Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance
Yu-Cheng QIU
Introduction
Setup
Landscape?
Summary and Discussion

Wiggly dilaton/radion: a landscape of spontaneously broken scale invariance

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2411.16304 with S. Girmohanta, Y. Nakai, Z. Zhang

Introduction

Setup

Landscape?

Summary and Discussion

I. Introduction

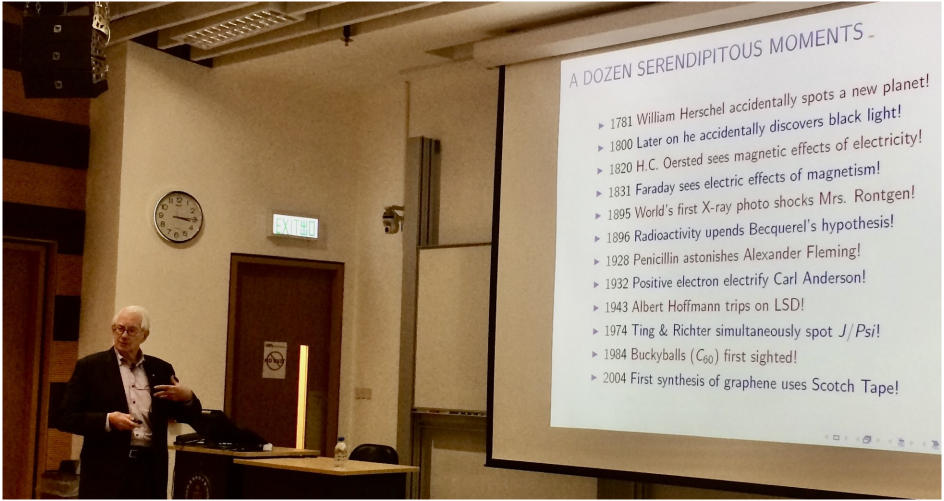

Figure: Sheldon Lee GLASHOW at HKUST, 2018.

On a quest of a naturally light dilaton,
we accidentally found a wiggly dilaton potential.

Naturally light dilaton

- "*These are transformations that would be exact invariances of the world if all elementary particle masses vanishes.* $\cdots$ " (*Dilatations* by Coleman (1971))

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- Dilaton naturally arise in the string theory.
- Naturally light dilaton is tightly related to the cosmological constant. (Sundrum (2003))
- The existence of a naturally light dilaton, as the pNGB of spontaneous symmetry breaking of dilatation, is the question.

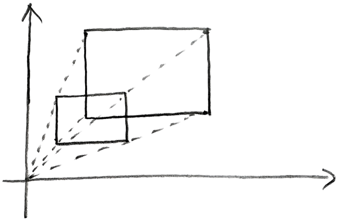
Spontaneous breaking of scale invariance (SBSI)

Dilatation is scale transformation.
The Goldstone theorem indicates that the SI manifest itself nonlinearly after SBSI by

$$\tau(x) \to \tau(\lambda x) + \log \lambda \ . \tag{1}$$

The SI potential of a canonically normalized dilaton $\chi = fe^{\tau}$ is

$$V = \lambda\chi^4 \ . \tag{2}$$



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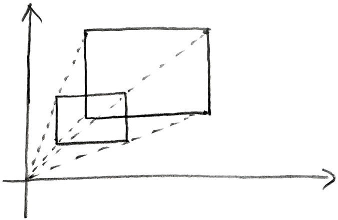

The SI potential of a canonically normalized dilaton $\chi = fe^{\tau}$ is

$$V = \lambda \chi^4 \,. \tag{2}$$

If $\lambda \neq 0$, χ cannot be stabilized unless $f \to 0$.
Thus, the exact SBSI only happens if $\lambda \to 0$.

- Dilaton mass measures the explicit breaking of SI.

Naturally light dilaton

- Dilatation is part of conformal transformations.
- SBSI in 4D $\rightarrow$ Stabilization of the *radion* in 5D. (AdS/CFT Correspondence Maldacena (1997))
- *Radion* is the size of the compactified 5th dim.
- A naturally light dilaton $\rightarrow$ a naturally light radion

5D Radion stabilization

- The 5D model is based on RS1 model. (Randall & Sundrum (1999))
  $V(\chi) = 0$ under the tuning $\Lambda_{\rm UV} = -\Lambda_{\rm IR} = -\Lambda_{\rm bulk}/k$.
- The RS1 geometry can be stabilized via GW mechanism.
  (Goldberger & Wise (1999))
  A bulk scalar ϕ with Dirichlet boundary conditions has a nontrivial profile $\phi(y)$ along the 5th-dim.
  Backreation to the metric is neglected.
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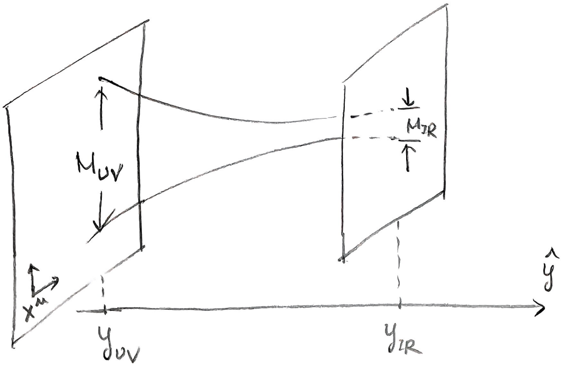
- The backreaction of the GW scalar to the metric can be included.
  (Csaki, Erlich, Grojean, Hollowood (2000))
- A holographic formulation of a naturally light dilaton emerge (?).
  Named CPR framework.
  ([unpublished] Contino, Pomarol, Rattazzi (2010))
  (Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale (2013))
  (Bellazzini, Csaki, Hubisz, Serra, Terning (2013))

5D Radion stabilization

$$ds^2 = e^{-2ky} g_{\mu\nu} dx^\mu dx^\nu - dy^2$$



$$M_{IR} \sim M_{UV}\, e^{-k(y_{IR}-y_{UV})}$$

Dilaton/radion potentials

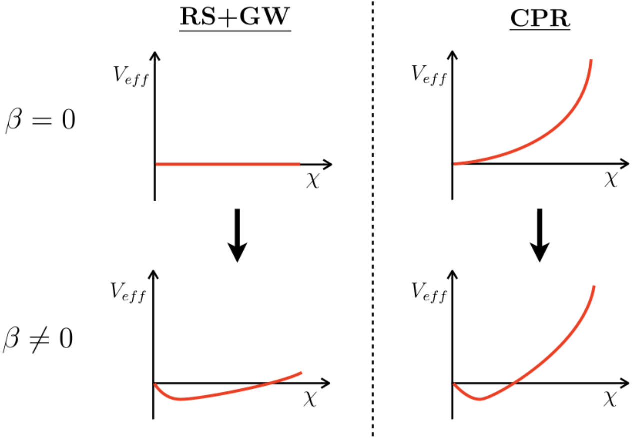


Figure: From 1305.3919 (Bellazzini, Csaki, Hubisz, Serra, Terning).

The CPR framework

	CFT4		AdS5	
marginal	λ	$\longleftrightarrow$	ϕ	GW scalar with ϵ mass
near marginal	$\beta(\lambda) \ll 1$	$\longleftrightarrow$	$\lvert\delta\phi(y)\rvert \ll 1$	slow-varying profile
	$F[\lambda]$	$\longleftrightarrow$	$V_{\rm IR}(\phi)$	IR brane tension

The CPR framework in 5D

The 5D formulation of CPR scenario includes:

1. a bulk scalar ϕ whose mass is parametrically small ϵ,
2. Dirichlet boundary conditions $\phi(y_{\mathrm{UV/IR}}) = v_{\mathrm{UV/IR}}$, and
3. it backreacts to the metric.

The resulting dilaton/radion χ has

$$\frac{m_\chi}{\langle\chi\rangle} \propto \sqrt{\epsilon}\,, \quad \langle\chi\rangle = \left(\frac{v_{\mathrm{UV}}}{v_{\mathrm{IR}}+\xi}\right)^{1/\epsilon} + \mathcal{O}(\epsilon) \tag{3}$$

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Two questions/comments:

1. what is the origin of ϵ?
   ϕ can be a pNGB.
2. one has to fine-tune the v_{UV} and v_{IR} to get a reasonable $\langle\chi\rangle$ for small ϵ.

Our journey

The idea:

The bulk profile back-reacts to the boundary.
Hopefully, some relaxation may happen to resolve the fine-tuning in $\langle\chi\rangle$ between $v_{\rm UV}$ and $v_{\rm IR}$.

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The result:

A wiggly dilaton !

II. Setup

The setup

The topology of the compactified extra dimension is S^1/Z_2.
The UV/IR brane locates at y_0/y_1.
The 5D action is

$$S = \overbrace{\int d^4xdy \left(-\frac{R}{2\kappa^2} + \frac{1}{2\kappa^2}(\partial a)^2 - V(a)\right)}^{\text{bulk}} - \underbrace{\int d^4x\sqrt{g_0}V_0(a)}_{\text{UV brane}} - \underbrace{\int d^4x\sqrt{g_1}V_1(a)}_{\text{IR brane}} . \tag{4}$$

Warped metric ansatz is

$$ds^2 = e^{-2T(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 . \tag{5}$$

The EOMs

The equations of motion are

$$4T'^2 - T'' + \frac{2\kappa^2}{3}V = 0 \tag{6}$$

$$T'^2 - \frac{1}{12}a'^2 + \frac{\kappa^2}{6}V = 0 \tag{7}$$

$$a'' - 4T'a' - \kappa^2\frac{\partial V}{\partial a} = 0 \tag{8}$$

The boundary condition for a are

$$2T'|_{y_0,y_1} = \pm\frac{\kappa^2}{3}V_{0,1}(a)|_{y_0,y_1} \ , \quad 2a'|_{y_0,y_1} = \pm\kappa^2\frac{\partial V_{0,1}}{\partial a}|_{y_0,y_1} \tag{9}$$

The dilaton/radion potential

The bulk and brane potentials for the axion are

$$V = \Lambda_5 - \epsilon \frac{2k^2}{\kappa^2} a^2 \tag{10}$$

$$V_i = \Lambda_i + \epsilon_i \frac{k}{\kappa^2} [1 - \cos(a - v_i)] \tag{11}$$

Note that

- For $\epsilon_i \to \infty$, one recovers Dirichlet boundary conditions.
- For $\epsilon_i \to 0$, one has Neumann boundary conditions.

The dilaton/radion is $\chi = e^{-ky_1}$.
The dilaton/radion potential is obtained by performing $\int dy(\cdots)$.

The boundary condition

The effective dilaton/radion potential is determined by the IR boundary potential

$$V_{\text{eff}} = \chi^4 F \,, \quad F = F(V_1(a(y_1))) \tag{12}$$

Suppose at the boundaries $a(y_0) = \tilde{v}_0$ and $a(y_1) = \tilde{v}_1$, the BCs give

$$2\epsilon\tilde{v}_0 = \epsilon_0 \sin(\tilde{v}_0 - v_0) \tag{13}$$

$$-4\sqrt{3}\sinh(2\beta) = \epsilon_1 \sin(\tilde{v}_1 - v_1) \tag{14}$$

$$\beta = \frac{1}{\sqrt{3}}\left(\tilde{v}_1 - \tilde{v}_0\chi^{-\epsilon}\right)$$

The dialton/radion potential

$$V_{\text{eff}}(\chi) = \chi^4 F[\beta(\chi)] \tag{15}$$

$$F[\beta(\chi)] = [-1 + \xi\Delta(\beta) + \xi \cosh(2\beta)] \operatorname{sech}^2\beta$$

$$\Delta(\beta) = \frac{\epsilon_1}{6}\left[1 - \eta\sqrt{1 - \frac{48}{\epsilon_1^2}\sinh^2(2\beta)}\right]$$

The function β can be given in the small ϵ_1 limit,

$$\beta(\chi) = \frac{\epsilon_1}{8\sqrt{3}} \sin\left(v_1 - \tilde{v}_0\chi^{-\epsilon}\right) + \mathcal{O}(\epsilon_1^2)\,. \tag{16}$$

- $|\tilde{v}_0| \leq |\epsilon_0/2\epsilon|$ is essentially a free parameter.

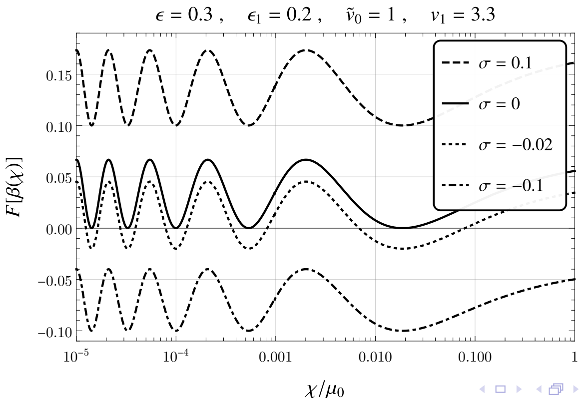
The potential is determined by parameters $\{\epsilon, \epsilon_1, v_1, \tilde{v}_0\}$.

III. Landscape ?

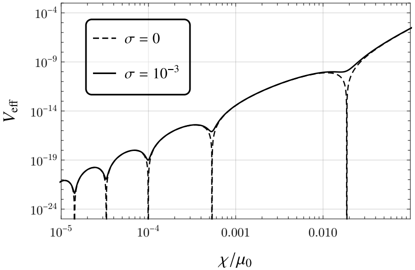
A detailed investigation on the potential

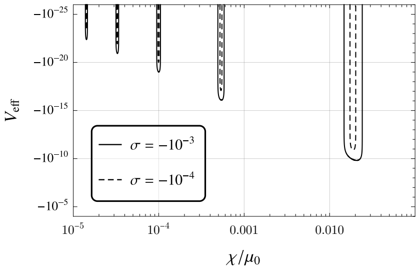
The mistune of IR brane tension and the bulk CC can be parametrized by $\xi = \Lambda_5/k\Lambda_1 \equiv 1 + \sigma$.

$$V_{\text{eff}}(\chi) = \chi^4 F[\beta(\chi)]$$



Dilaton landscape



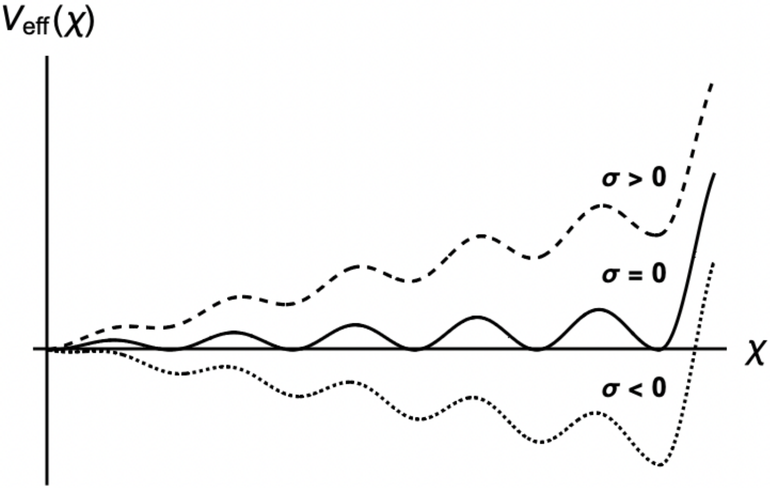


$$\langle\chi\rangle^{(p)} \simeq \left(\frac{\tilde{v}_0}{v_1 - 2p\pi}\right)^{1/\epsilon}\left[1 - \frac{24\sigma}{\epsilon^2\epsilon_1(v_1-2p\pi)^2} + \mathcal{O}(\sigma^2)\right] \quad (17)$$

$$\left(\frac{m_\chi}{\langle\chi\rangle}\right)^{(p)} \simeq \frac{\epsilon\epsilon_1^{1/2}}{\sqrt{6}}|v_1 - 2p\pi| + \mathcal{O}(\sigma) \quad (18)$$

$$\langle V_{\rm eff}\rangle^{(p)} \simeq \sigma\left(\frac{\tilde{v}_0}{v_1 - 2p\pi}\right)^{4/\epsilon} + \mathcal{O}(\sigma^2) \qquad p \in \mathbf{Z} \quad (19)$$

Dilaton landscape



Summary

1. We constructed a wiggly dilaton potential $V_{\text{eff}}(\chi)$.
2. For $\sigma = 0$, it has infinite number of degenerate ground states, thus a landscape.
3. For $\sigma > 0$, the only true ground state is $\chi = 0$.
4. For $\sigma < 0$, the only true ground state (if exists) is the local minimum that is closest to $\chi \to 1$.
5. The limitation of its application is our imagination.
   5.1 relaxion
   5.2 PBH production
   5.3 $\cdots$

Thank you

The CPR framework

$$V(\chi) = \chi^4 F[\lambda(\chi)] \tag{20}$$

To have a naturally light dilaton:

1. The CFT should be able to sample a direction with $F = 0$.
2. The coupling λ should stays 'naturally' close to marginality throughout the RG evolution.

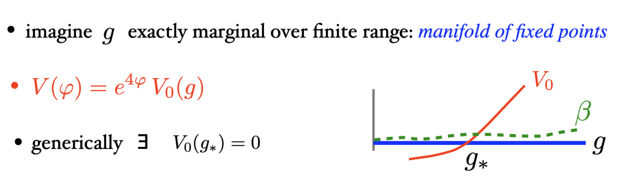


Figure: From Rattazzi's talk (2010)

$\sigma_{\rm crit}$