pseudo-Nambu-Goldstone-boson as a Dark Matter Candidate

A Model with Three Complex Scalars under $\mathbb{Z}_3$ Symmetry

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Work in Progress

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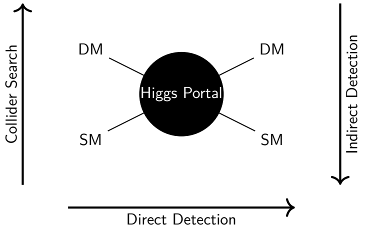
What are pNGBs?

The main features are:

- Arise when a symmetry is *spontaneously* $+$ *softly* broken.
- Produces Derivative Interaction i.e., *velocity suppressed*.
- Direct detection *naturally* suppressed (Cancellation Mechanism).

Higgs Portal for search!



- A way to connect the unknown[1].
- Higgs bosons as mediators.

The mixing of the higgs and scalar:

$$\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = \mathcal{O} \begin{pmatrix} 2v^2\,\lambda_\Phi & v_s v \lambda_{\Phi S} \\ v_s v \lambda_{\Phi S} & v_s^2 \lambda_S/2 \end{pmatrix} \mathcal{O}^T \tag{1}$$

where, the higgs basis and mass eigenbasis are related as:

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \mathcal{O} \begin{pmatrix} h \\ s \end{pmatrix}, \qquad \mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2}$$

The *mixing angle* is:

$$\tan 2\theta = \frac{2 v_s v \lambda_{\Phi S}}{v^2\,\lambda_\Phi - v_s^2 \lambda_S/2} \tag{3}$$

[1] (Brian Patt and Frank Wilczek. *Higgs-Field Portal into Hidden Sectors*. May 2006. arXiv: hep-ph/0605188)

Cancellation Mechanism

Interested part of Lagrangian[2]:

$$\mathcal{L} \supset -\chi^2 \left(\frac{m_1^2}{v_s} s_\theta \; h_1 + \frac{m_2^2}{v_s} c_\theta \; h_2 \right) - \sum_f \left(h_1 c_\theta - h_2 s_\theta \right) \frac{m_f}{v_s} \bar{f} f \qquad (4)$$

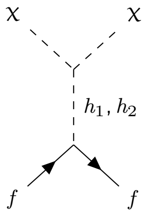


Figure: DM-matter interaction

Tree-level direct detection amplitude is:

$$\mathcal{A}_{dd} \propto \left(\frac{m_1^2}{t - m_1^2} - \frac{m_2^2}{t - m_2^2} \right) c_\theta \, s_\theta \qquad (5)$$

$$\simeq \frac{t \, (m_1^2 - m_2^2)}{m_1^2 m_2^2} c_\theta \, s_\theta \simeq 0 \qquad (6)$$

Reason: Momentum transfer is negligible i.e., $t \to 0$

[2] (Christian Gross, Oleg Lebedev, and Takashi Toma. "Cancellation Mechanism for Dark-Matter–Nucleon Interaction". In: *Physical Review Letters* 119.19 [Nov. 2017]. DOI: 10.1103/physrevlett.119.191801)

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Ingredients

- 2 Standard Model (SM) singlet *complex scalars*: S_1 and S_2
- Invariant under a gauged symmetry

$$U(1)_V: \qquad S_1 \to e^{i\theta_V(x)} S_1, \qquad S_2 \to e^{i\theta_V(x)} S_2 \tag{7}$$

- Impose a softly-broken global symmetry

$$U(1)_A: \qquad S_1 \to e^{i\theta_A} S_1, \qquad S_2 \to e^{-i\theta_A} S_2 \tag{8}$$

- Introduce a discrete $\mathbb{Z}_2$ *exchange symmetry*

$$S_1 \leftrightarrow S_2 \tag{9}$$

Scalar Potential

The scalar potential is[3]:

$$\mathcal{V}^{AHT}(S_1, S_2, \Phi) = \mu_S^2\left(|S_1|^2 + |S_2|^2\right) + \frac{\lambda_S}{2}\left(|S_1|^4 + |S_2|^4\right) + \lambda_S'\left(|S_1|^2|S_2|^2\right) - \mu_\Phi^2|\Phi|^2 + \lambda_\Phi|\Phi|^4 + \underbrace{\lambda_{\Phi S}|\Phi|^2\left(|S_1|^2 + |S_2|^2\right)}_{\text{Higgs portal}} - \underbrace{m_{12}^2\left(S_1^* S_2 + \text{h.c.}\right)}_{U(1)_A \text{ soft breaking}} \quad (10)$$

Vaccum expectation values (VEVs):

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad \langle S_1\rangle_0 = \langle S_2\rangle_0 = \frac{v_s}{2} \quad (11)$$

[3] Tomohiro Abe, Yu Hamada, and Koji Tsumura. *A Model of Pseudo-Nambu-Goldstone Dark Matter with Two Complex Scalars*. 2024.

Potential in Higgs basis

Introducing the quantum fluctuation:

$$\begin{pmatrix}\Sigma_1\\ \Sigma_2\end{pmatrix} \to R_2 \cdot \left\{ \frac{v_s}{2}\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}S_1\\S_2\end{pmatrix} \right\} = \frac{v_s}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}\Sigma_1\\ \Sigma_2\end{pmatrix}, \qquad R_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \tag{12}$$

Therefore, the potential in (10) becomes:

$$\begin{aligned}\mathcal{V}^{AHT}(\Sigma_1, \Sigma_2, \Phi) = \; & m_{12}^2|\Sigma_2|^2 + \lambda_{\Phi S}\left\{\left(|\Sigma_1|^2 + |\Sigma_2|^2 - \frac{{v_s}^2}{2}\right)\left(|\Phi|^2 - \frac{v^2}{2}\right)\right\} + \frac{\lambda_\Phi}{2}\left(|\Phi|^2 - \frac{v^2}{2}\right)^2 \\ & + \frac{\lambda_S}{4}\left\{\left(|\Sigma_1|^2 + |\Sigma_2|^2 - \frac{{v_s}^2}{2}\right)^2 + (\Sigma_1^*\Sigma_2 + \Sigma_2^*\Sigma_1)^2\right\} \\ & + \frac{\lambda_S'}{4}\left\{\left(|\Sigma_1|^2 + |\Sigma_2|^2 - \frac{{v_s}^2}{2}\right)^2 - (\Sigma_1^*\Sigma_2 + \Sigma_2^*\Sigma_1)^2\right\}\end{aligned} \tag{13}$$

It is clear that:

$$\mathbb{Z}_2: \qquad \Phi \to \Phi, \qquad \Sigma_1 \to \Sigma_1, \qquad \Sigma_2 \to -\Sigma_2 \tag{14}$$

$$(CP)_S: \qquad \Phi \to \Phi, \qquad \Sigma_1 \to \Sigma_1^*, \qquad \Sigma_2 \to \Sigma_2^* \tag{15}$$

Mass Spectrum

- Linear representation: $\Sigma_1 = (v_s + s_1' + iz_1)/\sqrt{2}$ and $\Sigma_2 = (s_2' + iz_2)/\sqrt{2}$
- $(CP, \mathbb{Z}_2)$ charges for $\begin{pmatrix} h & s_1' & z_1 & s_2' & z_2 \end{pmatrix}$:

$$(h_1, s_1')_{2\times 2} : (+,+)_{2\times 2}, \quad z_1 : (-,+), \quad s_2' : (+,-), \quad z_2 : (-,-)$$

- Seperated even sector:

$$M^2_{\text{even}} = \begin{pmatrix} 2v^2\lambda_\Phi & v_s v \lambda_{\Phi S} \\ v_s v \lambda_{\Phi S} & v_s^2(\lambda_S + \lambda_S')/2 \end{pmatrix}$$

- A Third Higgs: $m^2_{s_2'} = 2m^2_{12} + v_s^2(\lambda_S - \lambda_S')/2$
- 1 would-be NGB: $m^2_{z_1} = 0$
- 1 *real* pNGB: $m^2_{\text{DM}} = m^2_{z_2} = 2m^2_{12}$
- Stabilized by $\mathbb{Z}_2$

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Ingredients

- 3 SM singlet complex scalars: S_1, S_2 and S_3
- Symmetry configuration is same as earlier.

$$U(1)_V: \quad S_1 \to e^{i\theta_V(x)} S_1, \quad S_2 \to e^{i\theta_V(x)} S_2, \quad S_3 \to e^{i\theta_V(x)} S_3$$
$$U(1)_A: \quad S_1 \to e^{i\theta_A^1} S_1, \quad S_2 \to e^{i\theta_A^2} S_2, \quad S_3 \to e^{i\theta_A^3} S_3$$

- Introduce a discrete $S(3)$ *permutative exchange symmetry*

$$S_1 \leftrightarrow S_2 \qquad S_2 \leftrightarrow S_3 \qquad S_3 \leftrightarrow S_1 \tag{16}$$

Scalar Potential

The scalar potential of our model:

$$\begin{aligned}\mathcal{V}(S_1, S_2, S_3, \Phi) = {} & \mu_S^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right) + \frac{\lambda_S}{2}\left(|S_1|^4 + |S_2|^4 + |S_3|^4\right) \\ & + \lambda_S'\left(|S_1|^2|S_2|^2 + |S_2|^2|S_3|^2 + |S_3|^2|S_1|^2\right) - \mu_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4 \\ & + \underbrace{\lambda_{\Phi S}|\Phi|^2\left(|S_1|^2 + |S_2|^2 + |S_3|^2\right)}_{\text{Higgs portal}} \\ & \underbrace{- \frac{m_{12}^2}{3}\left(S_1^* S_2 + S_2^* S_3 + S_3^* S_1 + \text{h.c.}\right)}_{U(1)_A \text{ soft breaking}} \end{aligned} \tag{17}$$

VEVs:

$$\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad \langle S_1 \rangle_0 = \langle S_2 \rangle_0 = \langle S_3 \rangle_0 = \frac{v_s}{\sqrt{6}} \tag{18}$$

Potential in Higgs basis

Introducing the quantum fluctuation:

$$\begin{pmatrix}\Sigma_1\\ \Sigma_2\\ \Sigma_3\end{pmatrix} \to R_3 \cdot \left\{ \frac{v_s}{\sqrt{6}}\begin{pmatrix}1\\1\\1\end{pmatrix} + \begin{pmatrix}S_1\\S_2\\S_3\end{pmatrix} \right\} = \frac{v_s}{\sqrt{2}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}\Sigma_1\\ \Sigma_2\\ \Sigma_3\end{pmatrix} \tag{19}$$

Therefore, the potential in (17) becomes:

$$R_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \qquad R_3 = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{pmatrix}, \qquad \omega = e^{i2\pi/3} \tag{20}$$

You can actually generalize this as:

$$R_n = \frac{1}{\sqrt{n}}\begin{pmatrix}1 & 1 & 1 & \cdots & 1\\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1}\\ 1 & \omega^2 & \omega^4 & \cdots & \vdots\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & \omega^{n-1} & \cdots & \cdots & \omega^{(n-1)(n-1)}\end{pmatrix}, \qquad \omega = e^{i2\pi/n} \tag{21}$$

Potential in Higgs Basis (contd.)

Therefore, the potential in (17) becomes:

$$\mathcal{V}(\Sigma_1,\Sigma_2,\Sigma_3,\Phi) = m_{12}^2\left(|\Sigma_2|^2+|\Sigma_3|^2\right) + \frac{\lambda_\Phi}{2}\left(|\Phi|^2-\frac{v^2}{2}\right)^2 + \frac{\lambda_S}{6}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)^2 + 2\,|\Sigma_1^*\Sigma_2+\Sigma_2^*\Sigma_3+\Sigma_3^*\Sigma_1|^2\right\} + \frac{\lambda_S'}{3}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)^2 - |\Sigma_1^*\Sigma_2+\Sigma_2^*\Sigma_3+\Sigma_3^*\Sigma_1|^2\right\} + \lambda_{\Phi S}\left\{\left(|\Sigma_1|^2+|\Sigma_2|^2+|\Sigma_3|^2-\frac{3v_s^2}{2}\right)\left(|\Phi|^2-\frac{v^2}{2}\right)\right\} \tag{22}$$

It is clear that:

$$\mathbb{Z}_3: \quad \Phi\to\Phi, \quad \Sigma_1\to\Sigma_1, \quad \Sigma_2\to\omega\Sigma_2, \quad \Sigma_3\to\omega^2\Sigma_3 \tag{23}$$

$$(CP)_S: \quad \Phi\to\Phi, \quad \Sigma_1\to\Sigma_1^*, \quad \Sigma_2\to\Sigma_2^*, \quad \Sigma_3\to\Sigma_3^* \tag{24}$$

Mass Spectrum

- Linear representation: $\Sigma_1 = (v_s + s_1' + iz)/\sqrt{2}$
- $(CP, \mathbb{Z}_3)$ charges for $\begin{pmatrix} h & s_1' & z & \Sigma_\omega & a_\omega \end{pmatrix}$:

$$(h_1, s_1')_{2\times 2} : (+, 1)_{2\times 2}, \quad z : (-, 1), \quad \Sigma_\omega : (+, \omega), \quad a_\omega : (-, \omega)$$

- Seperated even sector:

$$M^2_{\text{even}} = \begin{pmatrix} v^2\,\lambda_\Phi & v_s v \lambda_{\Phi S} \\ v_s v \lambda_{\Phi S} & v_s^2(\lambda_S + 2\lambda_S')/3 \end{pmatrix}$$

- A Third Higgs: $m^2_\Sigma = m^2_{12} + v_s^2(\lambda_S - \lambda_S')/3$
- 1 would-be NGB: $m^2_z = 0$
- 1 *complex* pNGB: $m^2_{\text{DM}} = m^2_a = m^2_{12}$
- Stabilized by $\mathbb{Z}_3$

Lagrangian

The Lagrangian is given as:

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{SM(\text{w/o Higgs potential})} \\ & + |D_\mu S_1|^2 + |D_\mu S_2|^2 + |D_\mu S_3|^2 + |D_\mu \Phi|^2 \\ & - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} - \underbrace{\frac{\sin\epsilon}{2} V^{\mu\nu} Y_{\mu\nu}}_{\text{gauge kinetic mixing}} \\ & - \mathcal{V}(S_1, S_2, S_3, \Phi) \end{aligned} \tag{25}$$

- Field strength tensor
  - $Y^{\mu\nu}$: $U(1)_Y$ hypercharge gauge field Y_μ
  - $V^{\mu\nu}$: $U(1)_V$ dark gauge field V_μ
- $\sin\epsilon$ is the mixing parameter of the gauge kinetic mixing between V_μ and Y_μ.

Important interactions

the covariant derivatives:

$$D_\mu S_j = (\partial_\mu - i g_V V_\mu) S_j \qquad D_\mu \Phi = \left(\partial_\mu - i \frac{g}{2} W_\mu^a \sigma^a - i \frac{g_Y}{2} Y_\mu\right)\Phi \tag{26}$$

Interested Lagrangian:

$$\mathcal{L} \supset \kappa_1 \left(\Sigma_\omega \overleftrightarrow{\partial}_\mu a_\omega^* + a_\omega \overleftrightarrow{\partial}_\mu \Sigma_\omega^*\right) Z'^\mu - \kappa_2 \underbrace{\left(\Sigma_\omega^3 + \Sigma_\omega^{*3} - \Sigma_\omega a_\omega^2 - \Sigma_\omega^* a_\omega^{*2}\right)}_{\text{New cubic interactions}} - \kappa_3 |\Sigma_\omega|^2 h_2 \tag{27}$$

where,

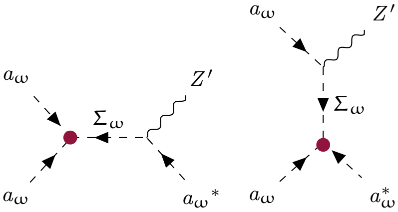
$$\kappa_1 \propto g_V, \qquad \kappa_2 \propto \frac{m_\Sigma^2 - m_a^2}{v_s}, \qquad \kappa_3 \propto \frac{m_2^2 + 2(m_\Sigma^2 - m_a^2)}{v_s} \cos\theta \tag{28}$$

which allows:

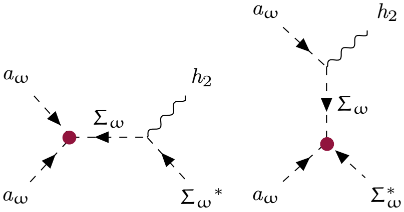
$$a_\omega a_\omega \to \Sigma_\omega \to a_\omega^* Z' \qquad a_\omega a_\omega \to \Sigma_\omega \to \Sigma_\omega^* h_2 \tag{29}$$

Feynman Diagrams

Interesting channels in our model:



(a) semi-annihilation channel



(b) semi-annihilation like channel

Parameters of our model

Dependent parameters:

$$\lambda_\Phi = \frac{m_1^2 c_\theta^2 + m_2^2 s_\theta^2}{v^2} \tag{30}$$

$$\lambda_{\Phi S} = \frac{(m_1^2 - m_2^2) s_\theta c_\theta}{v_s v} \tag{31}$$

$$\lambda_S = \frac{m_1^2 s_\theta^2 + m_2^2 c_\theta^2}{v_s^2} + \frac{2(m_\Sigma^2 - m_a^2)}{v_s^2} \tag{32}$$

$$\lambda_S' = \frac{m_1^2 s_\theta^2 + m_2^2 c_\theta^2}{v_s^2} - \frac{m_\Sigma^2 - m_a^2}{v_s^2} \tag{33}$$

Available free parameters:

$$m_1 (= 125\,\text{GeV}), \quad v (= 246\,\text{GeV}), \quad v_s, \quad m_2, \quad m_{\text{DM}}, \quad m_Z', \quad \sin\theta, \quad \sin\epsilon \tag{34}$$

Relic abundace

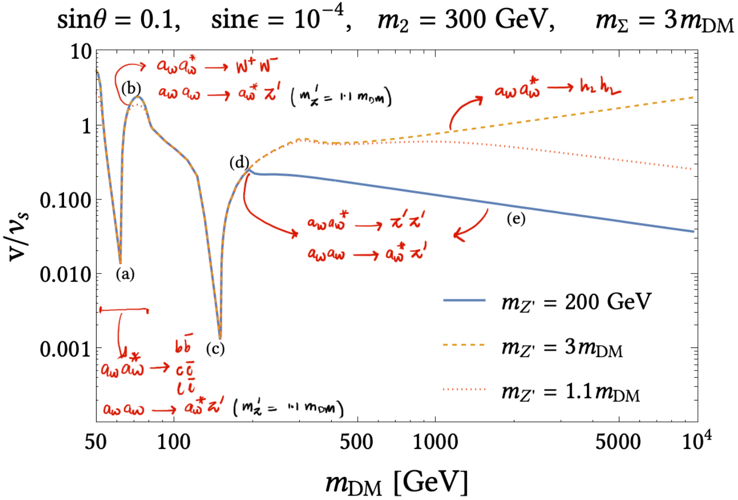


Figure: The cruves represent $\Omega h^2 \simeq 0.12 \pm 0.001$ by varying $m_{Z'}$ mass at $m_\Sigma = 3m_{\rm DM}$

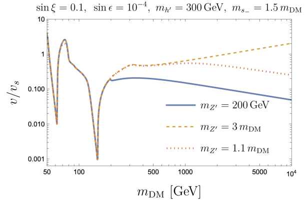
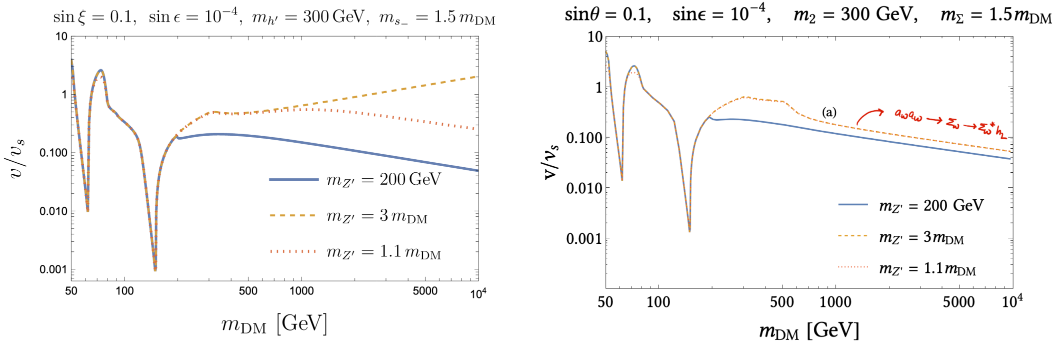


Figure: $\mathbb{Z}_2$ model reference

Relic abundance comparision



(a) AHT ($\mathbb{Z}_2$)   (b) This model ($\mathbb{Z}_3$)

Figure: $m_{s\_}$, $m_{h'}$ in (a) resembles m_Σ, m_2 in (b)

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Summary

- Comparison between the $\mathbb{Z}_2$ and $\mathbb{Z}_3$ model.

Model	Gauge	Global (w/o soft-breaking)	# of pNGB	DM Stability
AHT	$U(1)_V$	$U(1) \times U(1)$	1 real	$\mathbb{Z}_2$
This model	$U(1)_V$	$U(1) \times U(1)^2$	1 complex	$\mathbb{Z}_3$

- Availability of *semi-annihilation channels*.

Future directions

- Numerical calculation of *Direct Detection* cancellation.
- Signal search for semi-annihilation (*Boosted DM*)[4]

[4] (Mayumi Aoki and Takashi Toma. "Simultaneous Detection of Boosted Dark Matter and Neutrinos from the Semi-Annihilation at DUNE". In: *Journal of Cosmology and Astroparticle Physics* 2024.02 [2024], p. 033)