Axion Detection with Quantum Hall Effect

Nishogakusha Univ.　　Aiichi Iwazaki

arXiv: 2411.06038,   PTEP. (2024),   Phys. Lett. B(2023)

Axion dark matter is familiar to particle physicist, but quantum Hall effect (QHE) is not familiar.
So first I explain the basis of QHE. Then, examining previous experiments I show that
the axion effect has been observed. We show that the axion mass is nearly equal to $10^{-5}\mathrm{eV}$

contents

1) Microwaves are produced by axion under strong magnetic field

2)Quantum Hall effect
（there are inevitably axion induced microwaves）

3)Plateau-plateau transition in Quantum Hall effect

4)Examination of experiments with no axion effect

5)Examination of experiments with axion effect at low temperature <50mK
（we find axion mass ~$10^{-5}\mathrm{eV}$）
6)Axion effect V.S thermal effect

7)Propose experiment for the confirmation of the axion

7) Conclusion

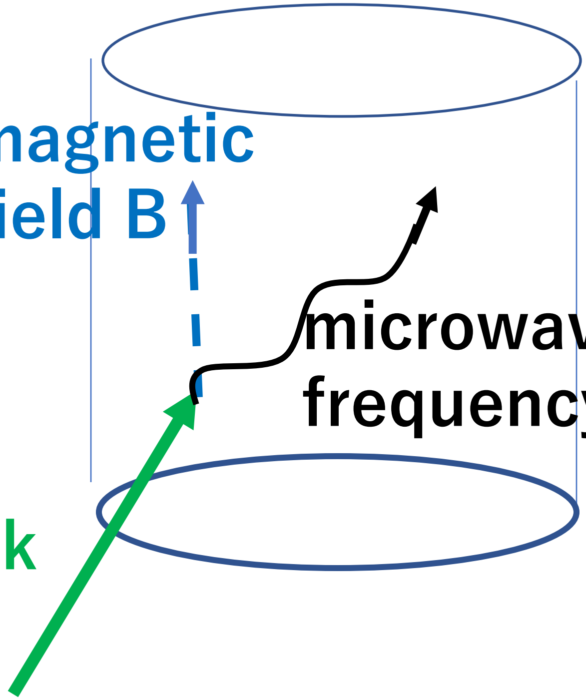
Axion produces microwave under strong magnetic field
The property is used for axion detection with resonant cavity

Electromagnetic coupling with axion

$$L_{aEB} = k_a \alpha \frac{a(\vec{x},t)\vec{E}\cdot\vec{B}}{f_a \pi}$$

$$a(\vec{x},t) \propto \cos(m_a t)$$

axion mass m_a

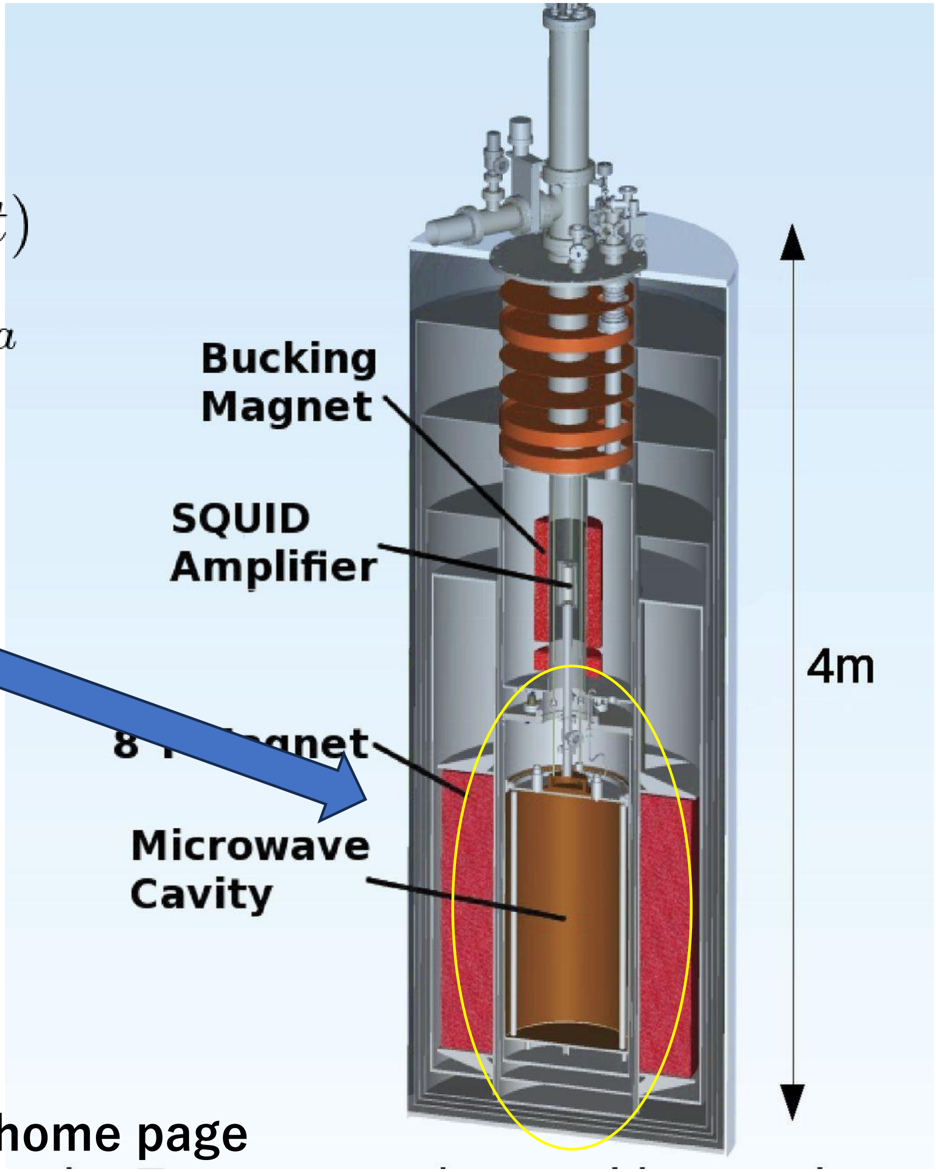


magnetic field B

microwave with frequency $f = \frac{m_a}{2\pi}$

axion dark matter

Resonant cavity amplifies weak axion microwave



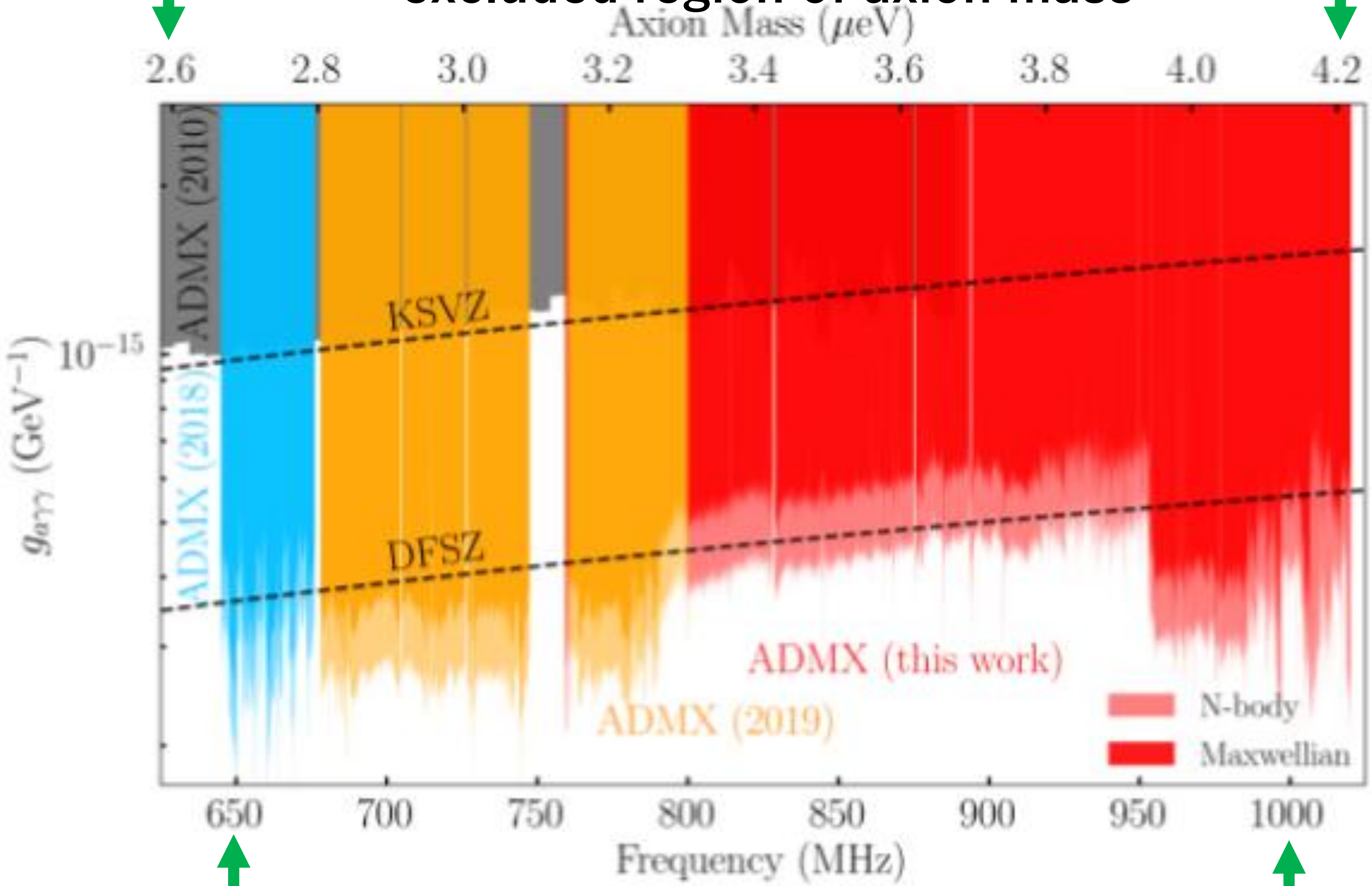
ADMX home page

ADMX(axion dark matter experiment)

$2.6 \times 10^{-6}\mathrm{eV}$

excluded region of axion mass

$4.2 \times 10^{-6}\mathrm{eV}$



QCD axion model

We show axion mass

$\sim 10^{-5}\mathrm{eV}$ **or frequency**

~2.4GHz

with quantum Hall effect

650MHz ADMX **(2021)**

1GHz

Quantum Hall effect

realized in two dimensional electrons under strong magnetic field

Electrons occupy states in Landau level

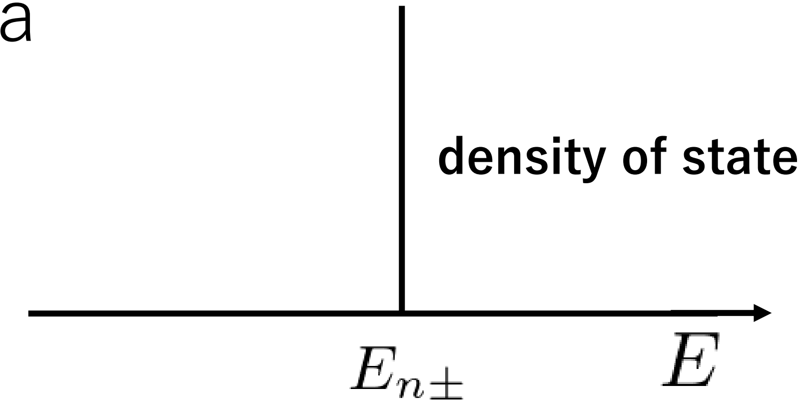
Electron states are characterized with integer n ≥ 0

Their energies $\boxed{E_{n\pm} = \frac{eB}{m_e} \times (n + \frac{1}{2}) \pm g\mu_B B}$

Each state is highly degenerate when there are no disorder potential

Degeneracy, i.e. number of states per unit area

$$\frac{eB}{2\pi} \sim 10^{11}\mathrm{cm}^{-2}(\mathrm{B}/10\mathrm{T})$$



Presence of strong magnetic field inevitably generates microwaves by axion dark matter.
It is similar to resonant cavity experiment, e.g. ADMX

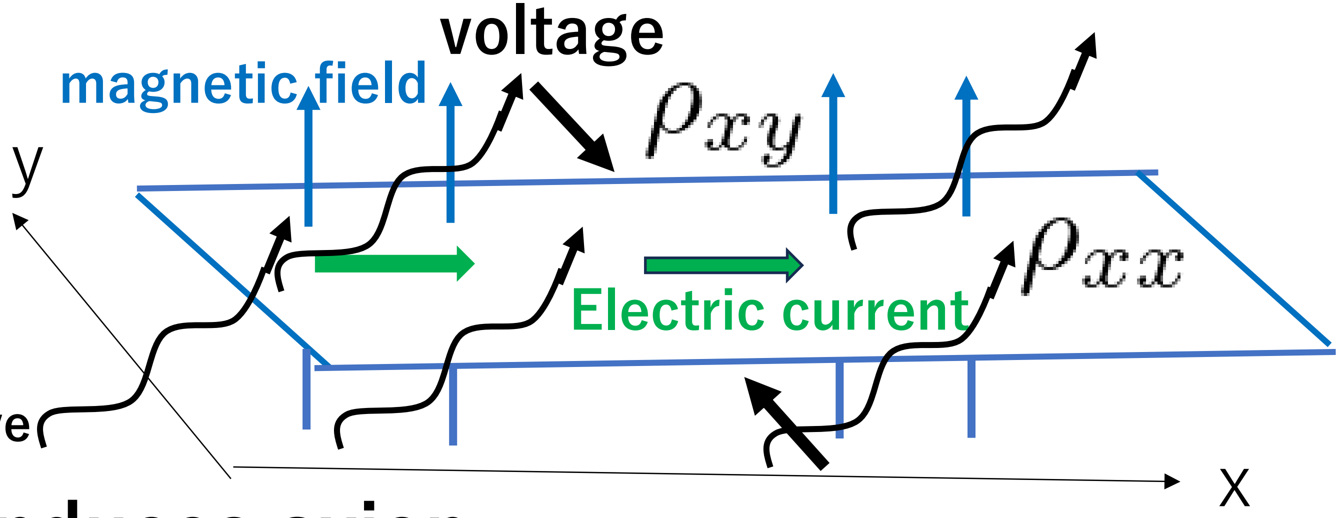
Experiment of quantum Hall effect

We measure Hall resistance (conductance)

$$\rho_{xy} \ (\sigma_{xy})$$

under strong magnetic field B~10T,

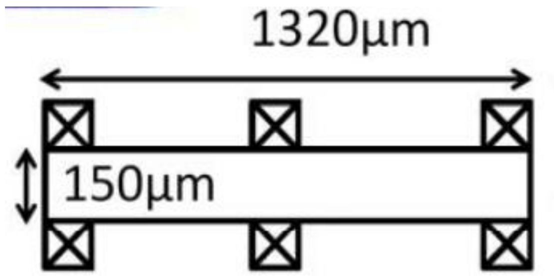
There are experiments imposing external microwave and measuring the response of Hall resistance



Because strong magnetic field induces axion microwave with frequency $f = \frac{m_a}{2\pi}$,

implicitly we have the axion microwave in the experiment

X. Wang, et.al.
Phys. Rev. B93 (2016)

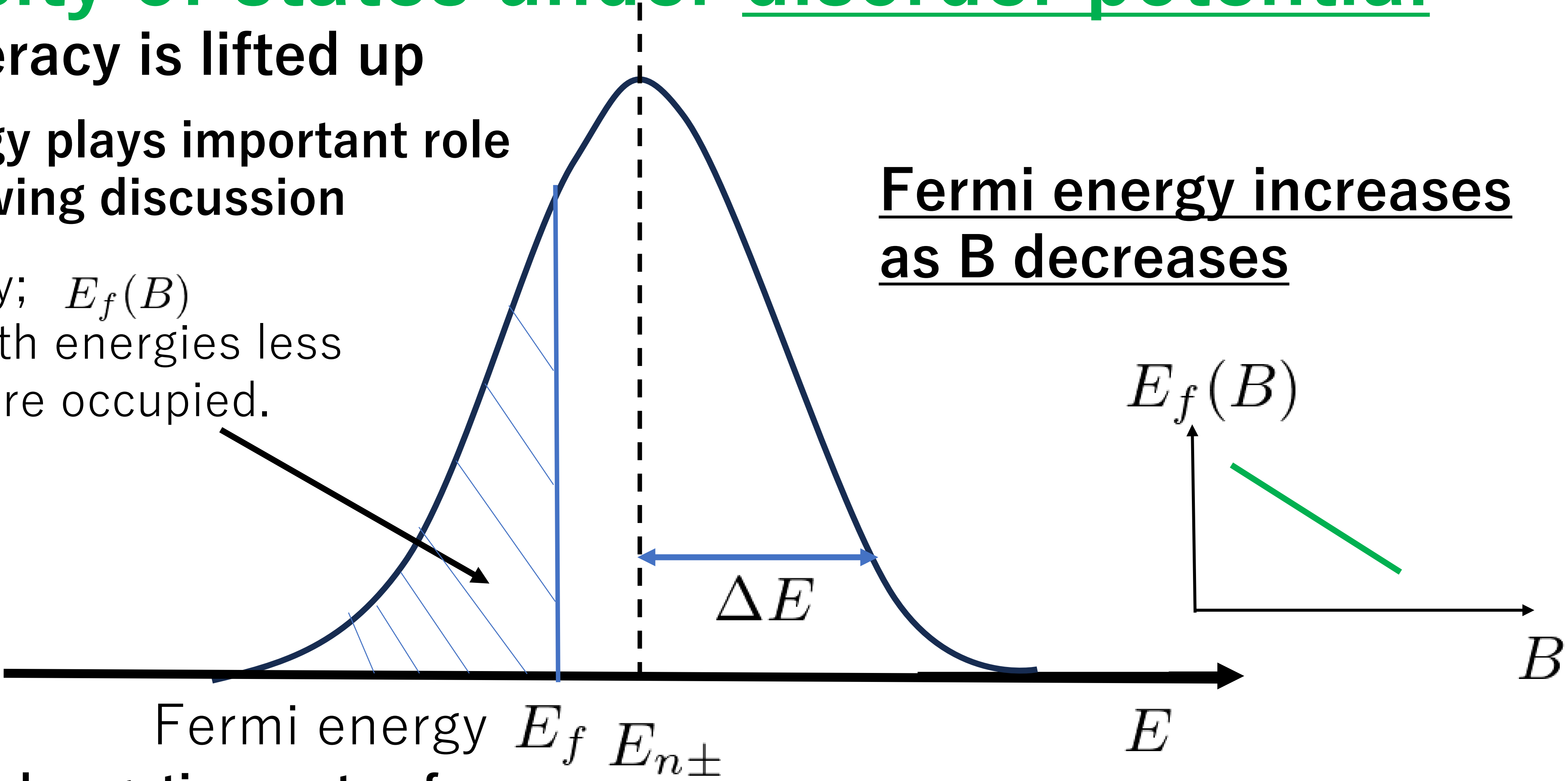


Density of states under disorder potential

degeneracy is lifted up

Fermi energy plays important role in the following discussion

Fermi energy; $E_f(B)$
All states with energies less than $E_f(B)$ are occupied.



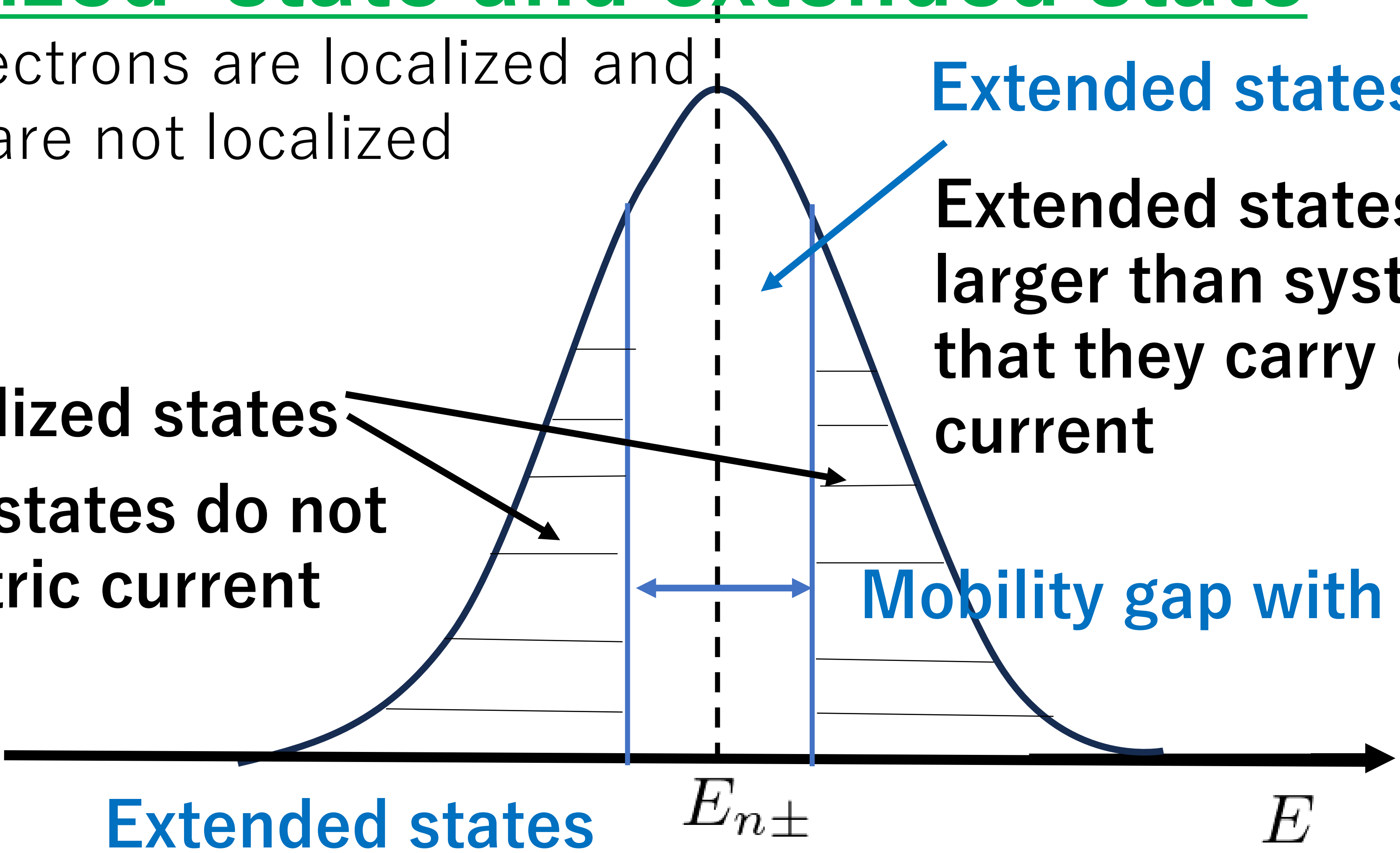
Fermi energy increases as B decreases

Positive and negative parts of disorder potential are included with the same amount

Number of states in the figure is proportional to B, while number of electrons is fixed.

localized state and extended state

Some of electrons are localized and the others are not localized



Extended states

Extended states are larger than system size so that they carry electric current

Localized states

Localized states do not carry electric current

Mobility gap with width 2δ

Extended states

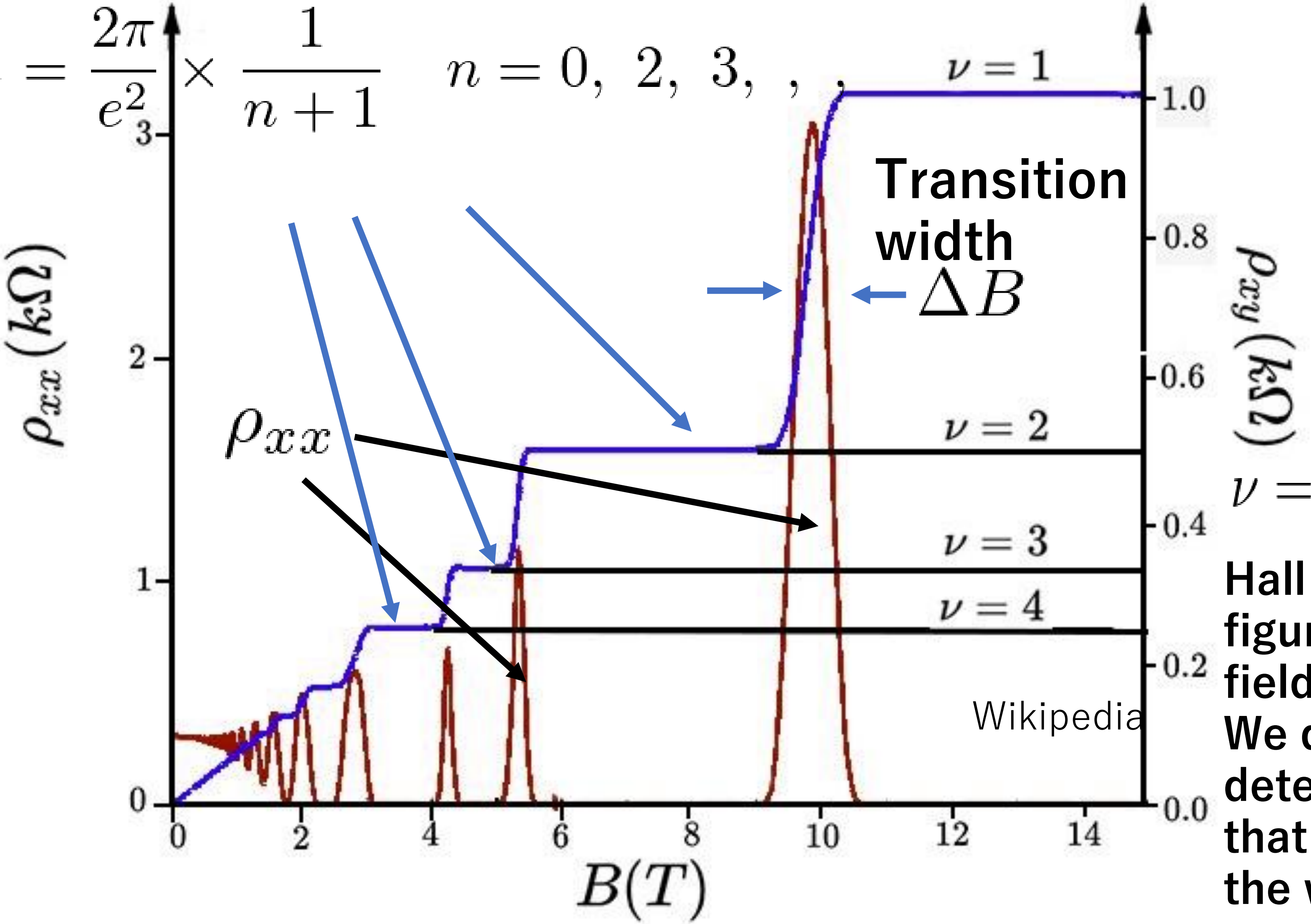
By noting these basic facts, we explain phenomena of quantum Hall effect

Integer quantum Hall effect

Hall resistance

$$\rho_{xy} = \frac{2\pi}{e^2} \times \frac{1}{n+1} \qquad n = 0,\ 2,\ 3,\ ,\ ,$$

We notice plateau-plateau transition



Transition width

ΔB

Wikipedia

filling; $\nu = \frac{\rho}{2\pi \mathrm{B}}$

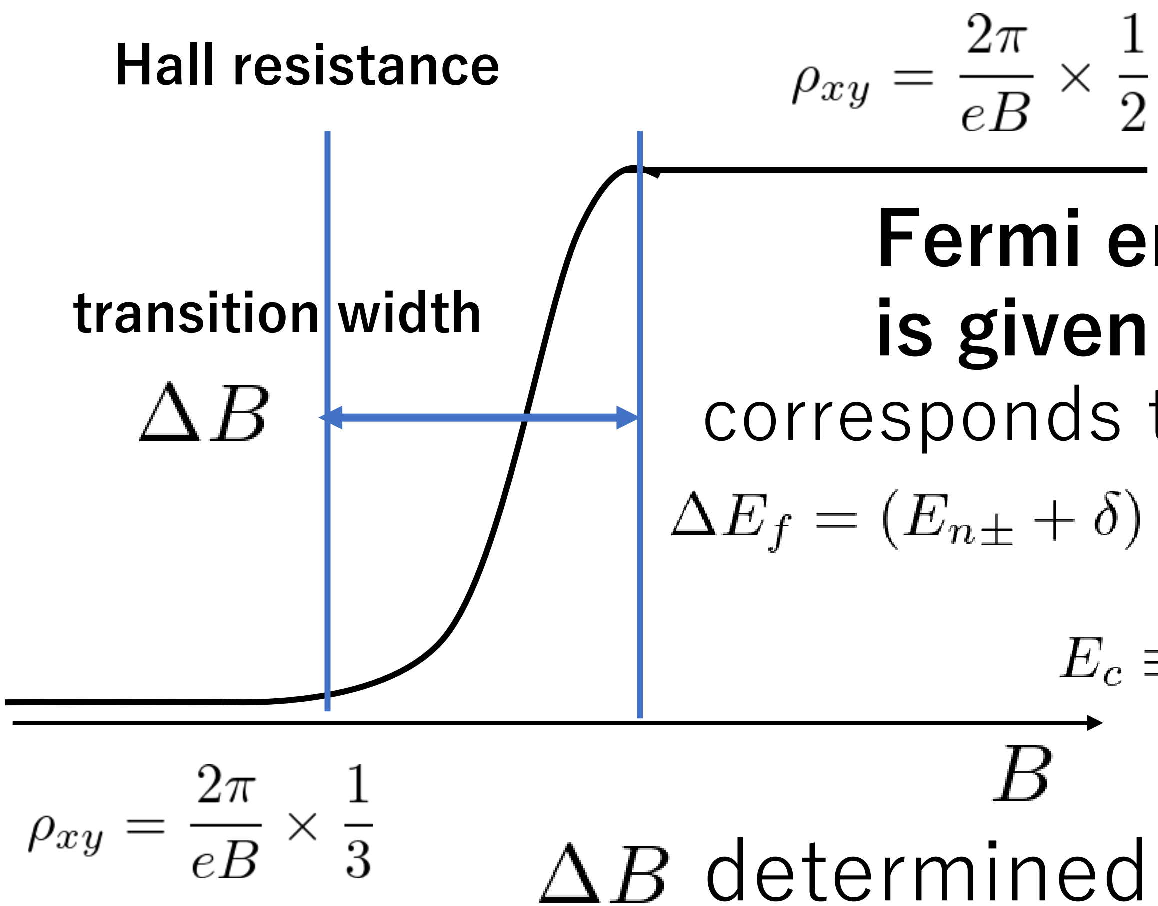
ρ electron density

$\frac{eB}{2\pi}$ density of state

$\nu = 1 \rightarrow$ lowest Landau level

Hall resistance behaves as in figure when we change magnetic field.
We discuss how the width ΔB is determined. It will turn out later that axion effect is observed in the width

Plateau-plateau transition at T=0

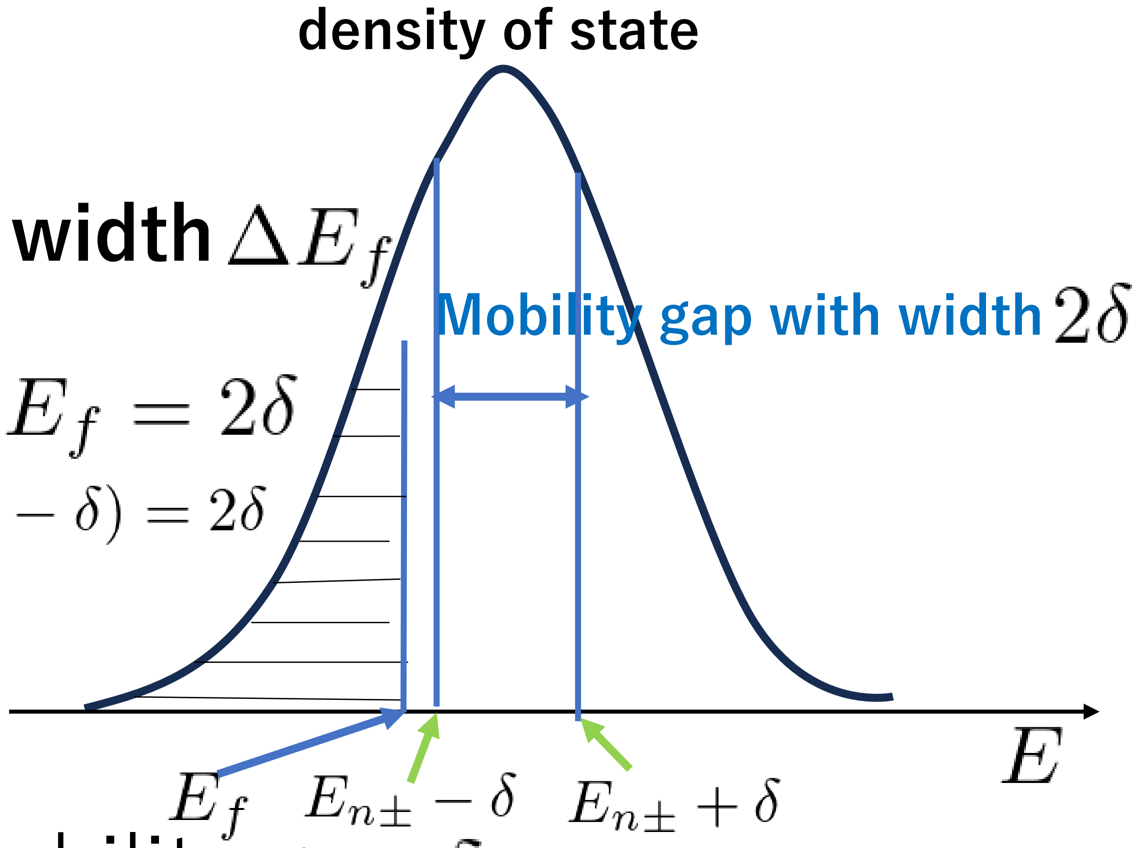


Fermi energy width ΔE_f is given by

corresponds to $\Delta E_f = 2\delta$

$$\Delta E_f = (E_{n\pm} + \delta) - (E_{n\pm} - \delta) = 2\delta$$

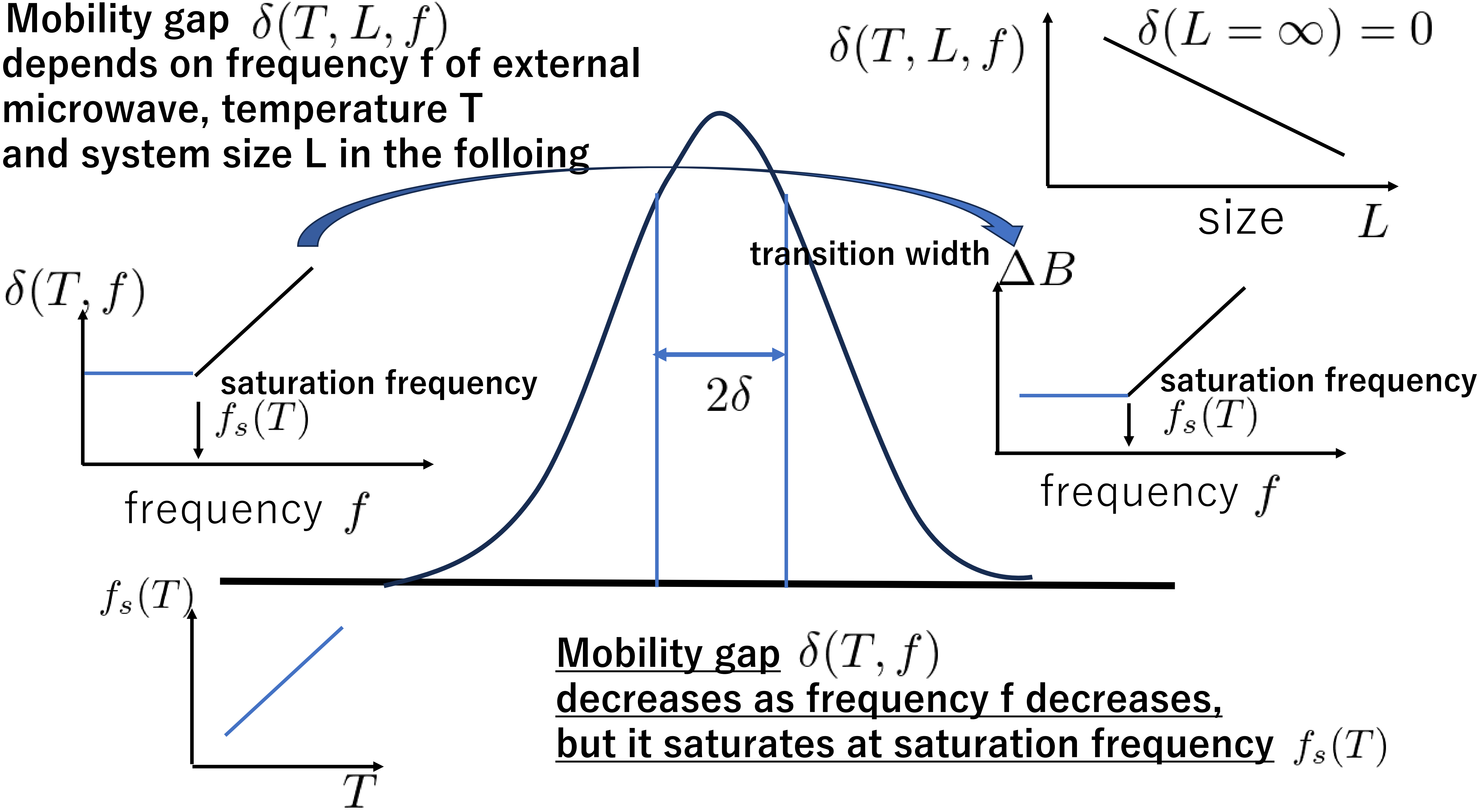
$$E_c \equiv E_{n\pm}$$



ΔB determined by mobility gap δ

Mobility gap depends on temperature T and frequency f of microwave, that is, oscillating voltage with frequency f

Mobility gap $\delta(T, L, f)$ depends on frequency f of external microwave, temperature T and system size L in the folloing



Mobility gap $\delta(T, f)$ decreases as frequency f decreases, but it saturates at saturation frequency $f_s(T)$

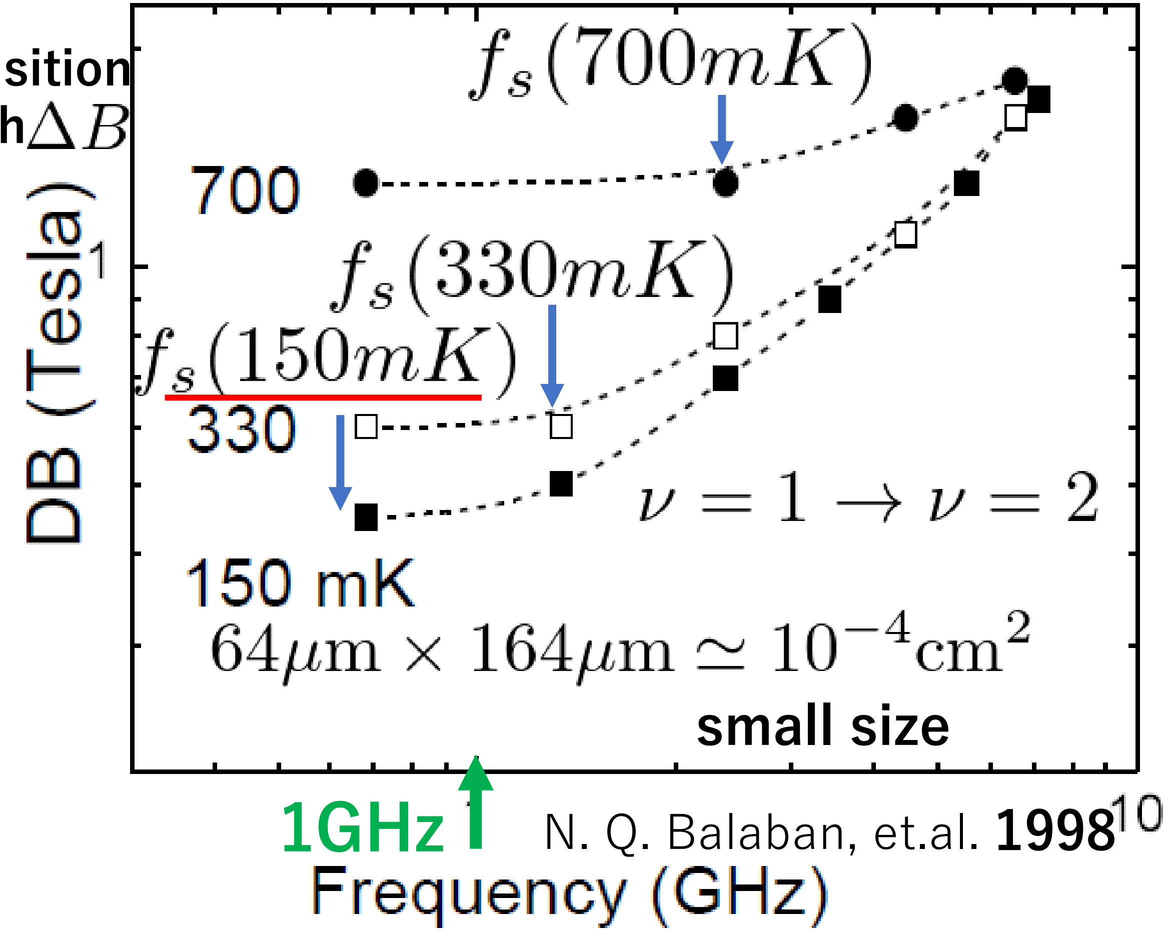
experiments of quantum Hall effect with no axion effect

as frequency f decreases, $\Delta E_f = 2\delta(T, f)$ → mobility gap saturates at f=$f_s(T)$ $\Delta E_f = 2\delta(T, f_s(T))$ for $f < f_s(100\mathrm{mK})$

Independent on frequency f

Transition width decreases as frequency decreases, but it saturates at $f = f_s(T)$

Transition widthΔB

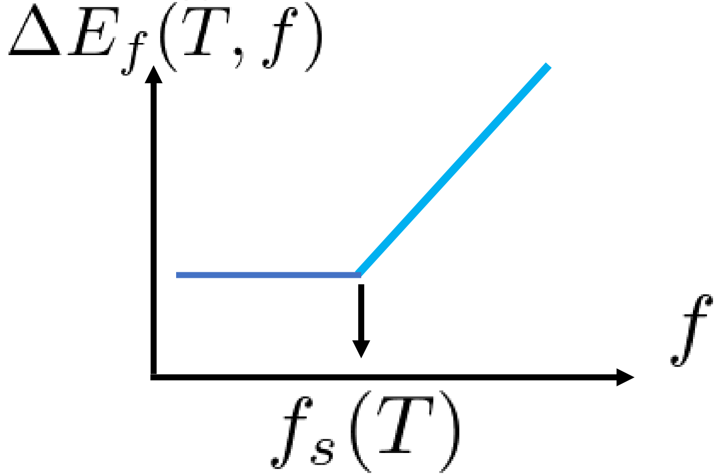


$64\mu\mathrm{m} \times 164\mu\mathrm{m} \simeq 10^{-4}\mathrm{cm}^2$

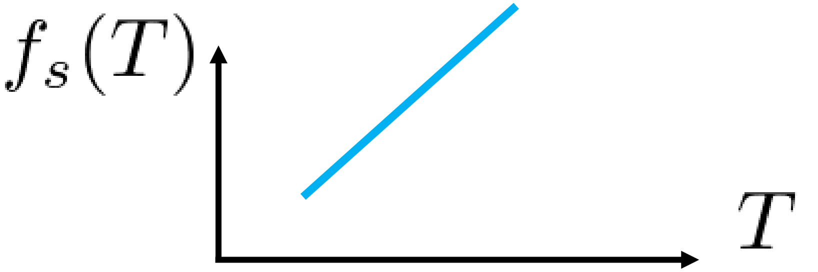
small size

N. Q. Balaban, et.al. 1998

Fermi energy width



Saturation frequency $f_s(T)$ decreases with decrease of temperature T



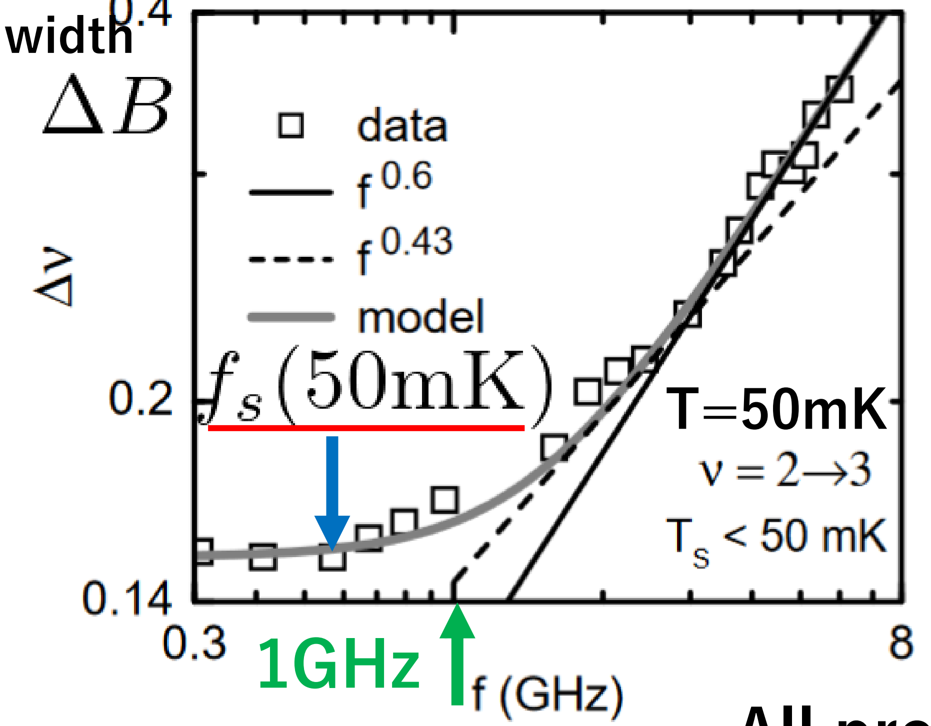
experiments of quantum Hall effect with no axion effect

as frequency f decreases → mobility gap saturates at f=$f_s(T)$

$$\Delta E_f = 2\delta(T,f) \longrightarrow \underline{\Delta E_f = 2\delta(T,f_s(T))} \quad \text{for } f < f_s(100\text{mK})$$

larger size

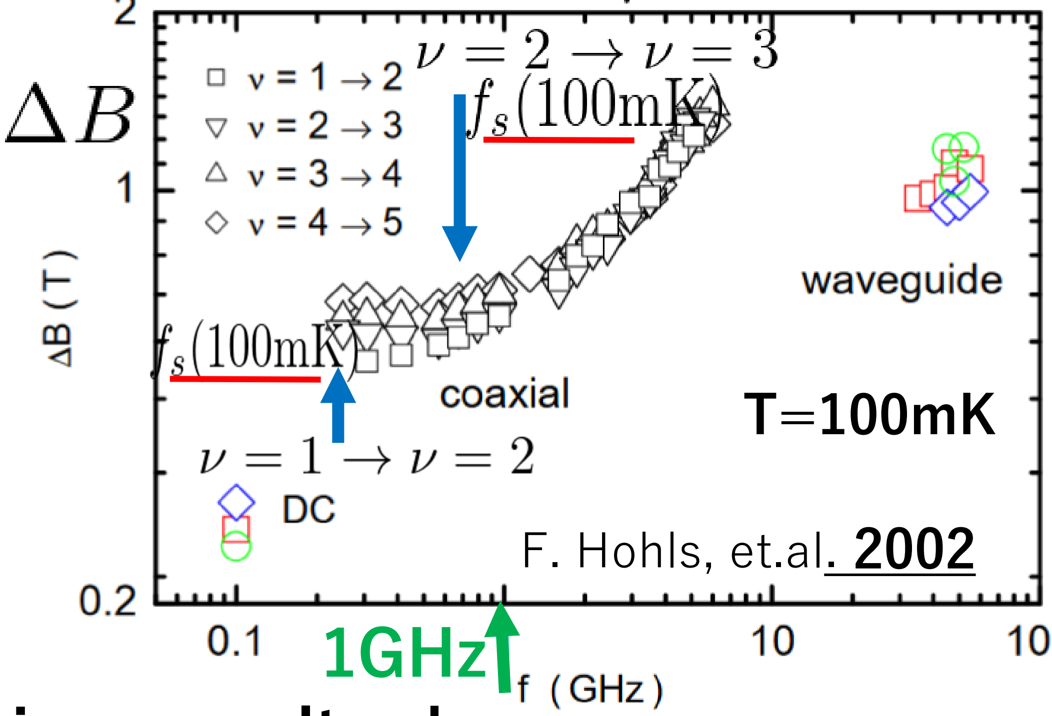
$5mm \times 20\mu m \simeq 10^{-3}\text{cm}^2$



F. Hohls, et.al. **2001**

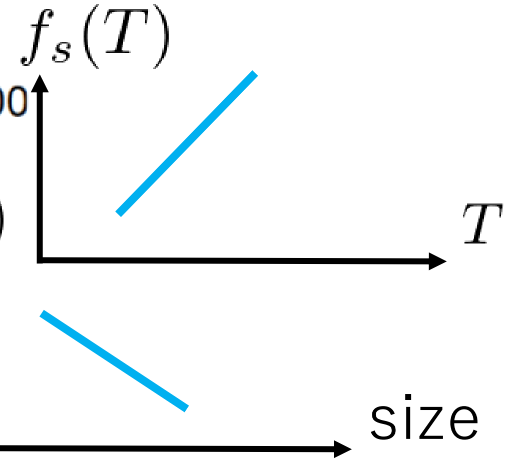
larger size

$5mm \times 20\mu m \simeq 10^{-3}\text{cm}^2$



F. Hohls, et.al. **2002**

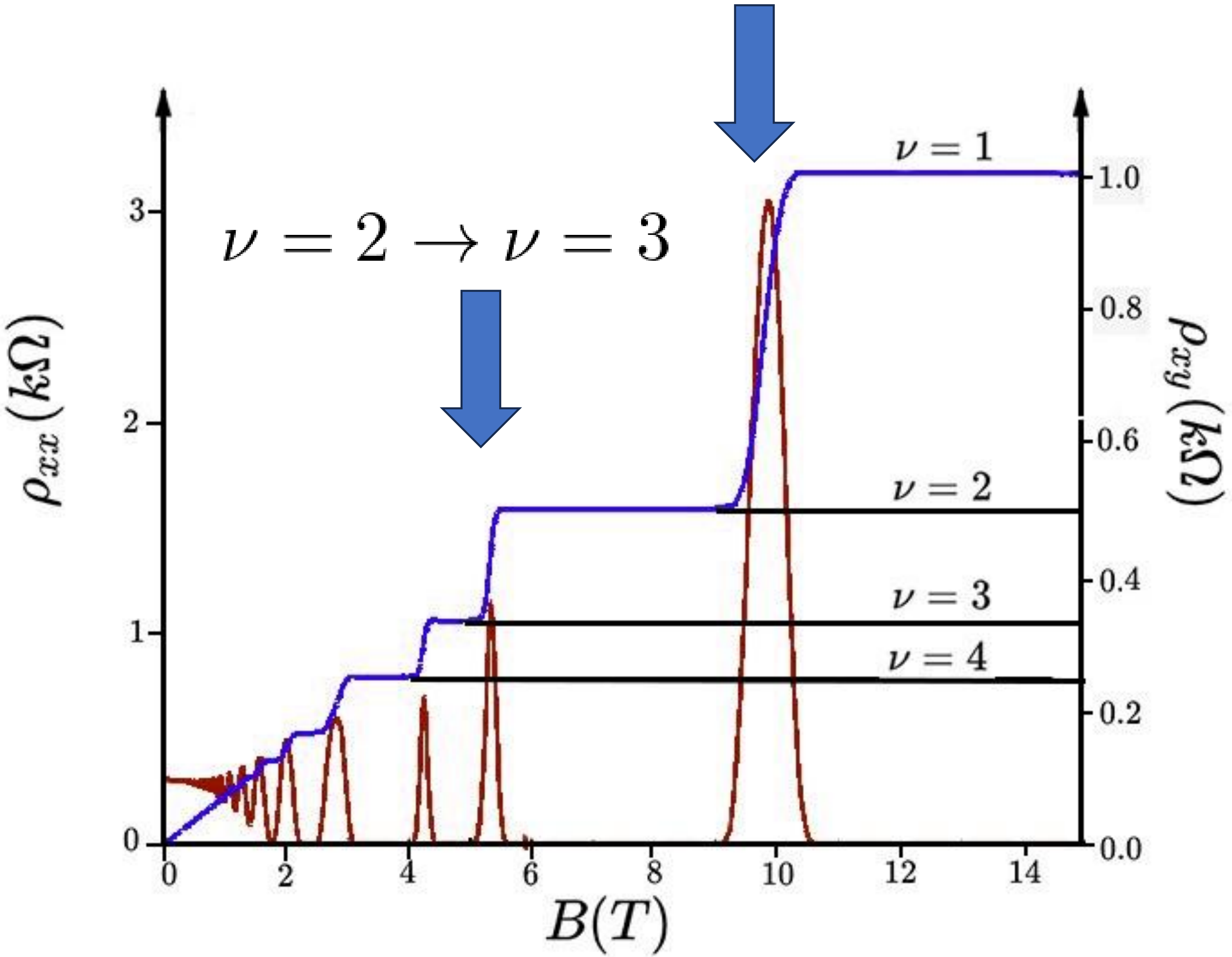
$f_s(T)$ decreases as size increases or T decreases



All previous results show

$$\underline{f_s(\text{size} > 10^{-4}\text{cm}^2, \text{T} < 200\text{mK}) < 1\text{GHz}}$$

All samples
GaAs/AlGaAs



$\nu = 1 \rightarrow \nu = 2$

$\nu = 2 \rightarrow \nu = 3$

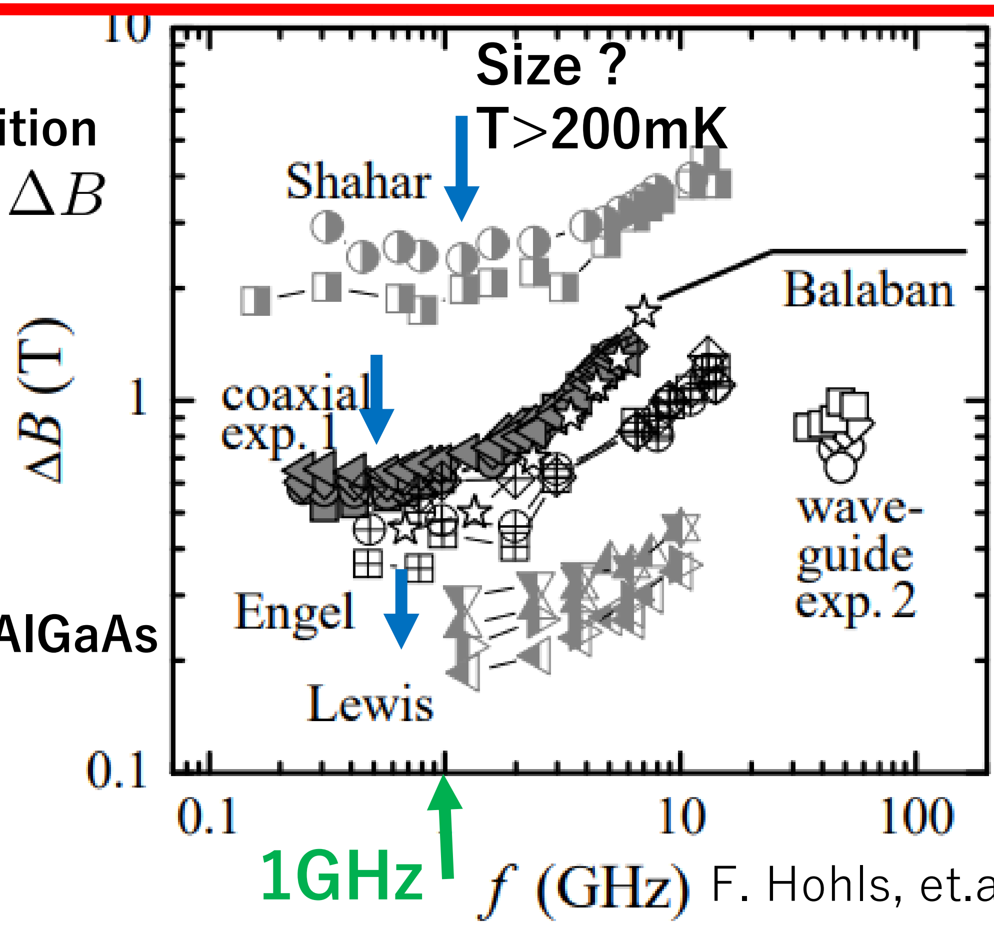
experiments of quantum Hall effect with no axion effect

All other results also show that saturation frequency is less than 1GHz, when T<200mK and large size

$$f_s(\mathrm{size} > 10^{-4}\mathrm{cm}^2, \mathrm{T} < 200\mathrm{mK}) < 1\mathrm{GHz}$$

Transition width ΔB

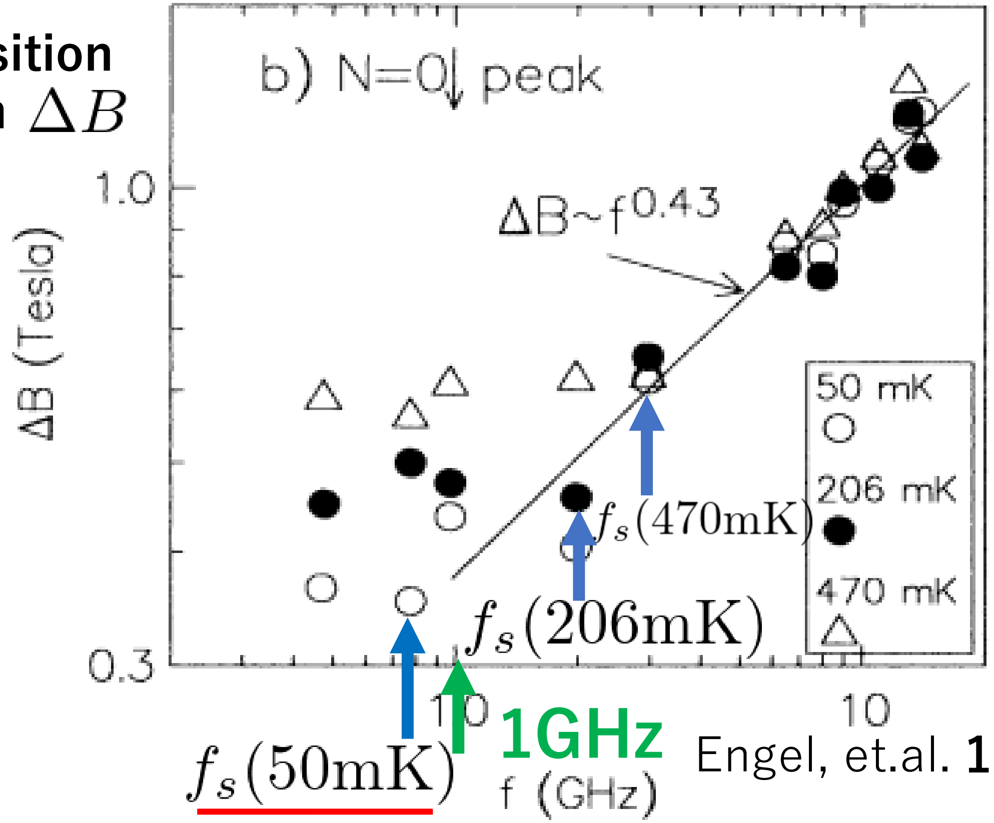
GaAs/AlGaAs



1GHz

F. Hohls, et.al. **2002**

Transition width ΔB



$f_s(470\mathrm{mK})$

$f_s(206\mathrm{mK})$

$f_s(50\mathrm{mK})$

1GHz

Engel, et.al. **1993**

$$\boxed{\mathrm{GaAs/Al_xGa_{1-x}As}, \quad \mathrm{x} > 0.015}$$

$$14mm \times 30\mu\mathrm{m} = 4.2 \times 10^{-3}\mathrm{cm}^2$$

much large size

All previous results show that saturation frequency of mobility gap is less than 1GHz when T<200mK and system size larger than 10^(-4)cm^2.

In general,
the saturation frequency of the mobility gap becomes smaller as temperature is smaller or system size is larger.

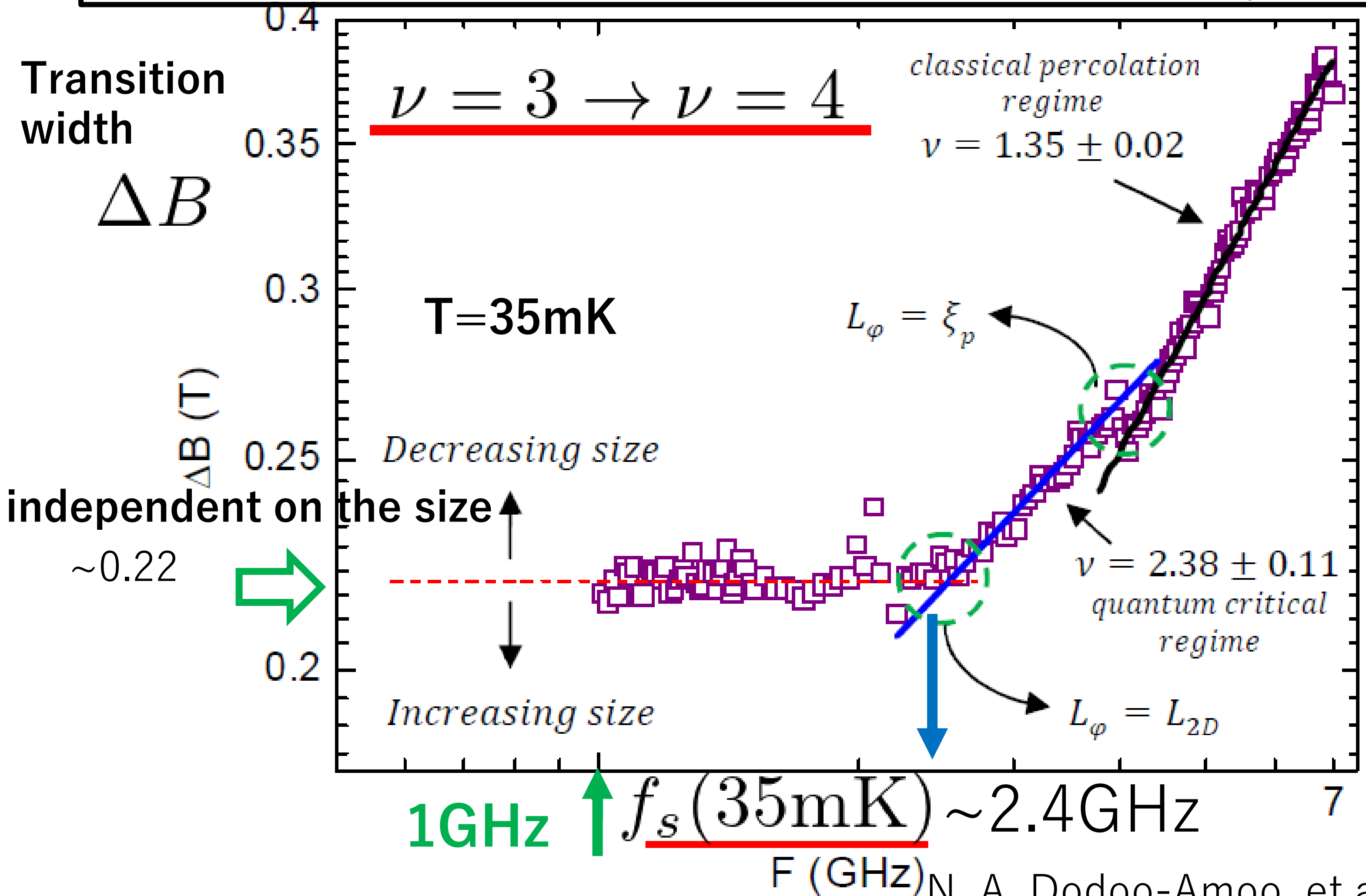
But we have an experiment which shows that saturation frequency is 2.4GHz in spite of lower temperature and larger system size than before. This is unexpected !!!

experiments of quantum Hall effect with axion effect

as frequency f decreases ⟹ axion microwave dominant, not saturation of mobility gap

$$\Delta E_f = 2\delta(35\mathrm{mK}, f) \Longrightarrow \Delta E_f = 2\delta(35\mathrm{mK}, m_a/2\pi) \quad \text{for } f \le m_a/2\pi$$

Lowest T=35mK, **largest size** $22mm \times 30\mu m \simeq 6.6 \times 10^{-3}\mathrm{cm}^2$



N. A. Dodoo-Amoo, et.al. **(2013)**.

Before mobility gap $\delta(f)$ saturates, axion microwave becomes dominant over external microwave at frequency $f = \frac{m_a}{2\pi}$

It is the saturation frequency

$$f_s \simeq 2.4\mathrm{GHz}$$

$$2\pi f_s(35\mathrm{mK}) = m_a$$

$$\simeq 10^{-5}\mathrm{eV}$$

We have two different microwaves; external microwave with frequency f and axion microwave with frequency $\frac{m_a}{2\pi}$

for high frequency f $m_a < 2\pi f$

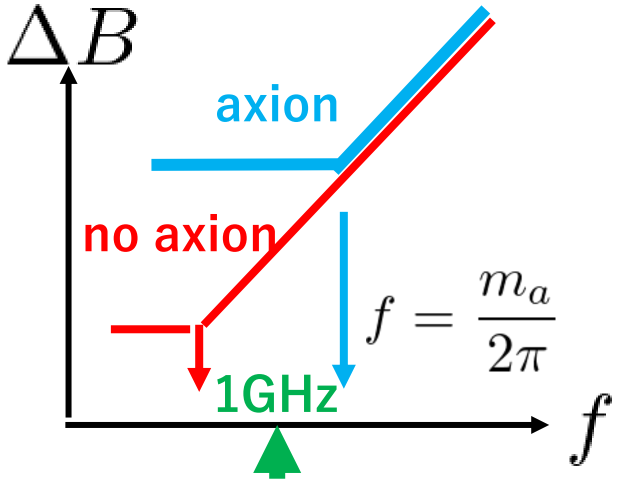
$$\Delta E_f = 2\delta(f)$$ （external microwave dominant）

transition at the frequency $f = \frac{m_a}{2\pi}$

for low frequency f $m_a > 2\pi f$

$$\Delta E_f = 2\delta(m_a/2\pi)$$ （axion dominant）

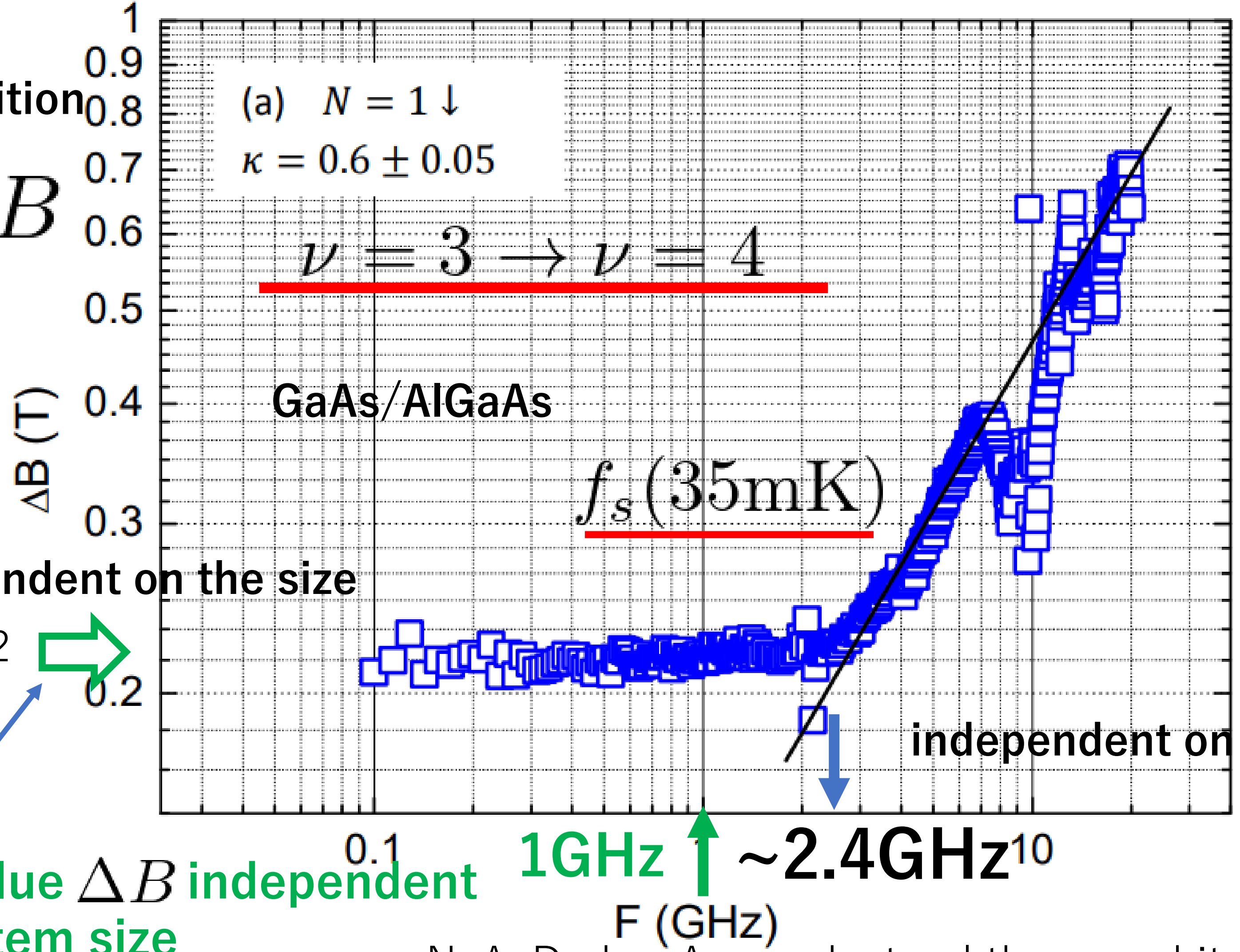
Mobility gap $\delta(f)$ saturates at much lower frequency than 1GHz when there is no axion microwave.



Other evidence of axion

Independence of $f_s(L) \simeq 2.4\text{GHz}$ on system size L

35mK, smaller $5.5\text{mm} \times 30\mu\text{m} \simeq 1.7 \times 10^{-3}\text{cm}^2$



Transition width ΔB

$\nu = 3 \rightarrow \nu = 4$

GaAs/AlGaAs

$f_s(35\text{mK})$

independent on the size

~0.22

independent on the size

The value ΔB independent on system size

1GHz

~2.4GHz

Change of system size does not make $f_s(L) \simeq 2.4\text{GHz}$ change

Saturation frequency is independent on the system size

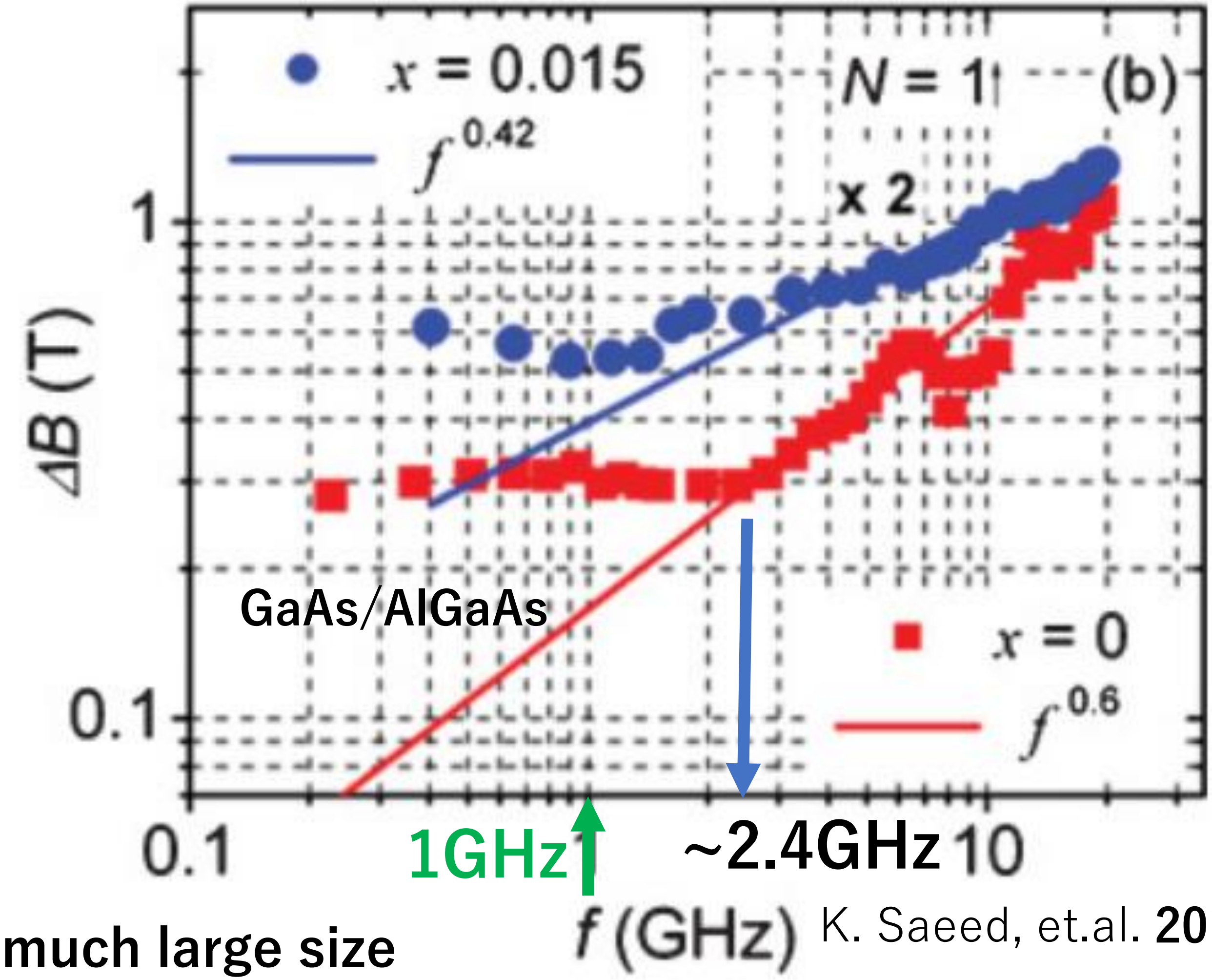
N. A. Dodoo-Amoo, doctoral theses, whiterose. ac. uk **(2013).**

further evidence of axion

$22mm \times 30\mu m \simeq 6.6\times 10^{-3}\mathrm{cm}^2$

Independence of $f_s(\nu=n\rightarrow n+1)\simeq 2.4\mathrm{GHz}$ on each plateau transitions

$\nu=4\rightarrow\nu=5$



GaAs/AlGaAs

1GHz

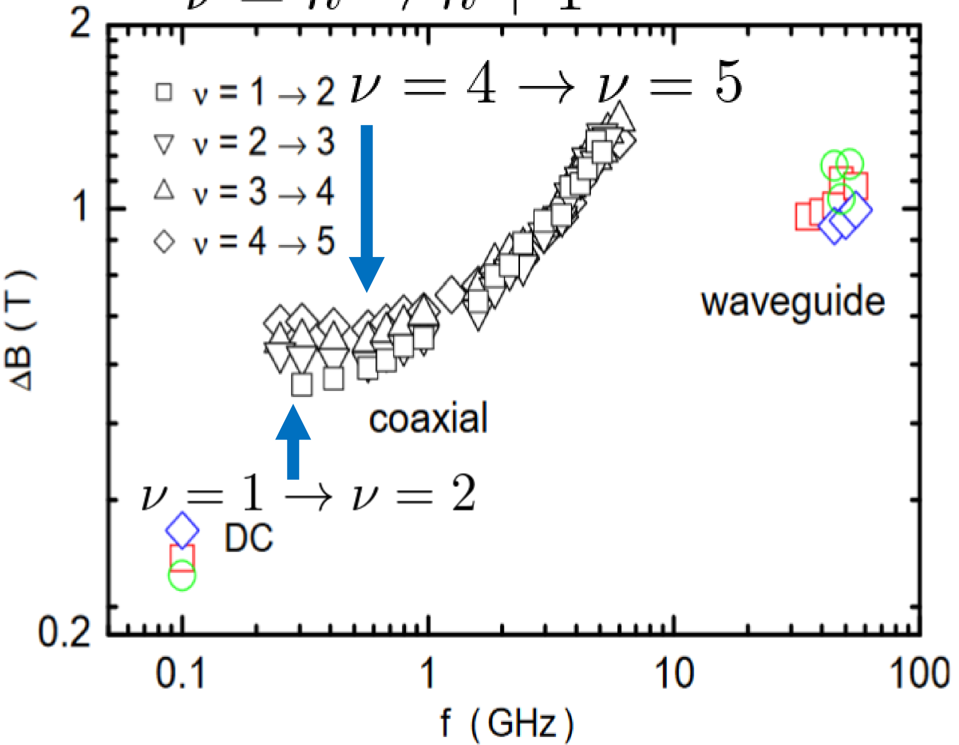
~2.4GHz

much large size

K. Saeed, et.al. **2011**

In general, saturation frequency depends on each plateau transition

$\nu=n\rightarrow n+1$



$\nu=4\rightarrow\nu=5$

$\nu=1\rightarrow\nu=2$

$\nu=3\rightarrow\nu=4$

$\nu=4\rightarrow\nu=5$

Saturation frequency is independent on each plateau transition

We show
independence of saturation frequency on system size and each plateau transition. Furthermore, transition width is also independence on system size.

These properties can be understood in the following. That is, as frequency f decreases, frequency dominance changes to axion microwave dominance at $2\pi f_s(35\mathrm{mK}) = m_a$
because microwave with higher frequency $\simeq 10^{-5}\mathrm{eV}$
is dominant for the determination of transition width ΔB

Such an axion effect can be observable in low temperature, that is, dominant over thermal effect.

We compare axion contribution with thermal contribution using QCD axion model

QCD axion parameter $g_{a\gamma\gamma} = g_\gamma \alpha/(\pi f_a)$, $f_a m_a \simeq 6 \times 10^{-3}\mathrm{GeV}^2$, dark matter density $\rho_d = 0.3\mathrm{GeVcm}^{-3}$

Number of electrons excited to mobility gap; $E_c - \delta < E < E_c - \delta + \delta E$
by axion microwaves

$$\delta N_a \sim 2.1 \times 10^7 s^{-1} \Big(\frac{A}{10^5}\Big)^2 \Big(\frac{S}{6.6 \times 10^{-3}\mathrm{cm}^2}\Big)^2 \Big(\frac{0.5 \times 10^{-4}\mathrm{eV}}{\Delta E}\Big)^2 \Big(\frac{\rho_d}{0.3\,\mathrm{GeVcm}^{-3}}\Big) \Big(\frac{B}{3.4\mathrm{T}}\Big)^3 \Big(\frac{m_a}{10^{-5}\mathrm{eV}}\Big) \Big(\frac{g_\gamma}{1.0}\Big)^2 \Big(\frac{\delta E}{m_a}\Big) \tau$$

Number of electrons thermally excited to mobility gap; $E_c - \delta < E < E_c - \delta + \delta E$

$$\delta N_t \simeq 7.4 \times 10^7 \exp(-\frac{m_a}{T}) \Big(\frac{S}{6.6 \times 10^{-3}\mathrm{cm}^2}\Big) \Big(\frac{B}{3.4T}\Big) \Big(\frac{0.5 \times 10^{-4}\mathrm{eV}}{\Delta E}\Big) \Big(\frac{\delta E}{m_a}\Big)$$

The ratio

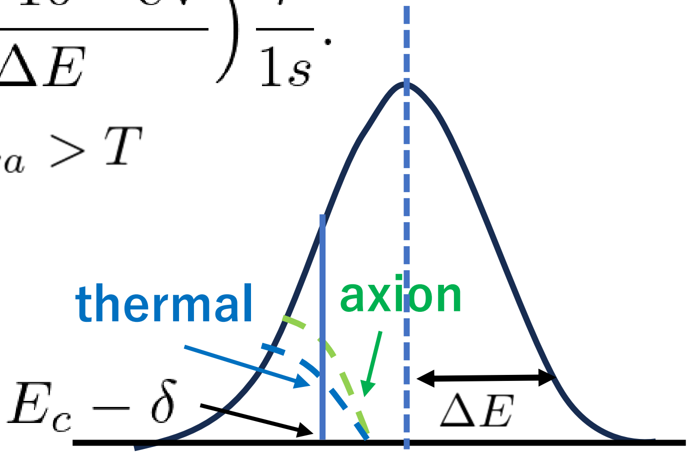
$$\frac{\delta N_a}{\delta N_t} \simeq 2.8 \times 10^{-1} \exp(\frac{m_a}{T}) \Big(\frac{S}{6.6 \times 10^{-3}\mathrm{cm}^2}\Big) \Big(\frac{B}{3.4T}\Big)^2 \Big(\frac{A}{10^5}\Big)^2 \Big(\frac{0.5 \times 10^{-4}\mathrm{eV}}{\Delta E}\Big) \frac{\tau}{1s}.$$

for $m_a > T$

very sensitive for the ratio m_a/T

axion dominant $\frac{\delta N_a}{\delta N_t} > 1$ for $T < 50\mathrm{mK}$

when $m_a \simeq 10^{-5}\mathrm{eV} \simeq 116\mathrm{mK}$

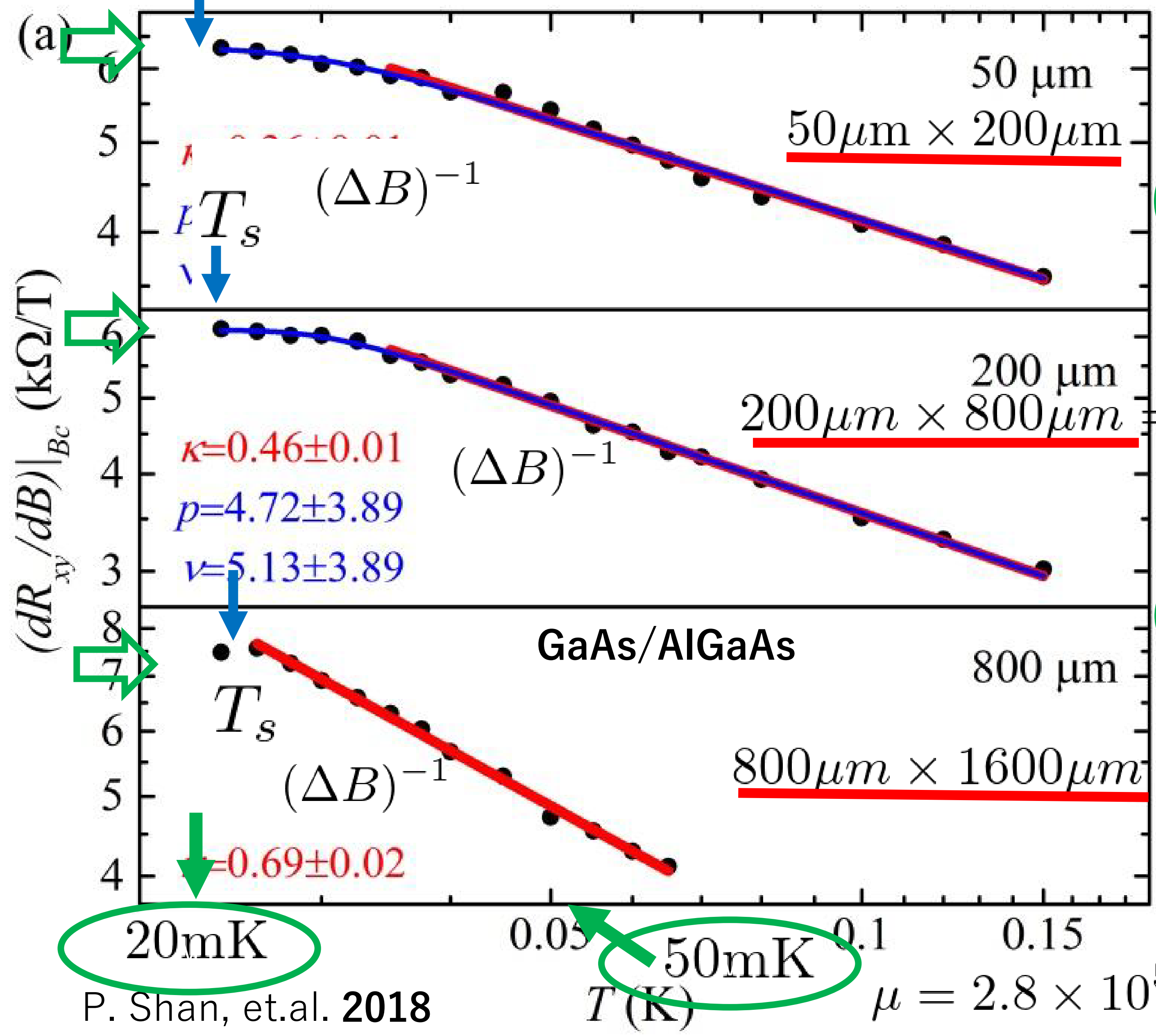


Although we have unknown parameters in the estimation such as extension ΔE of density of state , or life time τ of electrons staying in mobility gap, the result heavily depends on temperature.
So, when the temperature is in a range 10mK~30mK,
the axion contribution can be dominant over thermal one,
when system size is sufficiently large.

Similar axion dominance over temperature has been observed
in recent experiment at temperature ~20mK. It has been shown
that saturation temperature and saturated transition width are
independent on system size.

Axion dominates over thermal effect

$(\Delta B)^{-1}T_s$



$50\mu m \times 200\mu m$

$(\Delta B)^{-1}$

T_s

$200\mu m \times 800\mu m = 1.6 \times 10^{-3}\mathrm{cm}^2$

$(\Delta B)^{-1}$

GaAs/AlGaAs

T_s

$(\Delta B)^{-1}$

$800\mu m \times 1600\mu m = 1.28 \times 10^{-2}\mathrm{cm}^2$

20mK

50mK

P. Shan, et.al. **2018**

$\mu = 2.8 \times 10^5\mathrm{cm}^2/\mathrm{Vs}$

at $T_s \sim 20\mathrm{mK}$

Temperature dominant

$$\Delta E_f = 2\delta(T)$$

for $T > T_s \simeq 20\mathrm{mK}$

Axion dominant

$$\Delta E_f = 2\delta(T, m_a/2\pi)$$

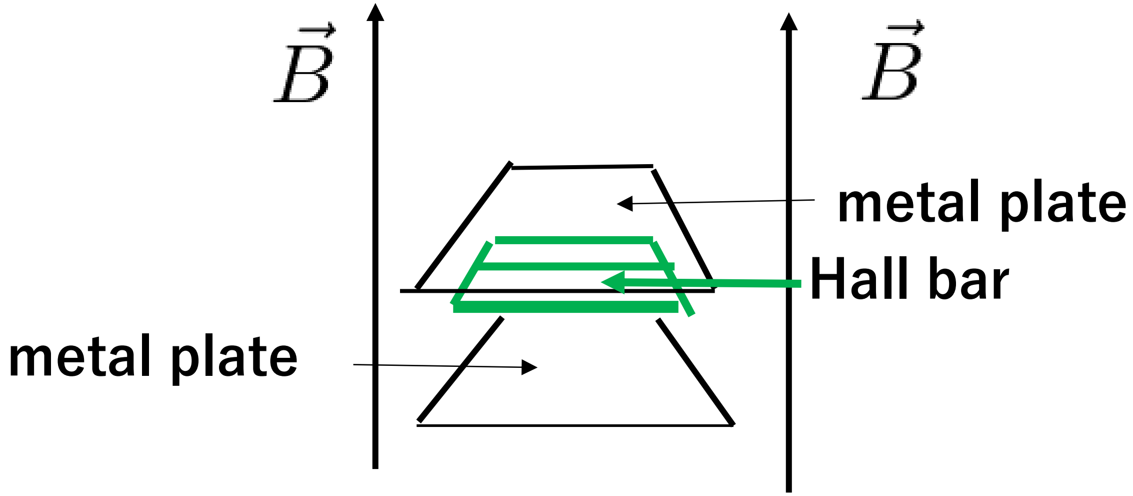
for $T \leq T_s \simeq 20\mathrm{mK}$

Transition width

ΔB **independent on the size**

Confirmation of existence of axion dark matter

Shielding axion microwave with metal plates



Saturation frequency $f_s \simeq 2.4\mathrm{GHz}$ and saturation temperature $T_s \sim 20\mathrm{mK}$ change or not ?

conclusion

We have shown that evidence of axion dark matter can be seen in quantum Hall effect when temperature is lower than 50mK and system size is larger than $10^{-3}\mathrm{cm}^2$

We have found that axion mass is nearly equal to $10^{-5}\mathrm{eV}$

To confirm the presence of the axion, we have proposed an experiment in which axion radiation is shielded and see whether or not saturation frequency is changed.