Spontaneous CP Violation in Supersymmetric QCD

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Warming up

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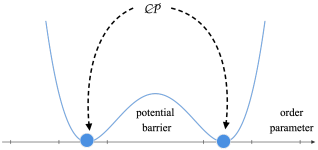
- The theory is **vector-like**.
- The theory at UV is **CP invariant**, i.e. $\bar{\theta} = 0$ (or π) and the fermion mass matrix $M_{ij} \in \mathbb{R}$
- SCPV means the appearance of multiple **CP violating vacua** separated by **potential barriers**.



Content

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SCPV sounds not surprising...?

Here are some **known examples** that can realize **spontaneous P/CP violation** in **vector-like** theories:

- $\not{P}$: QCD matter with finite isospin density [D. T. Son and M. A. Stephanov(2001)], with finite baryon density [A. B. Migdal (1971,1973,1978), A. A. Andrianov and D. Espriu (2007)], at finite temperature [T. D. Cohen (2001)] or in the superconducting phase[R. D. Pisarski and D. H. Rischke (1999)]
- $\not{P}, \not{CP}$: Dashen's phenomenon [R. F. Dashen(1971)]
- $\not{P}, \not{CP}$: Generalized Dashen's phenomenon [Di Vecchia and G. Veneziano (1980), E. Witten (1980), M. Creutz (1995), N. Evans, S.D. Hsu, A. Nyffeler and M. Schwetz (1997), A.V. Smilga (1999), M. H. G. Tytgat (2000)], which is about **SCPV in** $\bar{\theta} = \pi$ QCDs.

Motivation: Strong CP problem

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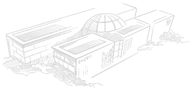
Spontaneous CP Violation (SCPV) as an alternative solution

- Let's anyway have a SCPV and then try to mediate those SCPV effects only to the **weak** but not the **strong** sector [A. Nilson (1984), S. M. Barr (1984)]
  - An $O(1)$ CKM phase is generated through SCPV
  - $\bar{\theta}$ is protected and remains (approximately) zero
- Practically, to manifest an $O(1)$ CKM phase, a **strongly coupled** supersymmetric system is needed [G. Hiller and M. Schmaltz(2001, 2002)]



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$\rightarrow$ Why not have a **QCD-like** thoery undergoing **spontaneous CP violation (SCPV)** at $\bar{\theta} = 0$?

The no-go theorem by Vafa and Witten

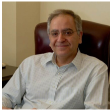
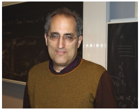
- A Lorentz-invariant $\not{P}$ operator[a] X picks up an i after **Wick rotation**
- The contribution of $\not{P}$ operator X only makes the **vector-like** theory's positive-definite measure[b] **less**

$$e^{-VE(\lambda)} = \int \mathcal{D}A \, \det(\not{D} + M) e^{-\frac{1}{4g^2}\int FF} e^{\int i\lambda X}$$

- The free energy $E(\lambda)$ always **minimizes** at $\langle X \rangle = 0$

[a] or a $\not{CP}$ operator containing **odd** number of $\epsilon_{\mu\nu\rho\sigma}$, e.g. $F\tilde{F}$. (A non-example is $F^a_{\mu\nu}F^b_{\mu\rho}F^c_{\nu\rho}f_{abc}$.)

[b] Euclidean path integral of fermions and gauge bosons

"P is not spontaneously broken in parity-conserving, vector-like theories such as QCD."

– Vafa and Witten (1984)

Scalar as a game changer

Desperate news!

Up to our knowledge, people have lost interest in SCPV with $\bar{\theta} = 0$ QCD-like theories thereafter[a].

[a] [D. E. Kharzeev, R. D. Pisarski and M. H. G. Tytgat (2000)] suggested there is still a possibility to have **metastable** $\not{CP}$ states

But wait, an elementary scalar ϕ can make a difference...[C. Vafa and E. Witten (1984[1], 1984[2])]

$$\not{D} + M \rightarrow \not{D} + M + y\phi$$

~~Positive definiteness~~

$\Rightarrow$ ~~Vafa-Witten thm.~~

Can we have an example?

Content

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What a supersymmetry buys us?

- Escape from the Vafa-Witten thm. (by introducing superpartners as elementary scalars)
- Protect $\bar{\theta}$ from SCPV (by non-renormalizationn thm.)

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- Escape from the Vafa-Witten thm. (by introducing super-partners as elementary scalars)
- Protect $\bar{\theta}$ from SCPV (by non-renormalizationn thm.)
- Simplify analysis of vacuum structure

What means supersymmetry? $\rightarrow$ We can take “square-root” for Hamiltonian!

$$H \sim \frac{QQ^\dagger + Q^\dagger Q}{2} \rightarrow H \geq 0$$

In seeking for vacua...

without supersymmetry

$V' = 0$ (differential equation),
$V'' > 0$ (metastable?)

with supersymmetry

$V = 0$ (algebraic equation!),
$V'' > 0$

The UV model: supersymmetric SU(5) + CP

$$
\begin{aligned}
W \supset\ & \frac{\widetilde{m}}{3}\bar{Q}^{pn}Q_{pn} + \widetilde{M}_j{}^{i}\bar{Q}^{jn}Q_{in} + \widetilde{\iota}_j{}^{i}\Psi_i{}^{p}\bar{Q}^{jn}Q_{pn} \\
& + \frac{\widetilde{\rho}}{3}\frac{(\bar{Q}^{pn}Q_{qn})(\bar{Q}^{qn}Q_{pn})}{\mu} + \widetilde{\lambda}^{ik}_{jl}\frac{(\bar{Q}^{jn}Q_{in})(\bar{Q}^{ln}Q_{kn})}{\mu}
\end{aligned}
$$

$n = 1, 2, \cdots, 5$: $SU(5)$ index
$\alpha = 1, \cdots, 6$: flavor index

- $p = 1, 2, 3$: in (anti-)fundamental Rep. of $SU(3)_c$
- $i = 4, 5, 6$: singlets of $SU(3)_c$.

$\widetilde{\iota}, \widetilde{\rho}, \widetilde{\lambda}$: dimensionless coupling constants

$\widetilde{m}, \widetilde{M}, \mu$: mass parameters

	Q_p	$\bar{Q}^p$	Q_i	$\bar{Q}^i$	$\Psi_i{}^q$
$SU(5)$	$\square$	$\overline{\square}$	$\square$	$\overline{\square}$	1
$SU(3)_c$	$\square$	$\overline{\square}$	1	1	$\overline{\square}$
$U(1)_Y$	$-2/15$	$2/15$	$1/5$	$-1/5$	$1/3$

s(mooth)-confinement and the IR EFT

- Smooth-confinement[C. Csaki, M. Schmaltz, and W. Skiba (1997, 1997)] as integrating out heavy flavors

$$\mathcal{M'}_\alpha{}^\beta \equiv \bar{Q}^{\beta n} Q_{\alpha n}\,,$$

$$B'^\alpha \equiv \epsilon^{\alpha_1,\alpha_2,\cdots,\alpha_N,\alpha} Q_{\alpha_1 n_1}\cdots Q_{\alpha_N n_N}\epsilon^{n_1,\cdots,n_N}\,,$$

$$\bar{B}'_\alpha \equiv \epsilon_{\alpha_1,\alpha_2,\cdots,\alpha_N,\alpha}\bar{Q}^{\alpha_1 n_1}\cdots\bar{Q}^{\alpha_N n_N}\epsilon_{n_1,\cdots,n_N}$$

$$\begin{aligned} W_{\text{eff}} \supset\ & \frac{1}{\Lambda^9}\left(B'^\alpha \mathcal{M'}_\alpha{}^\beta \bar{B}'_\beta - \det\mathcal{M'}\right) \\ & + \frac{\widetilde{m}}{3}\mathcal{M'}_p{}^p + \widetilde{M}_j{}^i \mathcal{M'}_i{}^j + \widetilde{\iota}_j{}^i \Psi_i{}^p \mathcal{M'}_p{}^j \\ & + \frac{\widetilde{\rho}}{3\mu}\mathcal{M'}_q{}^p \mathcal{M'}_p{}^q + \frac{1}{\mu}\widetilde{\lambda}_{jl}^{ik}\mathcal{M'}_i^j \mathcal{M'}_k{}^l \end{aligned}$$

Λ: dynamical scale

s(mooth)-confinement and the IR EFT

- Canonical normalization: $\mathcal{M}' \to \mathcal{M}$, $B' \to B$, $\bar{B}' \to \bar{B}$
- Effective field contents:

$$\frac{1}{3}\mathcal{M}_p{}^p \equiv \Phi\,, \quad \mathcal{M}_p{}^q - \Phi\delta_p{}^q \equiv \tilde{\Phi}_p{}^q\,, \quad \mathcal{M}_i{}^j \equiv \Sigma_i{}^j\,,$$

$$\mathcal{M}_p{}^j \equiv \Xi_p{}^j\,, \quad \mathcal{M}_i{}^q \equiv D_i{}^q\,,$$

$$B^\alpha \equiv \begin{pmatrix} \bar{T}^p \\ F^i \end{pmatrix}, \quad \bar{B}_\alpha \equiv \begin{pmatrix} T_p \\ \bar{F}_i \end{pmatrix}$$

	Φ	$\tilde{\Phi}$	Σ	Ξ	D	$\bar{T}$	T	F	$\bar{F}$	$\Psi_i{}^q$
$SU(3)_c$	1	**Ad**	1	$\square$	$\overline{\square}$	$\overline{\square}$	$\square$	1	1	$\overline{\square}$
$U(1)_Y$	0	0	0	$-1/3$	$1/3$	$1/3$	$-1/3$	0	0	$1/3$

Vacuum structure

$$\langle \Xi \rangle = \langle \Psi \rangle = 0$$
$$\langle T \rangle = \langle \bar{T} \rangle = \langle F \rangle = \langle \bar{F} \rangle = 0$$

$$\begin{aligned} W_{\text{eff}}(\Phi, \Sigma) \supset & -\frac{(4\pi)^4 c}{\Lambda^3} \Phi^3 \det \Sigma \\ & + \frac{\Lambda}{4\pi}\left(m\Phi + M_j{}^i \Sigma_i{}^j\right) \\ & + \frac{\Lambda^2}{\mu}\left(\rho \Phi^2 + \lambda_{jl}^{ik} \Sigma_i{}^j \Sigma_k{}^l\right) \\ & = W_0 + \delta W + \text{const.} \end{aligned}$$

- Σ: 3 by 3 chiral superfields, with i, j, k, l run over 1,2,3
- Φ: single chiral superfields

- Invariant part under $GL(3, \mathbb{C})$

$$W_0 = -\frac{(4\pi)^4 c}{\Lambda^3} \Phi^3 \left(\det \Sigma - \langle \tilde{\Sigma} \rangle \delta \Sigma\right)$$

$\langle \tilde{\Sigma} \rangle$: the adjugate of $\langle \Sigma \rangle$ the VEV of Σ

- Non-invariant part under $GL(3, \mathbb{C})$

$$\delta W = \frac{\Lambda^2}{\mu} \lambda_{jl}^{ik} \delta \Sigma_i{}^j \delta \Sigma_k{}^l$$

$\delta \Sigma = \Sigma - \langle \Sigma \rangle$

- $\langle \Sigma \rangle$ that minimize the potential W_0 (i.e. $\frac{\partial W_0}{\partial \Sigma} = 0$) can be **complex-valued** which **violates CP**
- δW is responsible to stabilize $\langle \Sigma \rangle$

A parameterization of W_0 moduli space

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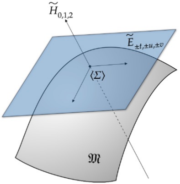
$$U(\theta) = e^{i\theta^a\sigma_a} \approx \mathbf{1} + i\theta^a\sigma_a\,.$$



$$\sigma_0 = H_0 = \frac{1}{\sqrt{3}}\mathbf{1}\,,$$

$$\sigma_1 = H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}, \qquad \sigma_2 = H_2 = \frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix},$$

$$\sigma_3 = E_{+t} = \begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}, \qquad \sigma_4 = E_{-t} = \begin{pmatrix}0&0&0\\1&0&0\\0&0&0\end{pmatrix},$$

$$\sigma_5 = E_{+u} = \begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix}, \qquad \sigma_6 = E_{-u} = \begin{pmatrix}0&0&0\\0&0&0\\0&1&0\end{pmatrix},$$

$$\sigma_7 = E_{+v} = \begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}, \qquad \sigma_8 = E_{-v} = \begin{pmatrix}0&0&0\\0&0&0\\1&0&0\end{pmatrix}.$$

$$\begin{aligned}\Sigma(\theta)_j{}^i &= U^{-1}(\theta)_j{}^{j'}\langle\Sigma\rangle_{j'}{}^{i'}U(\theta)_{i'}{}^i\\ &= \langle\Sigma\rangle_j{}^i - i\theta^a\left[\sigma_a,\langle\Sigma\rangle\right]_j{}^i + \frac{1}{2}\theta^a\theta^b\left[\sigma_a,\left[\sigma_b,\langle\Sigma\rangle\right]\right]_j{}^i\\ &= \langle\Sigma\rangle_j{}^i + i\theta^a D_a\Sigma_j{}^i + \frac{1}{2}\theta^a\theta^b D_a D_b\Sigma_j{}^i + \cdots\,,\end{aligned}$$

The existence of CP violating vacuum

$$\langle\Sigma\rangle = \begin{pmatrix} \Sigma_1 & \frac{i}{2}\Sigma_2\cos\phi & 0 \\ -\frac{i}{2}\Sigma_2\cos\phi & \Sigma_1+\Sigma_2\sin\phi & 0 \\ 0 & 0 & \Sigma_3 \end{pmatrix}$$

$$\lambda^{00}=\lambda_1\,,\quad \lambda^{11}=-\lambda_2\,,\quad \lambda^{22}=\lambda_3$$

$$\lambda^{tt}=-\lambda_2\left(1-\frac{2}{\sin\phi-1}\right),$$

$$\lambda^{-t,-t}=\lambda_2\left(1-\frac{2}{\sin\phi+1}\right),$$

$$\lambda^{uu}=-\lambda_4\,(1-\sin\phi)\,\Sigma_2/\Lambda\,,$$

$$\lambda^{-u,-u}=-\lambda_5(1+\sin\phi)\Sigma_2/\Lambda\,,$$

$$\lambda^{vv}=-\lambda_4\,(1+\sin\phi)\,\Sigma_2/\Lambda\,,$$

$$\lambda^{-v,-v}=-\lambda_5(1-\sin\phi)\Sigma_2/\Lambda\,,$$

$$\frac{\partial^2\delta W}{\partial\theta^t\partial\theta^t}=\frac{2\Lambda^2}{\mu}\lambda_2\left(\frac{2}{s_\phi+1}-1\right)(\Sigma_2)^2\,,$$

$$\frac{\partial^2\delta W}{\partial\theta^{-t}\partial\theta^{-t}}=\frac{2\Lambda^2}{\mu}\lambda_2\left(1+\frac{2}{1-s_\phi}\right)(\Sigma_2)^2\,,$$

$$\frac{\partial^2\delta W}{\partial\theta^u\partial\theta^u}=\frac{2\Lambda}{\mu}\lambda_5\,(1+s_\phi)\,\Sigma_2\left(\Sigma_1-\Sigma_3+\frac{1+s_\phi}{2}\Sigma_2\right)^2,$$

$$\frac{\partial^2\delta W}{\partial\theta^{-u}\partial\theta^{-u}}=\frac{2\Lambda}{\mu}\lambda_4\,(1-s_\phi)\,\Sigma_2\left(\Sigma_1-\Sigma_3+\frac{1+s_\phi}{2}\Sigma_2\right)^2$$

$$\frac{\partial^2\delta W}{\partial\theta^v\partial\theta^v}=\frac{2\Lambda}{\mu}\lambda_5\,(1-s_\phi)\,\Sigma_2\left(\Sigma_1-\Sigma_3-\frac{1-s_\phi}{2}\Sigma_2\right)^2,$$

$$\frac{\partial^2\delta W}{\partial\theta^{-v}\partial\theta^{-v}}=\frac{2\Lambda}{\mu}\lambda_4\,(1+s_\phi)\,\Sigma_2\left(\Sigma_1-\Sigma_3-\frac{1-s_\phi}{2}\Sigma_2\right)^2$$

→ We have a **QCD-like** thoery undergoing **spontaneous CP violation (SCPV)** (at $\bar{\theta}=0$)!

Take-away messages

- It is presumable to realize SCPV in $\bar{\theta} = 0$ QCD-like theories.
  - The Vafa-Witten theorem can be evaded by introducing scalar(s) coupling to fermions.
- If you are interested, in our paper (JHEP09(2024)105), we show how the above model helps addressing the strong CP problem. Please check!