DM relic abundance via multi-Higgs production in the early universe

Seishi Enomoto (Yokohama natl. Univ.
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Kohta Murase (Pennsylvania State Univ.)
Masato Yamanaka (Hosei Univ.)

[arXiv:XXXX.XXXXX] *...Coming soon!*

[Illustrated by *ChatGPT*]

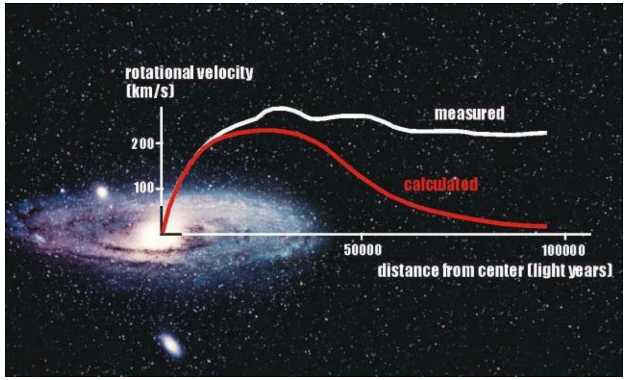
Outlook

1. Introduction
2. Application to DM relic abundance
    1. Model setup and formulation
    2. Numerical result via Boltzmann equation
3. Summary

1. Introduction

Evidences of dark matter

- Galaxy rotation curve
- Bullet cluster
- Cosmic microwave background



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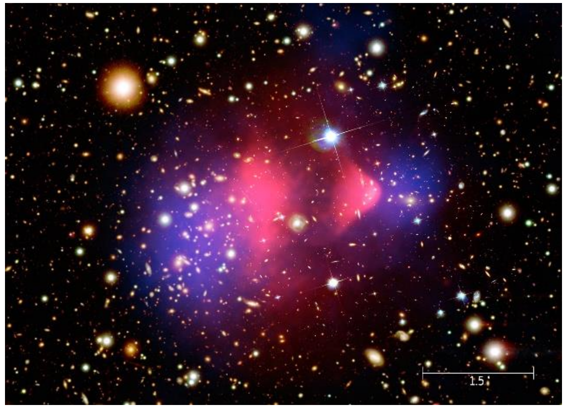
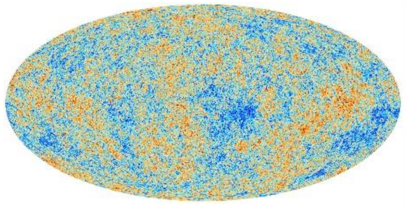
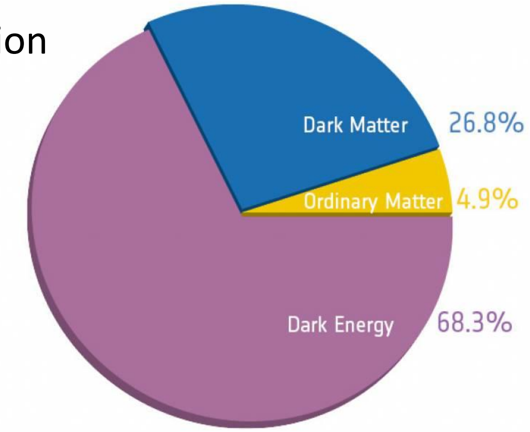


Mismatch of the mass distribution from X-ray and gravitational lensing

CMB observation



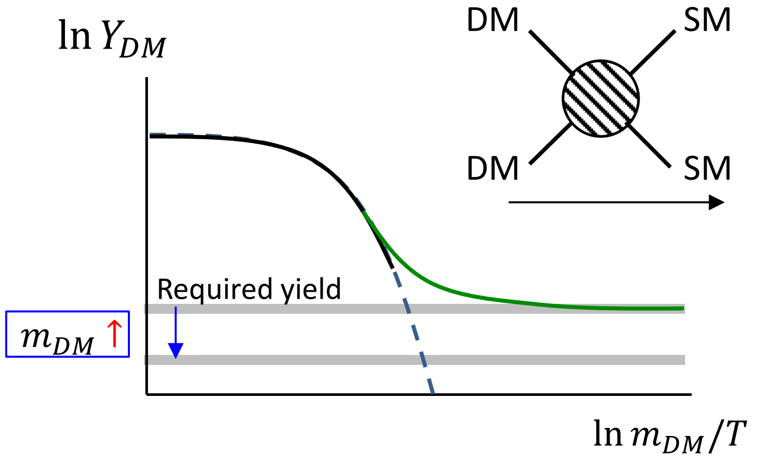


$$\Omega_{CDM} = 0.268$$

$$\Rightarrow \quad Y_{DM} \equiv \frac{n_{DM}}{s} = 4.36 \times 10^{-12} \cdot \frac{100\ \mathrm{GeV}}{m_{DM}}$$

$$m_{DM} \uparrow \quad \Rightarrow \quad Y_{DM} \downarrow$$

Thermal relic by freeze-out



$$\sigma \propto \frac{y^4}{m_{DM}^2}$$

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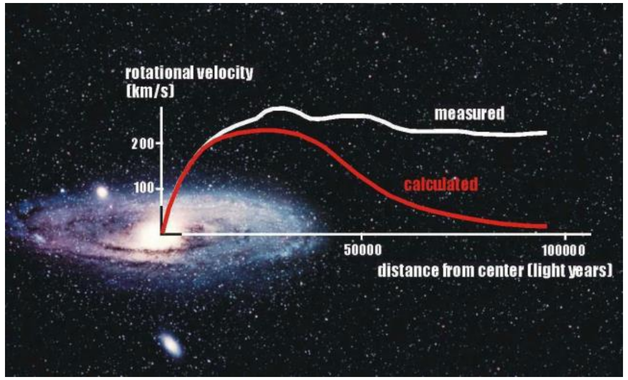

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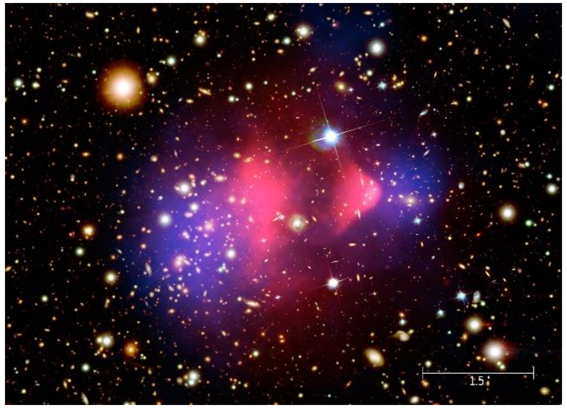

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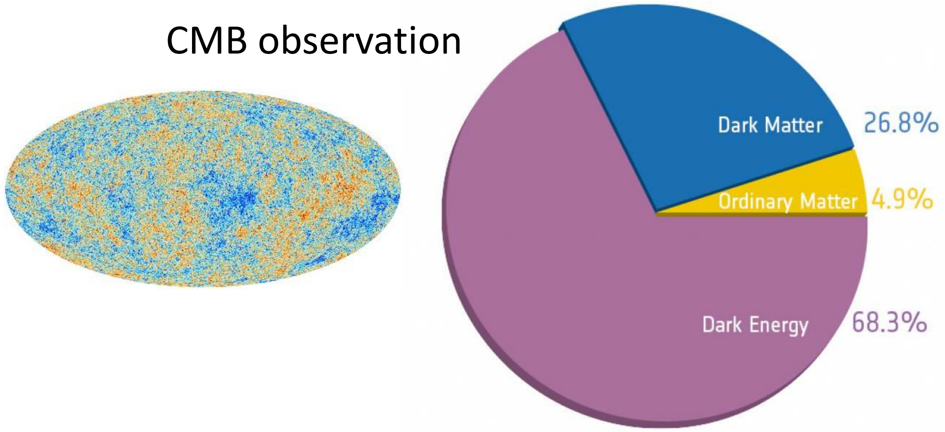

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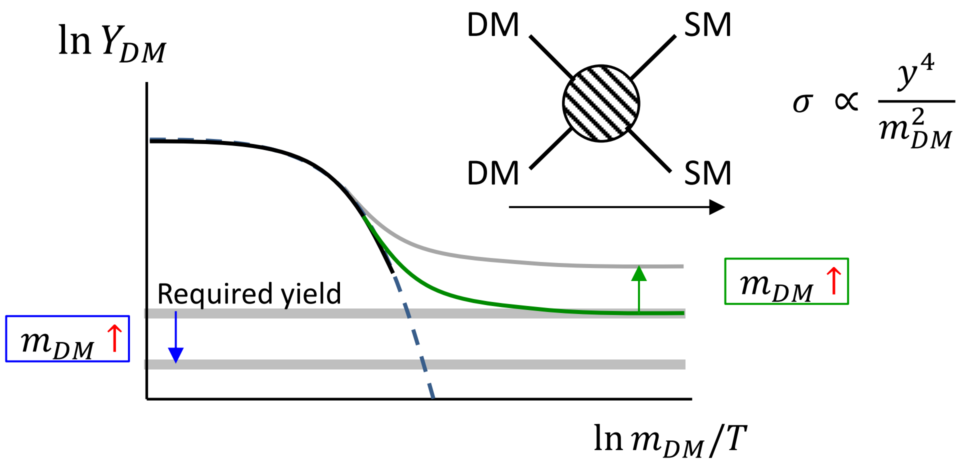

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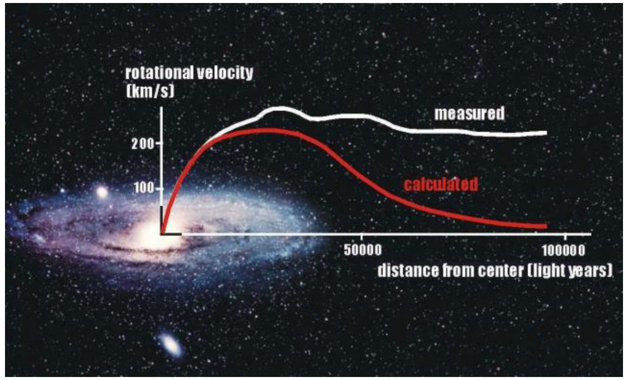

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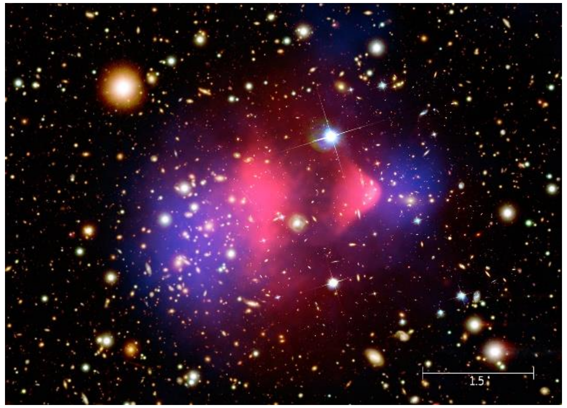

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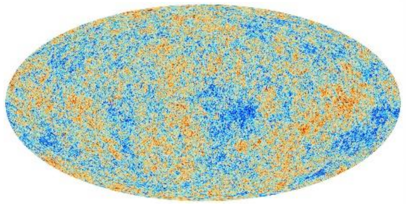

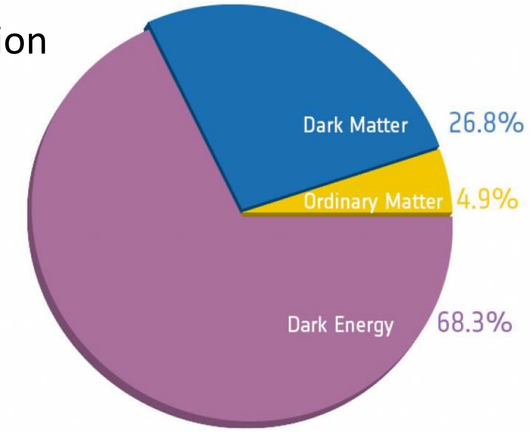

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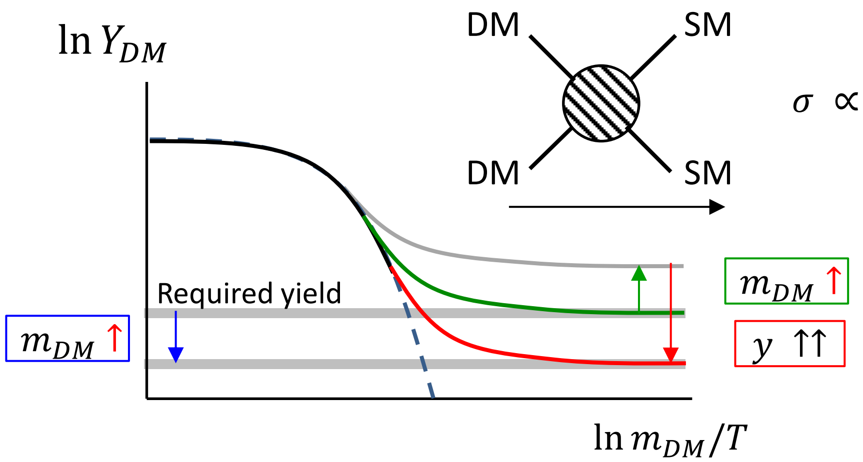

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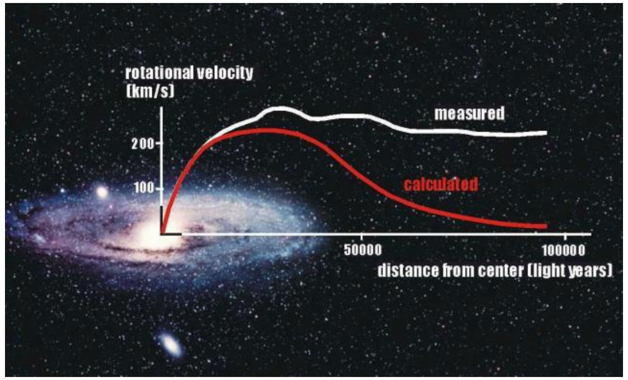

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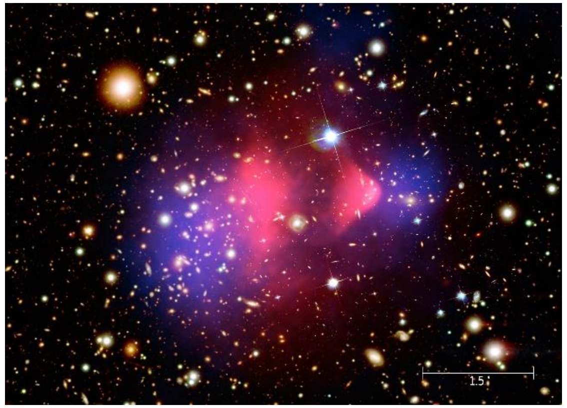

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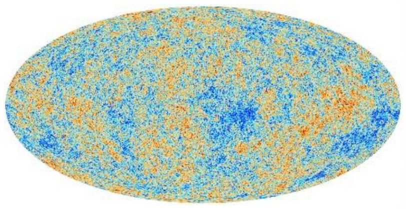

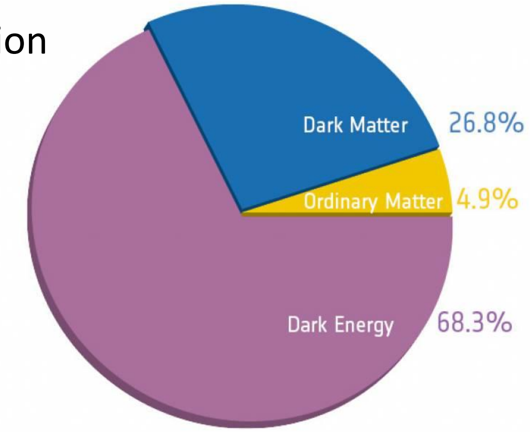

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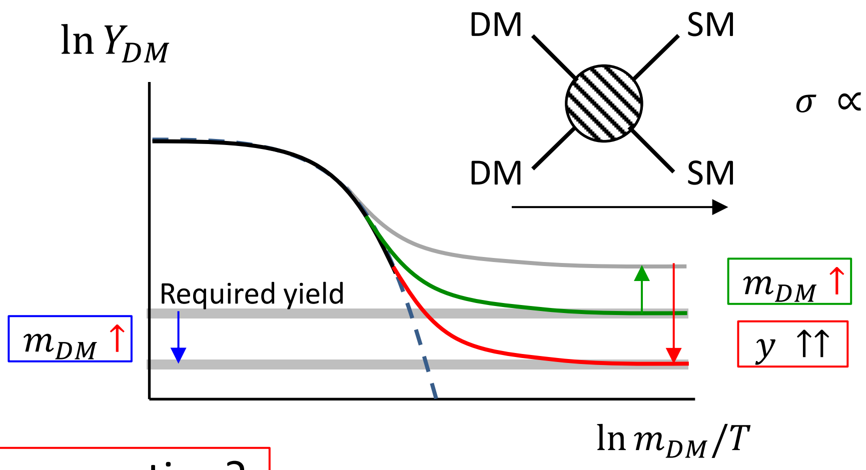


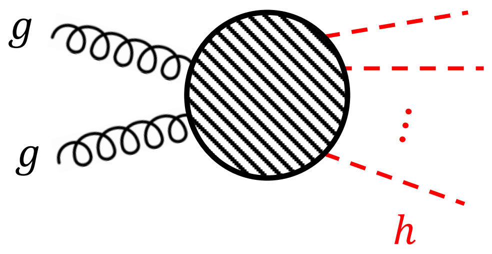
$$\sigma \propto \frac{y^4}{m_{DM}^2}$$

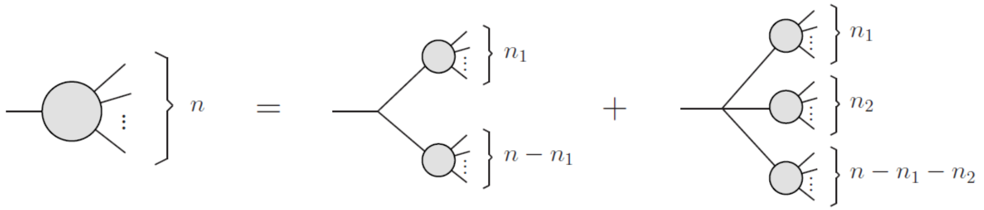
Are there other mechanism that can enhance the cross-section?

← Higgsplosion!

Higgsplosion [C. Degrande, V. V. Khoze, O. Mattelaer; PRD 94 (2016)]

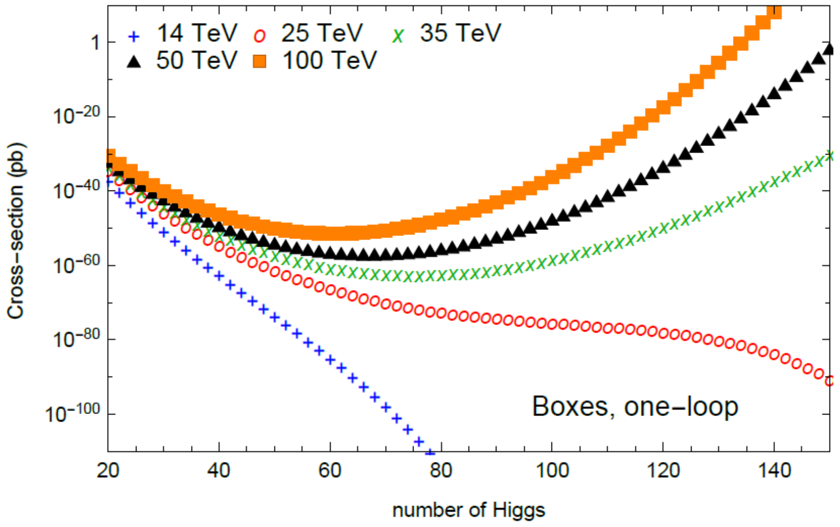
- a hypothesis that the explosive Higgs production is caused above a certain energy
- Essences:
  - bosonic statistical effect
  - (3- or 4-pt) self-interaction





$$
\begin{aligned}
i\mathcal{M}(1\to n) \;=\; & \sum_{n_1=1}^{n-1}\left(-i\sqrt{\frac{9}{2}\lambda m_\varphi^2}\right)\cdot\frac{n!}{n_1!(n-n_1)!}\times\frac{i}{s_{n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_1)\\
& \times\frac{i}{s_{n-n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n-n_1)\\
& +\sum_{n_1=1}^{n-2}\sum_{n_2=1}^{n-n_1-1}(-i\lambda)\cdot\frac{n!}{n_1!n_2!(n-n_1-n_2)!}\;\frac{i}{s_{n_1}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_1)\\
& \times\frac{i}{s_{n_2}-m_\varphi^2}\cdot i\mathcal{M}(1\to n_2)\;\;\frac{i}{s_{n-n_1-n_2}-m_\varphi^2}\cdot i\mathcal{M}(1\to n-n_1-n_2)
\end{aligned}
$$

$$\mathcal{M}(h^*\to nh) \;\sim\; \lambda^{n/2}\, n!\cdot f_n(s)$$



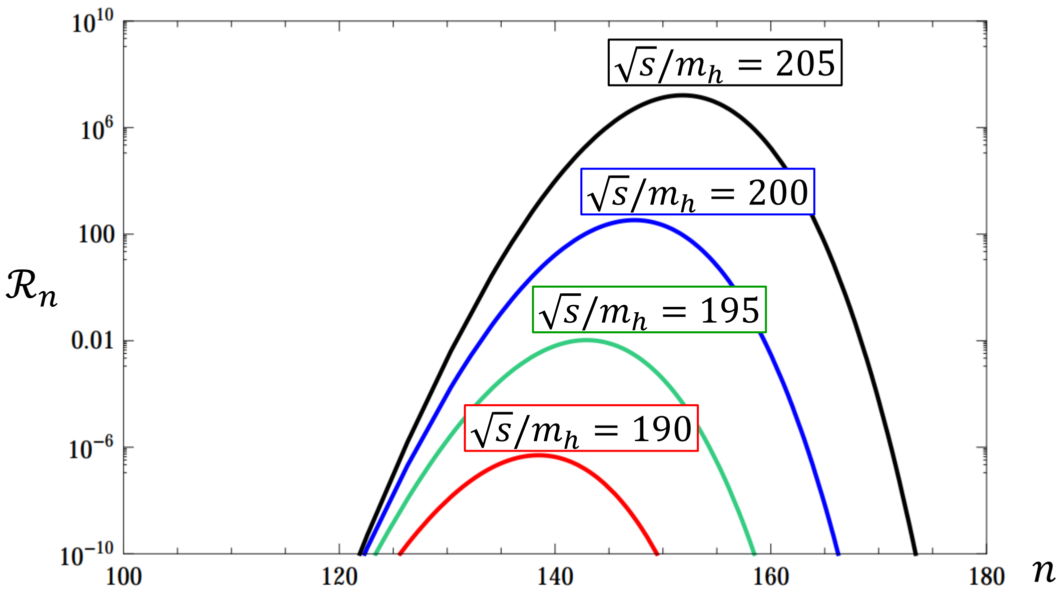
- Blow up around $\sigma(gg\to 60h)$
- $\sigma(gg\to h)\sim\sigma(gg\to 135h)$

"Decay rate" for $h^* \to nh$ near the n-particle mass threshold ($\sqrt{s} \sim n m_h$)

[V. V. Khoze and M. Spannowsky; NPB 926 (2018)]

$$\mathcal{R}_n \equiv \frac{\Gamma_{h^* \to nh}(s)}{m_n} = \exp\left[n\left(\ln\frac{\lambda n}{4e} + + \frac{3}{2}\ln\frac{e\epsilon}{3\pi} - \frac{25}{12}\epsilon + 3.02\sqrt{\lambda n}\right)\right]$$

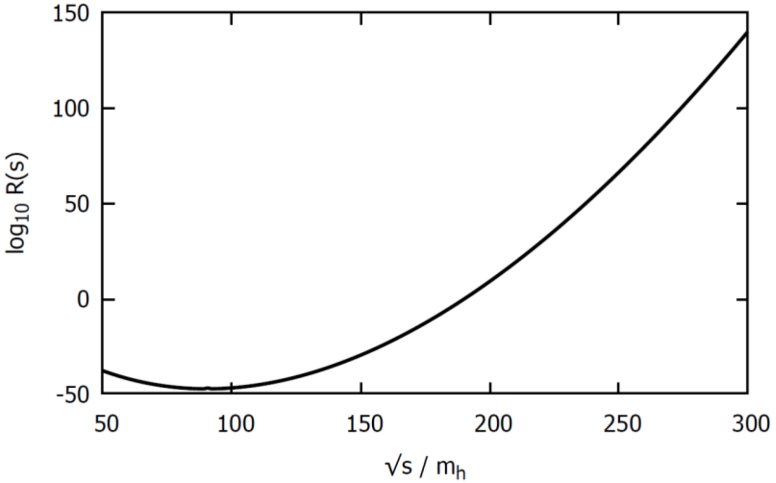
$$\left(\epsilon \equiv \frac{\sqrt{s} - n m_h}{n m_h} \ll 1\right)$$



Total rate

$$\mathcal{R}(s) \equiv \sum_n \mathcal{R}_n \cdot \theta(\sqrt{s} > n m_h)$$

Exp growing for high s



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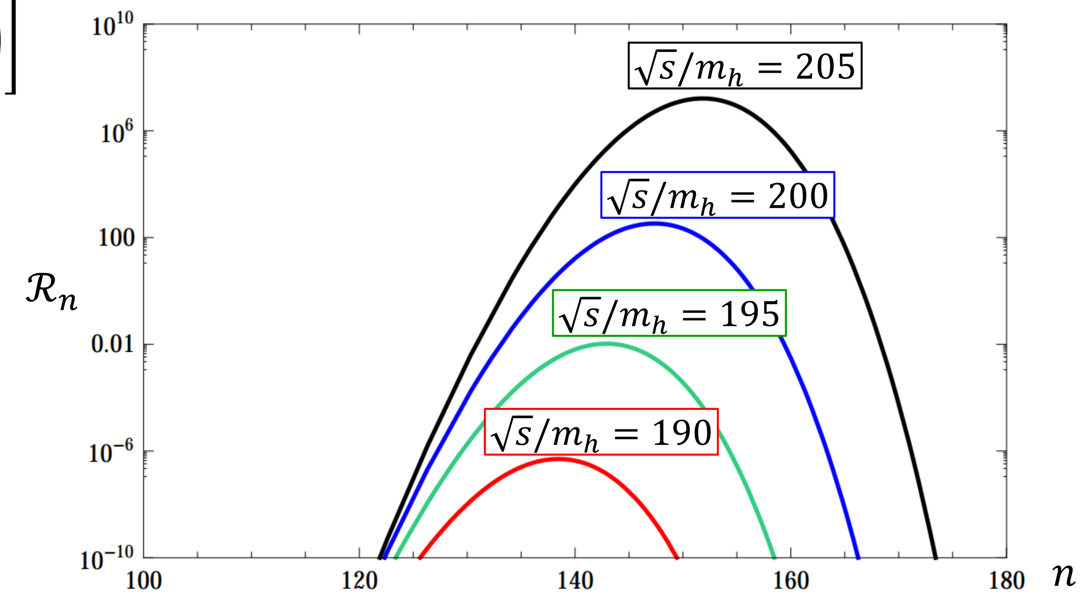

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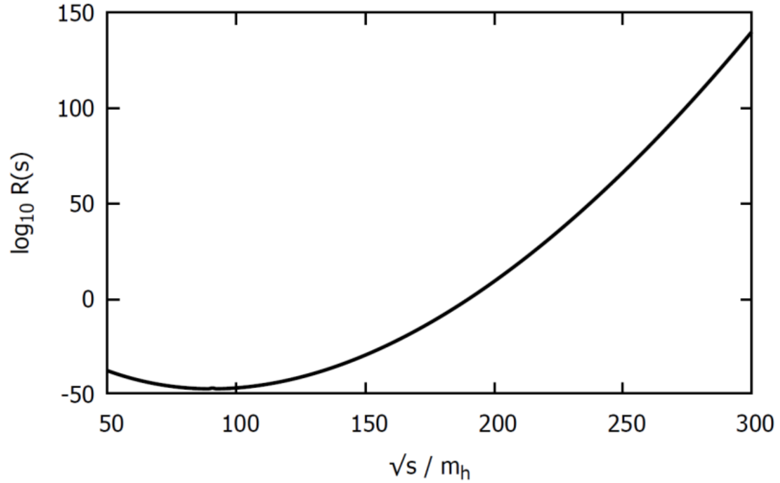


We consider the higgsplosion effect in the DM relic abundance!

2. Application to DM relic abundance

Model setup

Can we derive this naturally…?
→ Later

Lagrangian (ϕ: Higgs, χ_L, χ_R: DMs)

$$\mathcal{L} \supset \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda(\phi^2 - v^2)^2 + \bar{\chi}(\partial\!\!\!/ - m_\chi)\chi - \left(y_\chi \phi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

(y : complex Yukawa coupling)

$\Downarrow$ (SSB: $\phi = v + h$)

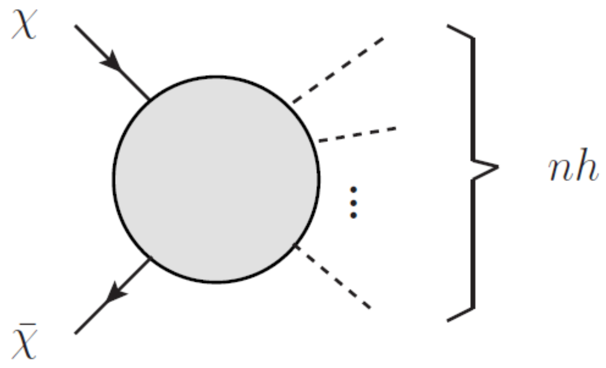
$$= \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \bar{\chi}(\partial\!\!\!/ - M_\chi)\chi - h\left(y_\chi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

($M_\chi \equiv m_\chi + y_\chi v$: complex)

$\Downarrow$ (Re-phasing: $\chi_L e^{i \arg M_\chi} \to \chi_L$)

$$= \frac{1}{2}(\partial h)^2 - \frac{1}{2}m_h^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \bar{\chi}(\partial\!\!\!/ - |M_\chi|)\chi - h\left(\tilde{y}_\chi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

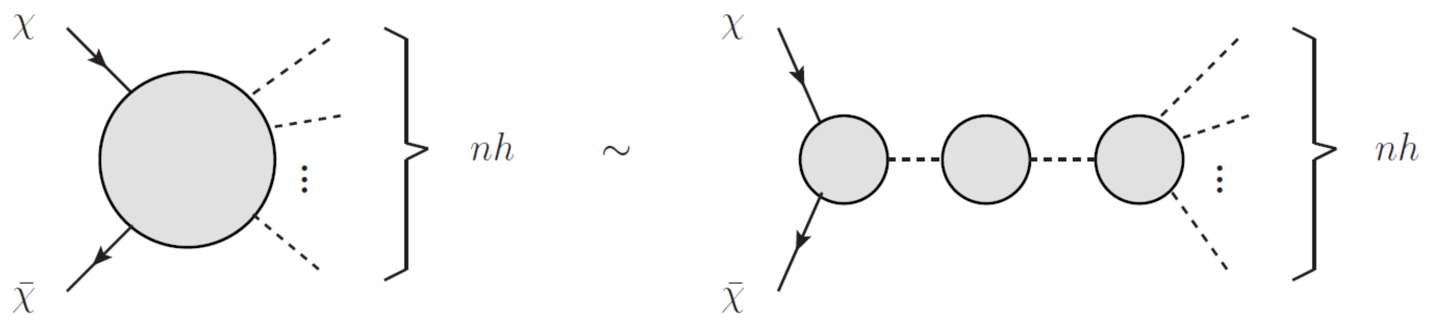
($\tilde{y}_\chi \equiv y_\chi e^{-i \arg M_\chi}$: complex)



Scattering amplitude

$$\mathcal{L} \supset -\tilde{y}_\chi h \overline{\chi_R} \chi_L$$

$$\theta_{\tilde{\chi}} \equiv \arg \tilde{y}_\chi$$



$$\sum_{\text{spins}} |\mathcal{M}(\chi\chi \to nh)|^2 \sim \sum_{\text{spins}} \left| \mathcal{M}(\chi\chi \to h) \frac{1}{s - M_h(s)^2 - iM_h(s)\Gamma_h(s)} \mathcal{M}(h \to nh) \right|^2$$

$$\sim 2|\tilde{y}_\chi|^2 (s - 4|M_\chi|^2 \cos^2\theta_{\tilde{y}}) \frac{1}{s^2 + m_h^2 \Gamma_h(s)^2} \underline{\underline{|\mathcal{M}(h \to nh)|^2}},$$

$$\sum_n \int d\Pi_n \times \ldots$$

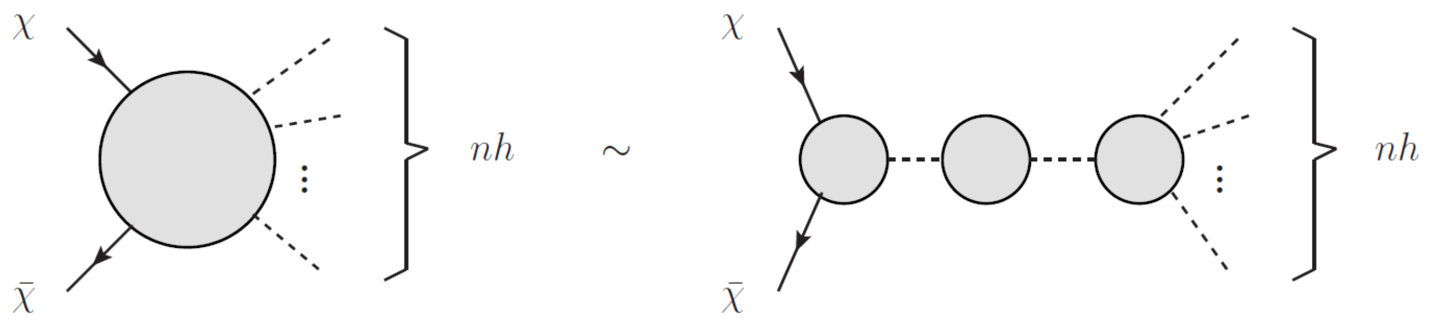
$$\sim \Gamma_h(s)$$

$\theta_{\tilde{\chi}} = 0$: p-wave

$\theta_{\tilde{\chi}} = \pm\pi/2$: s-wave

for the maximized cross section

Scattering amplitude



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$$\sim 2|\tilde{y}_\chi|^2 (s - 4|M_\chi|^2 \cos^2(\theta_{\tilde{y}}) \frac{1}{s^2 + m_h^2 \Gamma_h(s)^2} |\mathcal{M}(h \to nh)|^2 ,$$

$$\sum_n \int d\Pi_n \times \ldots$$

$\sim \Gamma_h(s)$

$\theta_{\tilde{\chi}} = 0$: p-wave

$\theta_{\tilde{\chi}} = \pm\pi/2$: s-wave

for the maximized cross section

Forms a "window"

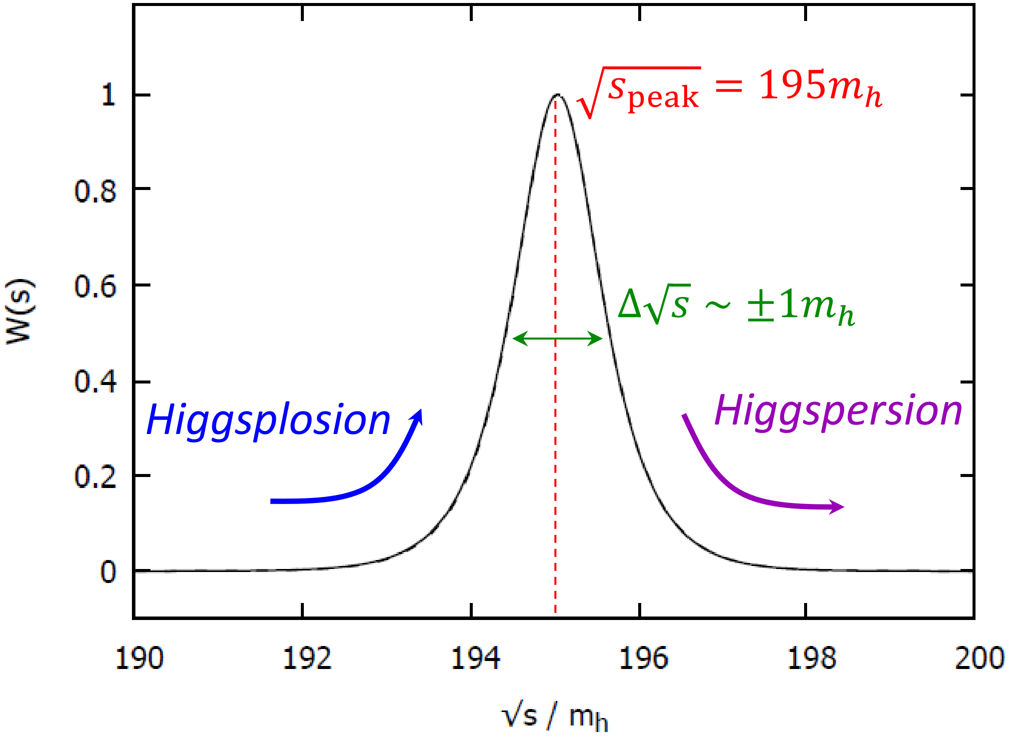
Cross section

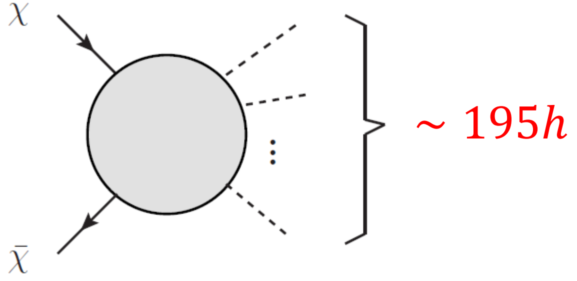
$$\sigma = \sum_n \sigma_{\chi\chi\to nh} = \frac{1}{\sqrt{s(s-4|M_\chi|^2)}} \cdot \frac{|\tilde{y}_\chi|^2}{4}\left(1 - \frac{4|M_\chi|^2\cos^2\theta_{\tilde{y}}}{s}\right) W(s)$$

"Window" function

$$W(s) = \frac{2sm_h^2\mathcal{R}(s)}{s^2 + m_h^4\mathcal{R}(s)^2}, \qquad 0 \le W(s) \le 1$$

$$\left(\mathcal{R}(s) \equiv \Gamma_h(s)/m_h \right)$$





$\sqrt{s} \sim 2M_\chi \sim 195 m_h$

$\therefore M_\chi \sim 100\, m_h$

for the maximized cross section

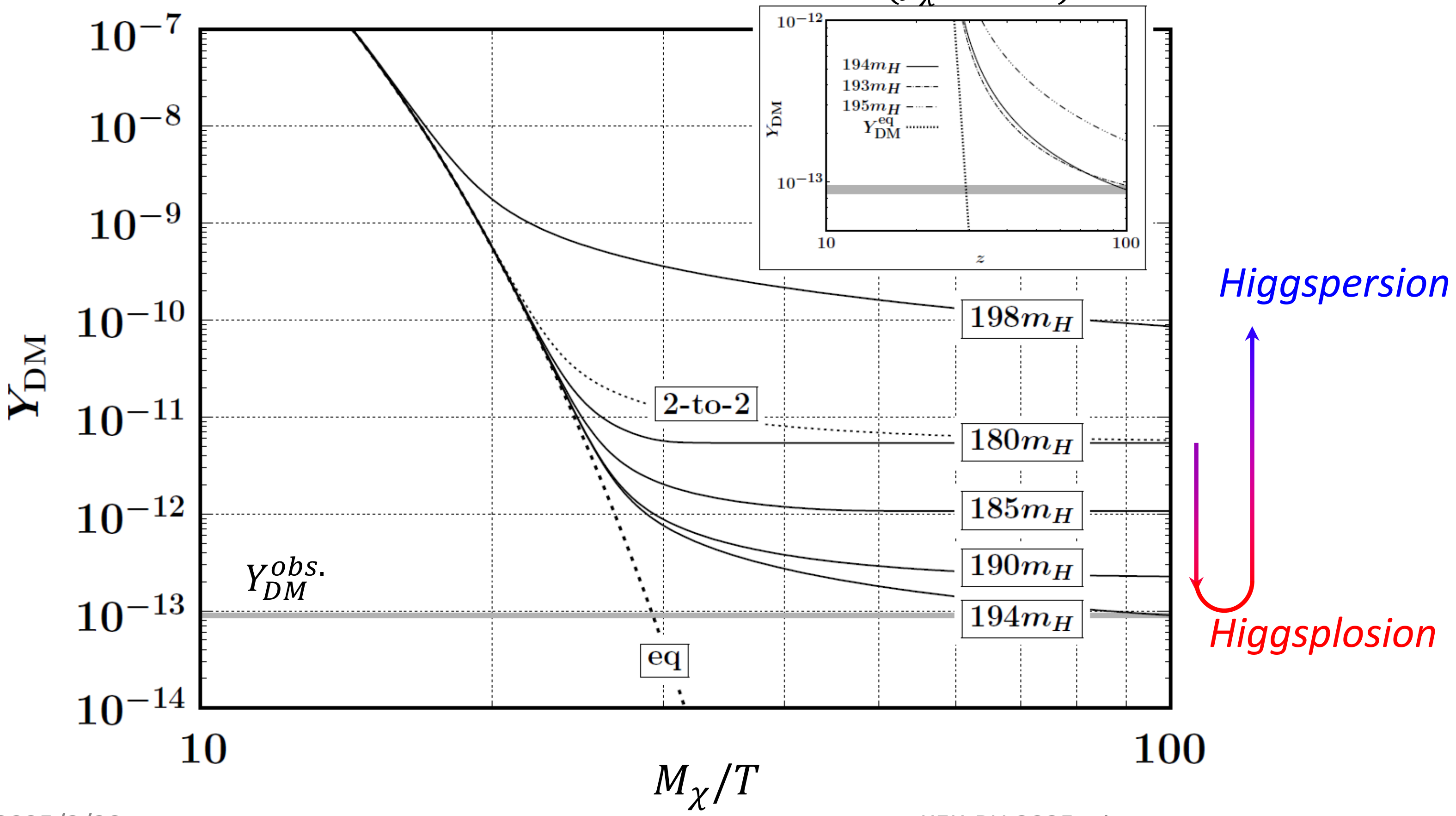
DM (thermal) relic abundance

$$\mathcal{L} \supset -\tilde{y}_\chi h \overline{\chi_R} \chi_L$$

- Boltzmann equation: $\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - (n_\chi^{eq})^2\right)$

- Numerical result

Evolution of DM abundance $(\tilde{y}_\chi = 1.3i)$



- Scale of freeze-out

$T \sim M_\chi/25$

$\sim 4m_h$ ($M_\chi \sim 100 m_h$)

~ 500 GeV $(m_h = 125$ GeV$)$

Failed (∵ before SSB)

$T \sim 200$ GeV $(m_h = 50$ GeV$)$

OK (during SSB)

$\Rightarrow M_\chi \sim 100 m_h \sim$ 5 TeV

A model to provide our setup

Simple model fails:

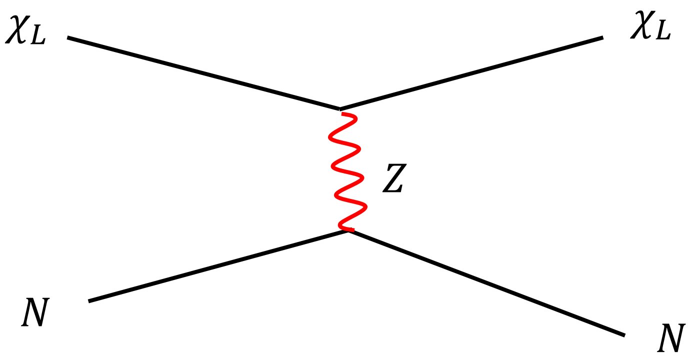
$$\mathcal{L} \supset \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda(\phi^2 - v^2)^2 + \bar{\chi}(\partial\!\!\!/ - m_\chi)\chi - \left(y_\chi \phi \cdot \overline{\chi_R}\chi_L + (h.c.)\right)$$

(ϕ: Higgs, χ_L, χ_R: DMs)

ϕ: $SU(2)$, χ_L: $SU(2)$

$$\Rightarrow \mathcal{L} \supset g\bar{\chi}_L Z\!\!\!/ \chi_L$$

Ruled out by the direct DM search



Singlet-doublet DM model (+ complex parameters) [T. Cohen, J. Kearney, A. Pierce, and D. Tucker-Smith; PRD85 (2012)]

$$\mathcal{L} \supset -\frac{1}{2}\overline{\Psi}(m_\psi P_L + m_\psi^* P_R)\Psi - m_D(\overline{f_R}f_L + \overline{f_L}f_R) - (\overline{\Psi}(y_L P_L + y_R P_R)H \cdot f + (h.c.))$$ (m_ψ: complex, others: real)

($\Psi = \Psi^c$: singlet fermion, $f_L = \begin{pmatrix} f_L^- \\ f_L^0 \end{pmatrix}$, $f_R = \begin{pmatrix} f_R'^- \\ f_R'^0 \end{pmatrix}$: $SU(2)_L$ doublet fermions)

$$\Rightarrow \mathcal{L} \supset -\frac{1}{2}(\overline{\Psi}\ \overline{f^0}\ \overline{f'^0})(\mathcal{M}P_L + \mathcal{M}^\dagger P_R)\begin{pmatrix} \Psi \\ f^0 \\ f'^0 \end{pmatrix}$$

($\mathcal{M} = \begin{pmatrix} m_\psi & m_L & m_R \\ m_L & 0 & m_D \\ m_R & m_D & 0 \end{pmatrix}$, $m_{L(R)} = \frac{1}{\sqrt{2}} y_{L(R)} v$)

$$= -\frac{1}{2}(\overline{\chi_1}\ \overline{\chi_2}\ \overline{\chi_3})\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$$

($m_1 \le m_2 \le m_3$)

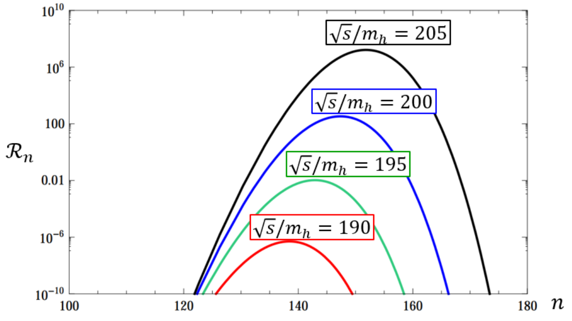
Case of $\mathrm{Re}\, m_\psi = 0$, $m_L = m_R$, and $m_D \gg |m_\psi|, m_L, m_R$

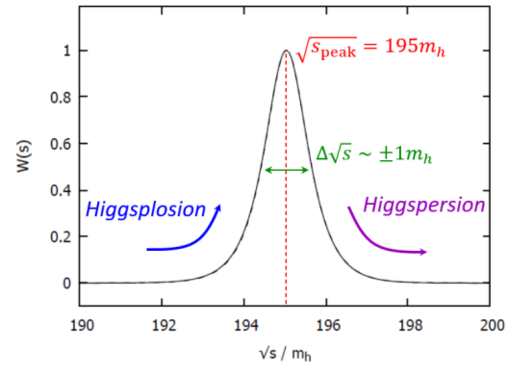
- $m_1 \ll m_2 \sim m_3$: χ_1 is LSP = DM
- Coefficients relating to the direct detection: $\mathcal{L} \supset c_h \cdot h\bar{\chi}_1\chi_1 + c_Z \cdot \bar{\chi}_1 \not{Z} \gamma^5 \chi_1$

$$c_h \sim \frac{y_L y_R v}{m_D}\frac{\mathrm{Re}\, m_\psi}{|m_\psi|} = 0, \qquad c_Z \sim \frac{g_2}{4\cos\theta_W}\frac{|y_L^2 - y_R^2|^2}{2m_D^2} = 0$$

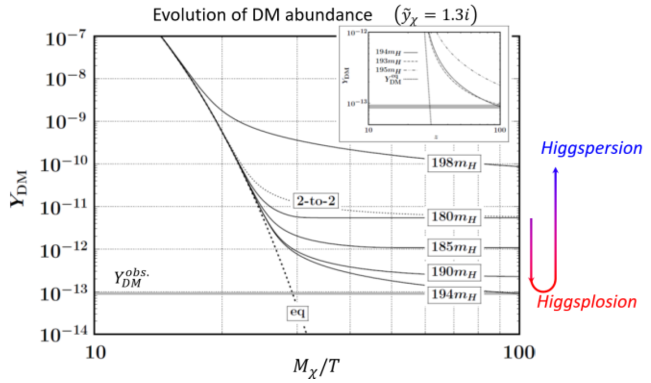
3. Summary

- Higgsplosion:
  - The explosive Higgs production at very high energy due to
    - Bosonic statistics
    - Self-interaction



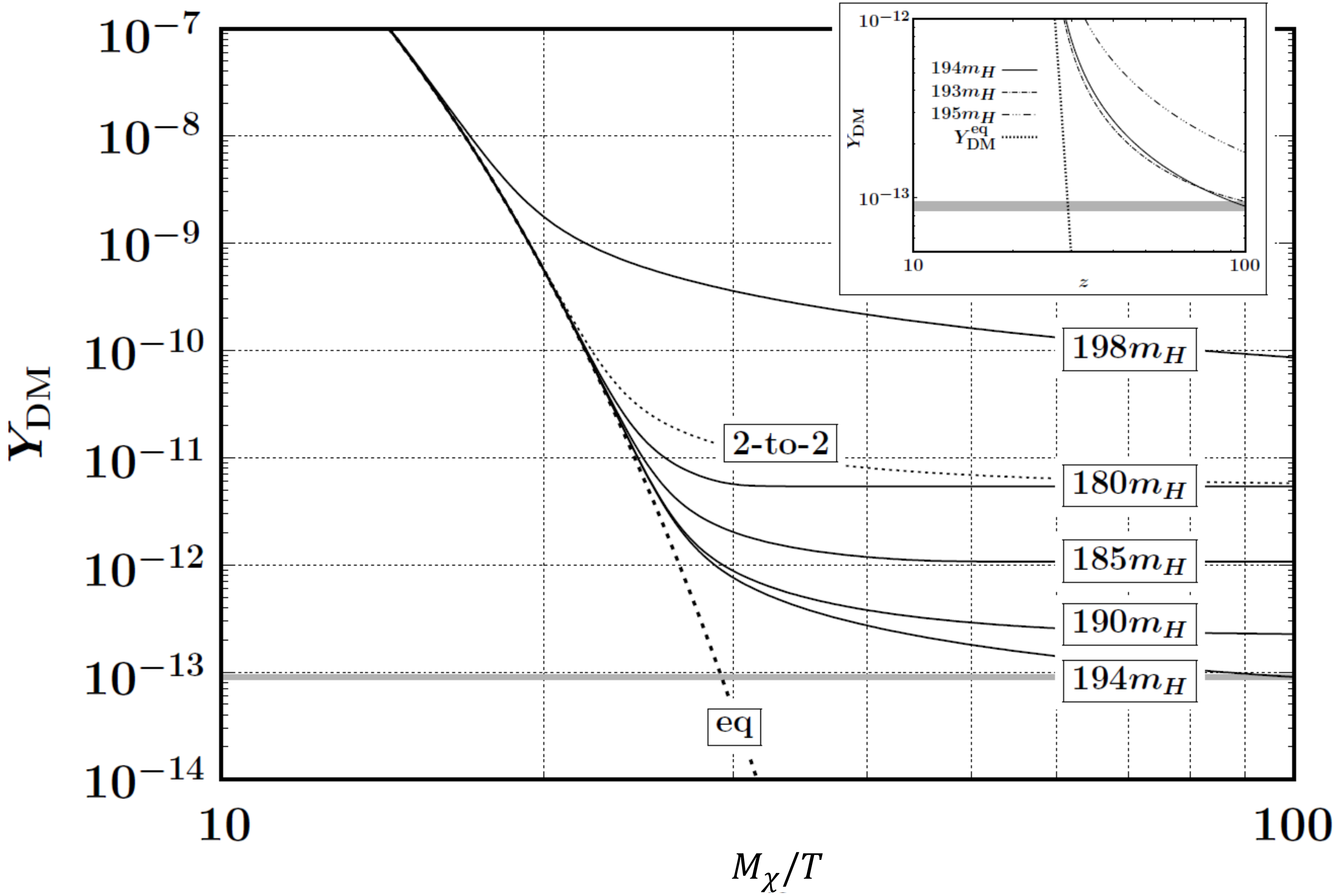


- Application to DM relic abundance
  - The decay rate including the higgsplosion forms a "window" in the cross section
  - Higgsplosion effect is maximized at $\sqrt{s} = 195m_h$
  - $M_{DM} \sim 5$ TeV higgs portal model is possible if $m_h \sim 50$ GeV at the freeze-out scale (during SSB, $T \sim 200$ GeV)
  - Singlet-doublet model is better to evade the direct detection

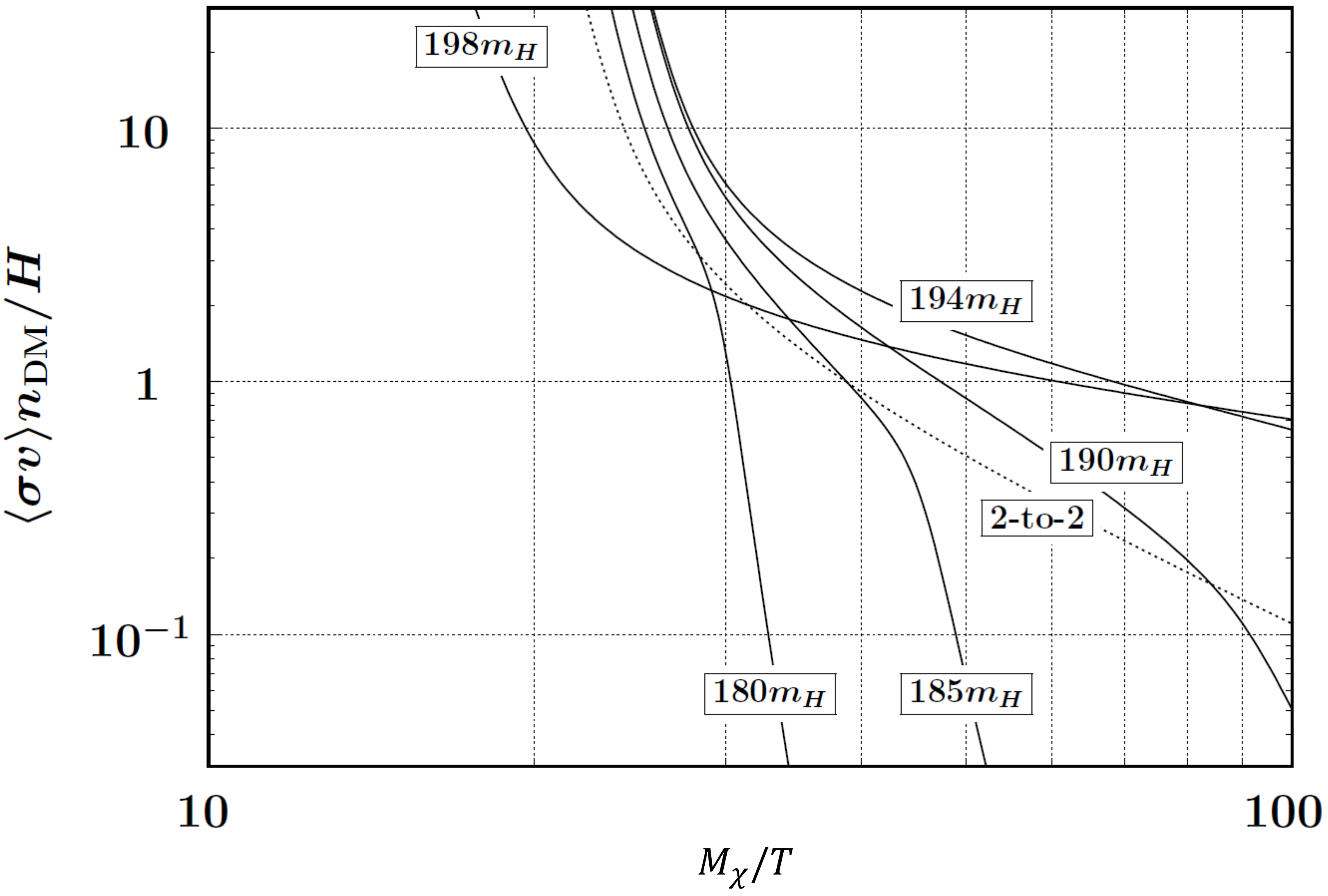


Backup

Evolution of relic abundance



Reaction rate v.s. Hubble



How do we evaluate analytically?

EOM for wave function