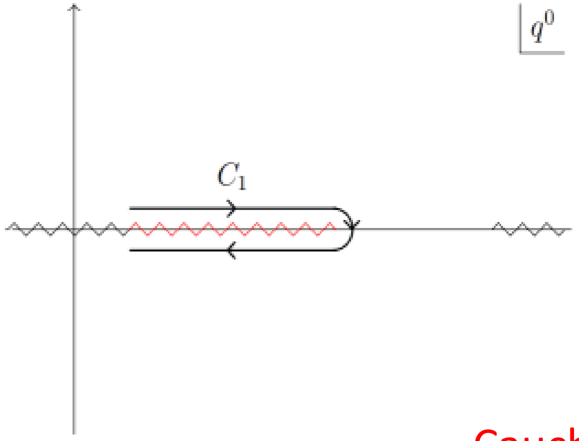
$$\Rightarrow \frac{d^2\Gamma}{dE_\ell d\alpha} \propto -|V_{cb}|^2 \mathrm{Im} \int_{E_\ell}^{q^0_{\max}} (q^0 - E_\ell) L_{\mu\nu} T^{\mu\nu} dq^0$$

The imaginary part of $T_{\mu\nu}$ can be calaulated by the discontinuity of the related branch cut:

$$\mathrm{Im}\, T^{\mu\nu} = -\frac{1}{2} \mathrm{Im}[\, T^{\mu\nu}(q^0 + i\varepsilon) - T^{\mu\nu}(q^0 - i\varepsilon)]$$

$$\int_{E_\ell}^{q^0_{\max}} \Rightarrow -\frac{1}{2}\int_{C_1}$$

The perturbative evaluation of $T^{\mu\nu}$ in the local OPE encounters an obstacle at the vicinity of the resonance region so that further deformation should be performed.



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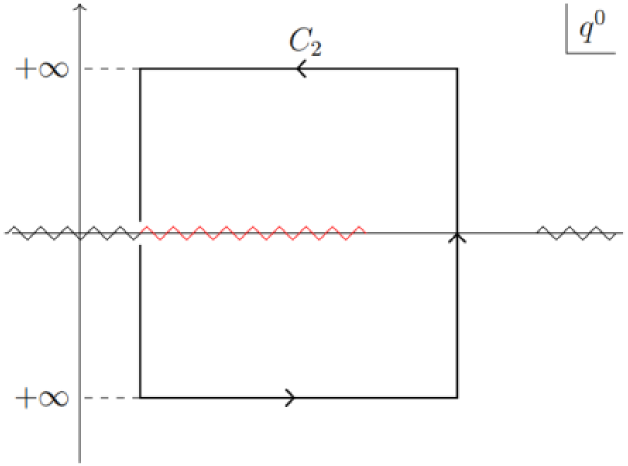


Cauchy's theorem

the perturbation theory gives a reliable prediction along the contour C_2,

$$T^{\mu\nu} \to T^{\mu\nu}_{pert.}$$

• Complex conjugate poles

Solving Dyson-Schwinger equation (DSE) for quark propagator in Euclidean space:

$$S_f(p) = -i\gamma \cdot p\sigma_V(p^2) + \sigma_S(p^2)\ .$$

By analytic continuation to Minkovski spacetime with a variant of Schlessinger point method, we obtain: [Zehao Zhu, et al., 2005.04181]

$$\sigma(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho(\omega)}{p^2+\omega}d\omega + \sum_i\left(\frac{R_i}{p^2-Q_i}+\frac{R_i^*}{p^2-Q_i^*}\right)$$

Complex parameters:
residue parameter *R*
pole position parameter *Q*

Complex Conjugate Poles (CCP)

with spectral density

$$\rho(\omega) = \mathrm{Im}[\,\sigma(-\omega - i\varepsilon)]$$

a possible indication of color confinement

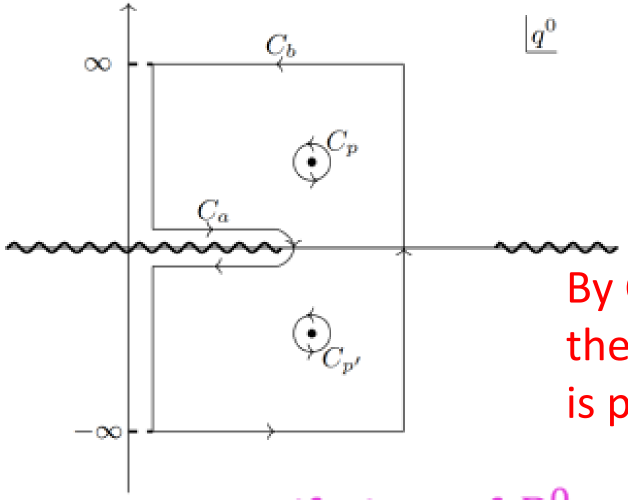
⇨ The usual Källén –Lehman spectral representation is changed to include the nontrival complex conjugate poles.

• Nonperturbative contribution to obsevables

Quark propagator: $S(x,y) \to S(x,y) + S(x,y)|_{\mathrm{CCP}}$

where $$S(x,y)|_{\mathrm{CCPs}} = \int \frac{d^4 p_c}{(2\pi)^4} i\not{p}_c \left(\frac{R}{p_c^2 + Q} + \frac{R^*}{p_c^2 + Q^*} \right) e^{-ip_c\cdot(x-y)}$$

$$T^{\mu\nu}_{\mathrm{pert.}} \to T^{\mu\nu}_{\mathrm{pert.}} + T^{\mu\nu}_{\mathrm{CCP}}$$



By Cauchy's theorem, the poles' contribution is picked up.

1. the lepton energy distribution for $B \to X_c \ell \bar{\nu}$,

$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell} = \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\bigg|_{pert.} + \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\bigg|_{\mathrm{CCP}}$$

↓ C_b ↓ $C_p + C_{p'}$

2. $|V_{cb}|$

$$|V_{cb}| = \frac{|V_{cb}^{\mathrm{OPE}}|}{\sqrt{1 + \frac{\tilde{\Gamma}^{\mathrm{CCPs}}}{\tilde{\Gamma}^{\mathrm{OPE}}}}}.$$

$$\tilde{\Gamma}^{\mathrm{CCPs}} = \Gamma^{\mathrm{CCPs}}/|V_{cb}|^2$$

$$\tilde{\Gamma}^{\mathrm{OPE}} \simeq G_F^2 (m_b^{\mathrm{kin}})^5/192\pi^3$$

[Gambino, et al., 2107.00604]

3. Lifetime of B_d^0 meson

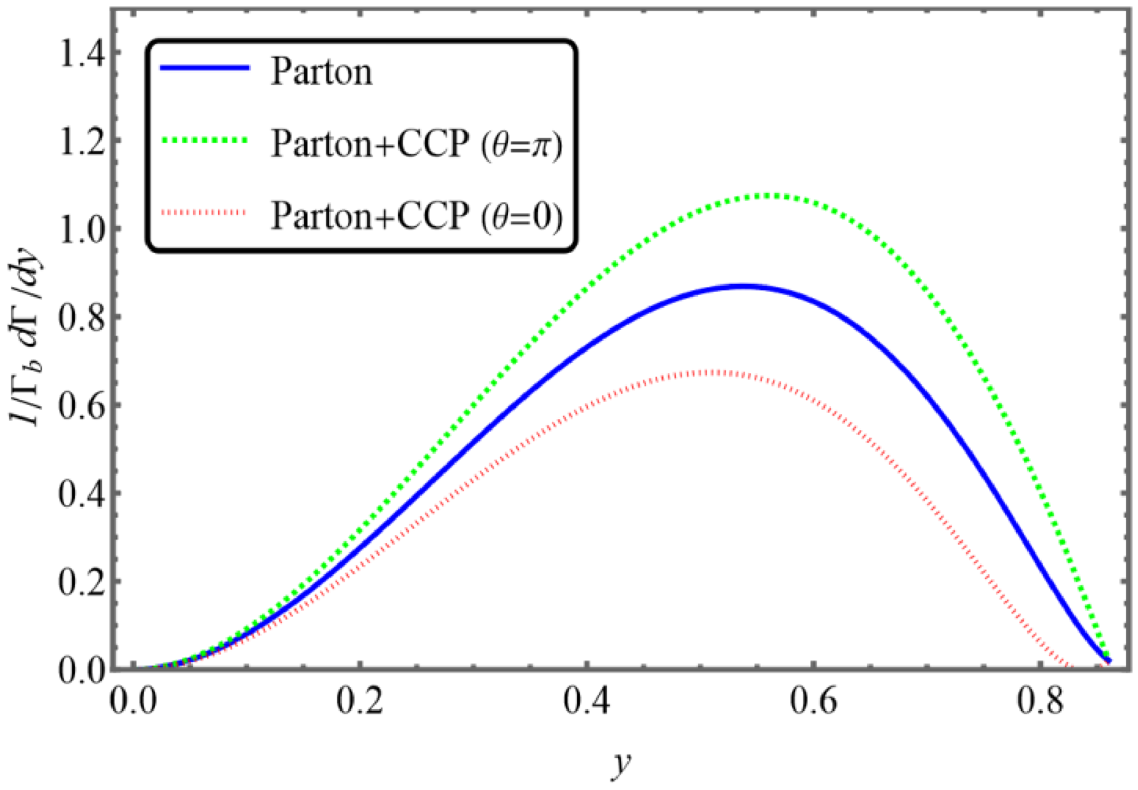
$$\tau(B_d^0) = \frac{1}{\Gamma_{\mathrm{all}}^{\mathrm{CCPs}} + \Gamma^{\mathrm{OPE}}},$$

- $b \to c\ell\bar{\nu}\ (\ell = e, \mu)$
- $b \to c\tau\bar{\nu}$,
- $b \to cq\bar{q}'\ (q = u, c;\ q' = d, s)$

To observe the novel analyticity behavior, the three of four real parameters, Q and $|R|$, are fixed properly.

$$R = |R|e^{i\theta} \text{ with } -\pi < \theta \le \pi$$

- Lepton energy distribution for $B \to X_c \ell \bar{\nu}$ decay



$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell} = \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\Big|_{pert.} + \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\Big|_{CCP}$$

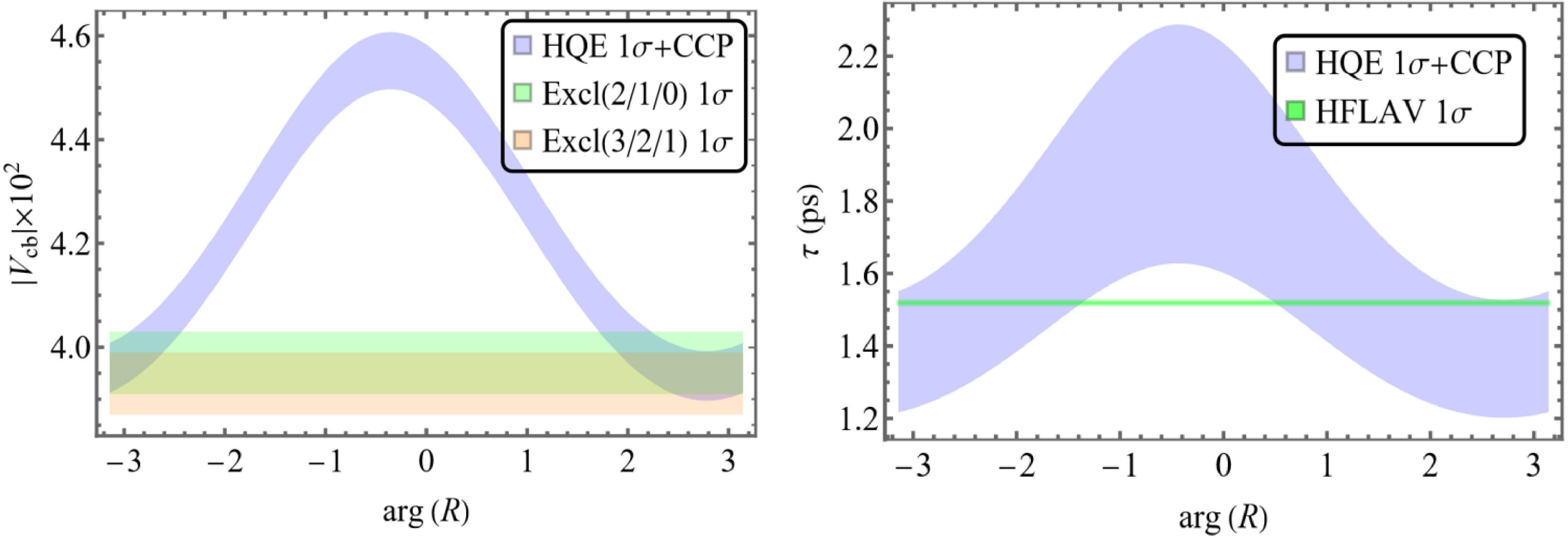
$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\Big|_{\mathrm{CCPs}} = 24\mathrm{Re}\Bigg(R\Big\{-\frac{1}{3}[1-(1-y_\ell)^{-3}] \times (1-y_\ell+\hat{Q}) + \frac{1}{2}[1-(1-y_\ell)^{-2}] \times (1+\hat{Q})\Big\} \times (1-y_\ell+\hat{Q})^2\Bigg).$$

$$y_\ell = 2E_\ell/m_b \; (\ell = e, \mu) \qquad \hat{Q} = Q/m_b^2$$

$$R = |R|e^{i\theta} \text{ with } -\pi < \theta \le \pi$$

- The case with θ = π (0) gives a positive (negative) contribution to the partonic rate.

- Plot $|V_{cb}|$ and lifetime of B_d^0 meson



There exists a parameter region around $arg(R) = \pm\pi$ explaining |Vcb| and τ (B_d^0) simultaneously within 1σ, which is exhibited with the particular values of the parameters.

Summary

- The nontrivial structure of quark propagator is discussed. It gives the additional contributions which can be extracted by the residue theorem.

- There is the possibility that the $|V_{cb}|$ puzzle is resolved by the CCPs through analyzing the width for $B \to X_c \ell \bar{\nu}$ and B_d^0-meson lifetime.

- If the mentioned puzzle is attributed to the CCPs, the usual Källén –Lehman spectrum should be corrected in the quark sector.

Thanks!

The indirect CP violation parameter in $K^0 - \overline{K}^0$ mixing: $\mathcal{E}_K \propto |V_{cb}|^4$

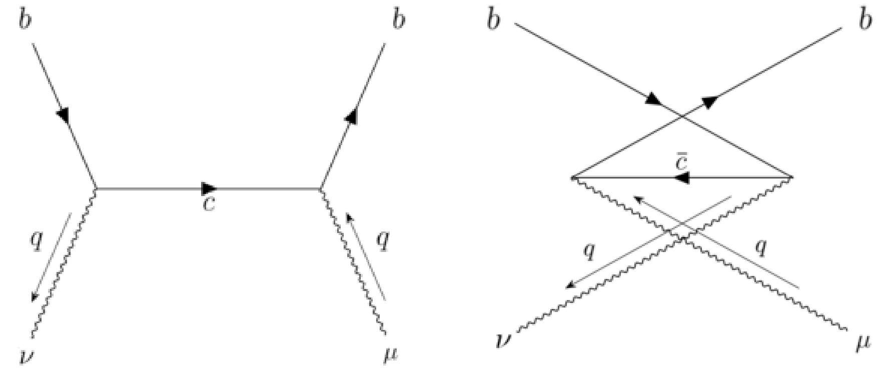
New physics interpretation is disfavored due to the constraint $Zb\bar{b}$ coupling.

One pair of CCP is considered. Two complex parameters, R and Q , are introduced because only σ_v survives while σ_s vanishes due to the chirality projection operator in B decay .

$T_{\mu\nu}$

$$\frac{d^2\Gamma}{dE_\ell d\alpha} = -\frac{G_F^2|V_{cb}|^2}{4\pi^3}\frac{2m_b}{\pi}\mathrm{Im}\int_{E_\ell}^{q^0_{\max}}(q^0 - E_\ell)L_{\mu\nu}T^{\mu\nu}dq^0$$

$$\frac{d^2\Gamma}{dE_\ell d\alpha} = \frac{G_F^2|V_{cb}|^2}{4\pi^3}\frac{m_b}{\pi}\mathrm{Im}\int_{C_1}(q^0 - E_\ell)L_{\mu\nu}T^{\mu\nu}dq^0$$



$$(p_B - q)^2 = M^2_{X_c}, \qquad (p_B + q)^2 = M^2_{X_{\bar{c}bb}},$$

Taking $b \to c\ell\bar{\nu}$ (massless lepton $\ell = e, \mu$) for an example.

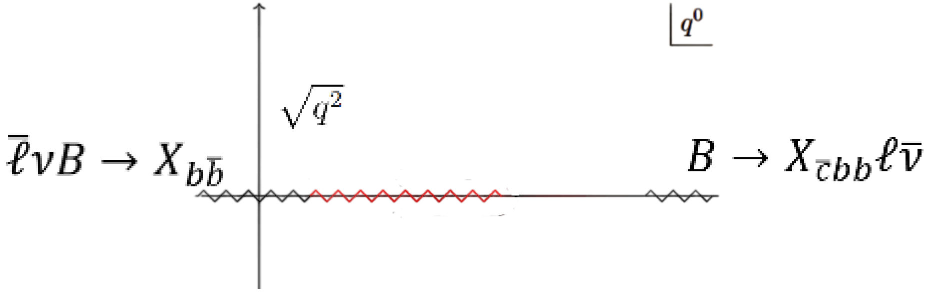
$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell} = \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\bigg|_{pert.} + \frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\bigg|_{CCP}$$

leading to,

$$-\infty < q^0 < \frac{M_B^2 + q^2 - M^2_{X_c^{\min}}}{2M_B}, \qquad \frac{M^2_{X^{\min}_{\bar{c}bb}} - M_B^2 - q^2}{2M_B} < q^0 < \infty.$$

$$\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell} = 2y_\ell[3x_\ell^2 y_\ell(2 - y_\ell) + x_\ell^3(y_\ell^2 - 3y_\ell)],$$

$$\begin{aligned}\frac{1}{\Gamma_b}\frac{d\Gamma}{dy_\ell}\bigg|_{\mathrm{CCPs}} = & \; 24\mathrm{Re}\left(R\left\{-\frac{1}{3}[1-(1-y_\ell)^{-3}]\right.\right. \\ & \times \; (1 - y_\ell + \hat{Q}) + \frac{1}{2}[1-(1-y_\ell)^{-2}] \\ & \times \left.\left.(1+\hat{Q})\right\} \times (1 - y_\ell + \hat{Q})^2\right).\end{aligned}$$



The effective Hamiltonian relevant for $\Delta B = 1$ non-leptonic decays reads [5]

$$\mathcal{H}_W = \frac{4G_F}{\sqrt{2}} V_{cb} V^*_{qq'} (C_1 O_1 + C_2 O_2),$$

$$O_1 = (\bar{c}^\alpha \gamma_\mu P_L b^\alpha)(\bar{q}'^\beta \gamma^\mu P_L q^\beta),$$

$$O_2 = (\bar{c}^\alpha \gamma_\mu P_L b^\beta)(\bar{q}'^\beta \gamma^\mu P_L q^\alpha).$$

$m_b^{\text{kin}} = 4.573$ GeV [Gambino, et al., 2107.00604]

$m_b = 4.78$ GeV

$m_c = 1.67$ GeV [PDG2025]

$\Gamma^{\text{OPE}} = (0.615^{+0.108}_{-0.069})\ \text{ps}^{-1}$ [Lenz, 2208.02643]

$Q = (-2.325 + 1.145i) GeV^2$ and $|R| = 0.115$.

1. Color confinement
2. Quark-hadron duality violation

Similar processing for

$b \to c\tau\bar{\nu}, b \to cq\bar{q}',$

$(q = u, c;\ q' = d, s)$

$$\frac{d\Gamma^{b\to c\bar{u}q'}}{dy_{q'}} = |V_{uq'}|^2 \tilde{C} \left. \frac{d\Gamma^{b\to c\ell\bar{\nu}}}{dy_\ell} \right|_{y_\ell \to y_{q'}},$$

$$\frac{d\Gamma^{b\to c\bar{c}q'}}{dy_{\bar{c}}} = |V_{cq'}|^2 \tilde{C} \left. \frac{d\Gamma^{b\to c\tau\bar{\nu}}}{dy_\tau} \right|_{y_\tau \to y_{\bar{c}},\ \rho_\tau \to \rho_{\bar{c}}},$$

The Wilson coefficient at the leading order

$$C_1(m_b) = 1.07$$

$$C_2(m_b) = -0.17$$

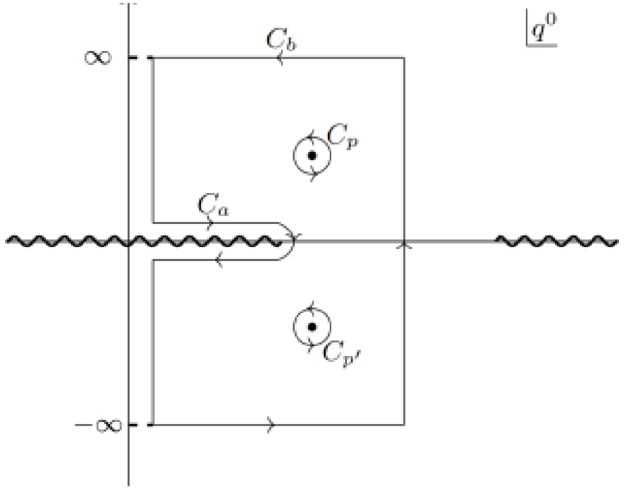
[Buchalla, et al.,9512380]

[Zehao Zhu, et al., 2005.04181]

$$\frac{1}{\Gamma_b}\Gamma^{\rm CCPs} = \sum_{m=-3}^{3} c_m F_m,$$

where $F_m = (1-\rho_c^{m+1})/(m+1)$ for $m \neq -1$, $F_{-1} = -\log(\rho_c)$, and

$$\begin{aligned}
c_3 &= -8{\rm Re}(R),\\
c_2 &= 12{\rm Re}[R(1-\hat{Q})],\\
c_1 &= 24{\rm Re}(R\hat{Q}),\\
c_0 &= -4{\rm Re}[R(1+3\hat{Q}-3\hat{Q}^2-\hat{Q}^3)],\\
c_{-1} &= -24{\rm Re}(R\hat{Q}^2),\\
c_{-2} &= 12{\rm Re}[R(1-\hat{Q})\hat{Q}^2],\\
c_{-3} &= 8{\rm Re}(R\hat{Q}^3).
\end{aligned}$$



$$\frac{d^2\Gamma}{dE_\ell d\alpha} = \left.\frac{d^2\Gamma}{dE_\ell d\alpha}\right|_{\rm pert} + \left.\frac{d^2\Gamma}{dE_\ell d\alpha}\right|_{\rm CCPs}.$$

the two terms read

$$\left.\frac{d^2\Gamma}{dE_\ell d\alpha}\right|_{\rm pert} = -\mathcal{F}(C_b,\tilde{T}),$$

$$\left.\frac{d^2\Gamma}{dE_\ell d\alpha}\right|_{\rm CCPs} = \mathcal{F}(C_p,T_{\rm CCP}) + \mathcal{F}(C_{p'},T_{\rm CCP'}),$$

where we defined

$$\mathcal{F}(\mathcal{C},\mathcal{T}) = \frac{G_F^2|V_{cb}|^2}{4\pi^3}\frac{m_b}{\pi}{\rm Im}\int_{\mathcal{C}}(q^0-E_\ell)L_{\mu\nu}\mathcal{T}^{\mu\nu}dq^0.$$