The time evolution of lepton numbers, Majorana (type) phases & the connection to CP violation for lepto-genesis

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(type I seesaw with two active neutrinos and two gauge-singlet heavy neutrinos)

Introduction: Majorana neutrinos and lepton numbers

Charged lepton flavor basis α=*e*, μ ***(two generation toy model)***

$$\begin{pmatrix}\nu_{L\alpha}\\ l_{L\alpha}\end{pmatrix}=\left\{\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix},\begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}\right\}$$

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu\nu_{L\alpha}W^- - \frac{1}{2}\overline{(\nu_{L\alpha})^c}m_{\nu\alpha\beta}\nu_{L\beta} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

$$m_{\nu\alpha\beta}=\begin{pmatrix}m_{\nu ee} & m_{\nu e\mu}\\ m_{\nu e\mu} & m_{\nu\mu\mu}\end{pmatrix}$$

From flavor basis to mass basis

$\nu_{L\alpha}=U_{\alpha i}\nu_{Li}$ $\quad i=1,2$ $\quad$ **U is two by two unitary matrix**

$$\mathrm{U}^T_{i\alpha}m_{\nu\alpha\beta}\,\mathrm{U}_{\beta j}=m_i\delta_{ij}$$

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu \mathrm{U}_{\alpha i}\nu_{Li}W^- - \frac{1}{2}\overline{(\nu_{Li})^c}\,m_i\nu_{Li} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

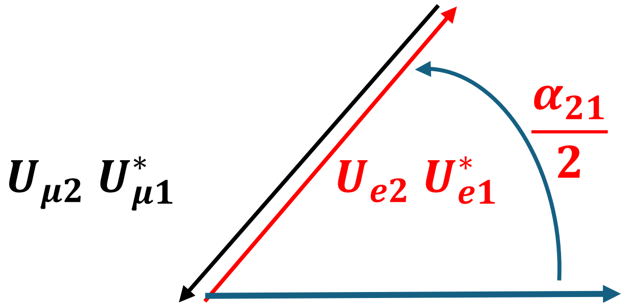
$l_\alpha \rightarrow e^{i\theta_\alpha}\, l_\alpha$:The rephasing is possible only for charged lepton field

$\mathrm{U}_{\alpha i}(\mathbf{3\ phases+1angle}) \rightarrow e^{i(-\theta_\alpha)}\,\mathrm{U}_{\alpha i} \rightarrow (\mathbf{1phase+1angle})$

PMNS matrix for two generation toy model

- 2 by 2 Unitary matrix can be parametrized with a mixing angle θ_{12} and a Majorana phase α_{21}.

- $U_{\alpha i} = \begin{pmatrix} c_{12} & s_{12} e^{i\frac{\alpha_{21}}{2}} \\ -s_{12} & c_{12} e^{i\frac{\alpha_{21}}{2}} \end{pmatrix}$, $s_{12} = \sin\theta_{12}$



$$U_{e2} U^*_{e1} + U_{\mu 2} U^*_{\mu 1} = 0 \longleftrightarrow c_{12} s_{12} e^{i\frac{\alpha_{21}}{2}} - c_{12} s_{12} e^{i\frac{\alpha_{21}}{2}} = 0$$

Lepton family numbers as a quantity sensitive to the lightest neutrino mass and the Majorana phase

- Lepton family numbers of Majorana neutrinos:

$$L_\alpha(x^0) = \int d^3x :\overline{\nu_{L\alpha}}\gamma^0\nu_{L\alpha}: \quad (\alpha=e,\mu)$$

$$= \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}\left(a_\alpha^\dagger(\boldsymbol{p},x^0)\, a_\alpha(\boldsymbol{p},x^0) - b_\alpha^\dagger(\boldsymbol{p},x^0)\, b_\alpha(\boldsymbol{p},x^0)\right)$$

$$0 = \int d^3x\, \nu_{L\alpha}(\mathbf{x},x^0)$$

$$\nu_{L\alpha}(x) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}\left(a_\alpha(\boldsymbol{p},x^0)\, u_L(\boldsymbol{p})e^{i\mathbf{p}\cdot\mathbf{x}} + b_\alpha^\dagger(\boldsymbol{p},x^0)\, v_L(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}\right)$$

$$\int' \rightarrow \mathbf{p}=0 \text{ is excluded from the momentum integration}$$

Time evolution of lepton family numbers and relativistic limit and non-relativistic limit

$$\langle \nu_e | L_e(x^0) | \nu_e \rangle = c_{12}^4\left(1-\frac{2m_1^2\sin^2(E_1 t)}{E_1^2}\right)+s_{12}^4\left(1-\frac{2m_1^2\sin^2(E_1 t)}{E_1^2}\right)$$
$$+ s_{12}^2 c_{12}^2\left\{\left(1+\frac{\boldsymbol{q}^2-m_1 m_2\cos(\alpha_{21})}{E_1 E_2}\right)\cos\{(E_1-E_2)t\}+\left(1-\frac{\boldsymbol{q}^2-m_1 m_2\cos(\alpha_{21})}{E_1 E_2}\right)\cos\{(E_1+E_2)t\}\right\}$$
$$|\boldsymbol{q}|\gg m_1, m_2 \rightarrow c_{12}^4+s_{12}^4+2s_{12}^2 c_{12}^2\cos\{(E_1-E_2)t\}$$

$$\langle \nu_e | L_\mu(x^0) | \nu_e \rangle = c_{12}^2 s_{12}^2\left\{\left(1-\frac{2m_1^2\sin^2(E_1 t)}{E_1^2}\right)+\left(1-\frac{2m_2^2\sin^2(E_2 t)}{E_2^2}\right)\right\}$$
$$- s_{12}^2 c_{12}^2\left\{\left(1+\frac{\boldsymbol{q}^2-m_1 m_2\cos(\alpha_{21})}{E_1 E_2}\right)\cos\{(E_1-E_2)t\}+\left(1-\frac{\boldsymbol{q}^2-m_1 m_2\cos(\alpha_{21})}{E_1 E_2}\right)\cos\{(E_1+E_2)t\}\right\}$$
$$|\boldsymbol{q}|\gg m_1, m_2 \rightarrow 2c_{12}^2 s_{12}^2\,(1-\cos\{(E_1-E_2)t\})$$

Relativistic limit:
Independent on Majorana phase, dependence of energy appears as the energy difference $E_1 - E_2$.

Time evolution of lepton family numbers and non-relativistic limit

$$\langle \nu_e | L_e(t) | \nu_e \rangle = c_{12}^4\left(1-\frac{2m_1^2\sin^2(E_1 t)}{E_1^2}\right)+s_{12}^4\left(1-\frac{2m_2^2\sin^2(E_2 t)}{E_2^2}\right)$$
$$+\ s_{12}^2 c_{12}^2\left\{\left(1+\frac{\boldsymbol{q}^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1-E_2)t\}+\left(1-\frac{\boldsymbol{q}^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1+E_2)t\}\right\}$$
$$0<|\boldsymbol{q}|\ll m_1,m_2 \ \rightarrow\ c_{12}^4(\cos 2E_1 t)+s_{12}^4(\cos 2E_2 t)+s_{12}^2c_{12}^2\{(1-\cos\alpha_{21})\cos(E_1-E_2)t+(1+\cos\alpha_{21})\cos(E_1+E_2)t\}$$

$$\langle \nu_e | L_\mu(x^0) | \nu_e \rangle = c_{12}^2 s_{12}^2\left\{\left(1-\frac{2m_1^2\sin^2(E_1 t)}{E_1^2}\right)+\left(1-\frac{2m_2^2\sin^2(E_2 t)}{E_2^2}\right)\right\}$$
$$-\ s_{12}^2 c_{12}^2\left\{\left(1+\frac{\boldsymbol{q}^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1-E_2)t\}+\left(1-\frac{\boldsymbol{q}^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1+E_2)t\}\right\}$$
$$0<|\boldsymbol{q}|\ll m_1,m_2 \ \rightarrow\ c_{12}^2s_{12}^2\{\cos 2E_1 t+\cos 2E_2 t-(1-\cos\alpha_{21})\cos(E_1-E_2)t-(1+\cos\alpha_{21})\cos(E_1+E_2)t\}$$

Non-relativistic limit:
dependent on Majorana phase, dependence of energy appears as the energy sum E_1+E_2 and, $2\ E_i$

Possibility for extracting Majorana phase and an absolute mass through $L_e \pm L_\mu$

$$\langle \nu_e | (L_e + L_\mu)(t) | \nu_e \rangle = c_{12}^4 \left(1 - \frac{2m_1^2 \sin^2(E_1 t)}{E_1^2}\right) + s_{12}^4 \left(1 - \frac{2m_2^2 \sin^2(E_2 t)}{E_2^2}\right) + c_{12}^2 s_{12}^2 \left\{\left(1 - \frac{2m_1^2 \sin^2(E_1 t)}{E_1^2}\right) + \left(1 - \frac{2m_2^2 \sin^2(E_2 t)}{E_2^2}\right)\right\}$$

$$\langle \nu_e | (\ddot{L}_e + \ddot{L}_\mu)(t=0) | \nu_e \rangle = -4\,(m_1^2 c_{12}^2 + m_2^2 s_{12}^2)$$

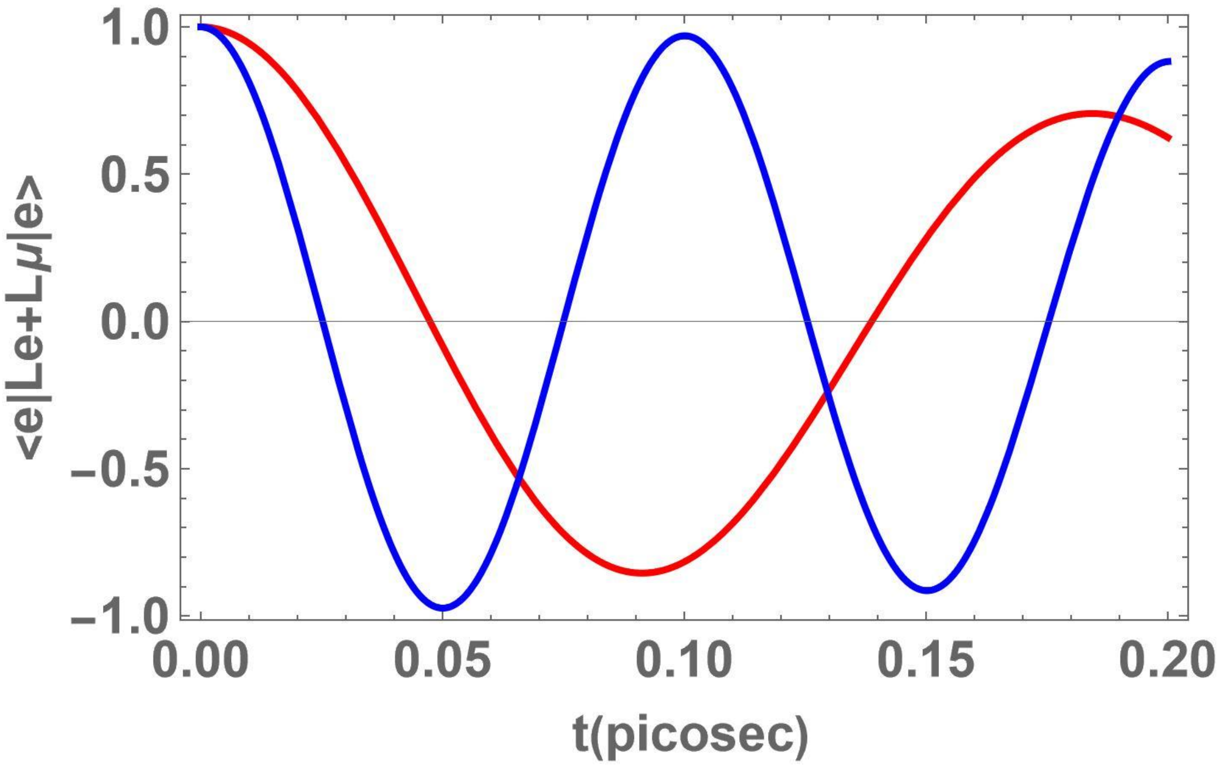
$$= -4\,\{m_1^2 + (m_2^2 - m_1^2)\, s_{12}^2\}$$

- From the total lepton number and its second time derivative at t=0, one can extract the lightest neutrino mass m_1^2 by knowing the mass squared difference $\Delta m^2{}_{21}$ and a mixing angle θ_{12}.

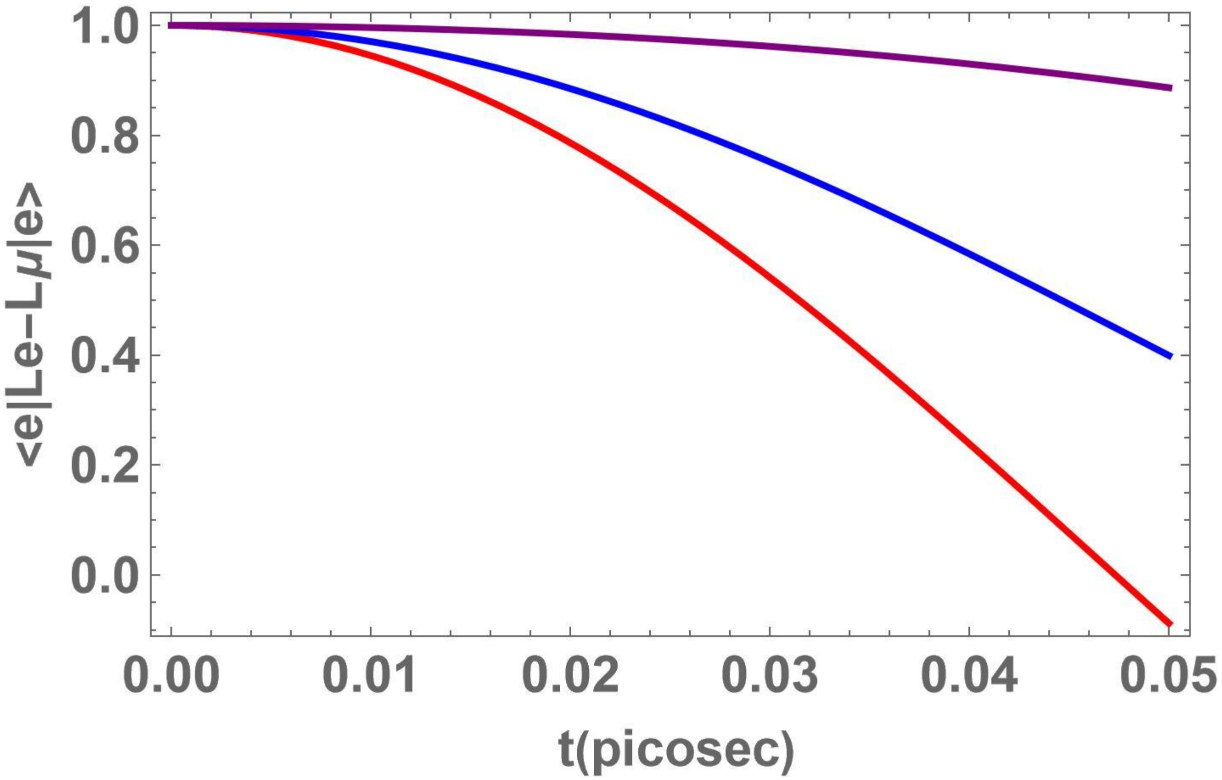
$$
\begin{aligned}
\langle \nu_e | L_e(t) - L_\mu(t) | \nu_e \rangle &= 2 s_{12}^2 c_{12}^2 \left\{ \left(1 + \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 - E_2)t\} + \left(1 - \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 + E_2)t\} \right\} \\
&+ c_{12}^2 \cos 2\theta_{12} \left(1 - \frac{2 m_1^2 \sin^2(E_1 t)}{E_1^2}\right) - s_{12}^2 \cos 2\theta_{12} \left(1 - \frac{2 m_2^2 \sin^2(E_2 t)}{E_2^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\langle \nu_e | \ddot{L}_e(t=0) - \ddot{L}_\mu(t=0) | \nu_e \rangle &= -4 |m_{\nu ee}|^2 \\
|m_{\nu ee}|^2 &= |m_1 c_{12}^2 + m_2 s_{12}^2 e^{-i\alpha_{21}}|^2 \\
&= m_1^2 c_{12}^4 + 2 m_1 m_2 \cos(\alpha_{21}) s_{12}^2 c_{12}^2 + m_2^2 s_{12}^4
\end{aligned}
$$

- From the electron number minus muon number and its second time derivative at t=0, one can extract the ee component of the effective Majorana mass matrix, $m_{\nu ee}$, then one may extract cosine of the Majorana phase α_{21}.



Red : m_1=0.01(eV) Blue : m_1=0.02(eV)



Red : $\alpha_{21}=0$ Blue : $\alpha_{21}=\frac{\pi}{2}$ Purple: $\alpha_{21}=\pi$

$$\frac{d^2}{dt^2}\langle\nu_e|L(t)|\nu_e\rangle\Big|_{t=0} = -4(m_1^2c_{12}^2 + m_2^2s_{12}^2)$$

$$\frac{d^2}{dt^2}\langle\nu_e|L_{e-\mu}(t)|\nu_e\rangle\Big|_{t=0} = -4|m_{ee}|^2$$

$\Delta m_{21}^2 = 7.42\times10^{-5}(eV^2)$

q=0.002(eV) $\sin\theta_{12} = 0.551$

$$|m_{ee}|^2 = |m_1c_{12}^2 + m_2s_{12}^2e^{-i\alpha_{21}}|^2 = m_1^2c_{12}^4 + m_2^2s_{12}^4 + 2m_1m_2c_{12}^2s_{12}^2\cos(\alpha_{21})$$

Effective Majorana mass matrix and mass eigenvalues, a mixing angle, and a Majorana phase

- **The 2 ×2 Majorana matrix has four parameters after changing weak basis.**
- $\left\{\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix},\begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}\right\}\to\left\{e^{-i\frac{\theta_{ee}}{2}}\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix},e^{-i\frac{\theta_{\mu\mu}}{2}}\begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}\right\}$ $\theta_{\alpha\beta}=arg(m_{\alpha\beta})$

$$\begin{pmatrix}m_{ee} & m_{e\mu}\\ m_{e\mu} & m_{\mu\mu}\end{pmatrix}\to\begin{pmatrix}|m_{ee}| & |m_{e\mu}|e^{i\theta'_{e\mu}}\\ |m_{e\mu}|e^{i\theta'_{e\mu}} & |m_{\mu\mu}|\end{pmatrix},\theta'_{e\mu}=\theta_{e\mu}-\frac{\theta_{ee}+\theta_{\mu\mu}}{2},$$

$$m^0_{\nu\alpha\beta}=\begin{pmatrix}|m_{ee}| & m_{e\mu}\\ m_{e\mu} & |m_{\mu\mu}|\end{pmatrix}:\theta'_{e\mu}=arg(m_{e\mu})$$

The four parameters determine the mass eigenvalues, a mixing angle and a Majorana phase

$$
\begin{aligned}
m_1 &= \sqrt{\frac{|m_{ee}|^2+|m_{\mu\mu}|^2+2|m_{e\mu}|^2-\Delta m_{21}^2}{2}} \\
m_2 &= \sqrt{\frac{|m_{ee}|^2+|m_{\mu\mu}|^2+2|m_{e\mu}|^2+\Delta m_{21}^2}{2}} \\
\Delta m_{21}^2 &= \sqrt{(|m_{ee}|^2-|m_{\mu\mu}|^2)^2+4|m_{e\mu}|^2 u} \\
u &= |m_{ee}|^2+|m_{\mu\mu}|^2+2|m_{ee}m_{\mu\mu}|\cos(2\theta'_{e\mu})
\end{aligned}
$$

$$
U_{PMNS} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} e^{i\frac{\alpha_{21}}{2}} \\ -\sin\theta_{12} & \cos\theta_{12} e^{i\frac{\alpha_{21}}{2}} \end{pmatrix}
$$

$$
\begin{aligned}
\sin 2\theta_{12} &= \frac{-2\,|m_{e\mu}|\sqrt{u}}{\Delta m_{21}^2} < 0 \\
\cos 2\theta_{12} &= \frac{|m_{\mu\mu}|^2-|m_{ee}|^2}{\Delta m_{21}^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\alpha_{21}}{2} &= \frac{\alpha_1-\alpha_2}{2}+\pi \\
\alpha_1 &= \arctan\frac{-2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{u-\Delta m_{21}^2} \\
\alpha_2 &= \arctan\frac{-2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{u+\Delta m_{21}^2}
\end{aligned}
$$

Implication on the high energy model generating Majorana mass matrix

- **Type I seesaw with two gauge-singlet neutrinos ($\boldsymbol{N_1}$, $\boldsymbol{N_2}$) & two active neutrinos ($\boldsymbol{\nu_e, \nu_\mu}$)**
- **8 parameters**
- $\boldsymbol{M_1, M_2}$
- **Dirac mass matrix** $\begin{pmatrix} \boldsymbol{m_{De1}} & \boldsymbol{m_{De2}} \\ \boldsymbol{m_{D\mu1}} & \boldsymbol{m_{D\mu2}} \end{pmatrix}$
- 6real +2 imaginary (CPV)

m_ν in the type I seesaw model with two gauge-singlet neutrinos and two active neutrinos

- $m_\nu = -m_D \begin{pmatrix} \frac{1}{M_1} & 0 \\ 0 & \frac{1}{M_2} \end{pmatrix} m_D^T = -\begin{pmatrix} \boldsymbol{u_{e1}} & \boldsymbol{u_{e2}} \\ \boldsymbol{u_{\mu 1}} & \boldsymbol{u_{\mu 2}} \end{pmatrix} \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{u_{e1}} & \boldsymbol{u_{\mu 1}} \\ \boldsymbol{u_{e2}} & \boldsymbol{u_{\mu 2}} \end{pmatrix}$

- $X_1 = \frac{m_{D1}^2}{M_1}, \quad X_2 = \frac{m_{D2}^2}{M_2}$

- $m_D = \begin{pmatrix} \boldsymbol{m_{De1}} & \boldsymbol{m_{De2}} \\ \boldsymbol{m_{D\mu 1}} & \boldsymbol{m_{D\mu 2}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{u_{e1}} & \boldsymbol{u_{e2}} \\ \boldsymbol{u_{\mu 1}} & \boldsymbol{u_{\mu 2}} \end{pmatrix} \begin{pmatrix} \boldsymbol{m_{D1}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{m_{D2}} \end{pmatrix},$

- $|u_{e1}|^2 + |u_{\mu 1}|^2 = 1, |u_{e2}|^2 + |u_{\mu 2}|^2 = 1$

One can go to the basis where the diagonal elements of Majorana matrix m_{ee} and $m_{\mu\mu}$ are given by real positive values by rephasing the weak basis

$$
\begin{aligned}
(m_\nu^0)_{ab} &= -\sum_{i=1,2} e^{i(\delta_a+\delta_b)} u_{ai} X_i u_{bi}, \\
&= -\sum_{i=1,2} W_{ai} W_{bi},
\end{aligned}
$$

$$
W_{ai} = e^{i\delta_a} u_{ai} \sqrt{X_i} = |W_{ai}| e^{i\frac{\alpha_{ai}}{2}},
$$

$$
\arg W_{ai} \equiv \frac{\alpha_{ai}}{2}, \quad |W_{ai}| = \sqrt{X_i} |u_{ai}|
$$

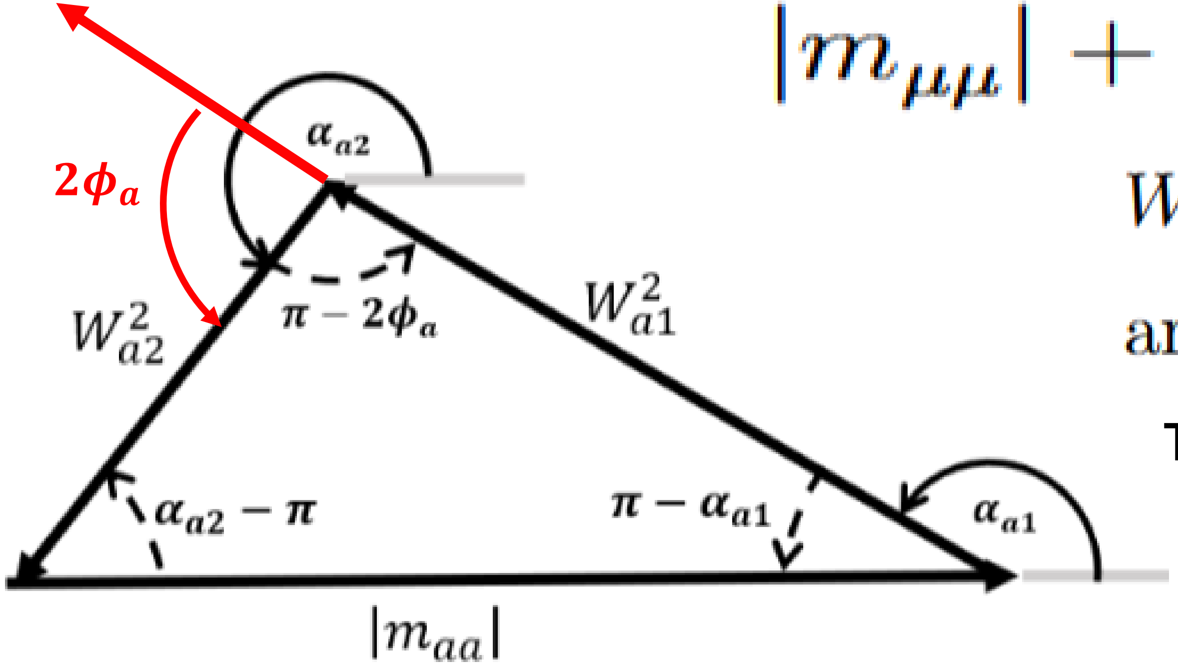
The triangles for Majorana mass matrix and CP violation in the two-generation toy model

Y.Kawakami, Bachelor thesis, Hiroshima university (2024)
Triangle for m_{ee} in a toy model

Δ_{aa} =area of triangle for $|m_{aa}|$

$2\,\Delta_{aa} = |W_{a1}^2 W_{a2}^2| \sin 2\phi_a$
$a = e, \mu$

$$|m_{aa}| + W_{a1}^2 + W_{a2}^2 = 0,$$
$$|m_{ee}| + (W_{e1})^2 + (W_{e2})^2 = 0,$$
$$|m_{\mu\mu}| + (W_{\mu1})^2 + (W_{\mu2})^2 = 0,$$



$$W_{ai} = e^{i\delta_a} u_{ai} \sqrt{X_i} = |W_{ai}| e^{i\frac{\alpha_{ai}}{2}},$$
$$\arg W_{ai} \equiv \frac{\alpha_{ai}}{2}, \quad |W_{ai}| = \sqrt{X_i}|u_{ai}|$$

Two CP violating phases in the toy model

$$2\phi_a = \arg\left(\frac{W_{a2}^2}{W_{a1}^2}\right) = \arg\left(\frac{u_{a2}^2}{u_{a1}^2}\right) = \arg\left(\frac{m_{Da2}^2}{m_{Da1}^2}\right), (a = e, \mu)$$

Triangles for diagonal elements

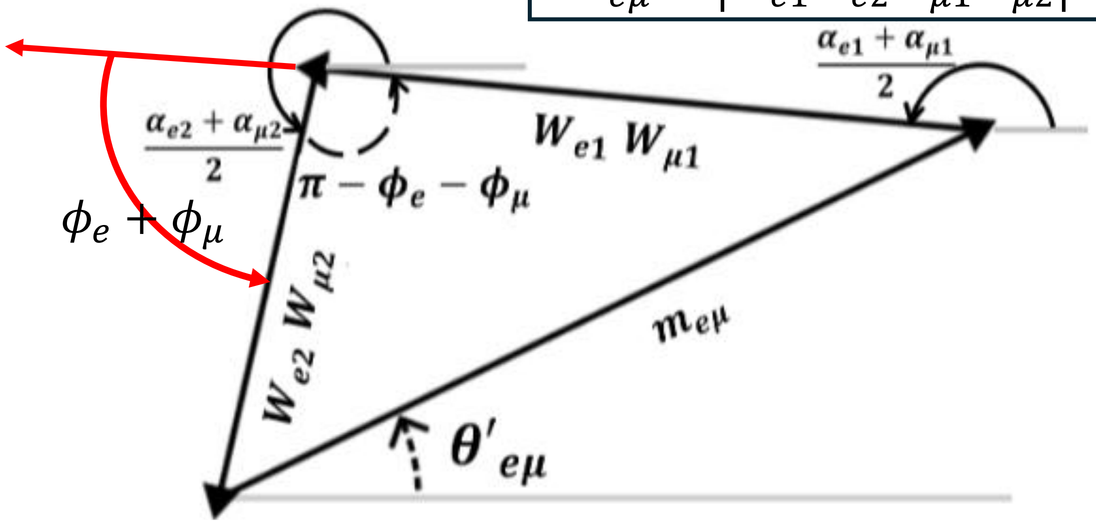
From Fig.6 of arxiv:2212.00142v4 [hep-ph]

Building the off-diagonal element $m_{e\mu}$

$$2\,\Delta_{e\mu} = |W_{e1}W_{e2}W_{\mu1}W_{\mu2}|\sin(\phi_e+\phi_\mu)$$

$$m_{e\mu} = -\sum_{i=1}^{2} W_{ei}W_{\mu i},$$

$$|m_{e\mu}|e^{i\theta'_{e\mu}} = \sum_{i=1}^{2} |W_{ei}||W_{\mu i}|e^{i\left(\frac{\alpha_{ei}+\alpha_{\mu i}}{2}-\pi\right)}$$



$$\theta'_{e\mu} = \arctan\frac{\sum_{i=1}^{2}|W_{ei}W_{\mu i}|\sin\left(\frac{\alpha_{ei}+\alpha_{\mu i}}{2}-\pi\right)}{\sum_{i=1}^{2}|W_{ei}W_{\mu i}|\cos\left(\frac{\alpha_{ei}+\alpha_{\mu i}}{2}-\pi\right)},$$

$$= \arctan\frac{\sum_{i=1}^{2}|u_{ei}u_{\mu i}|X_i\sin\left(\frac{\alpha_{ei}+\alpha_{\mu i}}{2}-\pi\right)}{\sum_{i=1}^{2}|u_{ei}u_{\mu i}|X_i\cos\left(\frac{\alpha_{ei}+\alpha_{\mu i}}{2}-\pi\right)}.$$

The phase for off-diagonal element relevant for a Majorana phase at low energy

$$\frac{\alpha_{e2}+\alpha_{\mu2}}{2}-\pi = \frac{1}{2}\sum_{a=e}^{\mu}\arctan\frac{|W_{a1}|^2\sin(2\phi_a)}{|W_{a2}|^2+|W_{a1}|^2\cos(2\phi_a)},$$

$$= \frac{1}{2}\sum_{a=e}^{\mu}\arctan\frac{|u_{a1}|^2X_1\sin(2\phi_a)}{|u_{a2}|^2X_2+|u_{a1}|^2X_1\cos(2\phi_a)},$$

$$\pi-\frac{\alpha_{e1}+\alpha_{\mu1}}{2} = \frac{1}{2}\sum_{a=e}^{\mu}\arctan\frac{|W_{a2}|^2\sin(2\phi_a)}{|W_{a1}|^2+|W_{a2}|^2\cos(2\phi_a)},$$

$$= \frac{1}{2}\sum_{a=e}^{\mu}\arctan\frac{|u_{a2}|^2X_2\sin(2\phi_a)}{|u_{a1}|^2X_1+|u_{a2}|^2X_2\cos(2\phi_a)}.$$

The phases obtained from the analysis of the diagonal triangles

Connection to lepton number asymmetries for lepto-genesis

Lepton Number asymmetries of heavy Majorana neutrinos (N_1, N_2) decays are given by:

$$\epsilon^k = \sum_{a=e}^{\mu} \epsilon_a^k \qquad \epsilon_a^k = \frac{\Gamma[N^k \to l_a \tilde{\phi}^\dagger] - \Gamma[N^k \to \overline{l_a} \tilde{\phi}]}{\sum_{a=e}^{\mu} (\Gamma[N^k \to l_a \tilde{\phi}^\dagger] + \Gamma[N^k \to \overline{l_a} \tilde{\phi}])},$$

$$\epsilon^1 = \frac{m_{D_2}^2}{4\pi v^2} I(x_{21}) [|u_{e1} u_{e2}|^2 \sin 2\phi_e + 2|u_{e1} u_{e2} u_{\mu 1} u_{\mu 2}| \sin(\phi_e + \phi_\mu) + |u_{\mu 1} u_{\mu 2}|^2 \sin 2\phi_\mu]$$

$$\epsilon^2 = \frac{-m_{D_1}^2}{4\pi v^2} I(x_{12}) [|u_{e1} u_{e2}|^2 \sin 2\phi_e + 2|u_{e1} u_{e2} u_{\mu 1} u_{\mu 2}| \sin(\phi_e + \phi_\mu) + |u_{\mu 1} u_{\mu 2}|^2 \sin 2\phi_\mu]$$

$$I(x) = \sqrt{x}[1 + \frac{1}{1-x} + (1+x) \log \frac{x}{1+x}]. \qquad x_{k'k} = \frac{M_{k'}^2}{M_k^2}$$

Discussion part: (discussion with Y. Kawakami)
Relation between CP asymmetries of lepto-genesis and the area of the triangles

$$\epsilon^1 = \frac{m_{D_2}^2}{4\pi v^2} I(x_{21})[|u_{e1}u_{e2}|^2 \sin 2\phi_e + 2|u_{e1}u_{e2}u_{\mu 1}u_{\mu 2}| \sin(\phi_e + \phi_\mu) + |u_{\mu 1}u_{\mu 2}|^2 \sin 2\phi_\mu]$$

$$2\,\Delta_{e\mu} = |W_{e1}W_{e2}W_{\mu 1}W_{\mu 2}| \sin(\phi_e + \phi_\mu) = X_1 X_2\, |u_{e1}u_{e2}u_{\mu 1}u_{\mu 2}| \sin(\phi_e + \phi_\mu)$$

$$2\,\Delta_{aa} = |W_{a1}^2 W_{a2}^2| \sin 2\phi_a = X_1 X_2 |u_{a1}^2 u_{a2}^2| \sin 2\phi_a$$

$$\begin{aligned}\epsilon^1 &= \frac{m_{D2}^2}{4\pi v^2} I\left(\frac{M_2^2}{M_1^2}\right) \frac{2}{X_1 X_2} [\Delta_{ee} + \Delta_{\mu\mu} + 2\,\Delta_{e\mu}] \\ &\simeq -3\,\frac{M_1^2}{m_{D1}^2} \frac{\Delta_{ee} + \Delta_{\mu\mu} + 2\,\Delta_{e\mu}}{4\pi v^2}\ (M_2 \gg M_1)\end{aligned}$$

Summary

1. **Time evolution of lepton number asymmetry and its second time derivatives are useful to determine the Majorana phase and the absolute value of active neutrinos.**

2. **Relation of effective Majorana mass matrix and mass eigenvalues , the mixing angle, and the Majorana phase is derived for two generation toy model.**

3. **In the type I seesaw model with two active and two gauge－singlet neutrinos, the effective Majorana matrix forms the three triangles in the complex plane.**

CP asymmetries for lepto-genesis are shown to be written in terms of the area of the triangles. (Discussion part)