Gauge symmetry breaking with two dimensional sphere as extra dimensions

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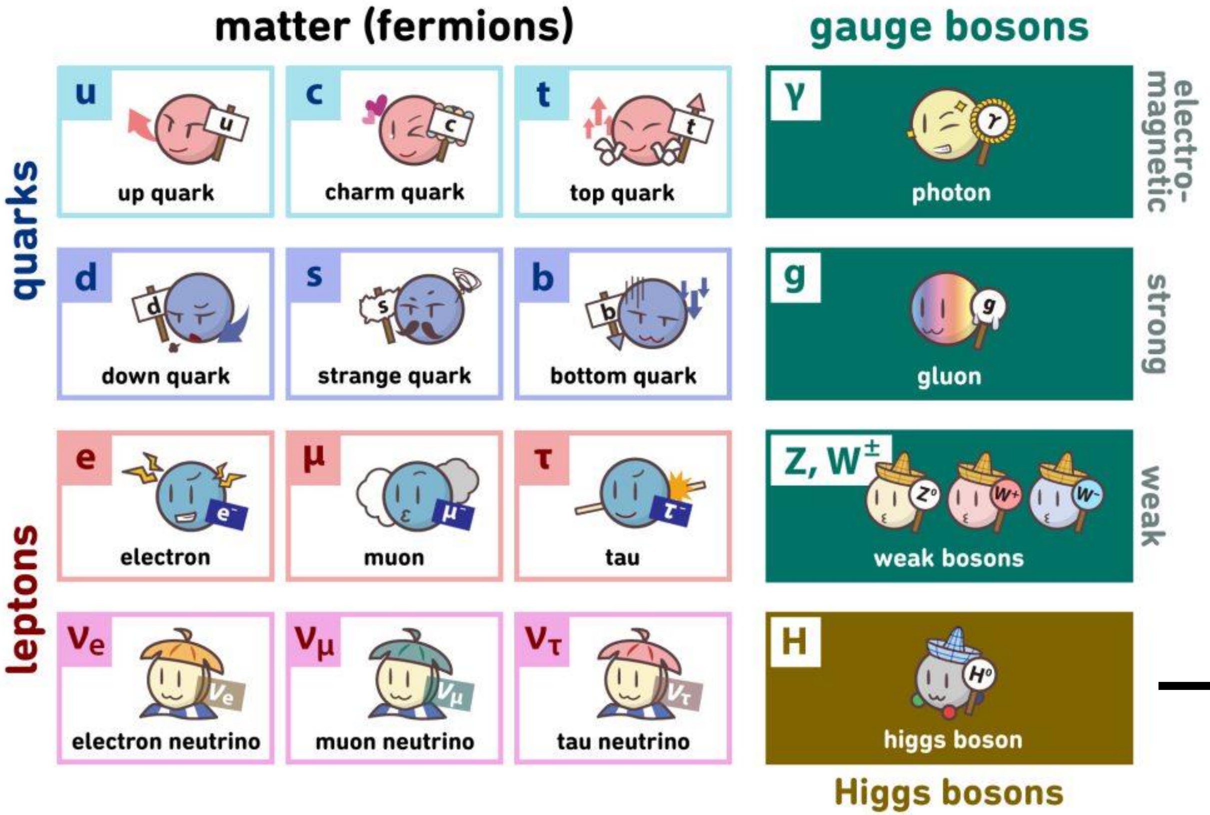
Introduction

Standard model(SM) explains many phenomena but still has problems



https://higgstan.com/standerd-model/

1. Why three coupling constants?

Gauge group of SM:

$$SU(3) \times SU(2) \times U(1)$$

2. Where did Higgs bosons come from?

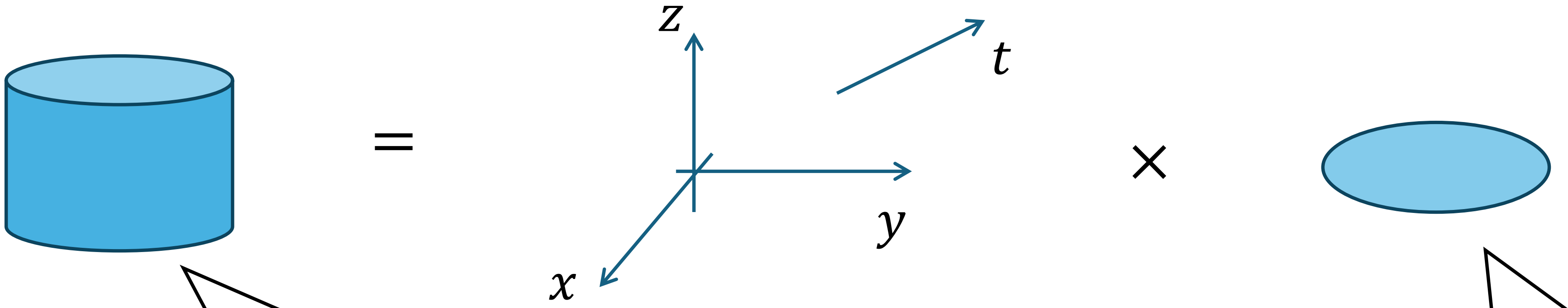
Higgs bosons are only scalar

Extra Dimensions

Spatial coordinates other than that of 4-dimensional Minkowski sp.

Our world = 4-dim. Minkowski sp. × extra dimensions

$$M^{4+d} = M^4 \times B$$



We live in higher dimensions!

Not observed yet→small

Gauge Higgs unified theory

Higher dimensional theory

Higher dimensional gauge field

$A_M = (A_\mu, A_{\hat{\alpha}})$

Dimensional reduction

4-dimensional theory

4-dimensional component A_μ

Extra dimensional component $A_{\hat{\alpha}} \equiv \phi_{\hat{\alpha}}$

Gauge symmetry breaking

Strong force

Electro magnetics

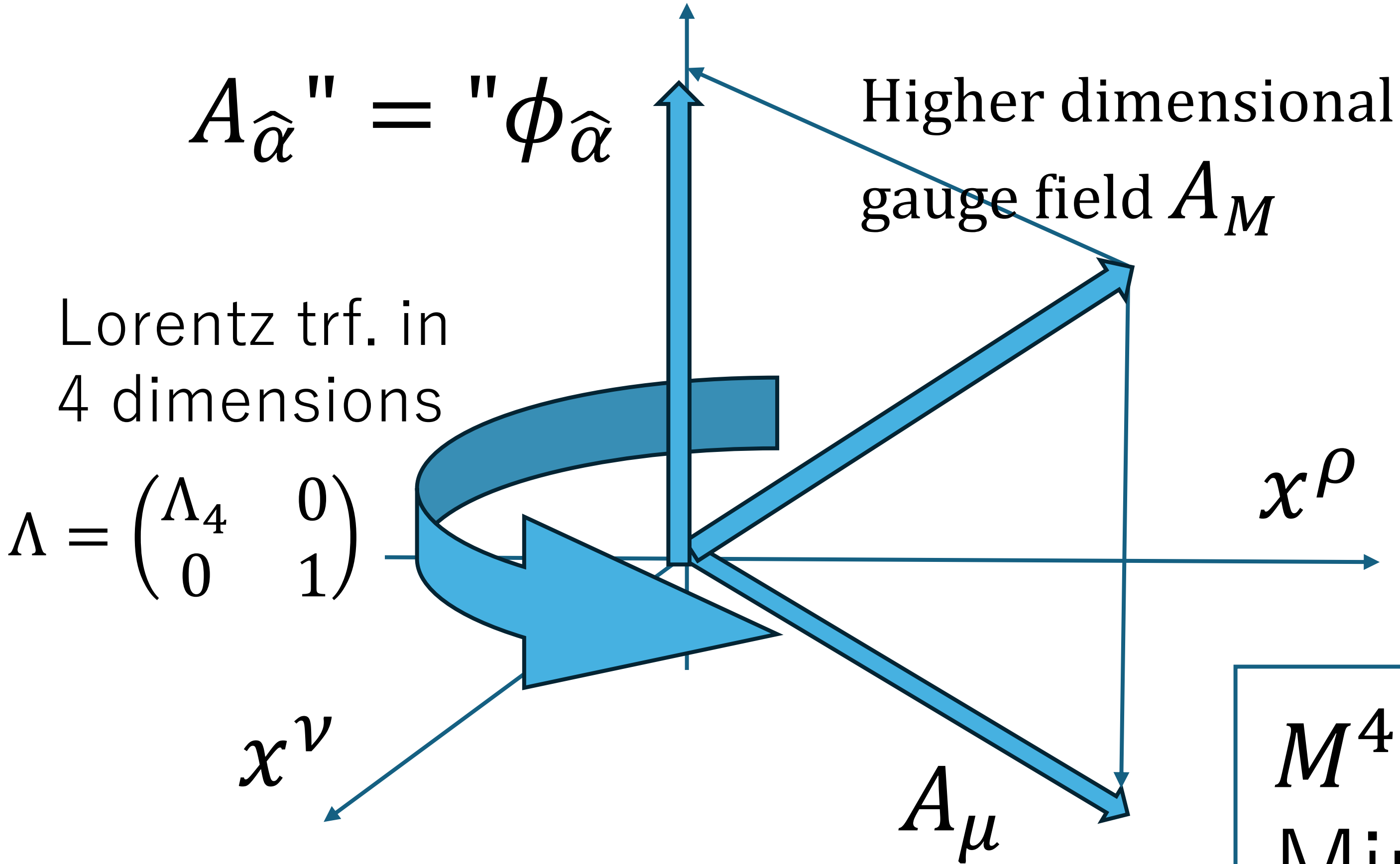
Weak force

$\phi_{\hat{\alpha}}$ appear as scalar in 4 dimensions

$A_{\hat{\alpha}}$ behaves as scaler in 4 dimensions

y^{α}(extra dimensional coordinates)

$M = 0, 1, \ldots, 4+d,$
$\mu, \nu, \rho = 0, 1, 2, 3,$
$\hat{\alpha} = 5, \ldots, 4+d$



4-dim Lorentz trf. doesn't change $A_{\hat{\alpha}}$

→ $A_{\hat{\alpha}}$ are scalars in 4-dim.

M^4(4-dim. Minkowski sp.)

Summary of introduction

- SM can’t explain following questions:
  - The reason why 3 types of forces exist
  - The origin of Higgs bosons
- Gauge Higgs unified theory is possible to explain them
  - A single force split into three in four dimensions
  - Extra dim. component of gauge field is candidate of Higgs

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1. Introduction
2. Dimensional reduction of S^2
3. Summary

Pure Yang-Mills Theory with S^2 extra dimensions

Pure Yang-Mills Theory with S^2 extra dimension:

$$S[A_M] = \int d^4x\, R^2 \sin\theta\, d\theta\, d\varphi \left(-\frac{1}{2} \mathrm{Tr}(F_{MN}F^{MN}) \right)$$

$M, N, \ldots = 0, \ldots, \theta, \varphi$
$\mu, \nu, \ldots = 0, 1, 2, 3$
R: radius of S^2

Extra-dimensional component have following background:

$$\langle A_\theta \rangle = 0$$
$$\langle A_\varphi \rangle = \Phi^i H_i \cos\theta$$

H_i: Cartan generator
Φ^i: constant

This satisfies the field equation of A_M: $\nabla_N F^{MN} - ig[A_N, F^{MN}] = 0$

Action of fluctuation around $\langle A_M \rangle$

We consider the fluctuation A_M around the background $\langle A_M \rangle$: $A_M \to A_M + \langle A_M \rangle$

The field strength is changed: $F_{MN} \to F_{MN} - ig[\langle A_M \rangle, A_N] + ig[\langle A_N \rangle, A_M]$

These terms affect the mass of A_μ

Then the action becomes as follows:

$$S = \int d^4x d\theta d\varphi \left(\frac{1}{2} R^2 \sin\theta A^A_\mu \left[\delta^B_A \Box + \frac{1}{R^2} \left\{ \delta^B_A L^2 - 2\frac{\cos\theta}{\sin^2\theta} \left(g\Phi^i H_i \right)_A{}^B i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta} \left(g\Phi^i H_i \right)^2_A{}^B \right\} \right] A^\mu_B \right.$$
$$- \frac{1}{2} \sin\theta A^A_\theta \left[\delta^B_A \Box + 2\frac{1}{R^2} \left\{ \frac{\cos\theta}{\sin^2\theta} \left(g\Phi^i H_i \right)_A{}^B i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta} \left(g\Phi^i H_i \right)^2{}_A{}^B \right\} \right] A_{\theta B}$$
$$- \frac{1}{2} \sin\theta A^A_\varphi \frac{1}{\sin^2\theta} \Box A_{\varphi A}$$
$$- \frac{1}{2R^2} \sin\theta \left\{ \frac{1}{\sin^2\theta} \left(\partial_\theta A_{\varphi A} - \partial_\varphi A_{\theta A} \right) \left(\partial_\theta A^A_\varphi - \partial_\varphi A^A_\theta \right) - A^A_\theta 2\frac{\cos\theta}{\sin^2\theta} i \left(g\Phi^i H_i \right)_A{}^B \left(\partial_\theta A_{\varphi B} - \partial_\varphi A_{\theta B} \right) \right.$$
$$\left. + A^A_\theta \frac{2}{\sin\theta} i \left(g\Phi^i H_i \right)_A{}^B A_{\varphi B} \right\}$$
$$+ \frac{1}{R^2} \sin\theta \left[-\frac{1}{\sin^2\theta} \left\{ g f^A{}_{BC} \left(\partial_\theta A_{\varphi A} - \partial_\varphi A_{\theta A} \right) A^B_\theta A^C_\varphi + \frac{1}{2} g^2 f_{ABC} f^A{}_{DE} A^B_\theta A^C_\varphi A^D_\theta A^E_\varphi \right\} \right.$$

Action of fluctuation around $\langle A_M \rangle$

$$\mathcal{L} \supset \left(\frac{1}{2} R^2 \sin\theta A_\mu^A \left[\delta_A^B \Box + \frac{1}{R^2} \left\{ \delta_A^B L^2 - 2 \frac{\cos\theta}{\sin^2\theta} \underline{\left(g\Phi^i H_i\right)_A{}^B} i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta} \underline{\left(g\Phi^i H_i\right)^2_A{}^B} \right\} \right] A_B^\mu \right.$$

Necessary to diagonalize

To evaluate $A_\mu^A H_i{}_A^B A_B^\mu$, we expand A_μ as

H_i, E_α: Cartan Weyl basis

$$A_\mu = \sum_{i:\text{Cartan}} A_\mu^i H_i + \sum_{\alpha:\text{root}} A_\mu^\alpha E_\alpha$$

$$[H_i, E_\alpha] = \alpha_i E_\alpha$$

Cartan Weyl basis is not Hermite($E_\alpha^\dagger = E_{-\alpha}$)

→ A_μ^α is not real field

Quadratic term of A_μ

Surface term

Quadratic term of A_μ : $S_{A,\mathrm{quad}} = \int d^4x\, d\theta\, d\varphi\, \mathcal{L}_{A,\mathrm{quad}} + \int d^4x d\varphi \left[\sin\theta\ \ A_{\mu A}\partial_\theta A^{\mu \mathrm{A}}\right]_{\theta=0}^{\pi}$

$$\mathcal{L}_{A,\mathrm{quad}} = \frac{1}{2}R^2 \sin\theta \left[A_\mu^i \left(\Box + \frac{1}{R^2}\boldsymbol{L}^2\right) A_{\mu i} + \sum_{\substack{\alpha:\text{all roots}\\ r=1,2}} A_\mu^{(\alpha)r}\left\{\Box + \frac{1}{R^2}\left(\boldsymbol{J}^{(\alpha)2} - k_\alpha^2\right)\right\} A^{\mu(\alpha)r}\right]$$

($A_\mu^{(\alpha)r}$ are real)

where

$$\boldsymbol{J}^{(\alpha)2} \equiv -\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{1}{\sin^2\theta}\partial_\varphi^2 + 2\frac{\cos\theta}{\sin^2\theta}k_\alpha i\partial_\varphi + \frac{1}{\sin^2\theta}k_\alpha^2$$
$$\equiv \boldsymbol{L}^2 + 2\frac{\cos\theta}{\sin^2\theta}k_\alpha i\partial_\varphi + \frac{1}{\sin^2\theta}k_\alpha^2$$

$, k_\alpha \equiv g \sum_{i:\mathrm{Cartan}} \Phi^i \alpha_i$

Same as usual angular momentum operator $\boldsymbol{L}^2$ if $k_\alpha = 0$

Algebraic structure of $\boldsymbol{J}^{(\alpha)2}$

$\boldsymbol{J}^{(\alpha)2}$ have algebraic structure of SU(2)

If we define $J_i^{(\alpha)}$ which satisfy $\boldsymbol{J}^{(\alpha)2} = J_1^{(\alpha)2} + J_2^{(\alpha)2} + J_3^{(\alpha)2}$ as

$$J_1^{(\alpha)} = L_1 - k_\alpha \frac{\cos\varphi}{\sin\theta}, \qquad J_2^{(\alpha)} = L_2 - k_\alpha \frac{\sin\varphi}{\sin\theta}, \qquad J_3^{(\alpha)} = L_3$$

then, the commutation relation of $J_i^{(\alpha)}$ is $\left[J_i^{(\alpha)}, J_j^{(\alpha)}\right] = i\varepsilon_{ijk} J_k^{(\alpha)}$

Eigenvalue of $\boldsymbol{J}^{(\alpha)2}$: positive half integer j
Eigenvalue of $J_3^{(\alpha)}$: $m = -j - j + 1, \dots, j$

Lagrangian should be single-valued → m, j are integer

Eigenfunction of $\boldsymbol{J}^{(\alpha)2}$

We can solve Eigenvalue equation: $\boldsymbol{J}^{(\alpha)2} f_{k_\alpha jm} = j(j+1) f_{k_\alpha jm}$

Eigenfunction of $\boldsymbol{J}^{(\alpha)2}$:

$$f_{kjm}(\theta,\varphi) = \left(\sin\frac{\theta}{2}\right)^{|m+k|} \left(\cos\frac{\theta}{2}\right)^{|m-k|} P^{(|m+k|,|m-k|)}_{j-\frac{|m+k|}{2}-\frac{|m-k|}{2}}(\cos\theta) e^{im\varphi}$$

(k means k_α)

$P_n^{(a,b)}(z)$ is a solution of the Jacobi polynomial:

$$\left[(1-z^2)\frac{d^2}{dz^2} - \{(a+b+2)z + a - b\}\frac{d}{dz} + n(n+a+b+1)\right] P_n^{(a,b)}(z) = 0$$

The surface term $\int d^4x d\varphi \left[\sin\theta \; A_{\mu A}\partial_\theta A^{\mu A}\right]_{\theta=0}^{\pi}$ should be zero

→ k_α is an integer and $j \geq |k_\alpha|$

Kaluza-Klein expansion of $A_\mu^{(\alpha)r}$

$$S_{A,\mathrm{quad}} = \int d^4x \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A^i_{\mu,lm} \left\{ \Box + \frac{l(l+1)}{R^2} \right\} A^{\mu i}_{lm}$$

$$+ \sum_{\substack{\alpha:\text{all roots} \\ r=1,2}} \sum_{\substack{j=|k_\alpha|+n \\ n=0,1,\dots}} \sum_{m=-j}^{j} A^{(\alpha)r}_{\mu,jm} \left\{ \Box + \frac{j(j+1)-k_\alpha^2}{R^2} \right\} A^{\mu(\alpha)r}_{jm}$$

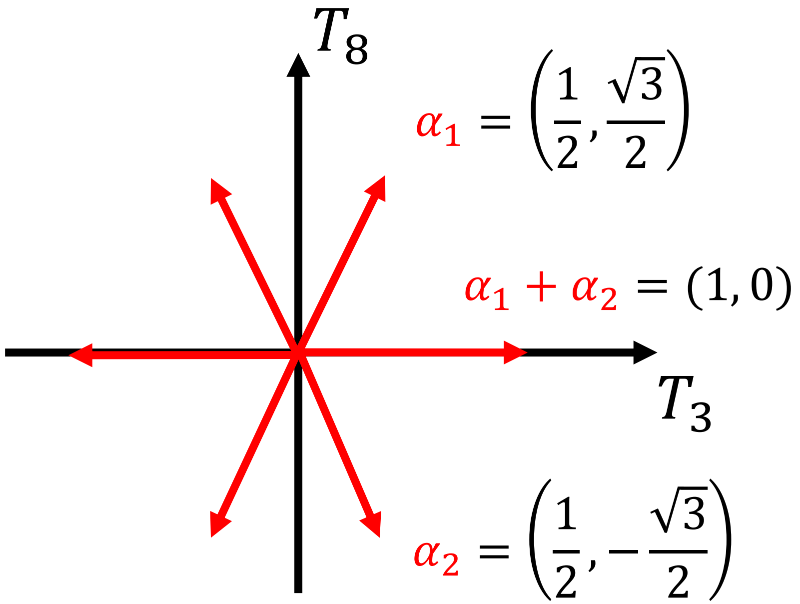
The least mass of $A^{(\alpha)r}_{\mu,jm}$ is $\frac{\sqrt{k_\alpha}}{R}$, since $j \geq |k_\alpha|$

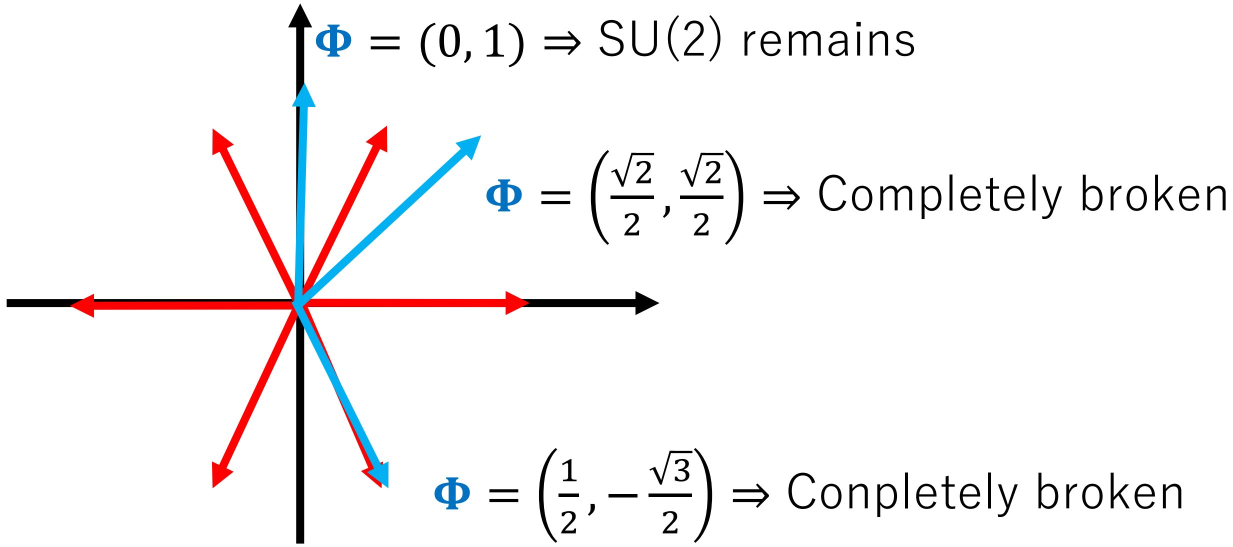
Only $A_\mu^{(\alpha)} E_\alpha$ which commute with background $\langle A_\varphi \rangle = \Phi^i H_i \cos\theta$ remains as a gauge group in 4 dimensions

Remained gauge symmetry

$k_\alpha = 0$ means $\boldsymbol{\alpha}$ is orthogonal to $\boldsymbol{\Phi}$ in root space:$k_\alpha = g\Phi^i \alpha_i = 0$

e.g.) SU(3)





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Summary

Summary

- Curvature of extra dimensions effects the mass of the gauge fields
- Gauge symmetry whose gauge fields commute with the background $\left(\langle A_\varphi \rangle = \Phi^i H_i \cos\theta\right)$ remains in 4 dimensions
- Scalar fields appears after dimensional reduction in the gauge theory with extra dimensions(HONDA san will talk details)

Future work

- Fermion can be coupled with gauge fields

Back up slides

About $J_i^{(\alpha)}$

Due to the background, covariant derivative is changed:

$$\partial_M \to \partial_M - ig[\langle A_M \rangle, \quad]$$

If $M = \varphi$, then $\partial_\varphi A_\mu^{(\alpha)} \to \partial_\varphi A_\mu^{(\alpha)} - ik\cos\theta A_\mu^{(\alpha)}$

$$L_1 = i\left(\sin\varphi\partial_\theta + \frac{\cos\theta}{\sin\theta}\cos\varphi\partial_\varphi\right)$$
$$L_2 = -i\left(\cos\varphi\partial_\theta - \frac{\cos\theta}{\sin\theta}\sin\varphi\partial_\varphi\right)$$
$$L_3 = -i\partial_\varphi$$

$$\longrightarrow$$

$$J_1^{(\alpha)} = i\left(\sin\varphi\partial_\theta + \frac{\cos\theta}{\sin\theta}\cos\varphi\partial_\varphi\right) - k_\alpha\frac{\cos\varphi}{\sin\theta}$$
$$J_2^{(\alpha)} = -i\left(\cos\varphi\partial_\theta - \frac{\cos\theta}{\sin\theta}\sin\varphi\partial_\varphi\right) - k_\alpha\frac{\sin\varphi}{\sin\theta}$$
$$J_3^{(\alpha)} = -i\partial_\varphi$$

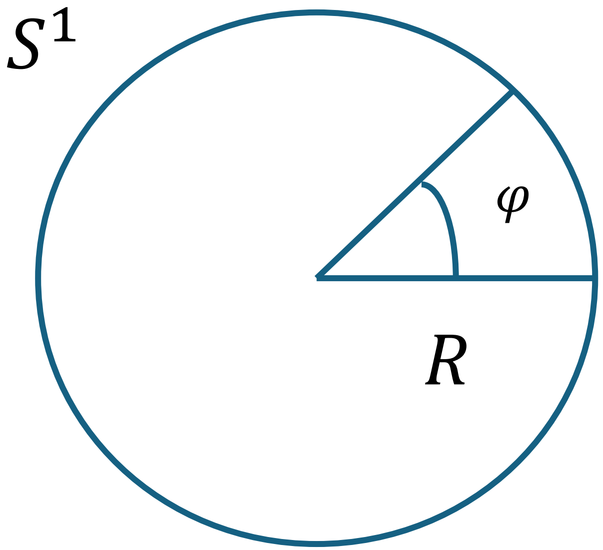
$$
\begin{aligned}
S = \int d^4x d\theta d\varphi \Bigg(& \frac{1}{2} R^2 \sin\theta A_\mu^A \left[\delta_A^B \Box + \frac{1}{R^2} \left\{ \delta_A^B L^2 - 2\frac{\cos\theta}{\sin^2\theta} \left(g\Phi^i H_i\right)_A{}^B i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta} \left(g\Phi^i H_i\right)_A^2{}^B \right\} \right] A_B^\mu \\
& - \frac{1}{2} \sin\theta A_\theta^A \left[\delta_A^B \Box + 2\frac{1}{R^2} \left\{ \frac{\cos\theta}{\sin^2\theta} \left(g\Phi^i H_i\right)_A{}^B i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta} \left(g\Phi^i H_i\right)^2{}_A{}^B \right\} \right] A_{\theta B} \\
& - \frac{1}{2} \sin\theta A_\varphi^A \frac{1}{\sin^2\theta} \Box A_{\varphi A} \\
& - \frac{1}{2R^2} \sin\theta \left\{ \frac{1}{\sin^2\theta} \left(\partial_\theta A_{\varphi A} - \partial_\varphi A_{\theta A}\right) \left(\partial_\theta A_\varphi^A - \partial_\varphi A_\theta^A\right) \right. \\
& \left. - A_\theta^A 2 \frac{\cos\theta}{\sin^2\theta} i \left(g\Phi^i H_i\right)_A{}^B \left(\partial_\theta A_{\varphi B} - \partial_\varphi A_{\theta B}\right) + A_\theta^A \frac{2}{\sin\theta} i \left(g\Phi^i H_i\right)_A{}^B A_{\varphi B} \right\} \\
& + \frac{1}{R^2} \sin\theta \left[- \frac{1}{\sin^2\theta} \left\{ g f^A{}_{BC} \left(\partial_\theta A_{\varphi A} - \partial_\varphi A_{\theta A}\right) A_\theta^B A_\varphi^C + \frac{1}{2} g^2 f_{ABC} f^A{}_{DE} A_\theta^B A_\varphi^C A_\theta^D A_\varphi^E \right\} \right. \\
& \left. - \frac{\cos\theta}{\sin^2\theta} \left(g\Phi^i H_i\right)_A{}^B i g f_{BCD} A_\theta^A A_\theta^C A_\varphi^D \right] \\
& + \frac{1}{2} R^2 \sin\theta \left\{ - \left(\partial_\nu A_A^\nu\right) + A_{\theta A}\partial_\theta + \frac{1}{\sin^2\theta} A_{\varphi A}\partial_\varphi + \frac{\cos\theta}{\sin^2\theta} i \left(g\Phi^i H_i\right)_A^B A_{\varphi B} \right\} \partial_\mu A^{\mu A} \\
& + \frac{1}{R^2} \Phi^i \left(\partial_\theta A_{\varphi i} - \partial_\varphi A_{\theta i}\right) - \frac{\Phi_a \Phi^i}{2R^2} \sin\theta \\
& + \frac{1}{2} R^2 \sin\theta \left\{ 2g \left(\partial^\mu A^{\nu A}\right) f_{ABC} A_\mu^B A_\nu^C + \frac{1}{2} g^2 f_{ABC} f^A{}_{DE} A_\mu^B A_\nu^C A^{\mu D} A^{\nu E} \right\} \\
& + \sin\theta \left\{ g f^A{}_{BC} \left(\partial_\mu A_{\theta A} - \partial_\theta A_{\mu A}\right) A^{\mu B} A_\theta^C + \frac{1}{2} g^2 f_{ABC} f^A{}_{DE} A_\mu^B A_\theta^C A^{\mu D} A_\theta^E \right\} \Bigg)
\end{aligned}
$$

Lagrangian contains A_μ cubic terms $\mathcal{L}_{A,\text{cubic}}$

Consider φ dependence of $\mathcal{L}_{A,\text{cubic}}$,
Then $\mathcal{L}_{A,\text{cubic}}$ is not single-valued

Example: S^1 dimensional reduction

$$S = \int d^4x\, R\, d\varphi \left(-\frac{1}{4} F_{MNA} F^{MNA} \right)$$



Expand $A_M(x, \varphi)$ w.r.t. S^1 coordinate φ :

$$A_{\mu A}(x, \varphi) = \sum_{m=-\infty}^{\infty} \sqrt{\frac{R}{2}} A_{\mu A, m}(x) \frac{e^{im\varphi}}{\sqrt{2\pi}}$$

$$A_{\varphi A}(x, \varphi) = \sum_{m=-\infty}^{\infty} \sqrt{\frac{2}{R}} \phi_{A, m}(x) \frac{e^{im\varphi}}{\sqrt{2\pi}}$$