Black String in the Standard Model

Yu Hamada[1], Yuta Hamada[2,3], **Hayate Kimura**[3]

[1]DESY, [2]KEK, [3]SOKENDAI

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Short summary

Our work is ...

constructing black string solutions in the SM numerically.

Its existence is predicted by the swampland conjecture.

There is a no-go theorem prohibiting the existence of black strings in 4d theory ...

$\rightarrow$ by considering Casimir energy in energy-momentum tensor, the no-go theorem can be avoided.

The black strings are intrinsically "quantum" object.

$*$ based on arXiv:2501.05678 [hep-th]

Motivation

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- Many charged objects should be contained in QG theory in order to be coupled with the gravity in the consistent way.
- However, we do not know the full QG theory. The possible and reasonable argument is that we impose this conjecture on effective theories.
- This kind of argument predicts $\mathbb{Z}_2$ charged string-like objects in SM.

The horizon supported by Casimir energy

How is a black string horizon supported ?

It is known that only spherical topology is allowed as horizon topology in 4D theory at least classically.

Then, how we can construct a black string solution as vacuum solution . . . ?

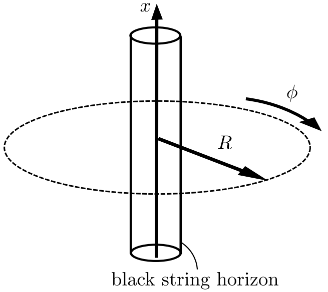
Actually, this theorem is valid under dominant energy condition.

The quantum collection for vacuum, Casimir energy, can violate the dominant energy condition !

Casimir energy

Casimir energy associated the S_1 surrounding the string is below.

$$V_{\text{Casimir}} = -\sum_{\text{particle}} (-1)^{2s_p} n_p \frac{m_p^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\theta_p)}{(2\pi R m_p n)^2} K_2(2\pi R m_p n)$$



θ_p : corresponds to the boundary condition of the particle when going around the string.

Fermions have periodic boundary condition $(\theta_p = 0)$ instead of anti-periodic, because this BS is $\mathbb{Z}_2$ charged. (corresponding flipping the sign of the fermion fields)

What we have to do

$$Rm_p \ll 1 \;\Rightarrow\; V_{\text{Casimir}} \sim (-1)^{2s_p}\cos(2\pi\theta_p)\frac{1}{(Rm_p)^4}.$$

$$Rm_p \gg 1 \;\Rightarrow\; K_2(Rm_p) \text{ exponentially dump} \;\Rightarrow\; V_{\text{Casimir}} \text{ also dump.}$$

In our low energy case, only light particles are dominant $\longrightarrow$ graviton, photon, neutrino

assumption : neutrinos are Majorana / Normal hierarchy / lightest mass is 0 ($m_3 = 0.05$[eV])

What we have to do is . . .

Solving Einstein eq. with Casimir energy-momentum tensor with appropriate metric ansatz and boundarty conditions.

Metric ansatz and boundary conditions

Extremal ansatz

The simplest string-like metric ansatz is

$$ds^2 = A^2(z)(-dt^2 + dx^2) + \left(\frac{M_P}{m_3^2}\right)^2 dz^2 + R(z)^2 d\phi^2.$$

z : parametrize the radial direction (proper distance). R is now a function of z.

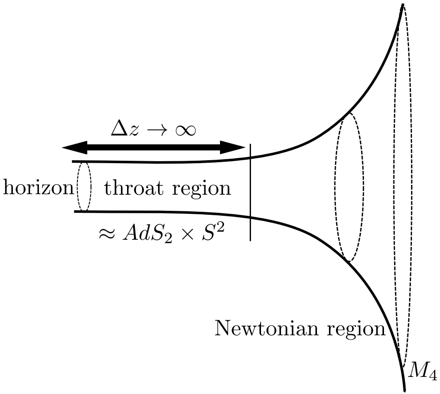
How about the boundary condition ?

Actually, the good candidate for such solution was suggested by Arkani-Hamed et. al. in 2007 (but they didn't construct it specifically).

They asserted that the extremal charged black string interpolate 4D vacua.

Reissner-Nordstrom BH interpolating 4D vacua (review)

Recall the extremal Reissner-Nordstrom BH : an exact solutoion of Einstein-Maxwell theory.



This BH interpolates between $AdS_2 \times S^2$ and M_4.

What is the $AdS_2 \times S^2$?

$\longrightarrow$ The vacuum solution of KK compactified

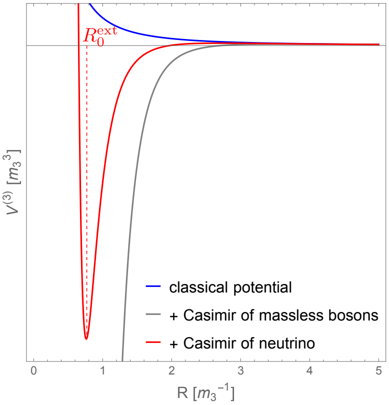
2-dim effective theroy :

$$g^{(4)}_{\mu\nu} \xrightarrow{\text{decomposed}} g^{(2)}_{ij},\ \chi \text{ (area of } S^2\text{, moduli)}$$

Its moduli is stabilized by electric flux.

The black string interpolating 4D vacua (review)

They expected the same thing is true for extremal $\mathbb{Z}_2$ charged black string.



We consider 1-dim compactified 4d SM

: the spacetime is $N_3 \times S^1$.

$$S = \int d^4x \sqrt{-g^{(4)}} \left(\frac{1}{2} M_P^2 \mathcal{R}^{(4)} - \Lambda^{(4)} - V_{\text{Casimir}}(R) \dots \right).$$

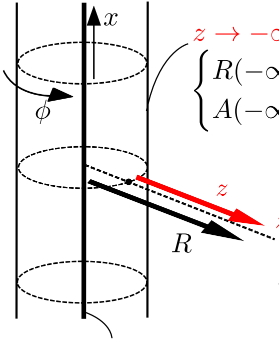
$g^{(4)}_{\mu\nu} \xrightarrow{\text{decomposed}} g^{(3)}_{ij}$, R (radius of S^1, moduli).

Actually the Casimir energy (periodic neutrinos) in the action works as the radion potential.

In this case, $N_3 = AdS_3$.

Boundary conditions for extremal BS

$$ds^2 = A^2(z)(-dt^2 + dx^2) + \left(\frac{M_P}{m_3^2}\right)^2 dz^2 + R^2(z)d\phi^2$$



$z \to -\infty$

$$\begin{cases} R(-\infty) = R_0^{\text{ext}} \\ A(-\infty) \sim e^{(M_P/m_3^2)z/l_{\text{AdS}}} \to 0 \end{cases}$$

$:$ horizon, $AdS_3 \times S^1$

$z \to \infty$

$$\begin{cases} R(\infty) \propto \dfrac{M_P}{m_3^2} z \to \infty \\ A(\infty) = \text{const.} \end{cases}$$

$R = 0 :$ conical singularity

$:$ infinite distance, M_4

Expected boundary conditions :

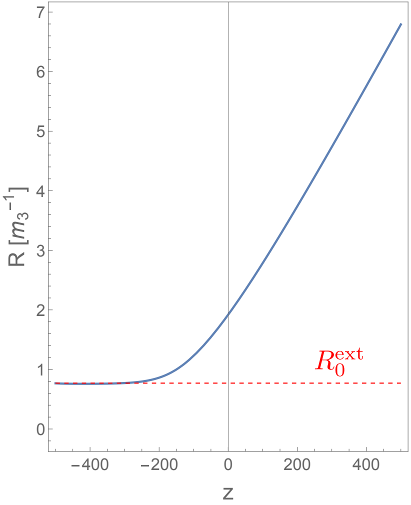
- $AdS_3 \times S^1$ near horizon
- M_4 at infinity.

infinite throat region of extremal RN-BH

$\Leftrightarrow$ the horizon is put at $z = -\infty$

Numerical Results

Extremal solution : R



$$ds^2 = A^2(z)(-dt^2 + dx^2) + \left(\frac{M_P}{m_3^2}\right)^2 dz^2 + R(z)^2 d\phi^2.$$

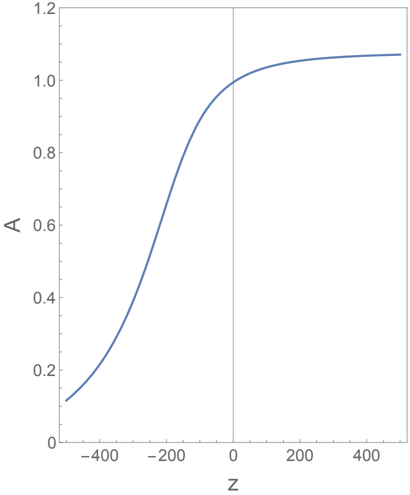
- $z \to -\infty$ (horizon) : $R \to R_0^{\rm ext}$ (S^1).
- $z \to \infty$ (infinity) : $R \sim (M_P/m_3^2)z/\sqrt{a}$ (flat).

Especially,

$$\frac{1}{\sqrt{a}} = \mathcal{O}(10^{-2}) \times \frac{m_3}{M_P}$$

This means the deficit angle $\theta = 2\pi(1 - 1/\sqrt{a})$ is $\theta \sim \mathcal{O}(1)$.

Extremal solution : A



$$ds^2 = A^2(z)(-dt^2 + dx^2) + \left(\frac{M_P}{m_3^2}\right)^2 dz^2 + R(z)^2 d\phi^2.$$

- $z \to -\infty$ (horizon)

  $: \ A \sim e^{(M_P/m_3^2)z/l_{\text{AdS}}} \ (AdS_3) \to 0.$

  where

$$l_{\text{AdS}} = 6.2 \times 10^{-3} \times \frac{M_P}{m_3^2}$$

- $z \to \infty$ (infinity) : $A \to$ const (flat).

Conclusion

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- The cobordism conjecture predicts a $\mathbb{Z}_2$ charged string-like object in 4D SM.
- We could actually construct this object as the “quantum” black string in SM.

Future direction

- checking the stability of the string
- cosmological construction ?
- $E_8 \times E_8$ heterotic SUGRA