Are Fractals Behind Flavour Structures in SM?

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$SU(3) \times SU(2)_L \times U(1)_Y$

SM Generations

Quantum numbers

q_L	u_R	d_R
(3,2,1/3)	(3,1,4/3)	(3,1,-2/3)

L_L	e_R	? ν_R
(1,2,-1)	(1,1,-2)	(1,1,0)

"Simple"

Particles / Masses

Q_1, u_R, d_R ~ (O(1)Mev, O(1)Mev)

Q_2, c_R, s_R ~ (O(1)Gev, O(0.1)Gev)

Q_3, t_R, b_R ~ (O(100)Gev, O(1)Gev)

L_1, e_R, ν_{eR}? ~ (O(0.5)Mev, O(0.1)ev)

$L_2, \mu_R, \nu_{\mu R}$ ~ (O(0.1)Gev, O(0.1)ev)

$L_3, \tau_R, \nu_{\tau R}$ ~ (O(1)Gev, O(0.1)ev)

Mixing

V_{CKM} wolfenstein parametrization

$$\begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

$$\lambda = 0.22500 \pm 0.00067\,, \quad A = 0.826^{+0.018}_{-0.015}\,,$$
$$\bar\rho = 0.159 \pm 0.010\,, \quad \bar\eta = 0.348 \pm 0.010\,.$$

CKM mixing

U_{PMNS} matrix

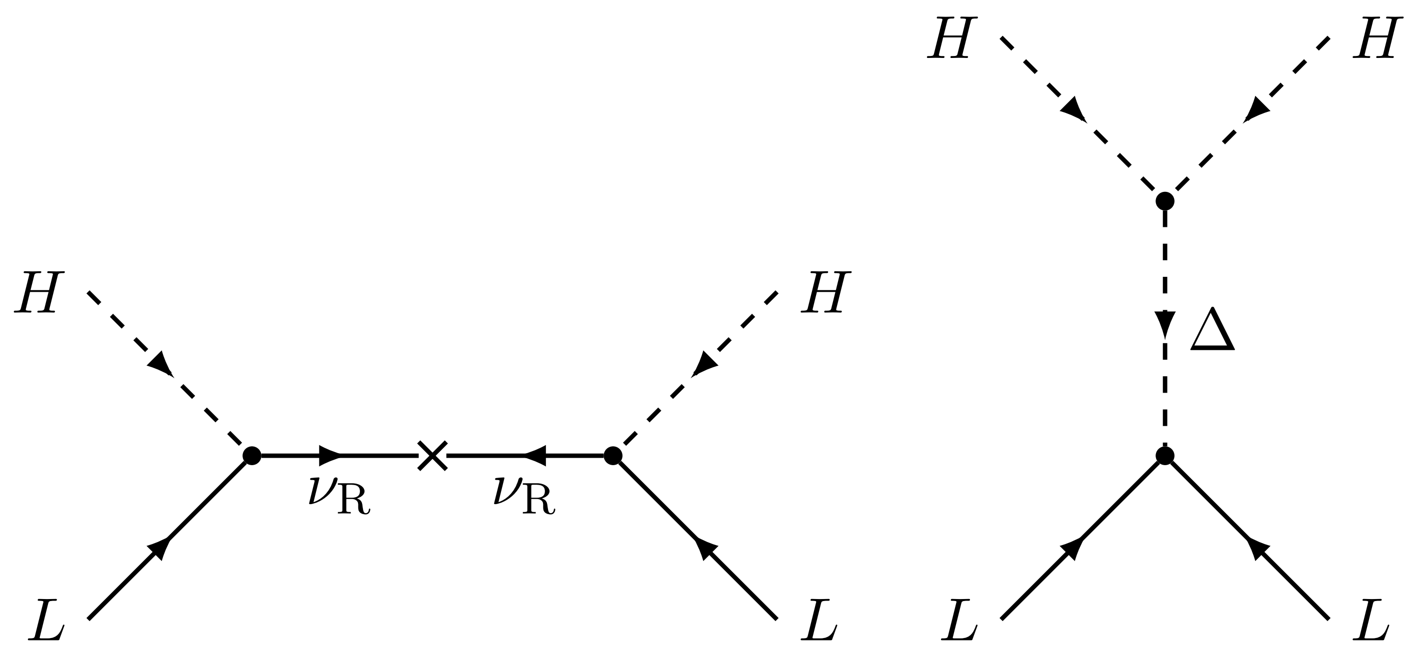
$$\begin{bmatrix} 0.803 \sim 0.845 & 0.514 \sim 0.578 & 0.142 \sim 0.155 \\ 0.233 \sim 0.505 & 0.460 \sim 0.693 & 0.630 \sim 0.779 \\ 0.262 \sim 0.525 & 0.473 \sim 0.702 & 0.610 \sim 0.762 \end{bmatrix}$$

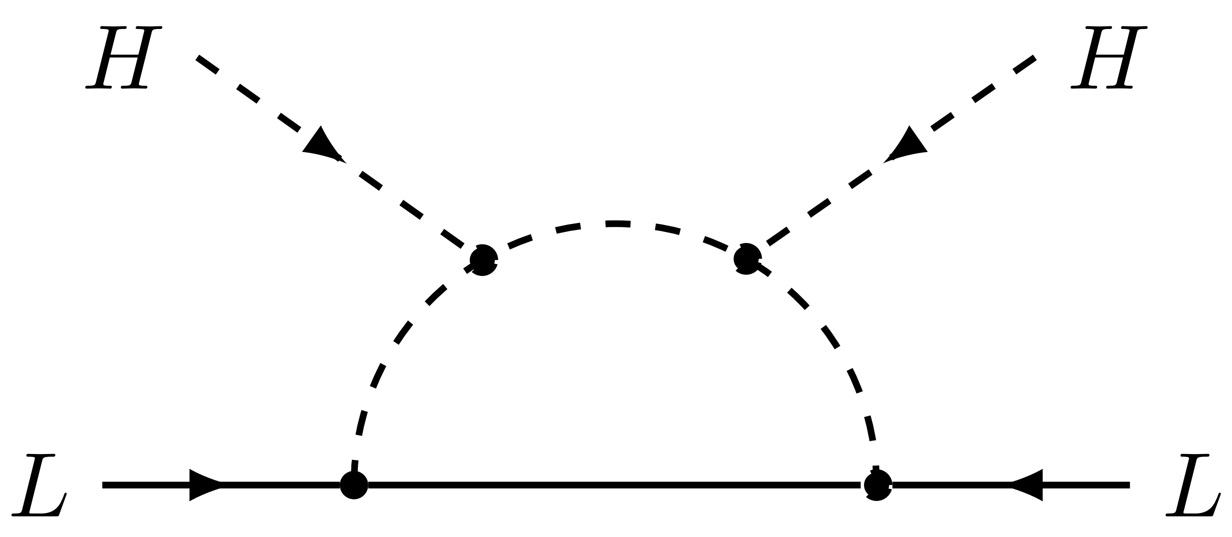
PMNS mixing

"Complex"

Neutrino Masses

There are several models in literature to explain different mass scale.





Minkowski
Senjanovic, Mohapatra,
GellMann, Ramond, Slansky
Yanagida

E. Ma,
Babu
Zee

Seesaw models

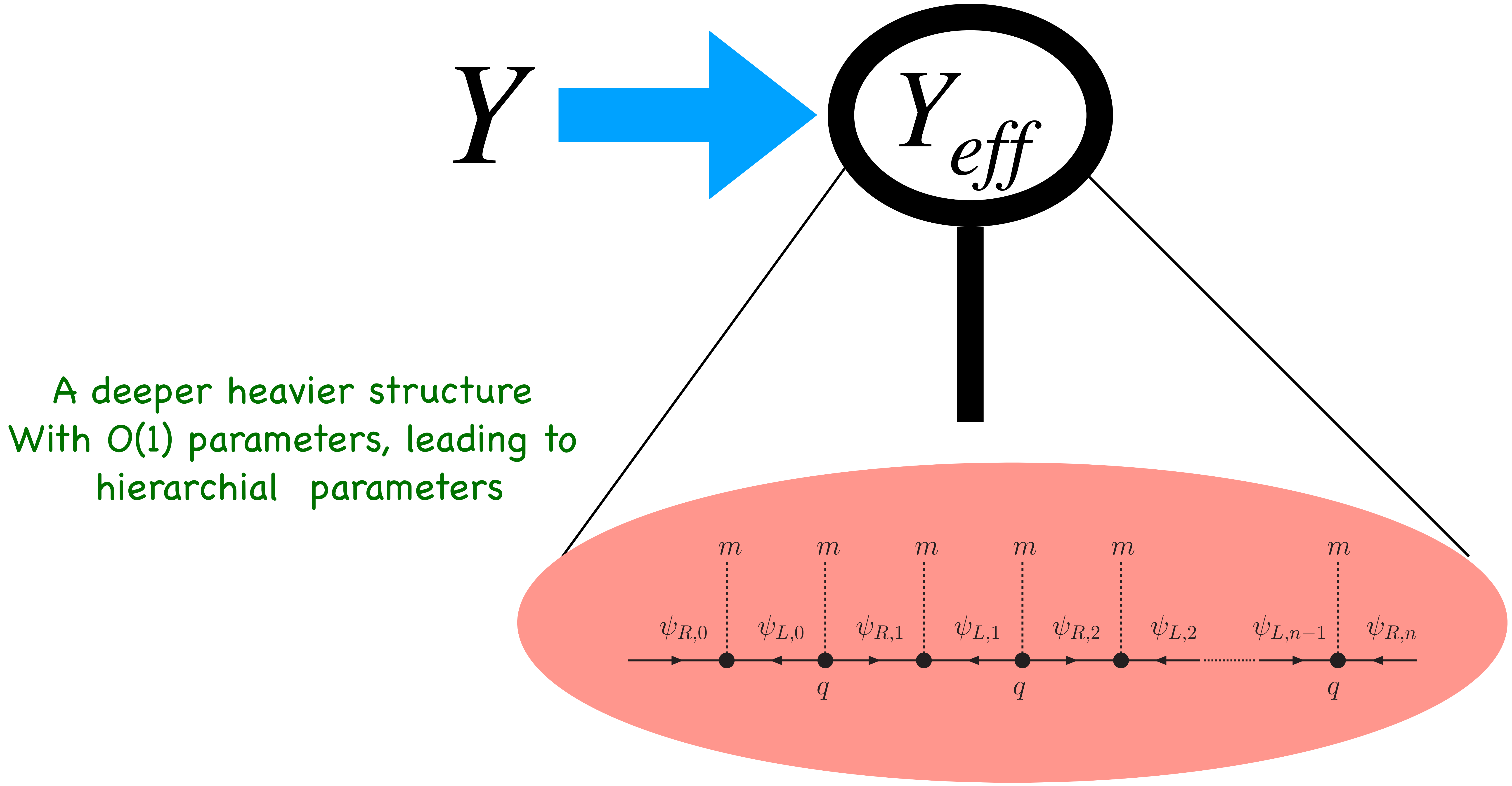
Radiative correction models

Other models also exist to explain the number of generations problem.

- String models
- UED models
- etc

Consider Dirac Masses

$$\mathcal{L}_{SM} + Y \bar{\nu}_L \nu_R \tilde{H}$$

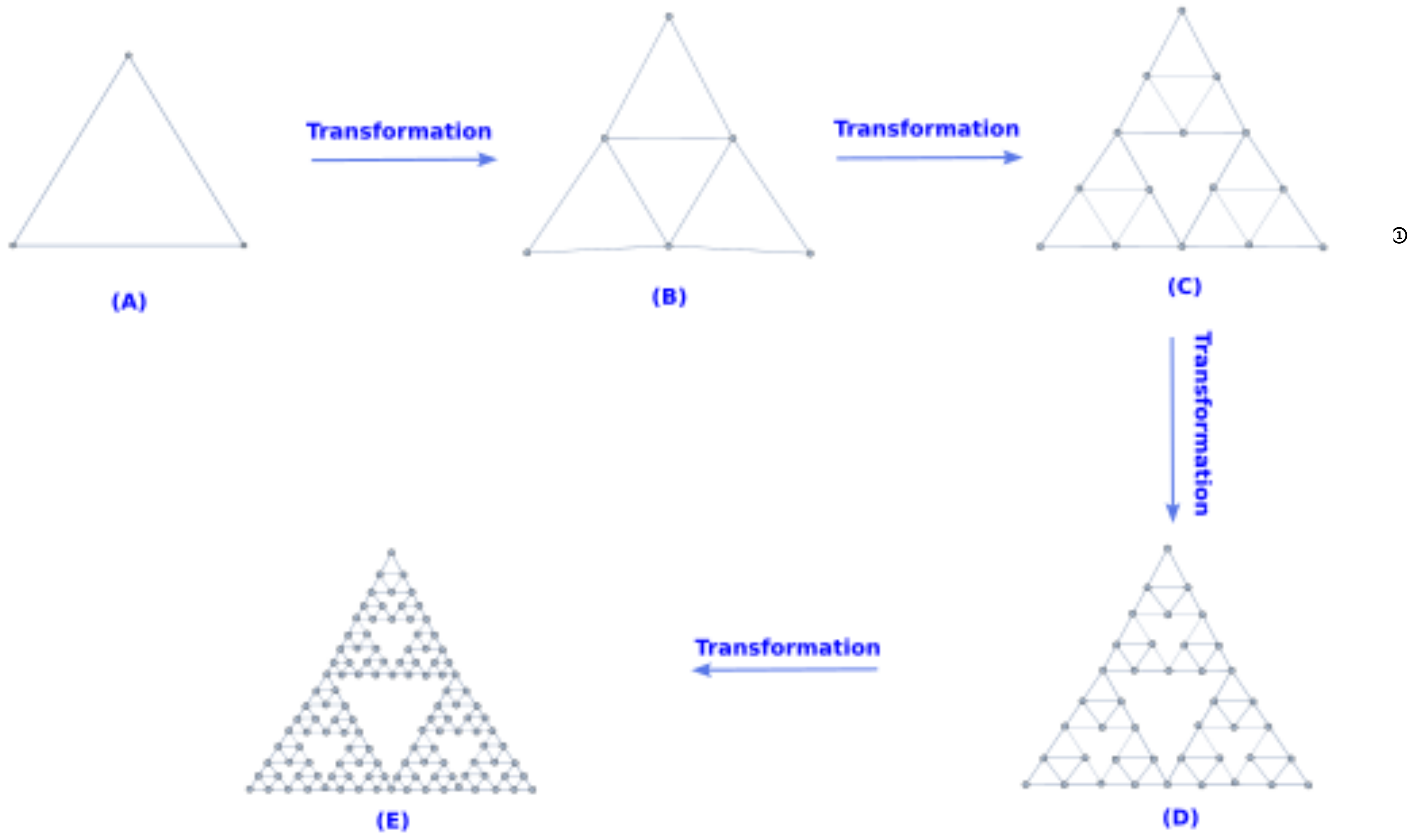


Fractals - Self-similiar objects

Fractals are self-similar i.e, they have similar properties at different scales.

"inspiration"

CT Hill - 0210076



- Self-similar
- Non-integer dimensions
- often formed by recursive process
- found in nature such as coastline, snowflake
- useful in various domains such as bio[1], quantum computing[2] etc.

(1) - nature 628, 894-900 (2024)

(2) - nature physics 20, 1421-1428 (2024)

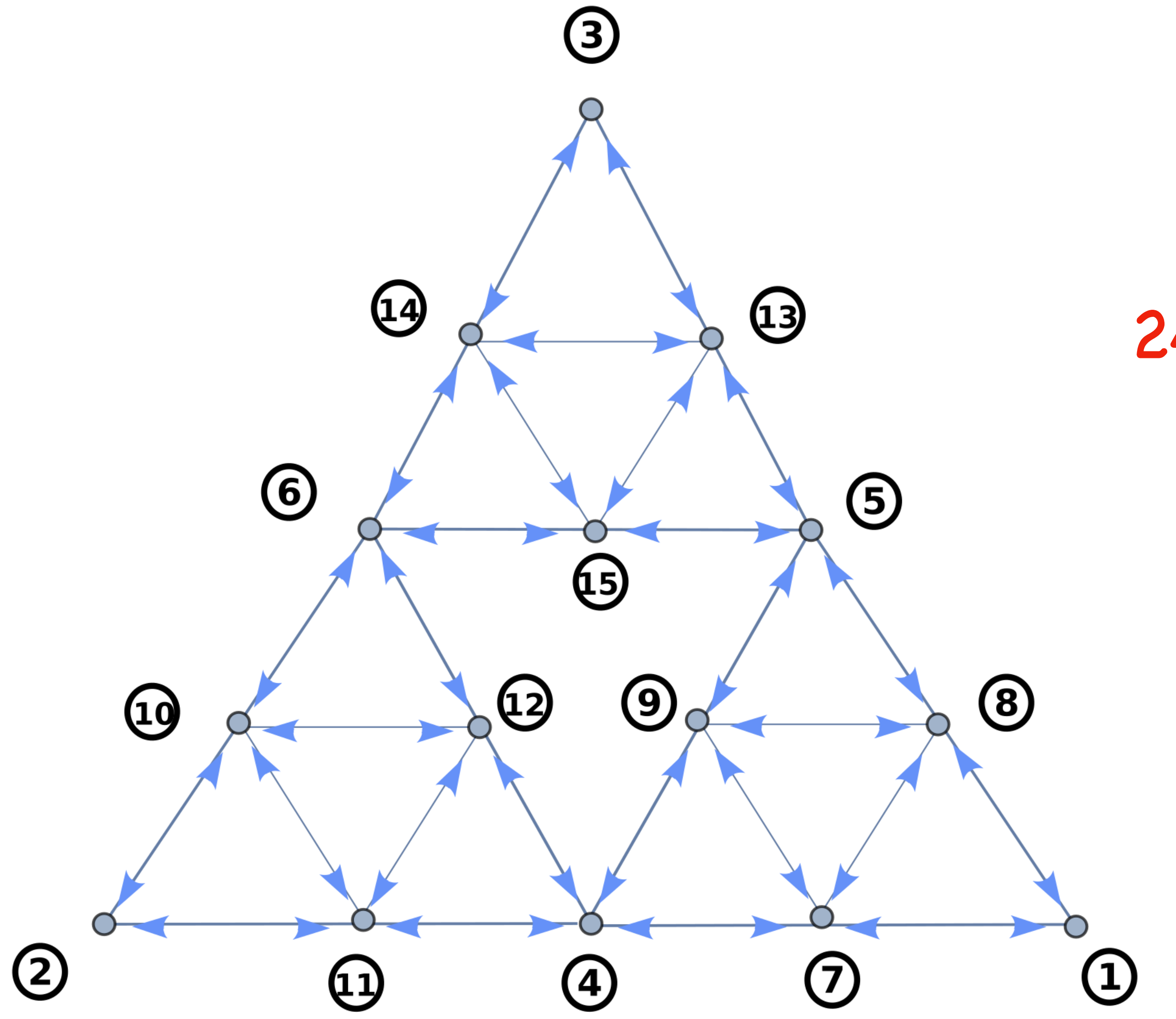
The idea is in "theory space"

Example with 15 vertices :

- three zero modes $\Rightarrow$ three generations !

-localisation of the zero modes !!!

One graph for all the three generations !!



CW Review

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- **Link Field Between L_i and R_j & L_j and R_i**

$$\mathcal{L}_{NP} = \mathcal{L}_{kin} - \sum_{i,j=1}^{15} m_i \overline{L_i}\delta_{i,j}R_j + m\Big(\overline{L_1}q_{1,7}R_7 + \overline{L_1}q_{1,8}R_8 + \overline{L_7}q_{7,4}R_4 + \overline{L_7}q_{7,9}R_9 + \overline{L_7}q_{7,8}R_8 + \overline{L_8}q_{8,5}R_5$$

$$+ \overline{L_8}q_{8,9}R_9 + \overline{L_4}q_{4,9}R_9 + \overline{L_4}q_{4,11}R_{11} + \overline{L_4}q_{4,12}R_{12} + \overline{L_9}q_{9,5}R_5 + \overline{L_5}q_{5,13}R_{13} + \overline{L_5}q_{5,15}R_{15} +$$

$$\overline{L_2}q_{2,10}R_{10} + \overline{L_2}q_{2,11}R_{11} + \overline{L_{10}}q_{10,6}R_6 + \overline{L_{10}}q_{10,12}R_{12} + \overline{L_{10}}q_{10,11}R_{11} + \overline{L_{11}}q_{11,12}R_{12} + \overline{L_6}q_{6,12}R_{12}$$

$$+ \overline{L_6}q_{6,14}R_{14} + \overline{L_6}q_{6,15}R_{15} + \overline{L_3}q_{3,13}R_{13} + \overline{L_3}q_{3,14}R_{14} + \overline{L_3}q_{3,15}R_{15} + \overline{L_{13}}q_{13,14}R_{14} + \overline{L_{14}}q_{14,15}R_{15}\Big)$$

$$+ m\overline{L_i}q_{i\leftrightarrow j}R_j + h.c.$$

with $q_{i,j} = f^{i-j}$ **m is universal for all nodes, three zero modes are present for all f values.**

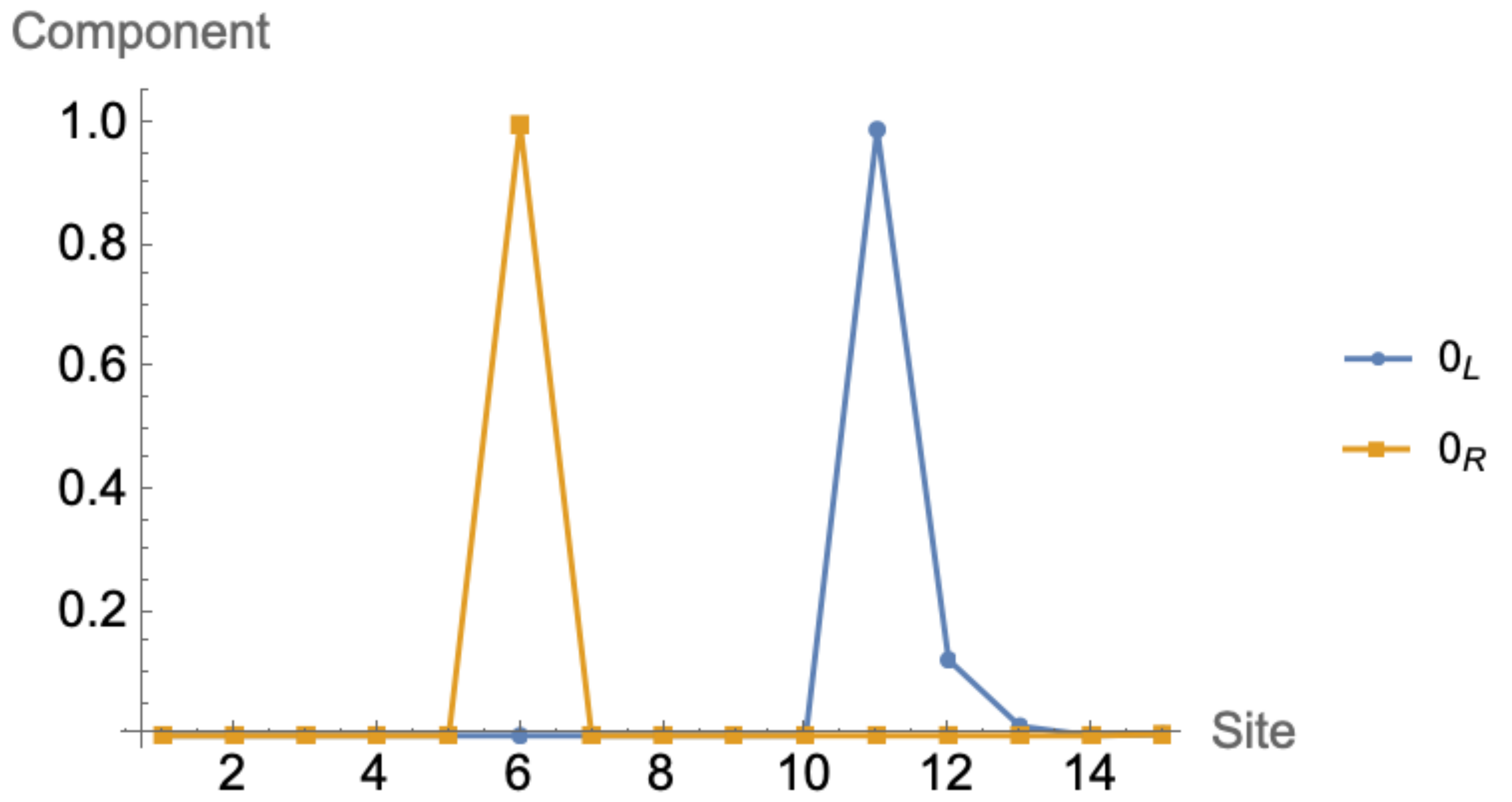
Zero Modes on the fractal graph/lattices

For f >1, 0-modes are localized

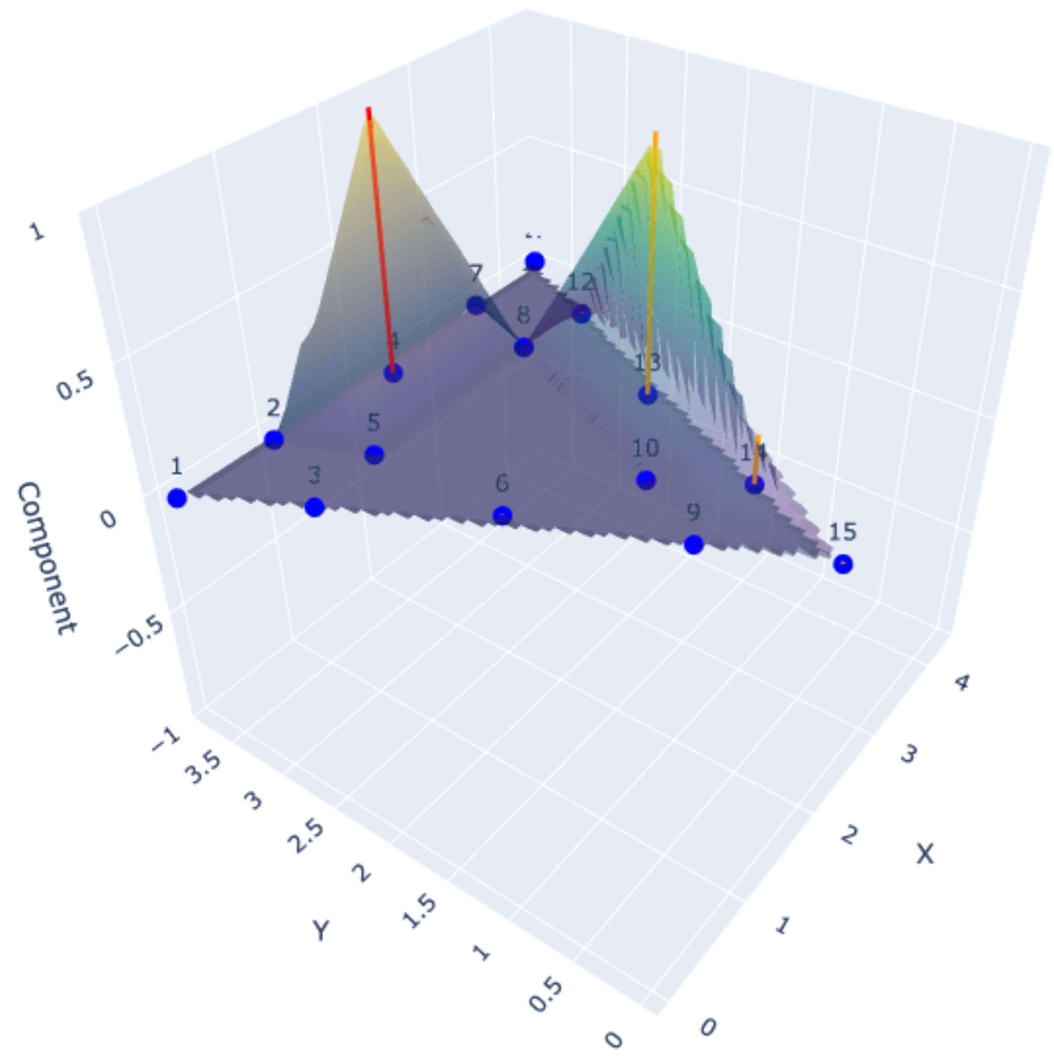
on the fractal nodes.

$$0_L = \{0 \quad f^{12} \quad -f^{11} \quad 0 \quad -f^9 \quad 2f^8 \quad 0 \quad 0 \quad -f^5 \quad -f^4 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0\}$$

$$0_R = \{0 \quad 0 \quad 0 \quad \frac{1}{f^9} \quad -\frac{1}{f^8} \quad \frac{1}{f^7} \quad 0 \quad -\frac{1}{f^5} \quad 0 \quad -\frac{1}{f^3} \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\}$$



Sierpiński Triangle Graph with Node Labels and zero modes



Higgs is coupled as per the localization of modes.

$$\mathscr{L}_{int} = -y_1 \bar{L}_4 \widetilde{H} R_4 - y_2 \bar{L}_9 \widetilde{H} R_9 - y_3 \bar{L}_{13} \widetilde{H} R_{13} + \text{ h.c.}$$

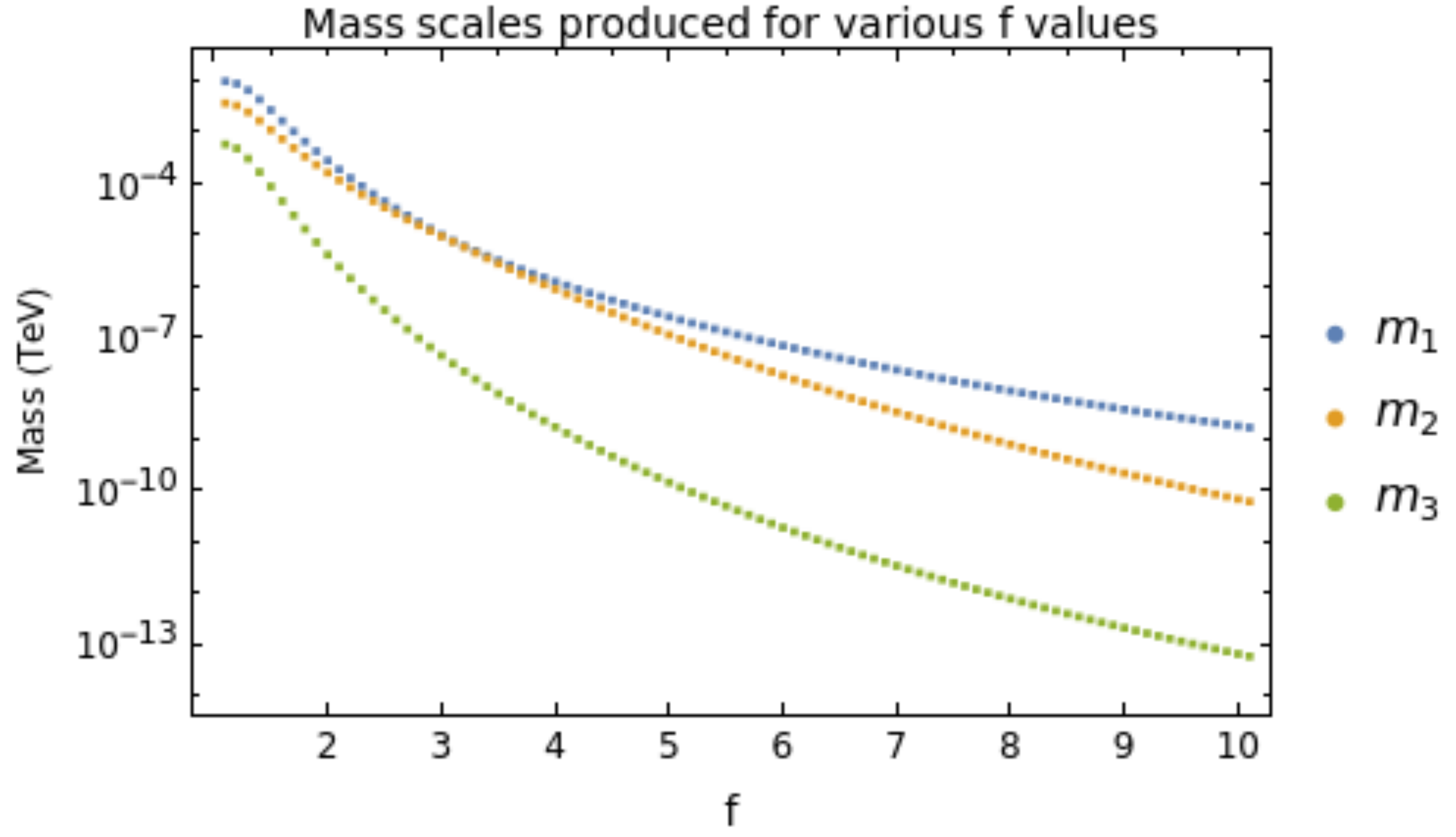
Can masses and flavour mixing be explained

Lepton masses and mixing

Charged Leptons — Masses $\mathcal{O}(1)GeV, \mathcal{O}(0.1)GeV, \mathcal{O}(0.5)MeV$

PMNS Mixing -

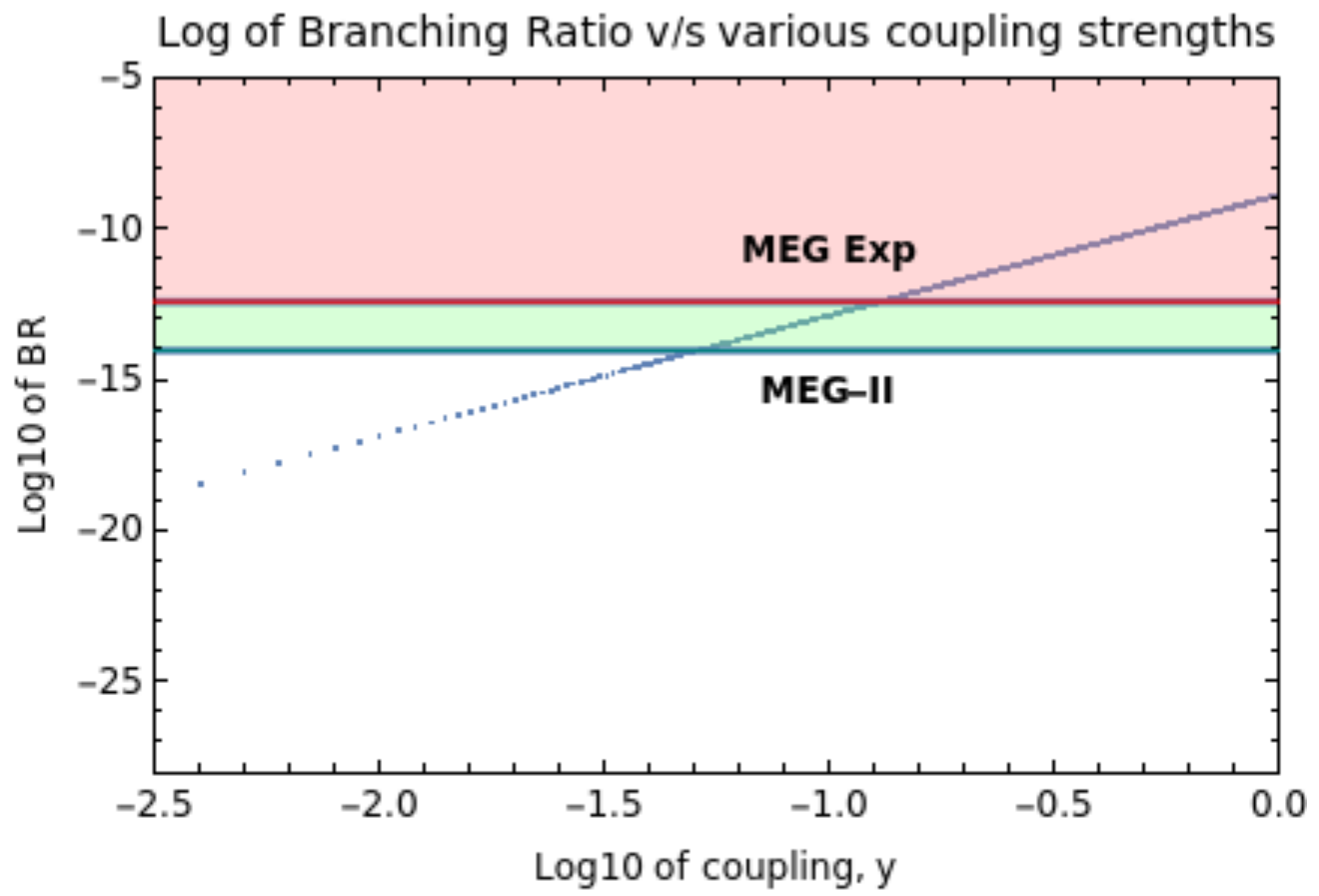
Neutral Leptons — Masses $\mathcal{O}(0.1)eV, \mathcal{O}(0.1)eV, \mathcal{O}(0.01)eV$

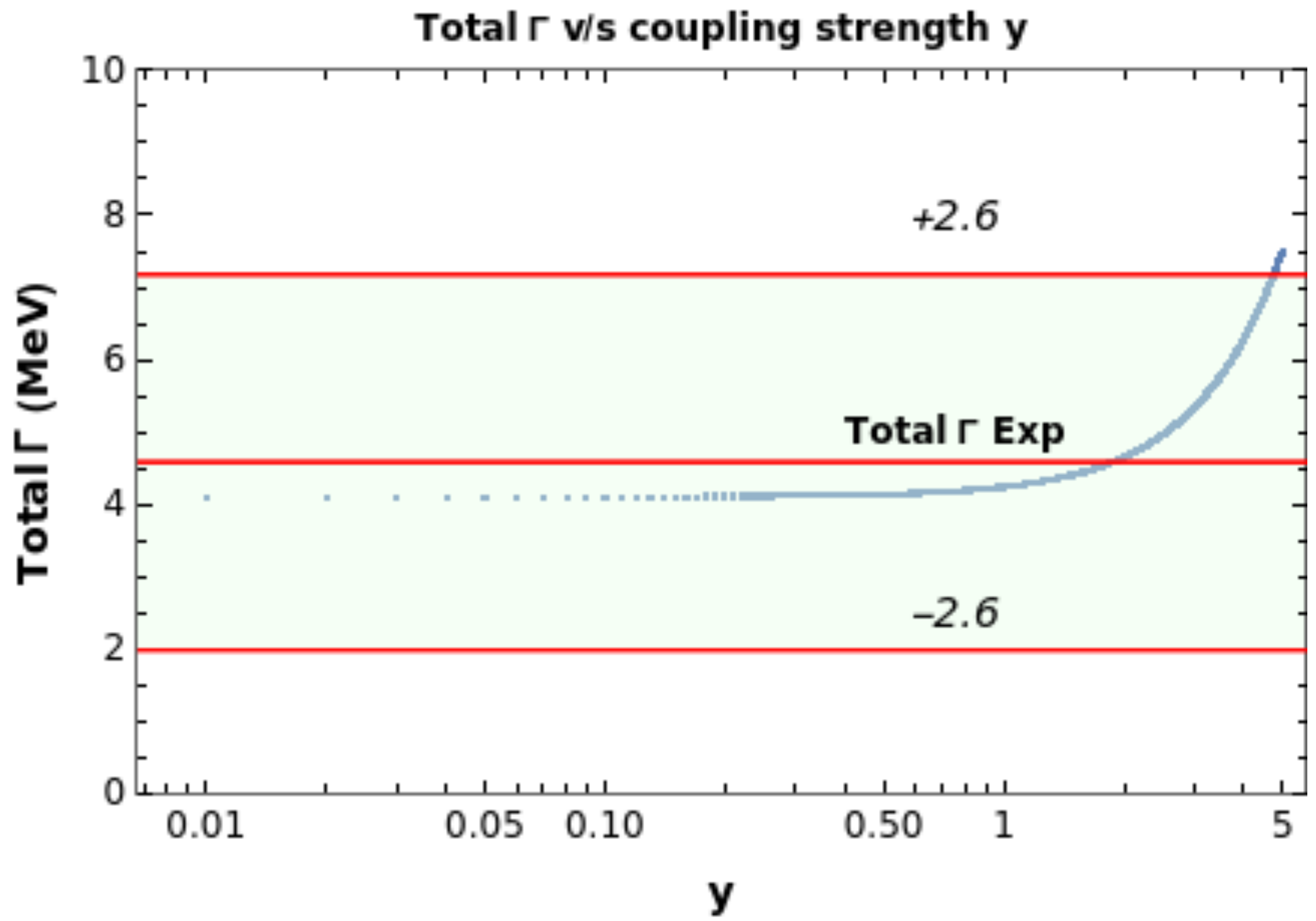


Masses produced as a function of f

What are the possible Signatures

FCNC BR ($\mu \to e\gamma$)





Higgs Decay Width

Summary

- SM has three generations of particles which are unexplained.

- Fractals can accommodate for intergenerational mixings due to complex connectivity along with different masses for three generations of particles due to different localizations.

- Sierpiński fractal with two iterations is used to account for leptons and quark masses and mixings.

THANK YOU

The Fractal Graphs and plots presented here are made using Mathematica and python.

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Fractal Graph Properties

$$M_0 = \begin{pmatrix} 2m & mf & mf^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{m}{f} & 2m & mf & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{m}{f^2} & \frac{m}{f} & 2m & 0 & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m}{f^2} & 0 & 2m & mf & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m}{f^3} & \frac{m}{f^2} & \frac{m}{f} & 2m & mf & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & mf^4 & mf^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & 0 & mf^4 & mf^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & 0 & mf^5 & mf^6 \\ 0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & 0 & 2m & mf & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & mf & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^6} & 0 & 0 & 0 & 0 & \frac{m}{f} & 2m \end{pmatrix}$$

Mass Matrix

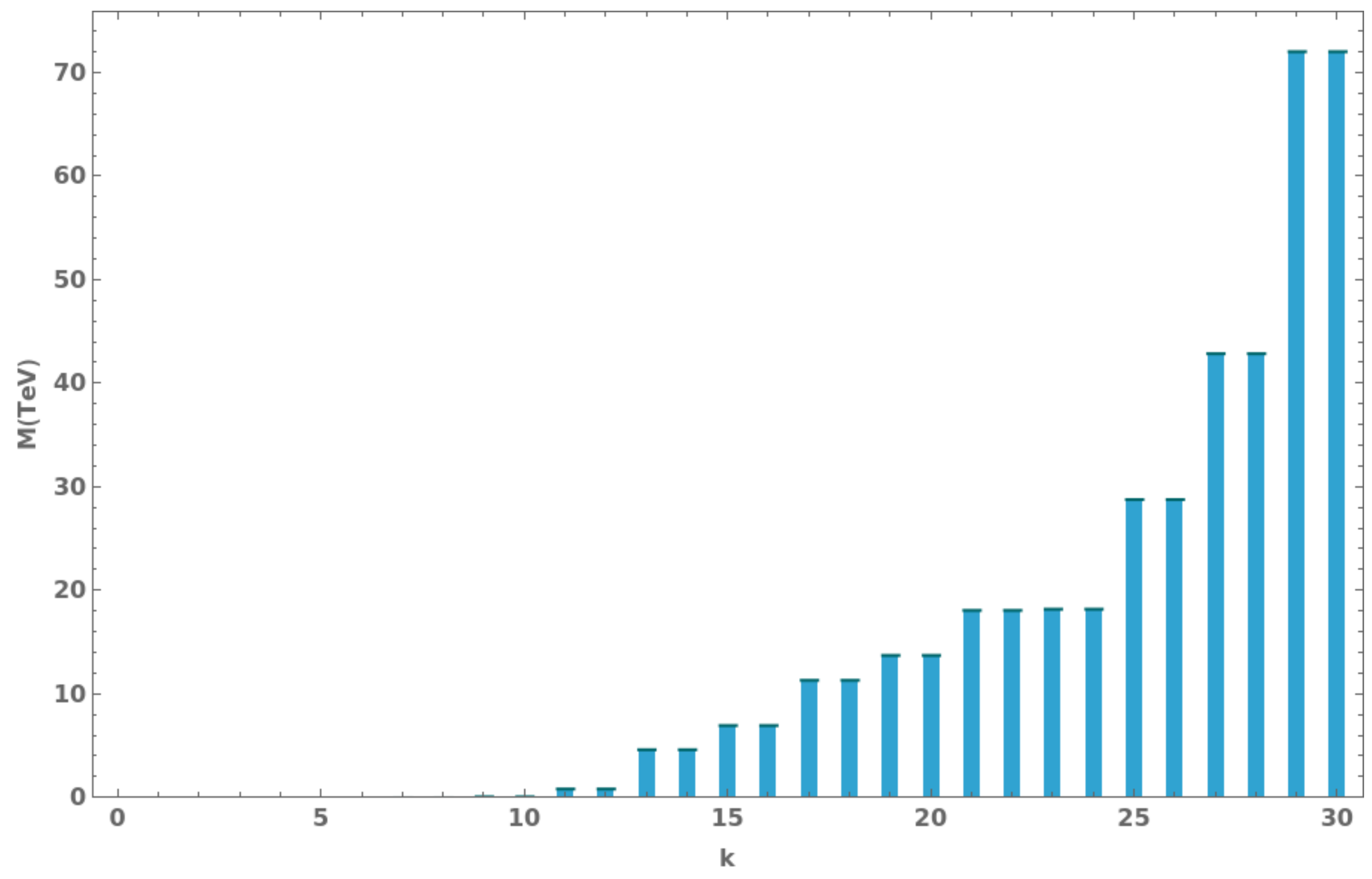
$$\Lambda_{iL} = \begin{pmatrix} 0 & f^{12} & -f^{11} & 0 & -f^9 & 2f^8 & 0 & 0 & -f^5 & -f^4 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & f^9 & -f^8 & f^7 & 0 & -f^5 & 0 & -f^3 & 0 & 0 & 1 & 0 & 0 \\ 0 & -f^{10} & f^9 & 2f^8 & -f^7 & 0 & -f^5 & -f^4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Three zero modes

$$\Lambda_{iR} = \begin{pmatrix} 0 & \frac{1}{f^{12}} & -\frac{1}{f^{11}} & 0 & -\frac{1}{f^9} & \frac{2}{f^8} & 0 & 0 & -\frac{1}{f^5} & -\frac{1}{f^4} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{f^9} & -\frac{1}{f^8} & \frac{1}{f^7} & 0 & -\frac{1}{f^5} & 0 & -\frac{1}{f^3} & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{f^{10}} & \frac{1}{f^9} & \frac{2}{f^8} & -\frac{1}{f^7} & 0 & -\frac{1}{f^5} & -\frac{1}{f^4} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_j = \{5.778m, 4.968m, 4.968m, 2.8418m, 2.8418m, 2.710m, 1.742m, 1.742m, m, 0.510m, 0.447m, 0.447m, 0, 0, 0\}$$

Mass modes of fractal



Mass modes spectrum

Parameter values

Parameters	Values	f	Mass Scales
$\{y_{e1}, y_{e2}, y_{e3}\}$	$y\{0.98, 0.01, 0.07\}$	1.91	$\mathcal{O}(1.7, 0.1, 0.0005)$ GeV
$\{y_{\nu 1}, y_{\nu 2}, y_{\nu 3}\}$	$y'\{0.23, 0.1, 0.025\}$	19	$\mathcal{O}(4.9, 4.83, 6 \times 10^{-5}) \times 10^{-2}$ eV

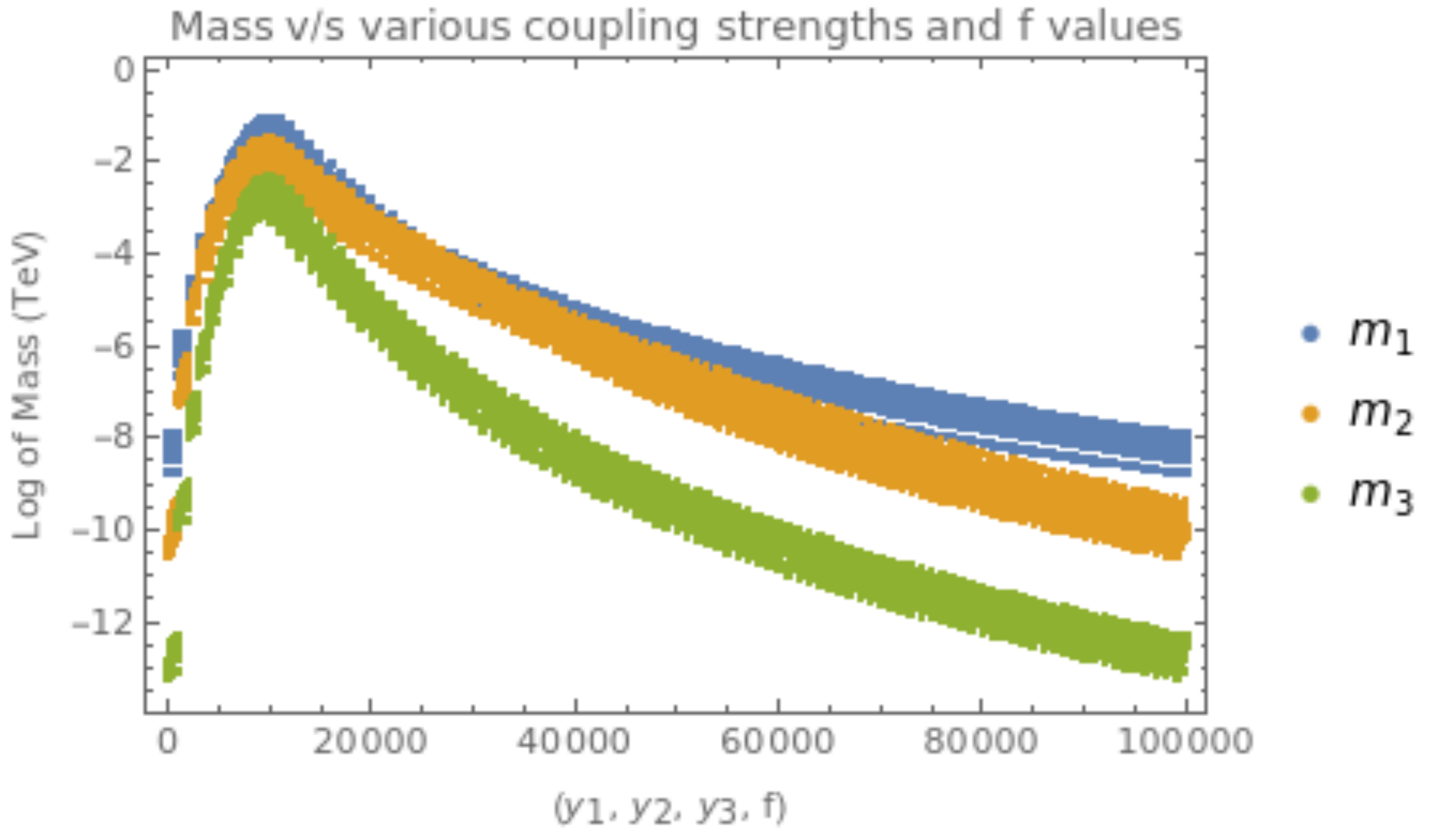
TABLE I: Possible choices of parameters that produce Dirac masses with scales similar to charged leptons and neutrinos, along with mixing similar to the observed PMNS matrix. Here $\log_{10}(y) = 0$ and $\log_{10}(y') = -2$.

Concretely, for { y1, y2, y3 } = {0.99, 0.0252, 0.03} and f = 1.7, the resulting masses are 4.8 MeV, 0.102 GeV, 4.2 GeV. For up-sector masses, and for { y'_1, y'_2, y'_3 } = {6.68, 0.001, 0.07 } and f' = 1.3, the up-type quark masses are 2.5 MeV, 1.22 GeV and 172.5 GeV.

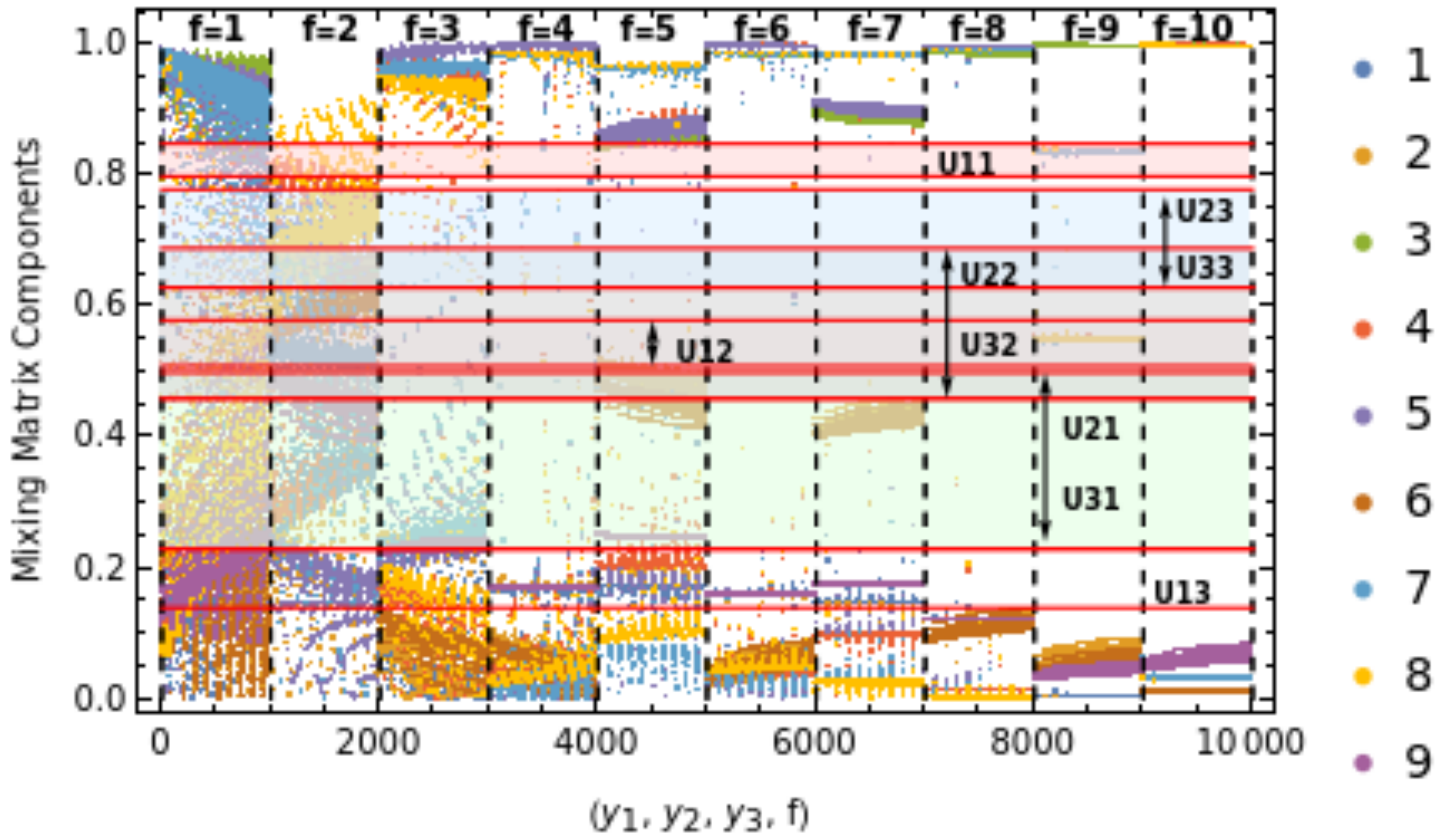
$$U_{PMNS} = \begin{pmatrix} 0.82196 & 0.55035 & -0.14602 \\ 0.31460 & -0.65324 & -0.68644 \\ 0.47164 & -0.51666 & 0.71236 \end{pmatrix}$$

$$U_{\mathrm{PMNS}} \approx \begin{pmatrix} -0.125f^{-4} - 2f'^{-8} + 1 & 0.5f^{-2} + 2f'^{-5} & f^{-5} + 2f^{-4} \\ 7f'^{-7} - 0.5f^{-2} & -0.125f^{-4} - 0.5f'^{-2} + 1 + f^{-3}f'^{-1} & f^{-3} - f'^{-1} + 0.5f'^{-3} \\ -0.5f^{-5} - 2f'^{-4} & -f^{-3} + f'^{-1} - 0.5f'^{-3} & -0.5f^{-6} - 0.5f'^{-2} + 1 \end{pmatrix}$$

Sierpiński Fractal Properties



$$\frac{Y_{yuk}}{Y} \approx \begin{bmatrix} 10f^{-12} & 4f^{-10} & f^{-7} \\ 4f^{-11} & 6f^{-9} & -4f^{-6} \\ -f^{-10} & 4f^{-8} & f^{-5} \end{bmatrix}$$



Masses produced as a function of f.

Mixing Matrix

Mixing as a function of f and y

Charged Leptons - f = 0.6 , {y1,y2,y3} = 0.1*{0.9,0.3,2.7}

Uncharged Leptons - f = 2.1 , {y1,y2,y3} = y{0.5,0.1,0.6}, y = O(10^{-10})

Down quarks - f = 1.9 , {y1,y2,y3} = {1, 0.1, 0.1}

Quarks down sector —— Masses $\mathcal{O}(2)GeV, \mathcal{O}(0.1)GeV, \mathcal{O}(5)MeV$

Linear Algebra Results

C1 - For any matrix A with a non-zero kernel space dimension, the nullity of matrix B, defined by the following operation, will be equal to the nullity of matrix A and hence rank of B will also be equal to the rank of A i.e., the original rank-nullity of A are preserved.

$$b_{i,j} = \frac{a_{i,j}}{f^{(i-j)}}, \forall f \in \mathbb{R} \backslash \{0\}$$

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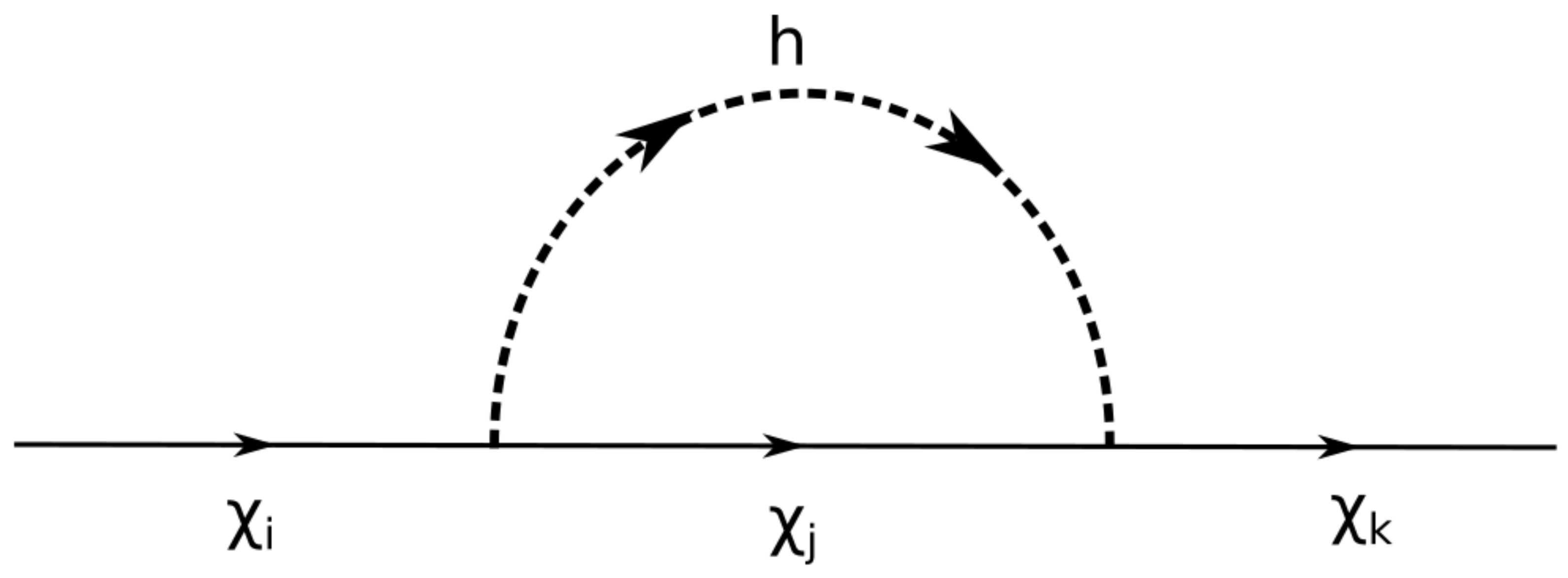
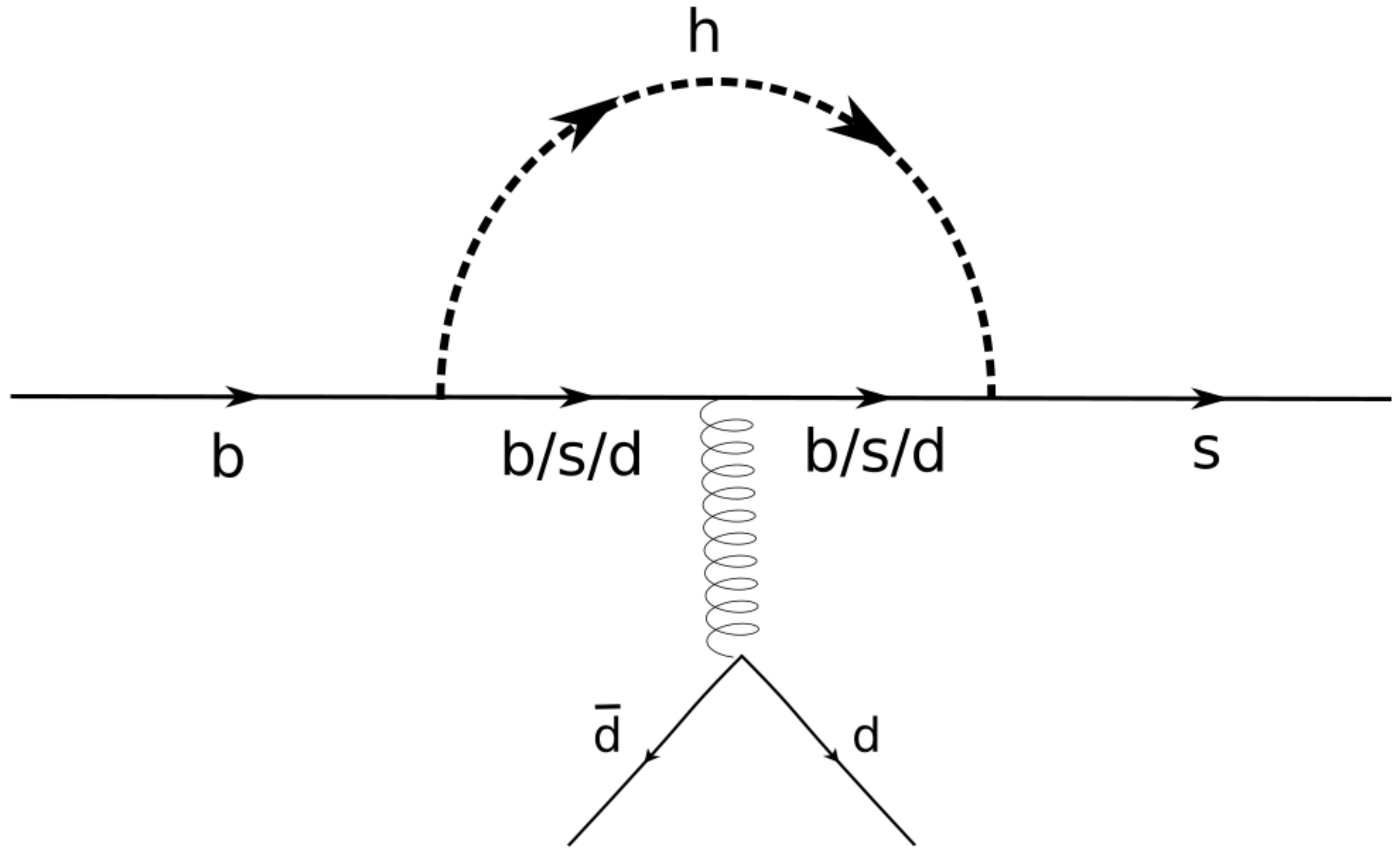
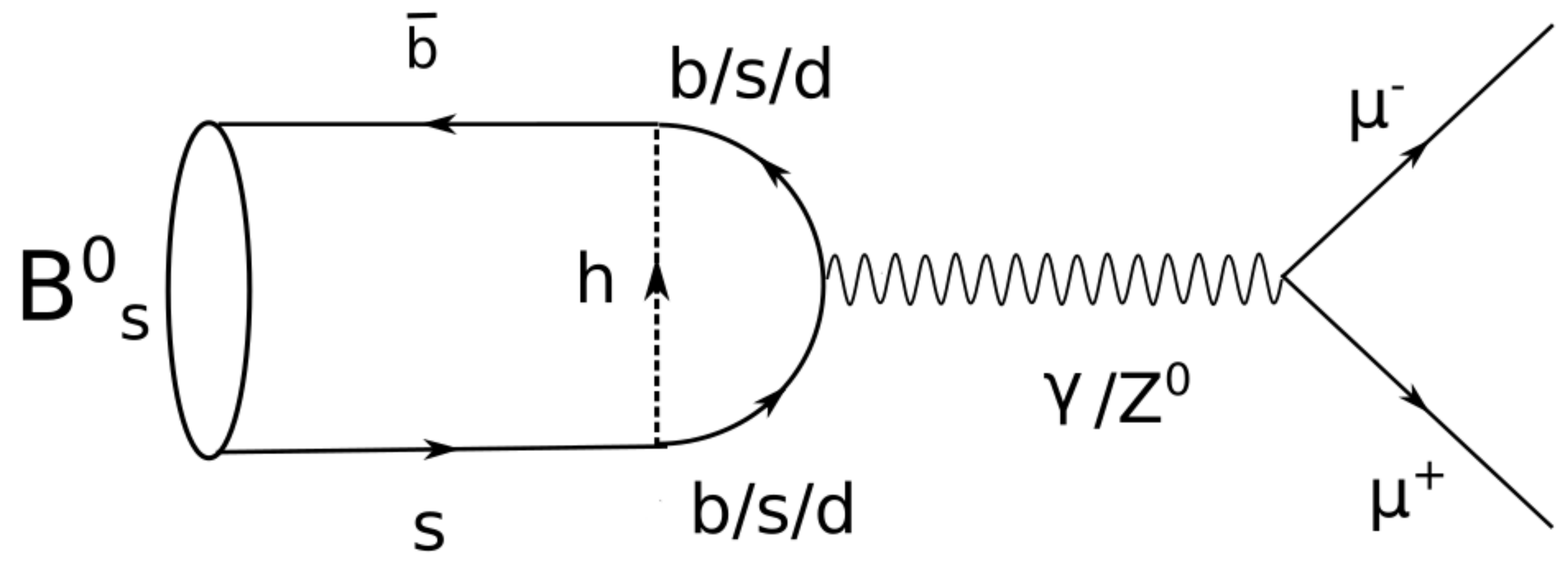
C2 - For any matrix A with $\{v^1, v^2, \ldots, v^n\}$ as eigenvectors of its nullspace, the corresponding eigenvectors for the nullspace of matrix B are given by $\{v'^1, v'^2, \ldots, v'^n\}$ with

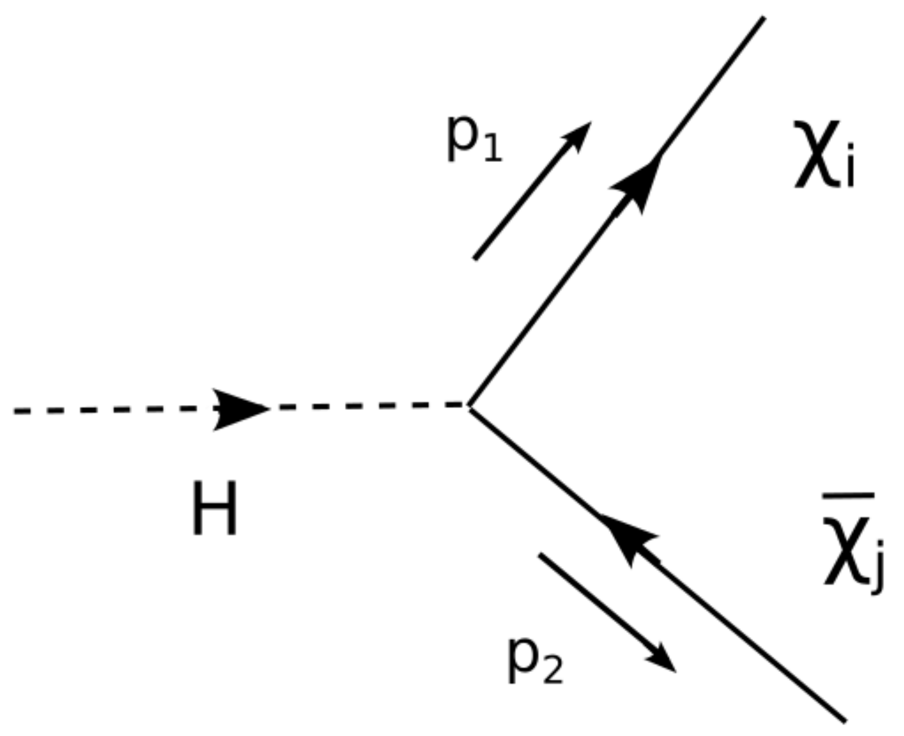
$$v'^i_j = v^i_j f^{(-j)}, \forall f \in \mathbb{R} \backslash \{0\}$$

Phenomenology Feynman Diagrams

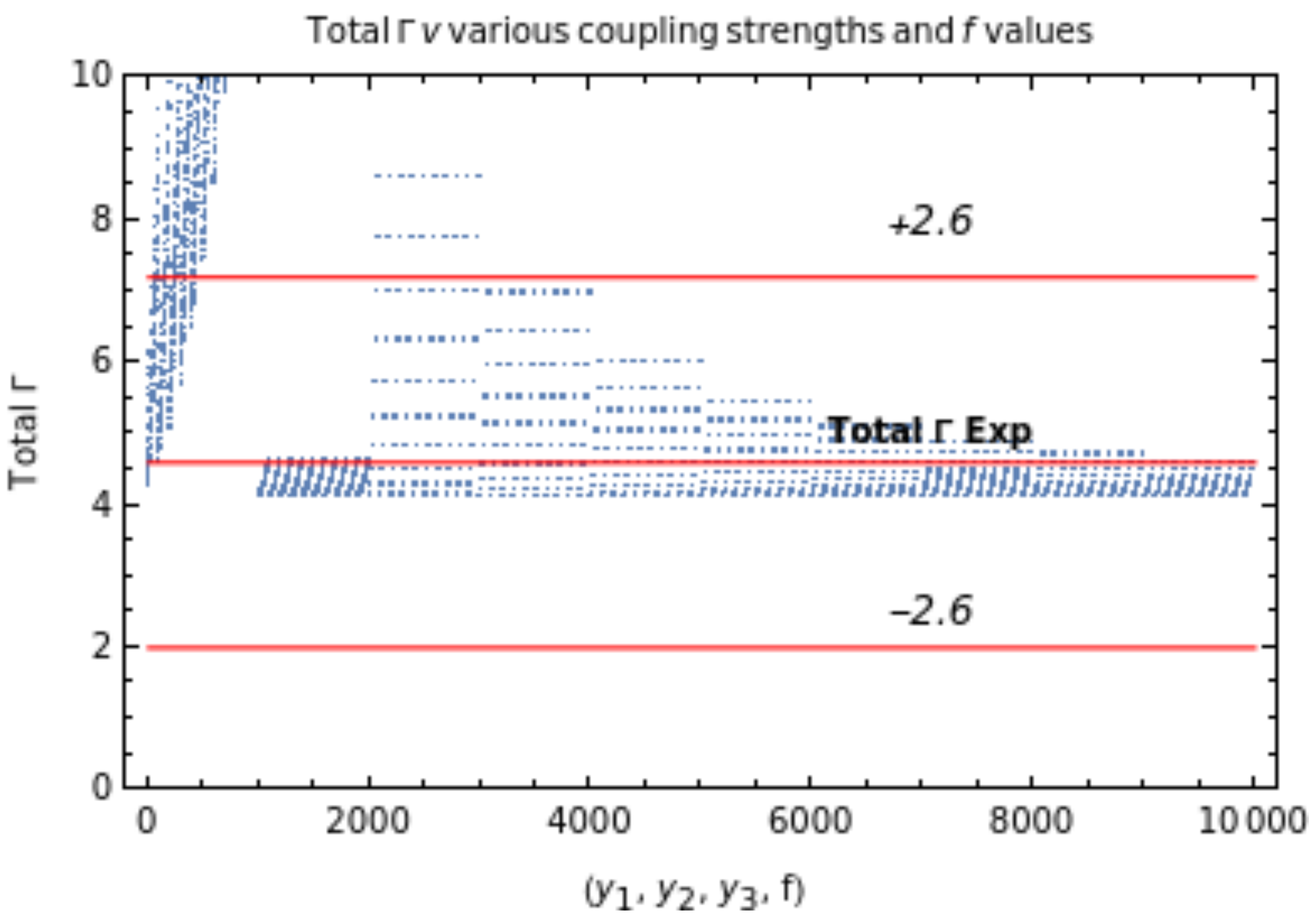




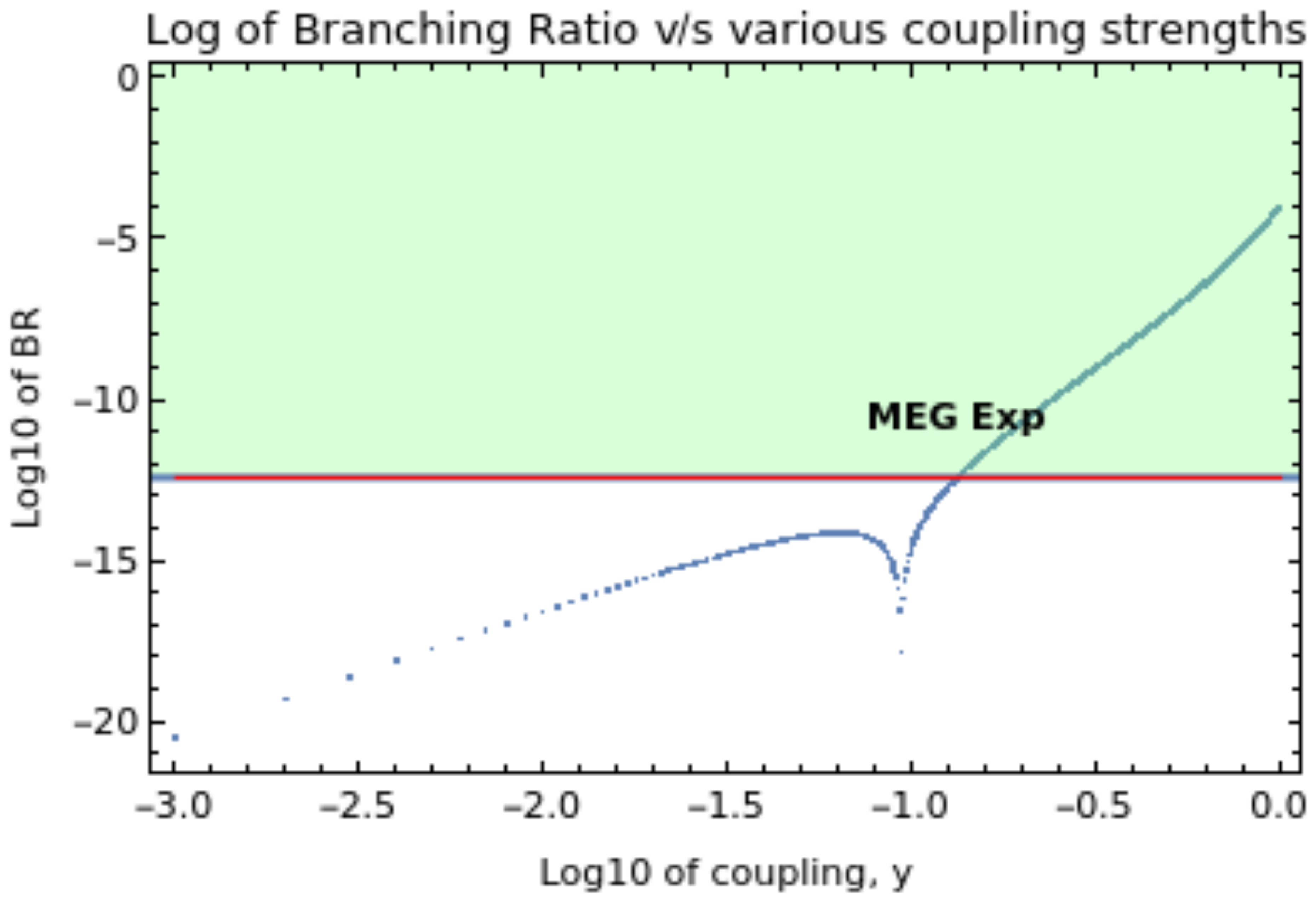




Signatures



Total Γ v various coupling strengths and f values
Total Γ
+2.6
Total Γ Exp
−2.6
(y1, y2, y3, f)



Log of Branching Ratio v/s various coupling strengths
Log10 of BR
MEG Exp
Log10 of coupling, y

Other Fractal created using Iterative Process on Graph

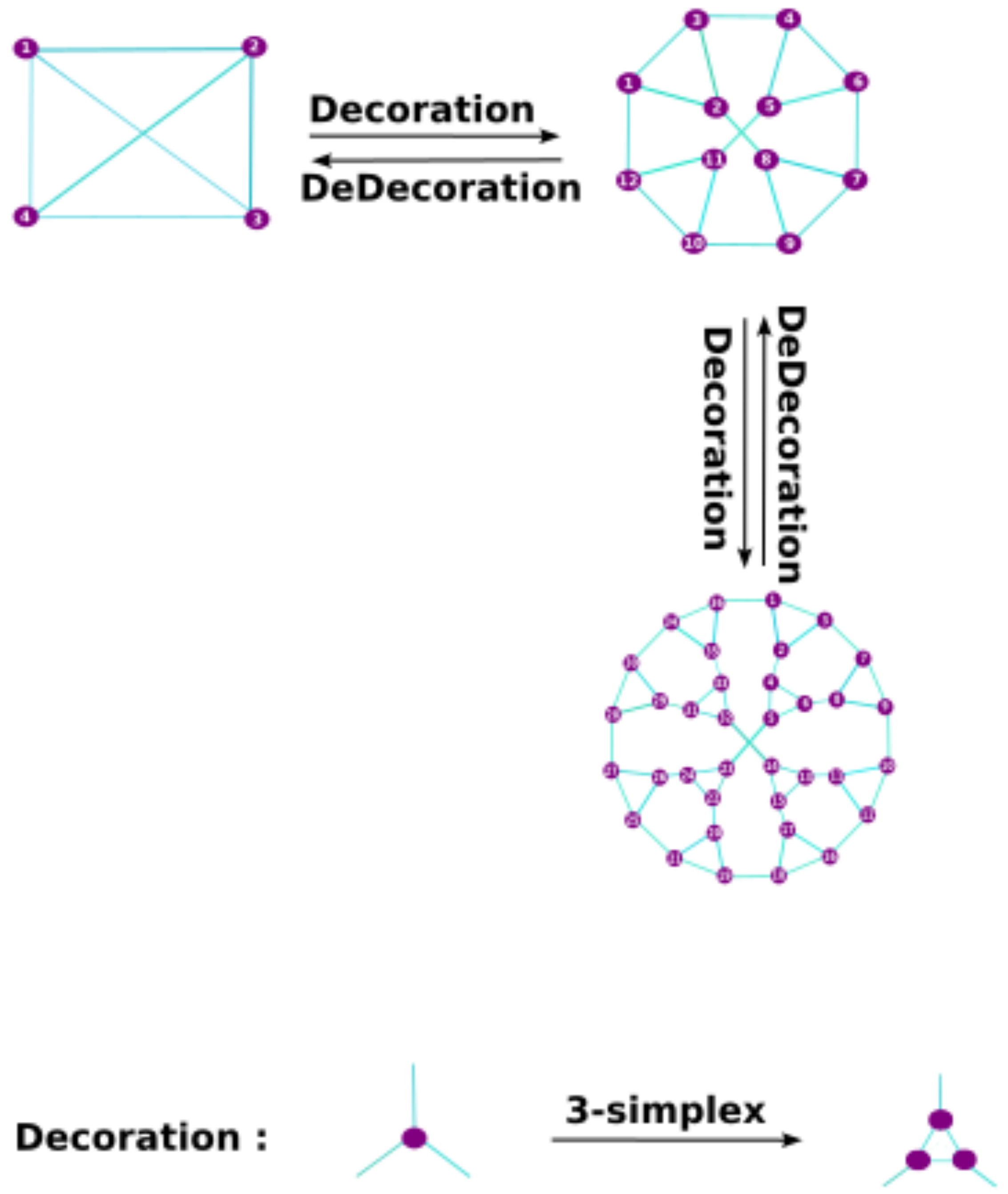


Fig. A

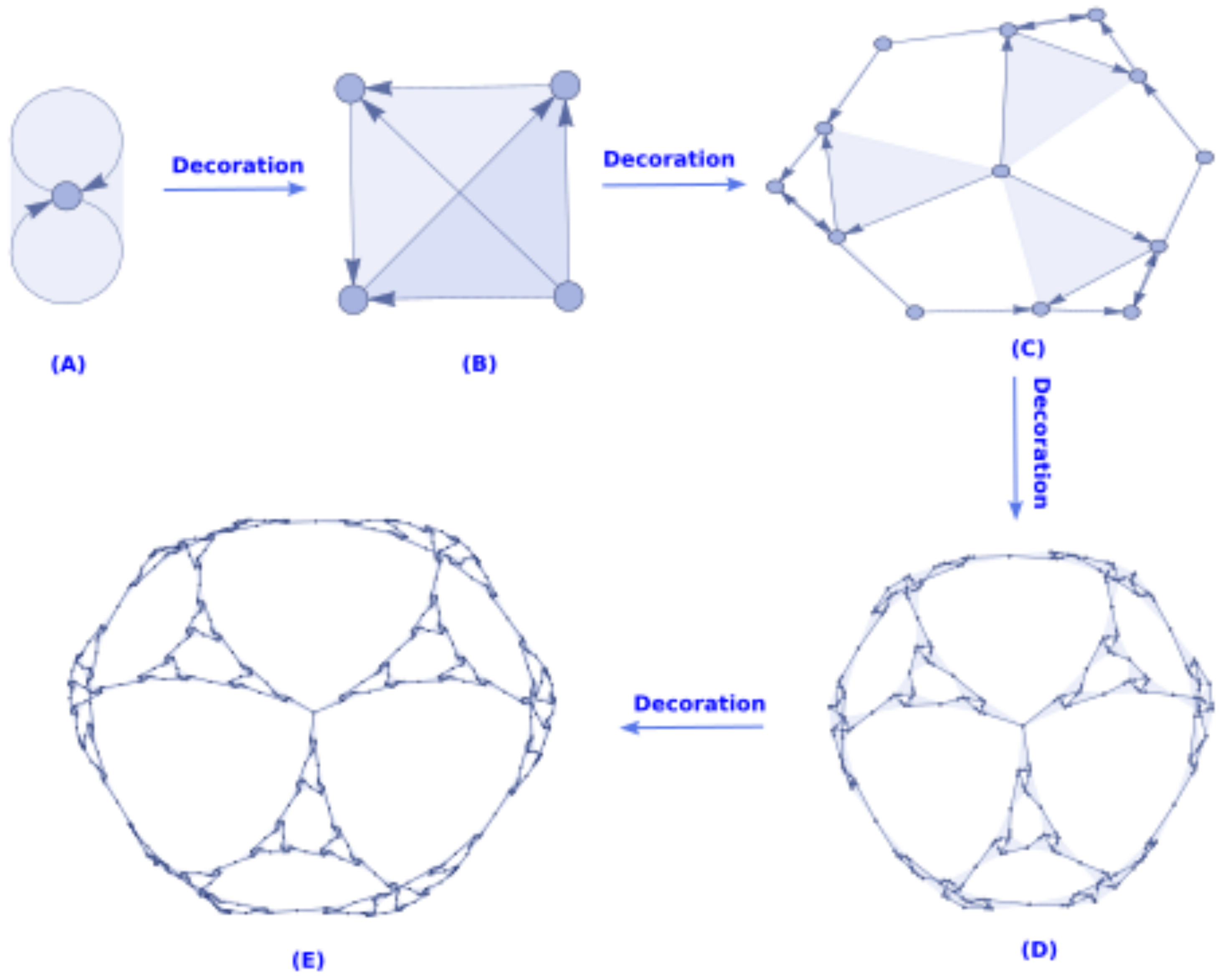


Fig. B

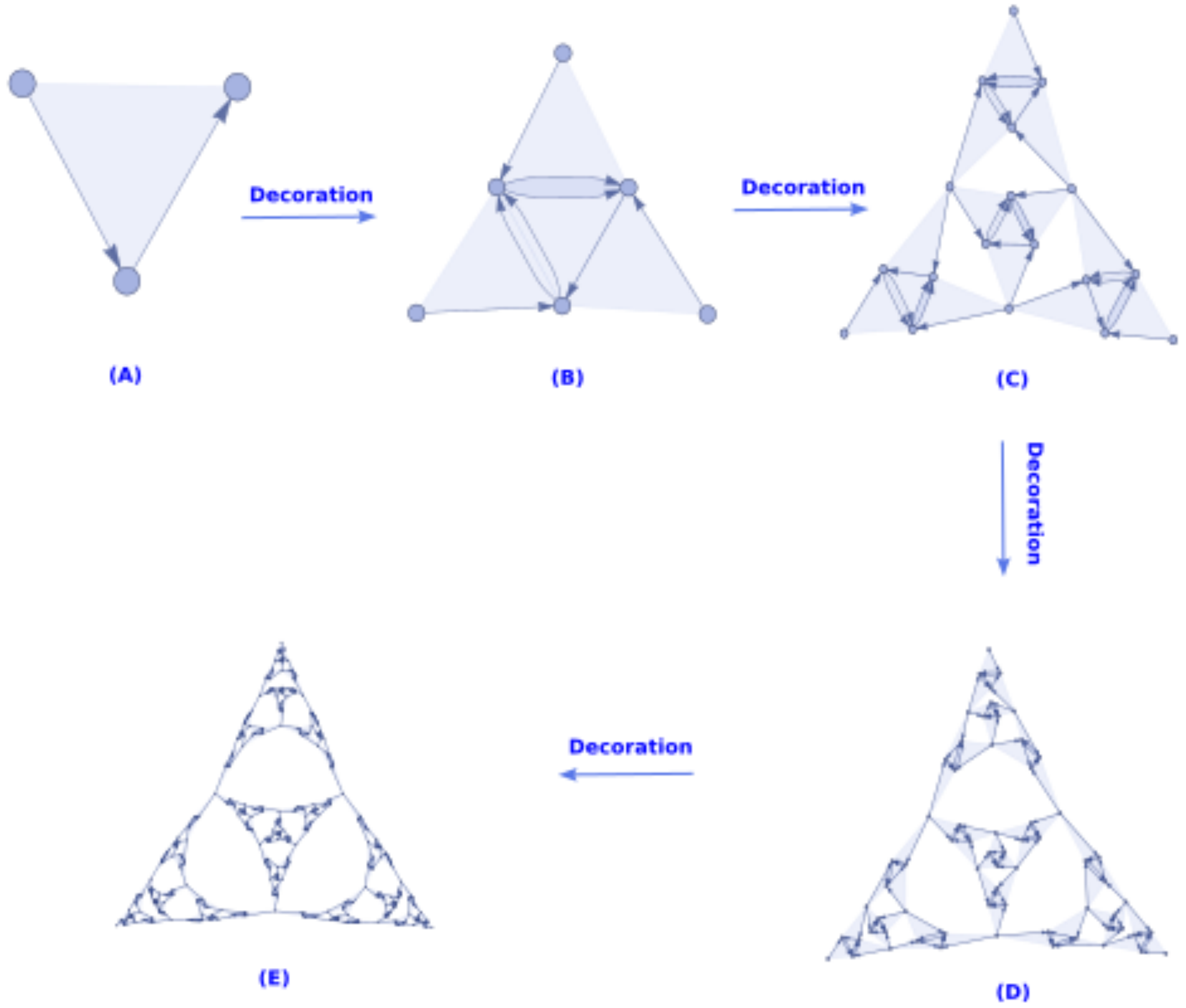


Fig. C

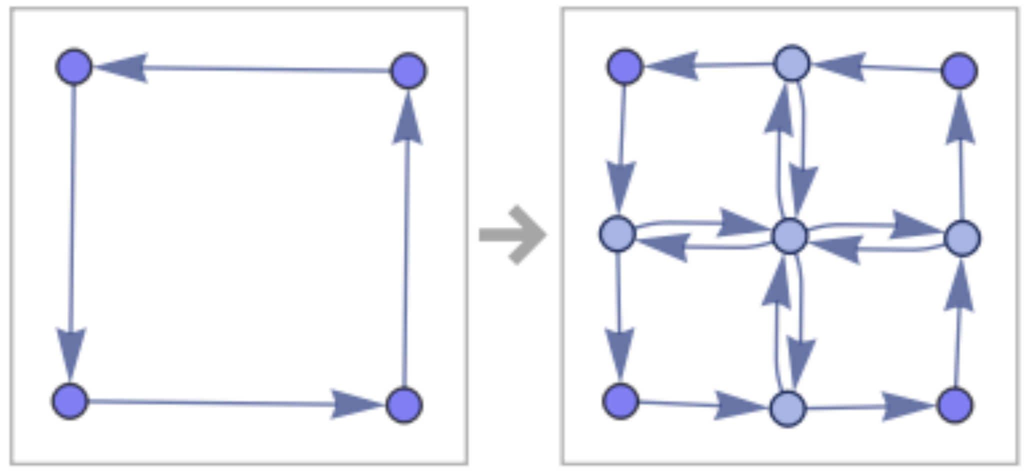


Fig. D

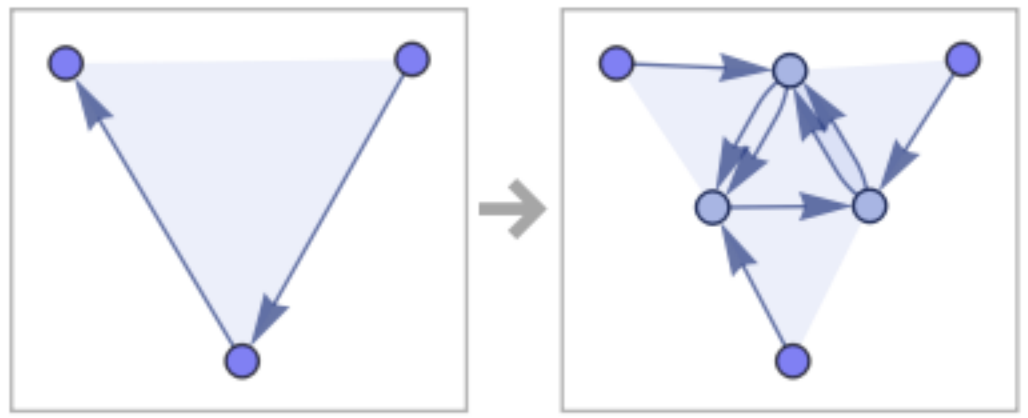


Fig. E