Composite Dark Matter with Forbidden Annihilation

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Based on the collaboration with T. Abe (Tokyo U. Sci.) and R. Sato (Osaka U.)

[JHEP **09** (2024) 064] and ongoing work

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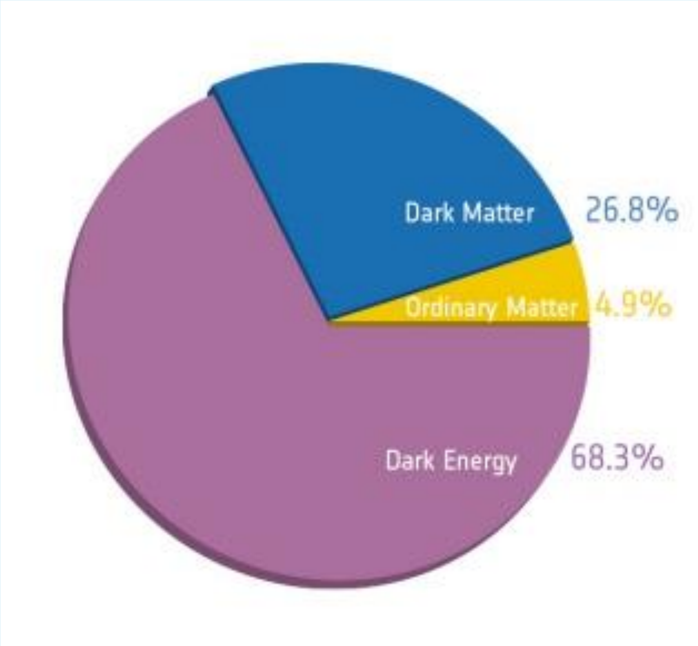
KEK Theory Meeting on Particle Physics Phenomenology (KEK-PH 2025winter)

Introduction

Dark Matter（DM）

observed energy density : $\Omega_{\rm DM}h^2 \simeq 0.12$

Planck Collaboration (2018)

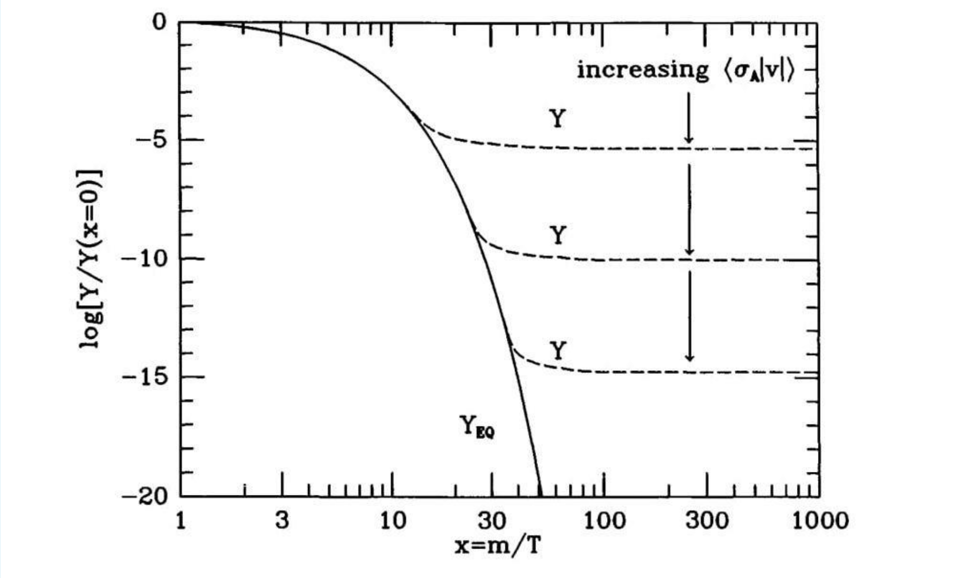


https://sci.esa.int/web/planck/-/51557-planck-new-cosmic-recipe

Thermal Freeze-out Scenario

Thermally produced DM decouples from thermal bath.

$$\Omega_{\rm DM}h^2 \sim 0.12\frac{2\times 10^{-26}\ {\rm cm}^3/{\rm s}}{\langle\sigma v\rangle_{\rm ann}}$$



Kolb, Turner *"The Early Universe"*

Motivation

Indirect Detection

Improving sensitivity to heavy DM

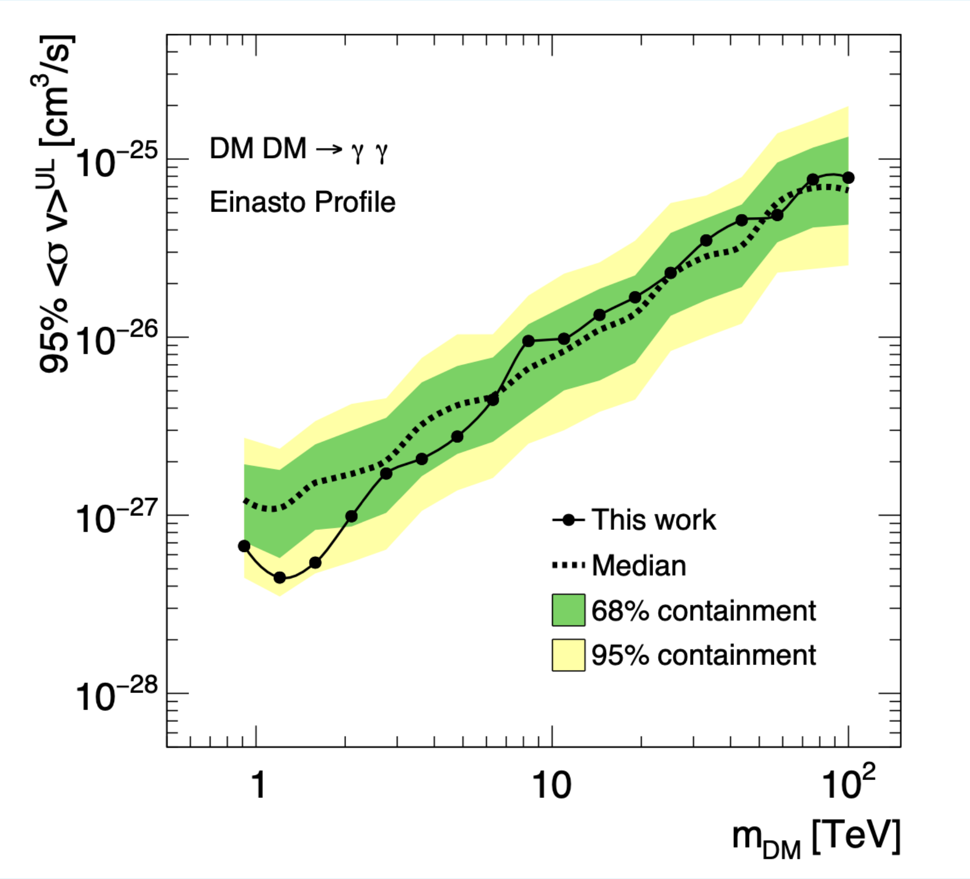
↓

Heavy thermal relic DM

$$\langle \sigma v \rangle_{\text{ann}} \sim \frac{\alpha_{\text{DM}}^2}{m_{\text{DM}}^2}$$

$\alpha_{\text{DM}} \sim \mathcal{O}(1)$ for $m_{\text{DM}} \sim \mathcal{O}(10)$ TeV

QCD-like DM model



MAGIC Collaboration [PRL **130** (2023)]

Composite DM Model

QCD-like $SU(N)_d$ gauge theory
w/ $SU(2)_W$ triplet dark quark (3-flavor)

Bai and Hill [PRD **82** (2010)]

	$SU(N)_d$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
ψ	N	$\mathbf{1}$	$\mathbf{3}$	0
$\bar{\psi}$	$\bar{N}$	$\mathbf{1}$	$\mathbf{3}$	0

Dark quark mass $m_q < \Lambda_d$

Chiral symmetry breaking: $SU(3)_L \times SU(3)_R \to SU(3)_V \supset SU(2)_W$ by $\langle \bar{\psi}\psi \rangle \sim v_d^3$

Dark pion $SU(3)$ octet $\to$ $SU(2)_W$ 3-plet χ $\oplus$ $SU(2)_W$ 5-plet π

Chiral Lagrangian

$$\mathcal{L} \supset \frac{f_d^2}{4}\mathrm{tr}[D_\mu U D^\mu U^\dagger] + v_d^3 \mathrm{tr}[MU + M^\dagger U^\dagger] + \mathcal{L}_{\mathrm{WZW}}$$

$$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right)$$

$$M = \mathrm{diag}(m_q, m_q, m_q)$$

Parameters of the model

dark pion decay const. f_d

dark quark mass m_q

Accidental Symmetry & Stability of DM

G-Parity $\Psi_i \to \exp(i\pi\, I_2)\Psi_i^c$

[Lee & Yang Nuovo. Cim. **10** (1956)]

	$SU(N)_d$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
ψ	N	$\mathbf{1}$	$\mathbf{3}$	0
$\bar{\psi}$	$\bar{N}$	$\mathbf{1}$	$\mathbf{3}$	0

$$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right)$$

$SU(2)_W$ 3-plet (anti-symmetric) $SU(2)_W$ 5-plet (symmetric)

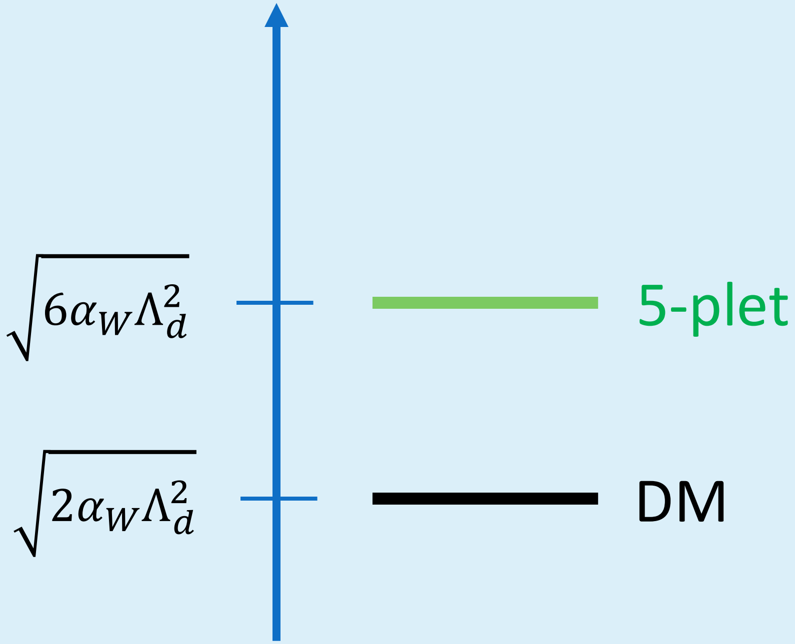
$$\Pi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i\chi^0 & \frac{\chi^- - \chi^+}{\sqrt{2}} \\ i\chi^0 & 0 & -i\frac{\chi^- + \chi^+}{\sqrt{2}} \\ -\frac{\chi^- - \chi^+}{\sqrt{2}} & i\frac{\chi^- + \chi^+}{\sqrt{2}} & 0 \end{pmatrix}, \quad \Pi_5 = \begin{pmatrix} \frac{\pi^0}{\sqrt{6}} - \frac{\pi^{++}+\pi^{--}}{2} & -i\frac{\pi^{++}-\pi^{--}}{2} & \frac{\pi^+ + \pi^-}{2} \\ -i\frac{\pi^{++}-\pi^{--}}{2} & \frac{\pi^0}{\sqrt{6}} + \frac{\pi^{++}+\pi^{--}}{2} & i\frac{\pi^+ - \pi^-}{2} \\ \frac{\pi^+ + \pi^-}{2} & i\frac{\pi^+ - \pi^-}{2} & -\sqrt{\frac{2}{3}}\pi^0 \end{pmatrix}$$

G-parity $\Pi_3 \to -\Pi_3$ (odd) $\Pi_5 \to \Pi_5$ (even)

DM Candidate : χ^0

Decay to EW gauge bosons via WZW

Mass Spectrum & Annihilation Processes

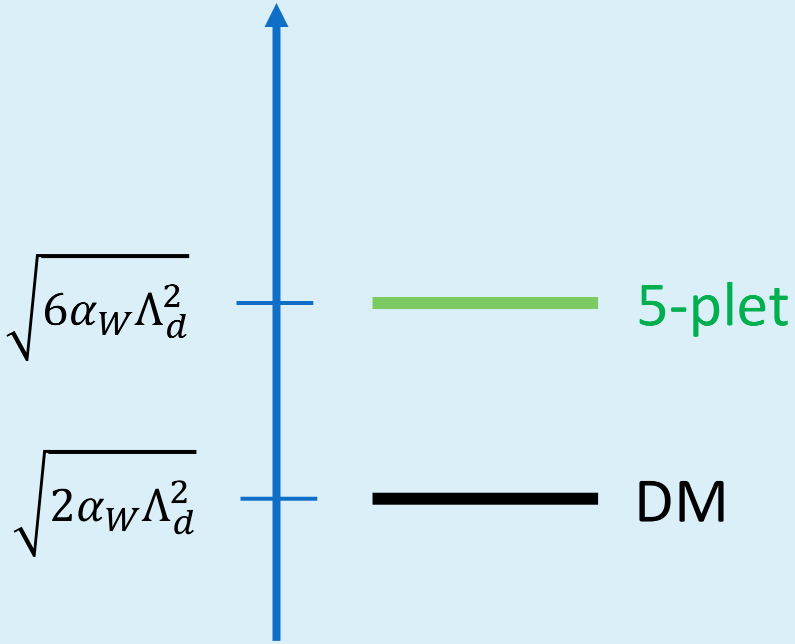


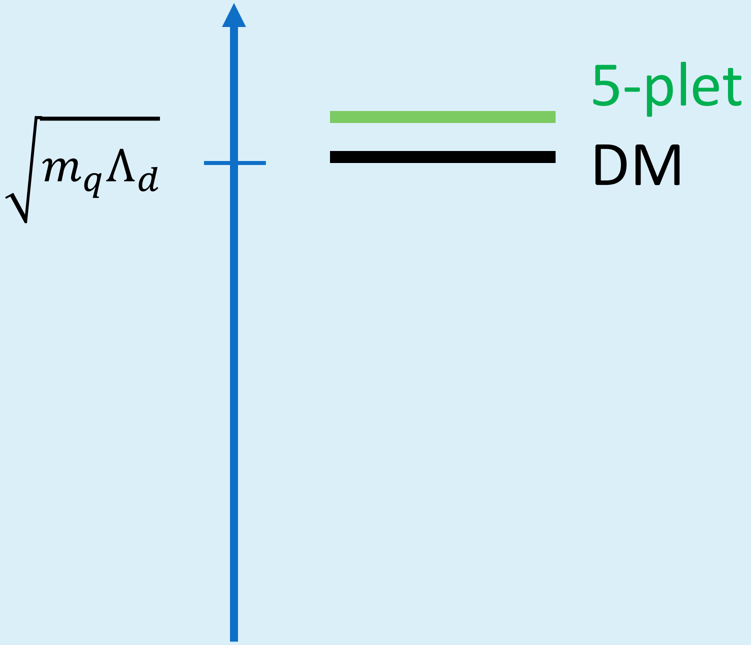
①large mass splitting : DM+DM→ EW gauge bosons

$$\langle \sigma v \rangle_{WW} \simeq \frac{4\pi\alpha_W^2}{m_\chi^2}$$

$m_\chi \sim 1.8\ \mathrm{TeV}$ (tree level calc.)

Mass Spectrum & Annihilation Processes





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$$m_\chi \sim 1.8\ \mathrm{TeV}\ \ \text{(tree level calc.)}$$

②small mass splitting: DM+DM→ 5-plet

Forbidden channel Abe, Sato, **TY** [JHEP **09** (2024)]

$$\langle \sigma v \rangle_{\pi\pi} \propto \frac{m_\chi^2}{f_d^4} \exp\left(-\frac{m_\pi - m_\chi}{T}\right)$$

$$m_\chi \sim \mathcal{O}(1-10)\ \mathrm{TeV}\ \text{(tree level calc.)}$$

Leading Order Calculation

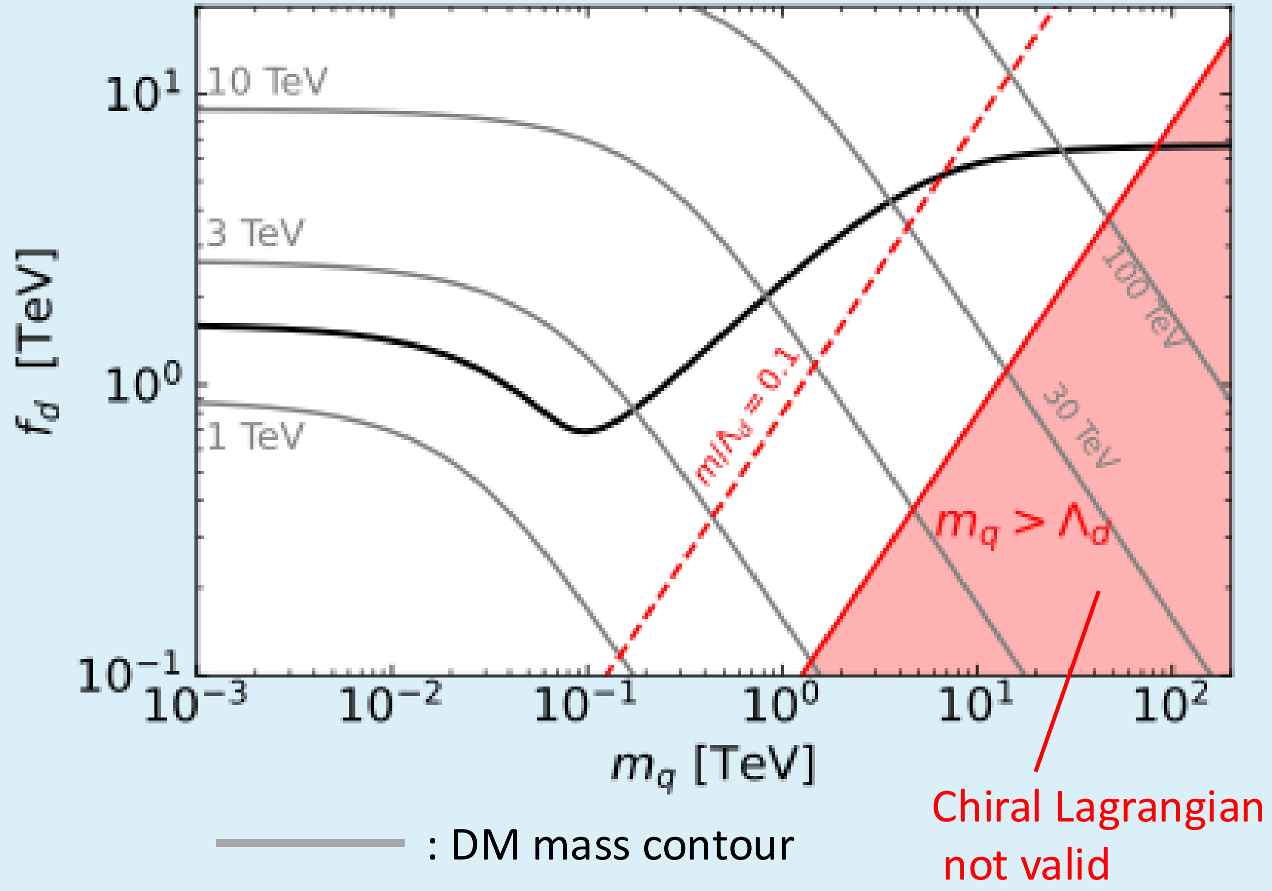
Abe, Sato, TY [JHEP **09** (2024)]

For small m_q, $m_\chi \sim 1.8$ TeV

For large m_q, $m_\chi \simeq m_\pi$, $m_\chi \sim O(1-10)$ TeV

→Forbidden channels contribute to DM abundance

$N = 3$



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→**Forbidden channels contribute to DM abundance**

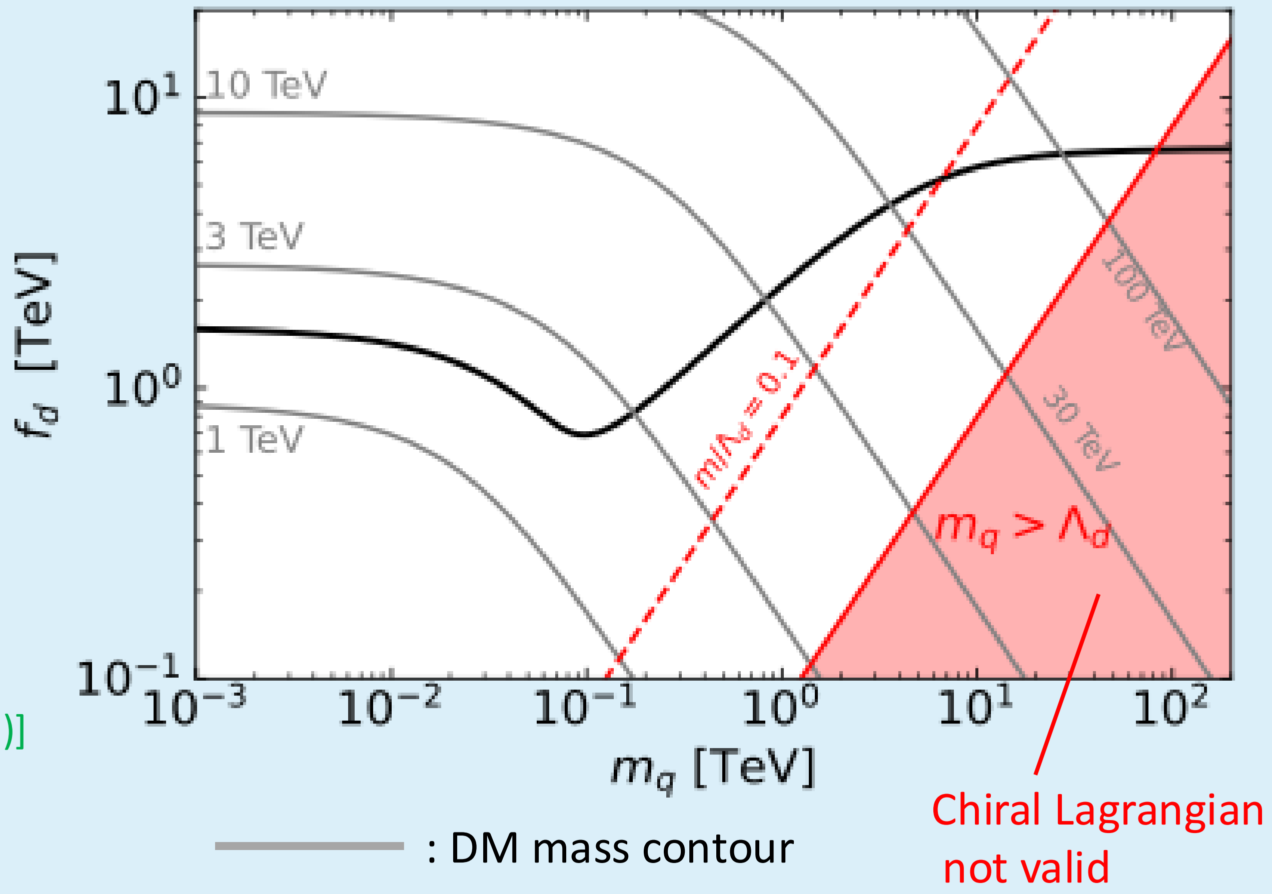
Next step…

→ **Sommerfeld Effect (SE)**

affects abundance of EW interacting heavy DM

Hisano, Matsumoto, Nojiri, Saito [PRD **71** (2005)]

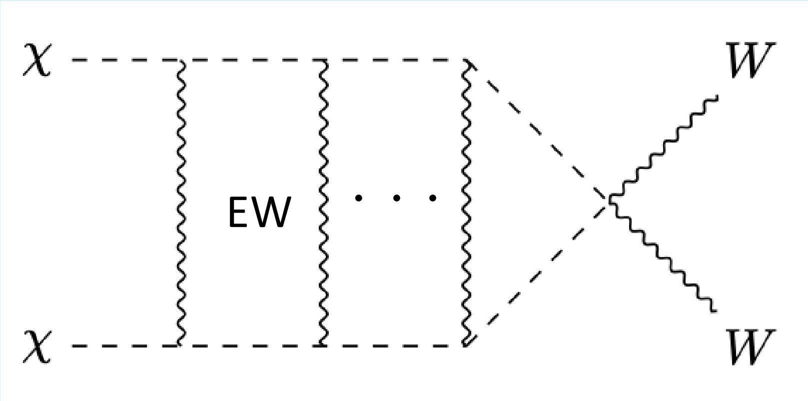
$N = 3$



Sommerfeld Effect

$m_\chi \gg m_{W,Z}$ $\rightarrow$ EW interaction behaves as a long-range force.

EW interaction affects non-relativistic two-body states



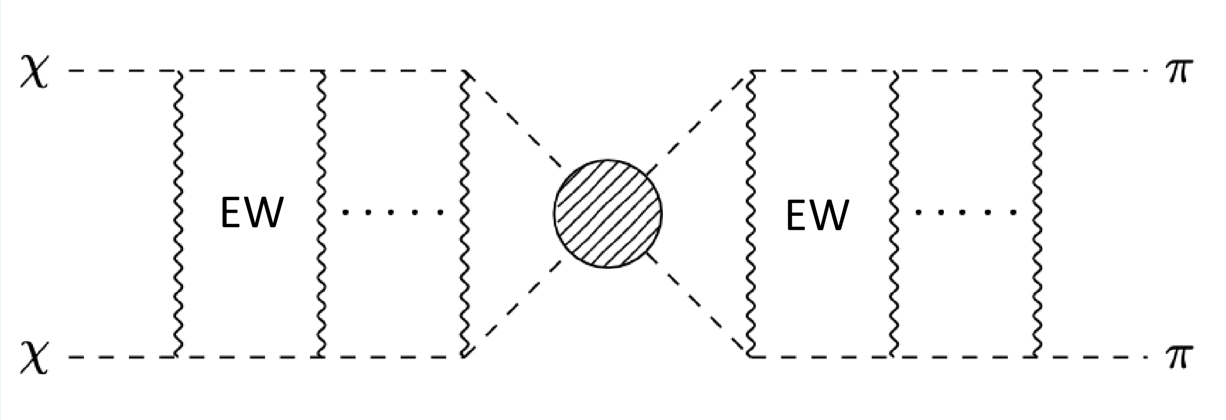
annihilation cross section

$$\sigma_{\text{ann}} = |\psi(0)|^2 \sigma_{\text{LO}}$$

↑obtained by solving Schroedinger eq.

Forbidden channel $\chi\chi \rightarrow$ 5-plet

$m_\pi \simeq m_\chi \gg m_{W,Z}$



SE in the final state？

Cui, Luo [JHEP **01** (2021)]

Toy Model

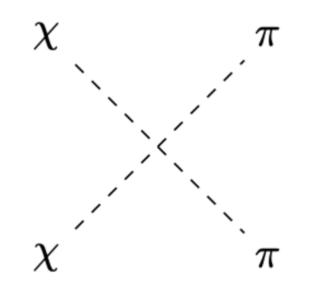
Annihilation process $\chi\chi \to \pi\pi$

$$\mathcal{L} \supset v_d^3 \mathrm{tr}[MU + \mathrm{h.c.}] \supset \frac{4mv_d^3}{3f_d^4}\left(\frac{1}{2}(\chi^0)^2 + \chi^+\chi^-\right)\left(\frac{1}{2}(\pi^0)^2 + \pi^+\pi^- + \pi^{++}\pi^{--}\right).$$

Amplitude w/ SE $\quad \mathcal{M} \propto \psi_i(0)\psi_f(0)\mathcal{M}_{\mathrm{LO}}$ $\quad \psi_{i,f}$: initial /final two-body states

SE factor $\quad S \sim |\psi_i(0)|^2|\psi_f(0)|^2$

$\mathcal{M}_{\mathrm{LO}} =$



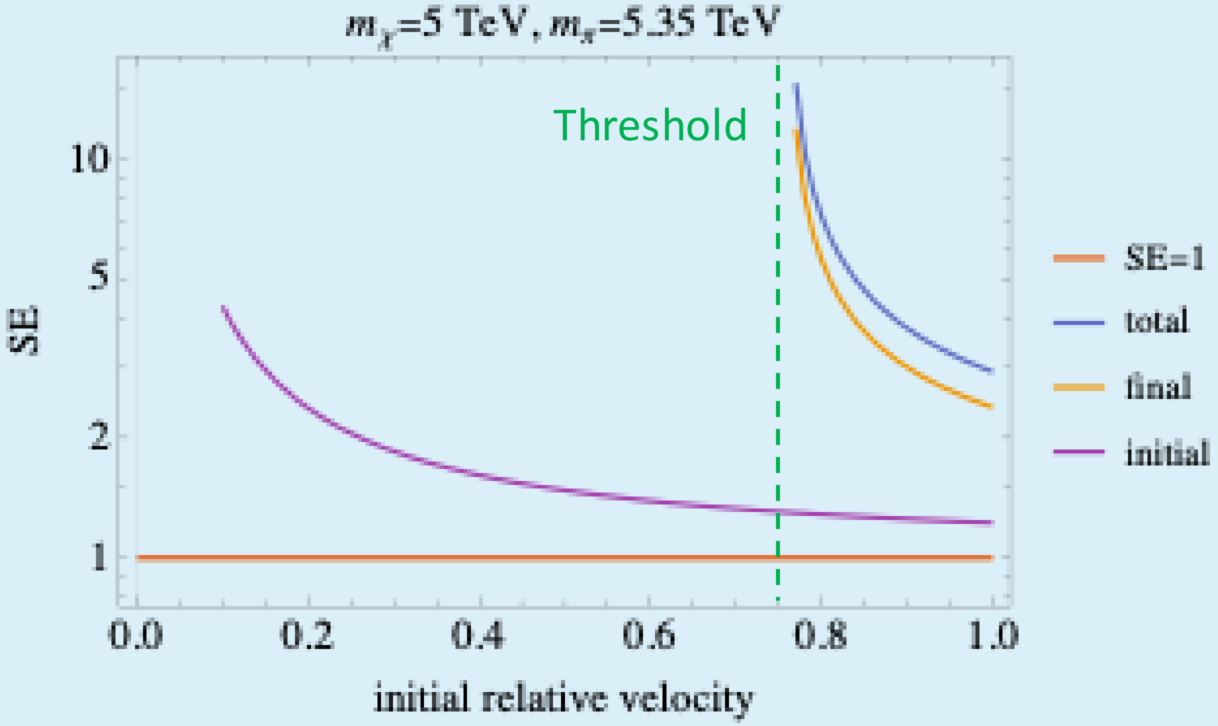
$W^{\pm}$ exchange : $\chi^0\chi^0 \leftrightarrow \chi^+\chi^-$、$\pi^0\pi^0 \leftrightarrow \pi^+\pi^- \leftrightarrow \pi^{++}\pi^{--}$

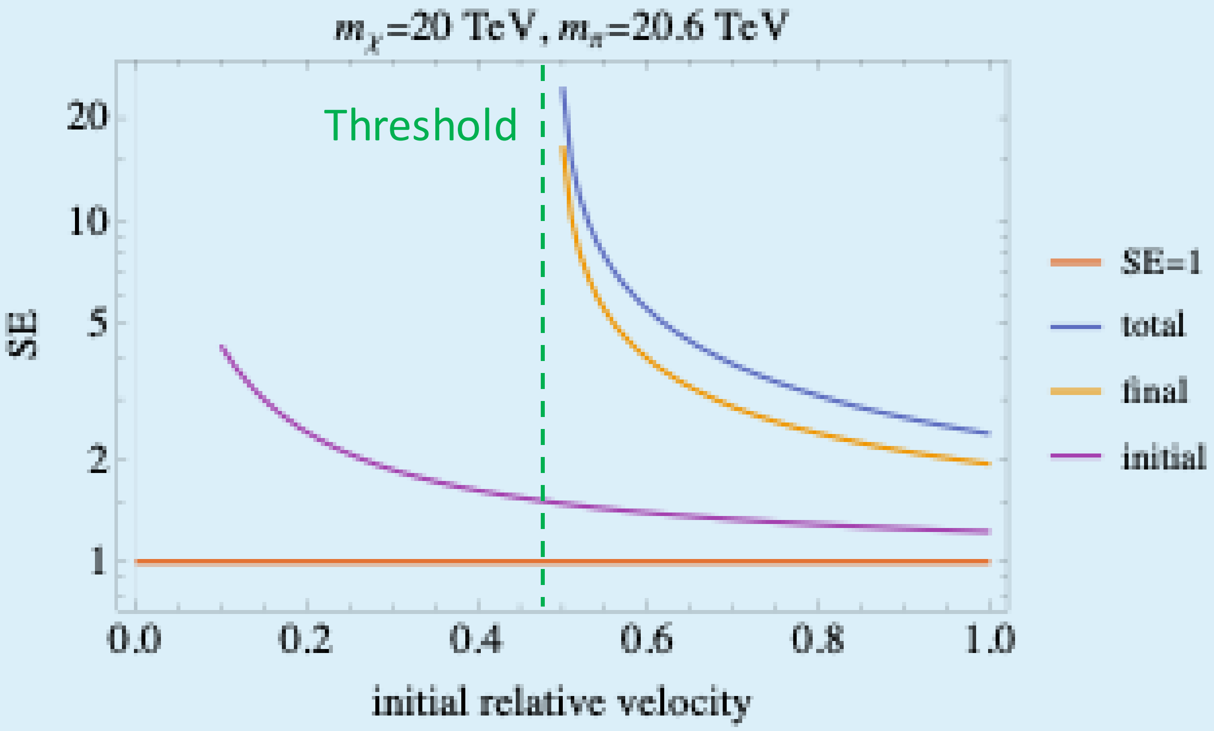
initial $\quad \left[-\frac{1}{m_\chi}\nabla^2 + V_\chi(r) - \frac{p^2}{m_\chi}\right]\Psi_\chi(r) = 0 \quad \Psi_\chi(r) = \begin{pmatrix}\psi_{\chi^0\chi^0}(r) \\ \psi_{\chi^+\chi^-}(r)\end{pmatrix}$ $\quad V_\chi$: 2×2 matrix

final $\quad \left[-\frac{1}{m_\pi}\nabla^2 + V_\pi(r) - \frac{p^2}{m_\pi}\right]\Psi_\pi(r) = 0 \quad \Psi_\pi(r) = \begin{pmatrix}\psi_{\pi^0\pi^0}(r) \\ \psi_{\pi^+\pi^-}(r) \\ \psi_{\pi^{++}\pi^{--}}(r)\end{pmatrix}$ $\quad V_\pi$: 3×3 matrix

Results

Threshold of relative velocity of initial state : $v_{rel} \geq 2\sqrt{\left(\frac{m_\pi}{m_\chi}\right)^2 - 1}$





SE in final states mainly contribute to the total SE factor

Summary & Future Prospect

Summary

- Composite DM model→DM with $O(1-10)$ TeV mass
- DM candidate：$SU(2)_W$ triplet dark pion χ

Forbidden channel $\chi\chi \to \pi\pi$ contributes to relic abundance

- EW interacting Heavy DM → **Sommerfeld Effect (SE)**

SE in final states also affect the relic abundance

Future prospect

Including all interaction terms

→Compare with current constraints from indirect detection experiments

Back Up

SU(N) Composite DM Model

Bai, Hill [Phys. Rev. D **82** (2010)], Antipin, Redi, Strumia, Vigiani [JHEP **07**(2015)]

$SU(N)_d$ gauge symmetry

$$\Psi_i \equiv \begin{pmatrix} \psi_i \\ \bar{\psi}_i^\dagger \end{pmatrix}$$

$\psi, \bar{\psi}$: dark quark, anti-dark quark (3-flavor)

Renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\Psi}_i(i\gamma^\mu D_\mu - m)\Psi_i - \frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + \frac{g_d^2\theta}{32\pi^2}G^A_{\mu\nu}\tilde{G}^{A\mu\nu}$$

Dark quark confines at the scale Λ_d→ dark baryons, dark pions

Accidental symmetries

U(1) global symmetry: $\Psi_i \to e^{i\alpha}\Psi_i$ Stability of dark baryons

G-parity: $\Psi_i \to \exp(i\pi I_2)\Psi_i^c$ Stability of dark pions

Dark Pion Matrices

$SU(2)_W$ triplet χ

$$\Pi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i\chi^0 & \frac{\chi^- - \chi^+}{\sqrt{2}} \\ i\chi^0 & 0 & -i\frac{\chi^- + \chi^+}{\sqrt{2}} \\ -\frac{\chi^- - \chi^+}{\sqrt{2}} & i\frac{\chi^- + \chi^+}{\sqrt{2}} & 0 \end{pmatrix}$$

$SU(2)_W$ quintuplet π

$$\Pi_5 = \begin{pmatrix} \frac{\pi^0}{\sqrt{6}} - \frac{\pi^{++} + \pi^{--}}{2} & -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^+ + \pi^-}{2} \\ -i\frac{\pi^{++} - \pi^{--}}{2} & \frac{\pi^0}{\sqrt{6}} + \frac{\pi^{++} + \pi^{--}}{2} & i\frac{\pi^+ - \pi^-}{2} \\ \frac{\pi^+ + \pi^-}{2} & i\frac{\pi^+ - \pi^-}{2} & -\sqrt{\frac{2}{3}}\pi^0 \end{pmatrix}$$

Dark pion fields $$U = \exp\left(\frac{\sqrt{2}i}{f_d}(\Pi_3 + \Pi_5)\right)$$

$U \in SU(3)_V$

G-parity

$$W_{\mu\nu} \to W_{\mu\nu}, \quad U \to U^T$$

$$\Pi_3 \to -\Pi_3, \quad \Pi_5 \to \Pi_5$$

***π* decay process** $$\mathcal{L}_{\rm WZW} \supset -\frac{g^2 N}{16\sqrt{2}\pi^2 f_d}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}[\Pi_5 W_{\mu\nu} W_{\rho\sigma}]$$

Charge Assignment for Dark Quarks

	$SU(N)_d$	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$
ψ	N	$\mathbf{1}$	$\mathbf{3}$	0
$\bar{\psi}$	$\bar{N}$	$\mathbf{1}$	$\mathbf{3}$	0

Vector-like : Anomaly cancelation

$Y = 0$: Escape from direct detection constraints & obtain G-parity

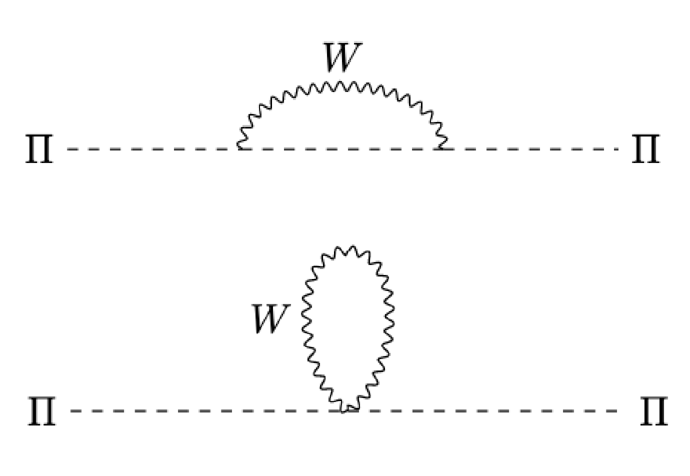
Minimal setup w/ forbidden channel

Dark Pion Mass

Bai, Hill [Phys. Rev. D **82** (2010)], Antipin, Redi, Strumia, Vigiani [JHEP **07**(2015)]

Explicit breaking of chiral symmetry→ Dark pion mass

① Dark quark mass term $m_\chi^2 = m_\pi^2 = \frac{4mv_d^3}{f_d^2}$



② $SU(2)_W$ radiative corrections

$$\delta m^2 \sim C^2(R)\alpha_2(\Lambda_d)\Lambda_d^2$$

$C^2(R)$: Casimir op. of rep. R $C^2(\mathbf{3}) = 2$, $C^2(\mathbf{5}) = 6$

mass splitting among multiplet

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]

$$m_Q - m_0 \simeq \alpha_2 Q^2 m_W \sin^2\frac{\theta_W}{2} \quad (\text{for} \quad m \gg m_W)$$

$$\simeq 166 \times Q^2 \ \mathrm{MeV}$$

Dark baryon

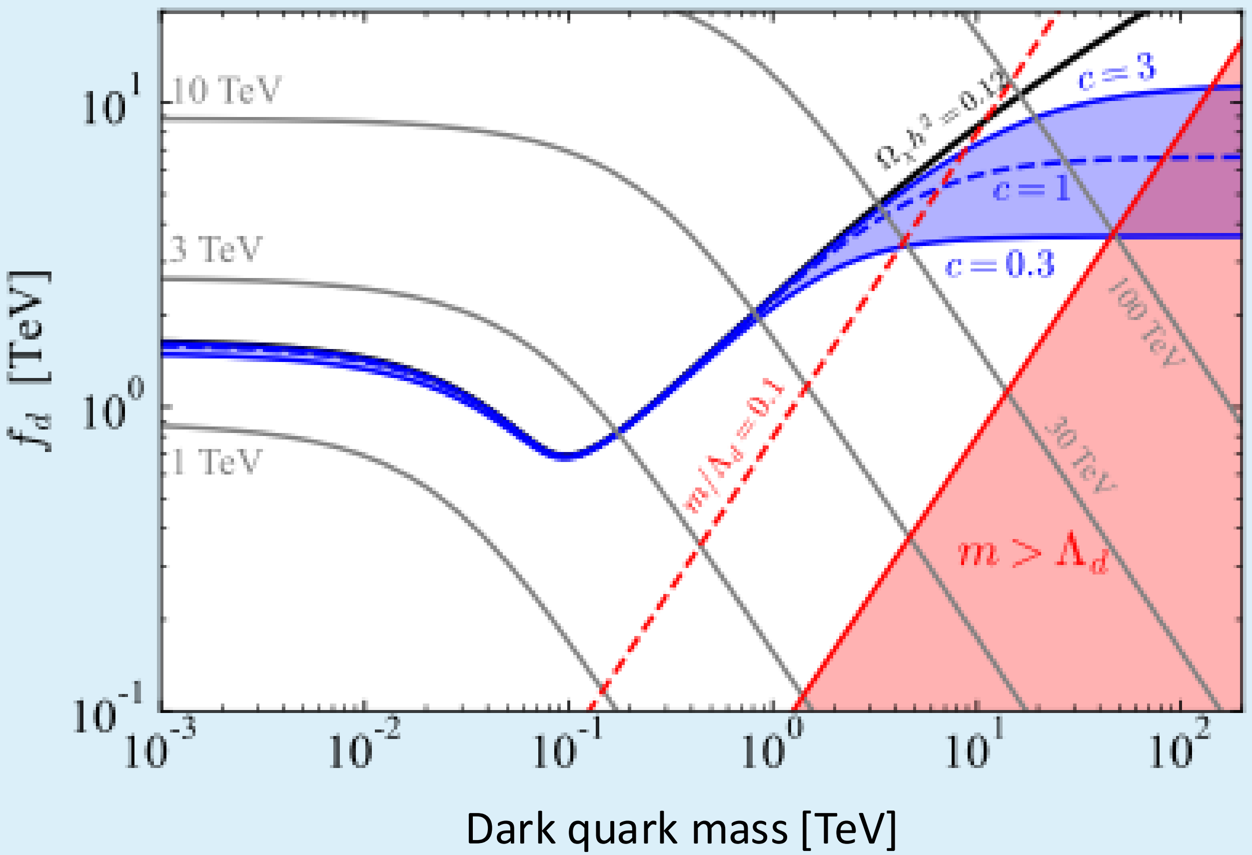
DM components $\Omega_{\rm DM}h^2 = \Omega_\chi h^2 + \Omega_B h^2$

energy density of dark baryon

annihilation cross section (s-wave)

$\langle \sigma_B v \rangle \simeq c \frac{4\pi}{m_B^2}, \quad c \sim O(1)$ $m_B \sim \Lambda_d \sim 4\pi f_d$

$\Omega_B h^2 \simeq 0.12 \left(\frac{f_d}{6.66\,{\rm TeV}} \right)^2 \left(\frac{1.0}{c} \right)$



CP phase

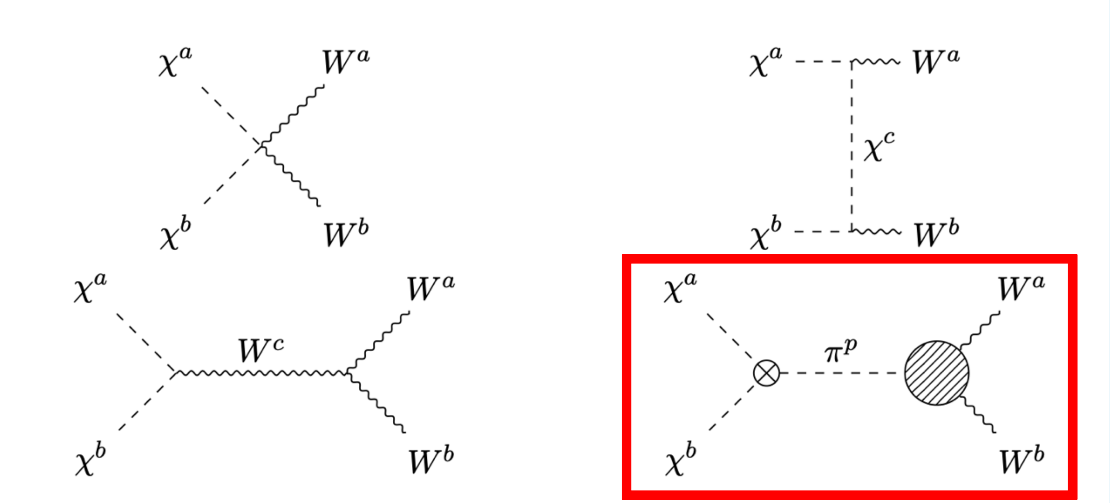
$\boldsymbol{\theta \neq 0}$

$$\chi\chi \to WW$$

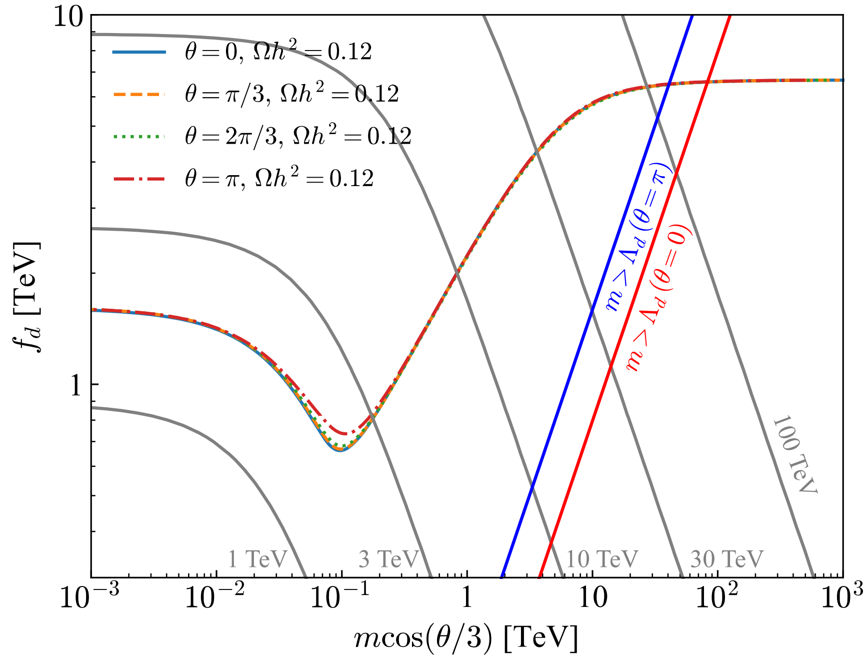
annihilation cross section

$$\langle \sigma v \rangle_{WW} = \langle \sigma v \rangle_{WW}^{\mathrm{MDM}} + \langle \sigma v \rangle_{WW}^{\mathrm{WZW}}$$

$$\langle \sigma v \rangle_{WW}^{\mathrm{MDM}} \simeq \frac{4\pi\alpha^2}{m_\chi^2} \qquad \langle \sigma v \rangle_{WW}^{\mathrm{WZW}} \propto \frac{g^4}{m_\chi^2}\left(\frac{m_q \sin\frac{\theta}{3}}{m_\chi}\right)^2 \frac{m_\chi^4}{m_\pi^4}$$



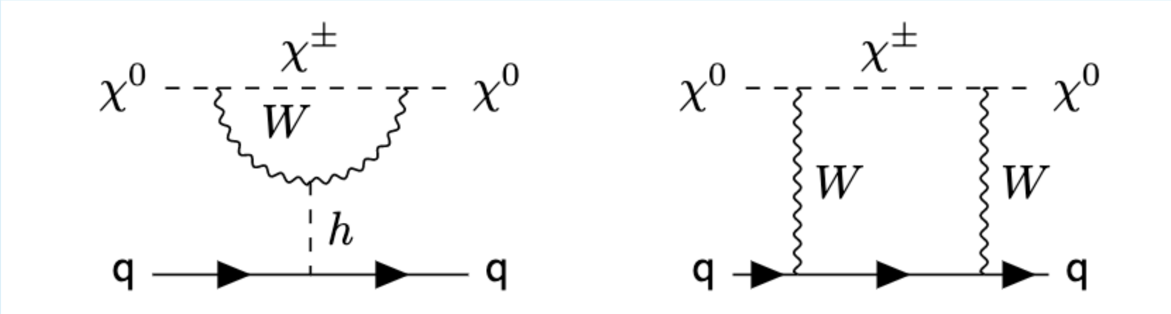
CP violation



Direct Detection

Elastic scattering suppressed at one-loop level

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]

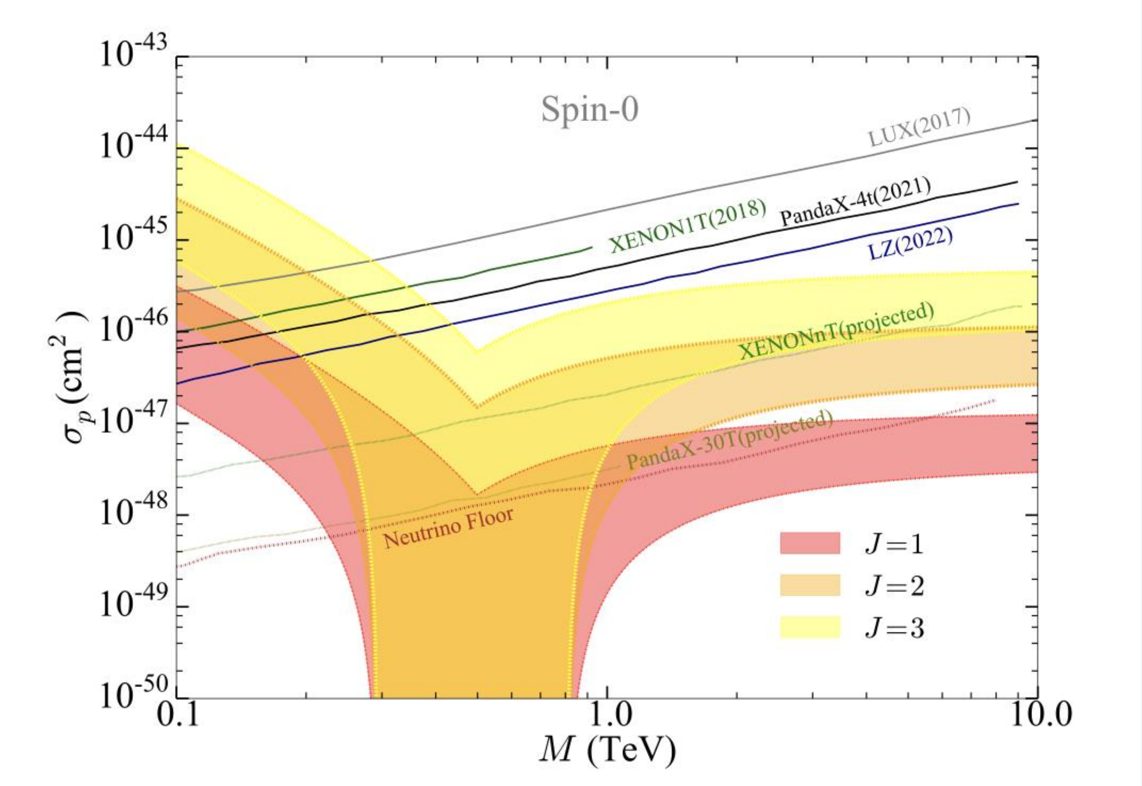


cross section: $\sigma_{\rm SI} \sim O(10^{-47})\ {\rm cm}^2$

Hisano, Ishiwata, Nagata [JHEP **06** (2015)]

Cheng, Ding, Hill, [Phys. Rev. D **108** (2023)]

Above the neutrino fog if $m_\chi \lesssim 4$ TeV



Leading Order analysis

Cheng, Ding, Hill, [Phys. Rev. D **108** (2023)]

Collider Experiments

①production of χ : $pp \to W^{\pm *} \to \chi^{\pm}\chi^{0}$ Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2005)]

How to detect Charged component decay $\chi^{\pm} \to \chi^{0}\pi^{\pm}_{\mathrm{QCD}}$ $\pi^{\pm}_{\mathrm{QCD}}$: soft

Disappearing tracks of $\chi^{\pm}$: $c\tau \sim 6\ \mathrm{cm}$

Might be tested by future 100 TeV pp-collider? [Chiang, Cottin, Du, Fuyuto, Ramsey-Musolf [JHEP **01** (2021) 198]

②production of π

$$pp \to W^{*} \to \pi W$$
$$pp \to \rho_d^{*} \to \pi\pi$$

Kilic, Okui, Sundrum [JHEP **02** (2010)], Draper, Kozaczuk, Yu [Phys. Rev. D **98** (2018)]

π decays into EW gauge bosons → clean signals

Quintuplet decay

Decay into EW gauge bosons

$$\mathcal{L}_{\mathrm{WZW}} \supset -\frac{g^2 N}{16\sqrt{2}\pi^2 f_d} \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}[\Pi_5 W_{\mu\nu} W_{\rho\sigma}]$$

Decay Width $\Gamma_\pi \sim \left(\frac{\alpha_W}{4\pi}\right)^2 \frac{m_\pi^3}{f_d^2}$ Lifetime $\tau_\pi = \frac{1}{\Gamma_\pi}$

Range of weak interaction $r_W \sim \frac{1}{m_W}$ $r_W \ll c\tau_\pi$

two-body π affected by SE before its decay

Schroedinger Equations

Cirelli, Fornengo, Strumia [Nucl. Phys. B **753** (2007)]

$$\left[-\frac{1}{m_\chi}\nabla^2 + V_\chi(r) - \frac{p^2}{m_\chi}\right]\Psi_\chi(r) = 0 \qquad \left[-\frac{1}{m_\pi}\nabla^2 + V_\pi(r) - \frac{p^2}{m_\pi}\right]\Psi_\pi(r) = 0$$

$$V_\chi(r) \equiv \begin{pmatrix} 0 & -\sqrt{2}B \\ -\sqrt{2}B & -A+2\Delta \end{pmatrix} \qquad V_\pi(r) \equiv \begin{pmatrix} 0 & -3\sqrt{2}B & 0 \\ -3\sqrt{2}B & -A+2\Delta & -2B \\ 0 & -2B & -4A+8\Delta \end{pmatrix}$$

$$A \equiv \frac{\alpha}{r} + \frac{\alpha_W c_W^2}{r} e^{-m_Z r} \qquad B \equiv \frac{\alpha_W}{r} e^{-m_W r} \qquad \Delta \equiv 166\ \mathrm{MeV}$$

Boundary conditions   Plain wave outside the potential

$$\frac{\psi_\chi^{i\prime}(r)}{\psi_\chi^i(r)} = i\sqrt{p^2 - m_\chi V_\chi(\infty)_{ii}} \qquad \frac{\psi_\pi^{i\prime}(r)}{\psi_\pi^i(r)} = i\sqrt{p^2 - m_\pi V_\pi(\infty)_{ii}}$$

Derivative Coupling

$$\mathcal{L} \supset \frac{f_d^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger] \supset \partial_\mu \chi \partial^\mu \chi \pi\pi, \chi\partial_\mu \chi \partial^\mu \pi\pi, \chi\chi\partial_\mu \pi \partial^\mu \pi$$

terms proportional to 3-momentum is included. $\mathcal{M}_{\mathrm{w/o}} \propto \lambda \mathbf{p}^2$

SE factor for ℓ-wave scattering (prop. to p^ℓ) $S_\ell \propto \left| \frac{\partial^\ell \psi(r)}{\partial r^\ell} \Big|_{r=0} \right|^2$ Cassel [J. Phys. G **37**(2010)]

momentum is replaced by derivatives of the wave func.