A novel approach to charm mixing

Hiroyuki Umeeda (Jilin Univ.)

Collaborators: Xiaotong Xie (Jilin Univ.)
Jinglong Zhu (Jilin Univ.)

(ongoing work)

February 18, 2025

KEK Theory Meeting on Particle Physics Phenomenology
(KEK-PH2025winter)

Introduction

◉ Charm quark mass: $m_c \approx 1.3 - 1.7$ GeV

-- too heavy to apply chiral perturbation theory (ChPT).

-- may be too light to apply the $1/m_c$ expansion.

} theoretically challenging

◉ Difficulty in $D^0 - \overline{D^0}$ mixing

-- Due to cancellation from the GIM mechanism, theoretical precision is uncontrollable.

◉ In flavor factory experiments, precise data are obtained for charmed mesons.

-- test of QCD / search for new physics

$D^0 \leftrightarrow \overline{D^0}$ transition via time evolution

Schrödinger Eq. $i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = H_{2\times 2}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$ with $H_{2\times 2} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$

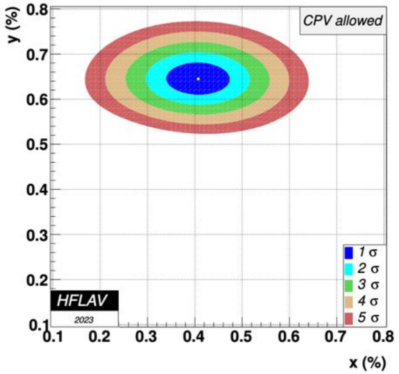
Mass eigenstates: $|D_{1,2}> = p|D^0> \pm q|\overline{D^0}>$ if $q/p \neq \pm 1 \longrightarrow$ CP violation

Observables $\begin{cases} \Delta m = m_2 - m_1 & \text{(mass difference)} \\ \Delta\Gamma = \Gamma_2 - \Gamma_1 & \text{(width difference)} \end{cases}$

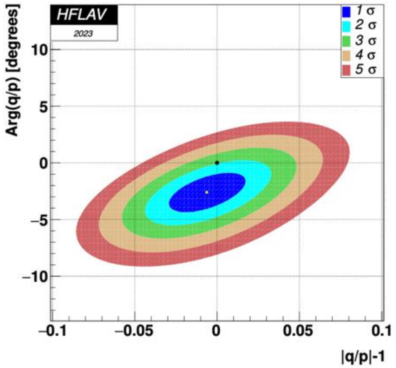
Dimensionless $\begin{cases} x = \Delta m/\Gamma_D \\ y = \Delta\Gamma/2\Gamma_D \end{cases}$

HFLAV:

x vs. y



$|q/p|$-1 vs. Arg(q/p)



Γ_D: total width of D^0

$\Gamma_D = \frac{\Gamma_1 + \Gamma_2}{2}$

Methods for $D^0 - \overline{D^0}$ mixing

Exclusive

Hadronic level analysis

Hard to calculable $\begin{cases} \Gamma[D \to \pi\pi] \\ \Gamma[D \to KK] \end{cases}$

$\longrightarrow$ Data are used

Previous works

Topological approach: Cheng and Chiang [2401.06316]

FAT approach: Jiang, Yu, Qin, Li and Lü [1705.07335]

Inclusive

Quark-level analysis

(no experimental input)

Operator product expansion (OPE)

$\longrightarrow$ $1/m_c$ & α_s series

Previous works

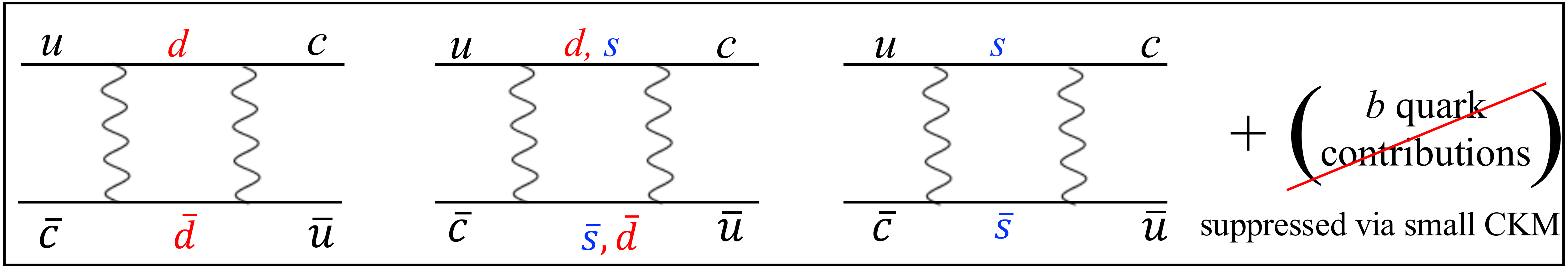
OPE

Golowich and Petrov [0506185]

Bobrowski, Lenz, Riedl and Rohrwild [0904.3971]

Box diagrams

$\lambda_i = V_{ci} V_{ui}^*$



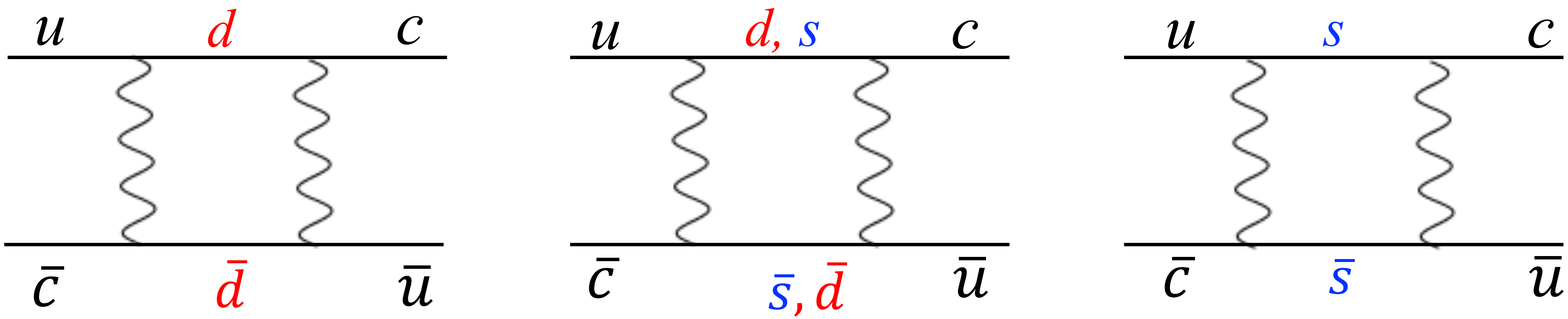
If $m_s = m_d$: $\text{sum} \propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2$

Unitarity of CKM matrix

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

Box diagrams

$\lambda_i = V_{ci} V_{ui}^*$



+ (~~b quark contributions~~)

suppressed via small CKM

If $m_s = m_d$: $\text{sum} \propto \lambda_d^2 + 2\lambda_d\lambda_s + \lambda_s^2 = 0$

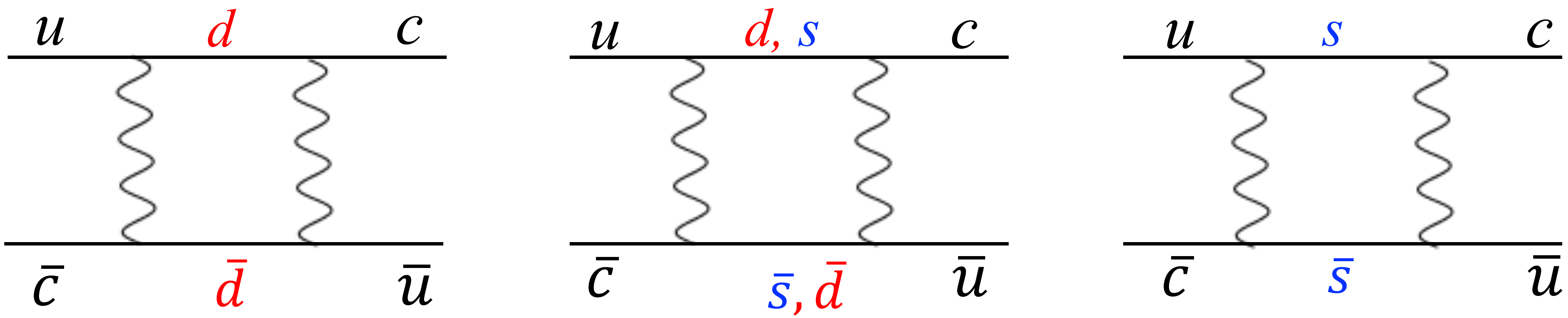
unitarity

Unitarity of CKM matrix

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

Box diagrams

$\lambda_i = V_{ci} V_{ui}^*$



+ (~~b quark contributions~~)

suppressed via small CKM

If $m_s = m_d$: $\text{sum} \propto \lambda_d^2 + 2\lambda_d \lambda_s + \lambda_s^2 = 0$

unitarity

Unitarity of CKM matrix

$\lambda_d + \lambda_s + \cancel{\lambda_b} = 0$

If $m_s \neq m_d$: $\text{sum} \propto \left(\frac{m_s^2 - m_d^2}{m_c^2} \right)^2 = \mathcal{O}(10^{-5})$

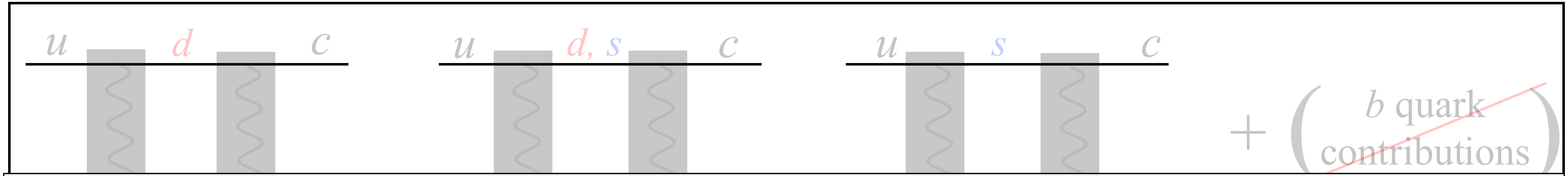
(box diagram case)

$\overline{\text{MS}}$ @ m_c scale

extreme suppression
for the theoretical side

$\lambda_i = V_{ci}V_{ui}^*$

Box diagrams



Theory @ NLO

$$\begin{cases} x = 6 \times 10^{-7} \\ y = 6 \times 10^{-7} \end{cases}$$

Golowich and Petrov [0506185]

Experiment (HFLAV)

$$\begin{cases} x = (4.07 \pm 0.44) \times 10^{-3} \\ y = (6.45^{+0.24}_{-0.23}) \times 10^{-3} \end{cases}$$

(all CPV allowed)

If $m_s \neq m_d$: sum $\propto \left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^2 = \mathcal{O}(10^{-5})$

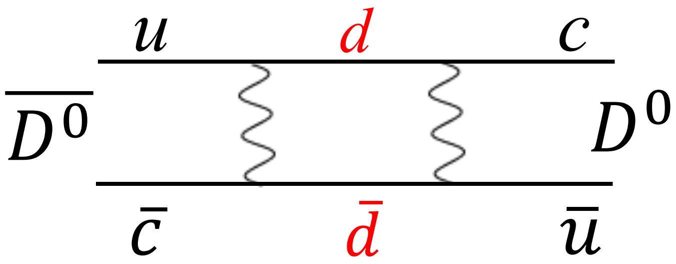
(box diagram case)

$\overline{\text{MS}}$ @ m_c scale

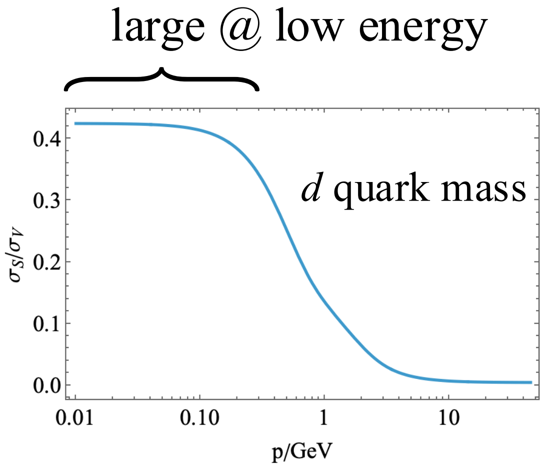
extreme suppression for the theoretical side

Box diagrams:

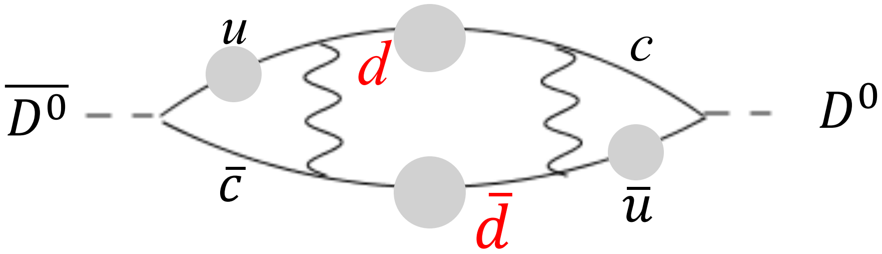
$$x \propto \left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^2$$



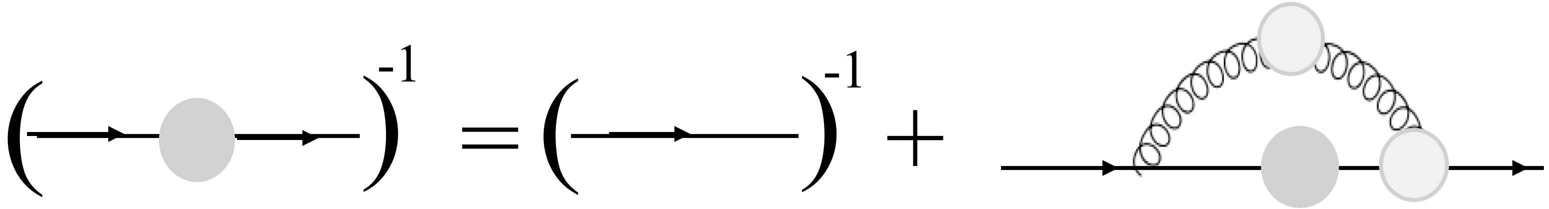
extreme suppression



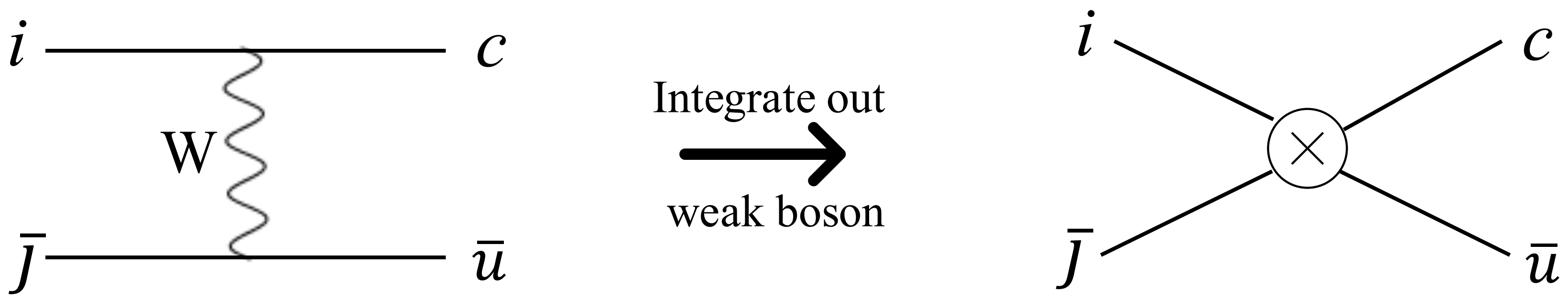
This work: Schwinger-Dyson approach



Dyson-Schwinger equation (DSE):



$x \propto$ [SU(3) breaking that takes account of chiral symmetry breaking]



Effective Hamiltonian for $\Delta C = 1$ processes

$$\mathcal{H}_{\text{eff}} = \sum_{i,j} \mathcal{H}^{ij} \quad \text{sum over } i, j = d, s$$

$$\mathcal{H}^{ij} = \frac{4G_F}{\sqrt{2}} V_{ci} V_{uj}^* (C_1 O_1^{ij} + C_2 O_2^{ij})$$

$$\begin{cases} O_1^{ij} = (\overline{c^\alpha} \gamma_\mu P_L i^\beta)(\overline{j^\beta} \gamma_\mu P_L u^\alpha) \\ O_2^{ij} = (\overline{c^\alpha} \gamma_\mu P_L i^\alpha)(\overline{j^\beta} \gamma_\mu P_L u^\beta) \end{cases}$$

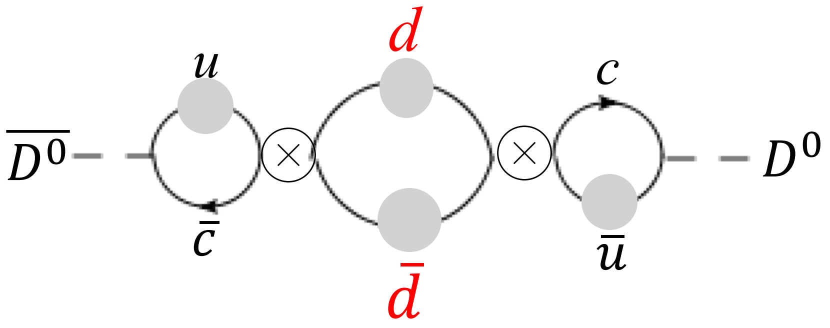
$$M_{12} = -\frac{i}{2!} \int d^4 z \left. \frac{< \overline{D^0} | T[\mathcal{H}_{\text{eff}}(z) \mathcal{H}_{\text{eff}}(0)] | D^0 >}{2M_D} \right|_{\text{off-shell}}$$

DSE (this work) or OPE

$$|x| = \frac{2|M_{12}|}{\Gamma_D}$$

(in the CP conserving limit)

$$\begin{cases} C_1(m_c) = -0.35 \\ C_2(m_c) = 1.13 \end{cases}$$



$$M_{12} \propto \underbrace{\lambda_d^2}_{\text{CKM}} (S_d \otimes S_d)$$

Quark propagator $\quad f = d, s$

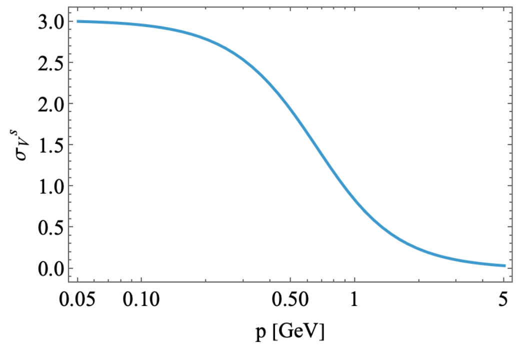
$$S_f(p) = -i\gamma \cdot p\, \sigma_V^f(p^2) + \sigma_S^f(p^2)$$

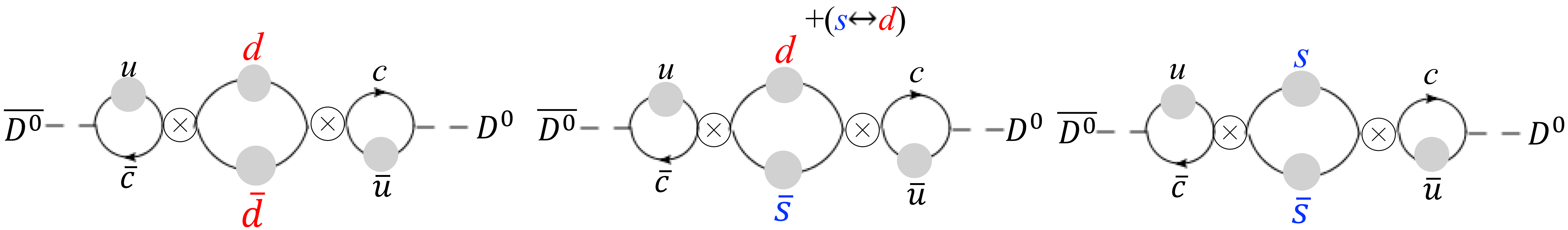
Parametrizations of σ_V^f, σ_S^f

Ivanov, Kalinovsky, Maris and Roberts [9711023]

$$\bar{\sigma}_S^f(x) := \sqrt{2D}\, \sigma_S^f(p^2)\,, \qquad \bar{\sigma}_S^f(x) = 2\bar{m}_f \mathcal{F}(2(x+\bar{m}_f^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left(b_0^f + b_2^f \mathcal{F}(\epsilon x)\right)\,,$$

$$\bar{\sigma}_V^f(x) := 2D\, \sigma_V^f(p^2)\,, \qquad \bar{\sigma}_V^f(x) = \frac{2(x+\bar{m}_f^2) - 1 + e^{-2(x+\bar{m}_f^2)}}{2(x+\bar{m}_f^2)^2}\,,$$





$$M_{12} \propto \lambda_d^2 (S_d \otimes S_d) + 2\lambda_d \lambda_s (S_d \otimes S_s) + \lambda_s^2 (S_s \otimes S_s)$$

Quark propagator $\quad f = d,\ s$

$$S_f(p) = -i\gamma \cdot p\, \sigma_V^f(p^2) + \sigma_S^f(p^2)$$

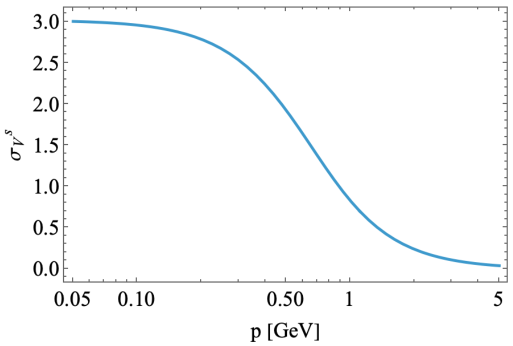
Parametrizations of σ_V^f, σ_S^f

Ivanov, Kalinovsky, Maris and Roberts [9711023]

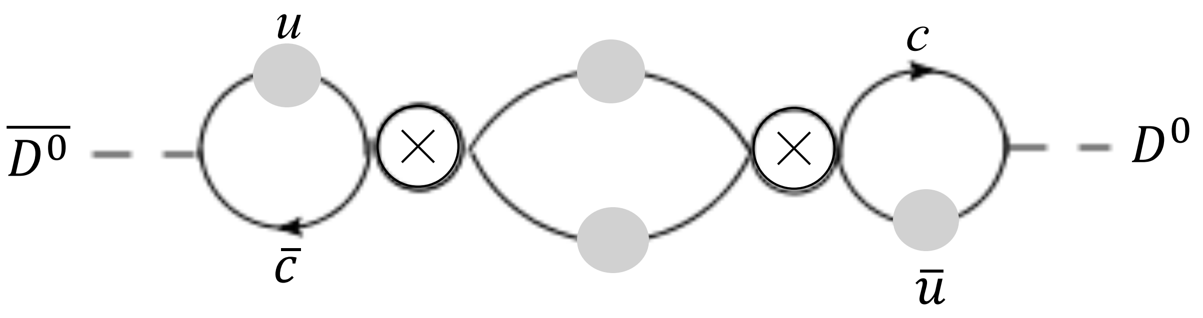
$$\bar{\sigma}_S^f(x) := \sqrt{2D}\, \sigma_S^f(p^2)\,, \qquad \bar{\sigma}_V^f(x) := 2D\, \sigma_V^f(p^2)\,,$$

$$\bar{\sigma}_S^f(x) = 2\bar{m}_f \mathcal{F}(2(x+\bar{m}_f^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left(b_0^f + b_2^f \mathcal{F}(\epsilon x)\right)\,,$$

$$\bar{\sigma}_V^f(x) = \frac{2(x+\bar{m}_f^2) - 1 + e^{-2(x+\bar{m}_f^2)}}{2(x+\bar{m}_f^2)^2}\,,$$

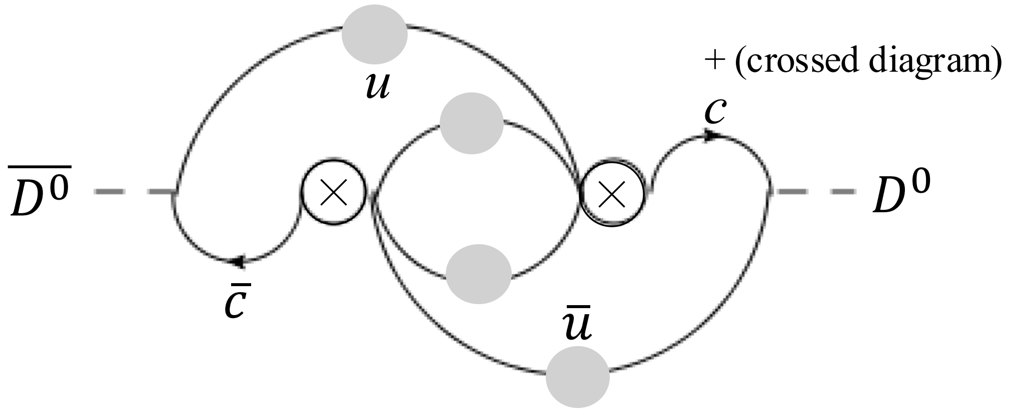


1st diagram

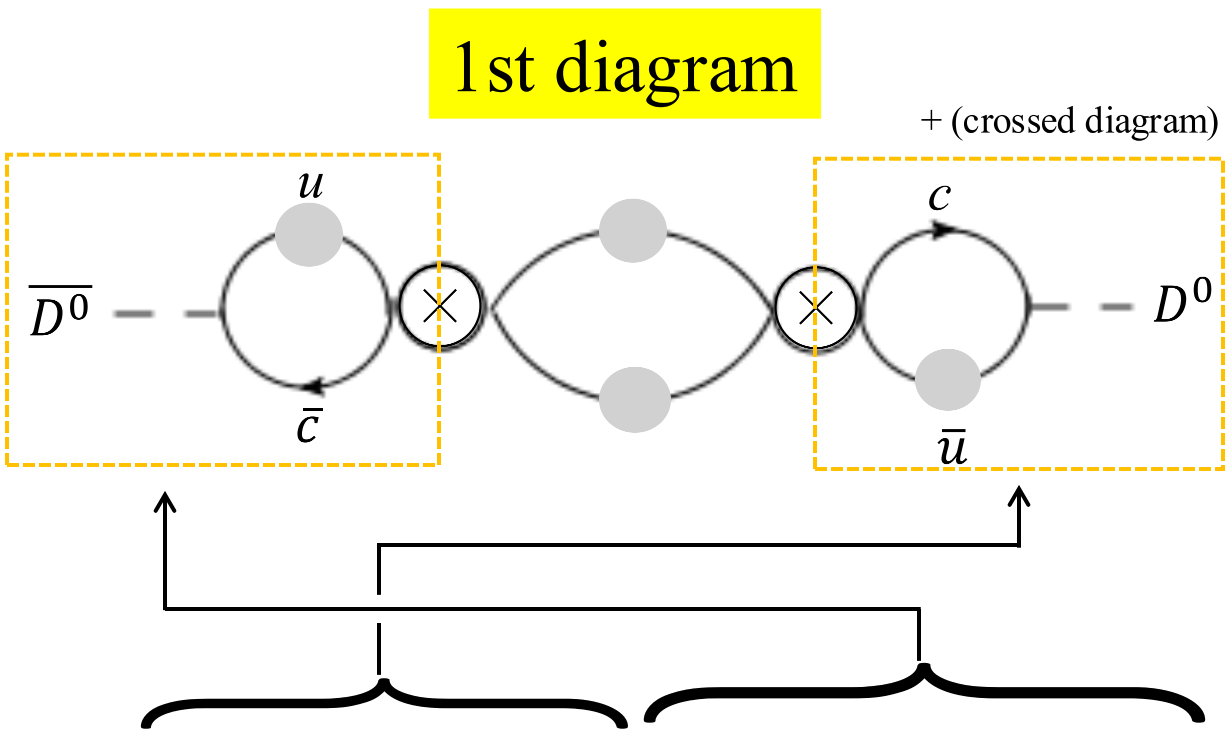


$$M_{12}^{(1st)} \propto < D^0|[\bar{c}(x)u(x)]_{V-A}|0 >< 0|[\bar{c}(0)u(0)]_{V-A}|\overline{D^0} >$$

2nd diagram

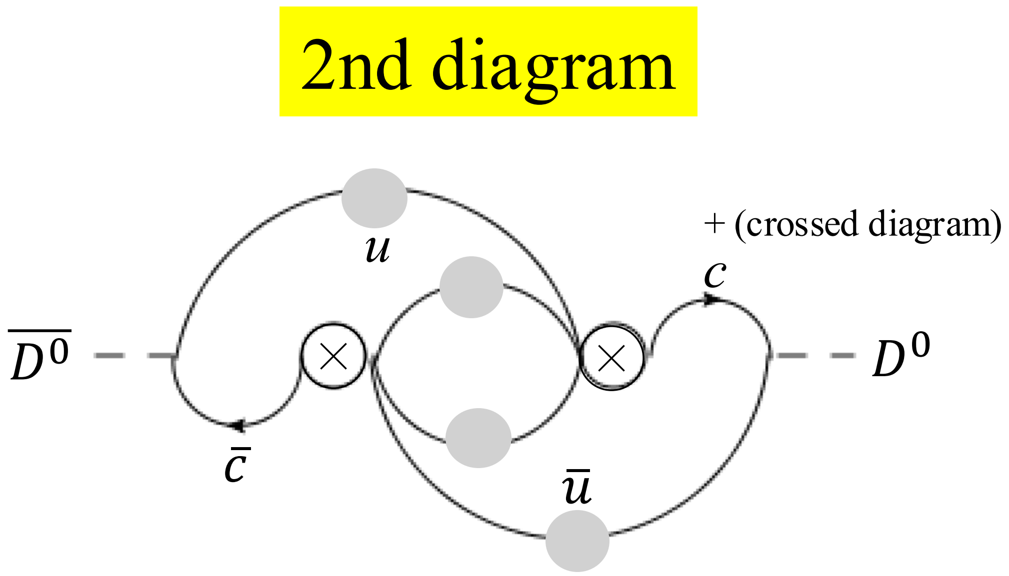


$$M_{12}^{(2nd)} \propto < D^0|[\bar{c}(x)u(0)]_{V-A}|0 >< 0|[\bar{c}(0)u(x)]_{V-A}|\overline{D^0} >$$



$$M_{12}^{(1st)} \propto < D^0|[\bar{c}(x)u(x)]_{V-A}|0 >< 0|[\bar{c}(0)u(0)]_{V-A}|\overline{D^0} >$$

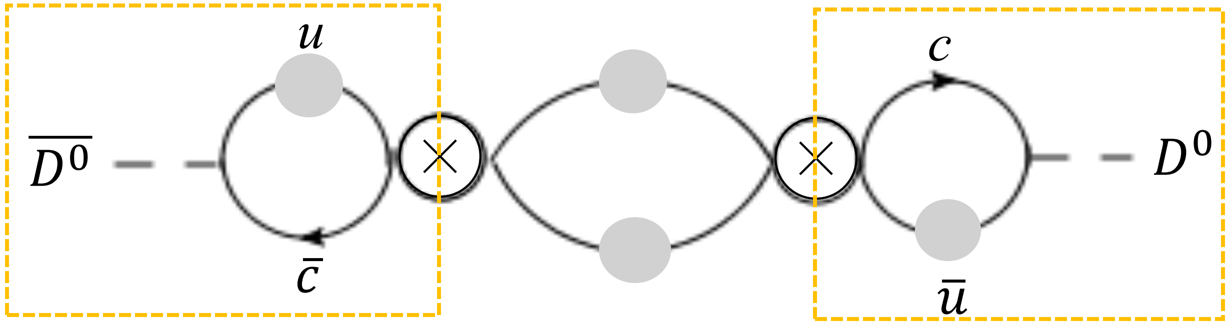
$$< 0|\bar{c}\gamma_\mu\gamma_5 u|\overline{D^0} > = i f_D p_\mu$$



$$M_{12}^{(2nd)} \propto < D^0|[\bar{c}(x)u(0)]_{V-A}|0 >< 0|[\bar{c}(0)u(x)]_{V-A}|\overline{D^0} >$$

$$u(x) \sim u(0)$$

1st diagram

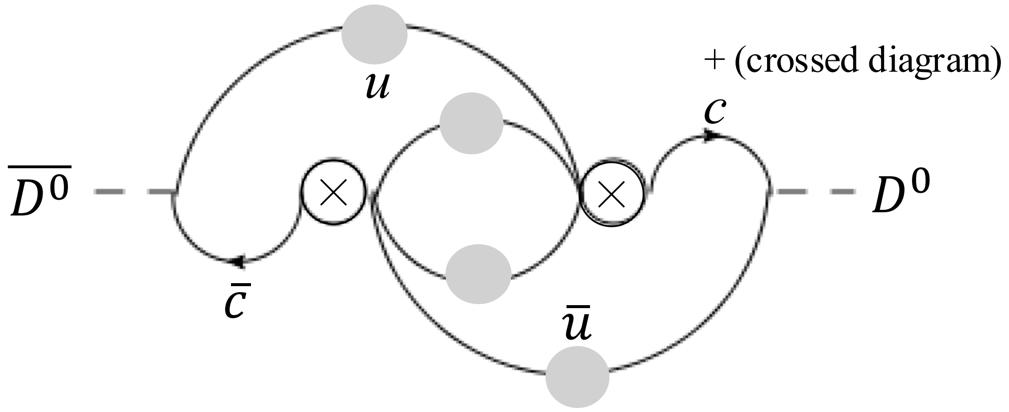


$$M_{12}^{(1st)} \propto < D^0|[\bar{c}(x)u(x)]_{V-A}|0 >< 0|[\bar{c}(0)u(0)]_{V-A}|\overline{D^0} >$$

$$< 0|\bar{c}\gamma_\mu\gamma_5 u|\overline{D^0} > = i f_D p_\mu$$

$$M_{12}^{(1st)} \propto f_D^2 \left(\frac{C_2^2}{N_c} + 2C_1C_2 + N_c C_1^2 \right)$$

2nd diagram



$$M_{12}^{(2nd)} \propto < D^0|[\bar{c}(x)u(0)]_{V-A}|0 >< 0|[\bar{c}(0)u(x)]_{V-A}|\overline{D^0} >$$

$$u(x) \sim u(0)$$

$$M_{12}^{(2nd)} \underset{\sim}{\propto} 2 f_D^2 \left(C_2^2 + \frac{2}{N_c} C_1C_2 + C_1^2 \right)$$

Numerical result

This work (preliminary)	$\lvert x \rvert = 1.7 \times 10^{-3}$	DSE
Golowich and Petrov [0506185]	$x = 6 \times 10^{-7}$	OPE
Melić, Dulibić and Petrov [2410.14382]	$x^{NLC} = 7.7 \times 10^{-6}$	Non-local condensate
HFLAV [2411.18639]	$x = (4.07 \pm 0.44) \times 10^{-3}$	Experiment

Separate results
$$\begin{cases} \left| x^{(1st)} \right| = 1 \times 10^{-6} \\ \left| x^{(2nd)} \right| = 1.67 \times 10^{-3} \text{ (dominant)} \end{cases}$$

Summary

◉ We studied $D^0 - \overline{D^0}$ mixing in the Schwinger-Dyson approach.

◉ The order of magnitude for the mass difference is improved,

compared with the OPE-based approaches.

◉ The preliminary result is based on the approximation, $u(x) \to u(0)$.

Future direction: we aim to include the *x*-dependence
and update the numerical result.

Backup slides

Input values

$$S_f(p) = -ip \cdot \gamma \sigma_V^f(p^2) + \sigma_S^f(p^2)$$

$$\bar{\sigma}_S^f(x) := \sqrt{2D}\,\sigma_S^f(p^2)\,,$$
$$\bar{\sigma}_V^f(x) := 2D\,\sigma_V^f(p^2)\,,$$

$$\bar{\sigma}_S^f(x) = 2\bar{m}_f \mathcal{F}(2(x+\bar{m}_f^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left(b_0^f + b_2^f \mathcal{F}(\epsilon x)\right)\,,$$
$$\bar{\sigma}_V^f(x) = \frac{2(x+\bar{m}_f^2) - 1 + e^{-2(x+\bar{m}_f^2)}}{2(x+\bar{m}_f^2)^2}\,,$$

$$x = p^2/(2D);\ \bar{m}_f = m_f/\sqrt{2D}; \qquad \mathcal{F}(y) := \frac{1-\mathrm{e}^{-y}}{y}$$

$D = 0.160\,\mathrm{GeV}^2$

	$\bar{m}_f$	b_0^f	b_1^f	b_2^f	b_3^f
u :	0.00897	0.131	2.90	0.603	0.185
s :	0.224	0.105	<u>2.90</u>	0.740	<u>0.185</u>

Ivanov, Kalinovsky, Maris and Roberts [9711023]

Accidental cancellation for C_1, C_2

$$\begin{cases} M_{12}^{(1st)} \propto f_D^2 \left(\frac{C_2^2}{N_c} + 2C_1C_2 + N_c C_1^2 \right) \\ M_{12}^{(2nd)} \propto 2f_D^2 \left(C_2^2 + \frac{2}{N_c} C_1C_2 + C_1^2 \right) \end{cases}$$

$$\begin{cases} \left(\frac{C_2^2}{N_c} + 2C_1C_2 + N_c C_1^2 \right) = 0.002 \\ 2\left(C_2^2 + \frac{2}{N_c} C_1C_2 + C_1^2 \right) = 2.2 \end{cases}$$

$$\begin{cases} C_1(m_c) = -0.35 \\ C_2(m_c) = 1.13 \end{cases}$$