



Decoding quantum information from chaos beyond the standard situation

Yoshifumi Nakata
(YITP, Kyoto University)



Quantum Information is an **interdisciplinary** field between **Physics** and **Computer Science**.

10-15 years ago

Computer
Science

Computing

Error Correction

Non-locality and security

Complexity problem



Topological order

Thermalization

Entanglement

Q field theory

Quantum Information

Physics



Quantum Information is an **interdisciplinary field** between **Physics** and **Computer Science**.

Currently....

Computer

Physics

Fault-Tolerant Quantum Computation

Quantum Communication

Quantum Internet

Quantum Cryptography



Quantum Information



Simulating quantum systems,
e.g. QCD, high energy physics, etc..

Quantum Information as a Tool.



Quantum Information is an interdisciplinary field between **Physics** and **Computer Science**.

Currently....

Physically-relevant
Quantum Error Correction

There are still many things in **Quantum Information** that can be explored based on the **physics intuition**.

Quantum Cryptography

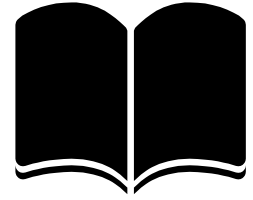
Quantum Information

Simulating quantum systems,
e.g. QCD, high energy physics, etc..

We may use quantum technology to
better understand physics.

Physics

Outline



1. **Introduction: Quantum Error Correction & physics**
2. **Decoding the Hayden-Preskill protocol**
3. **Conclusion**

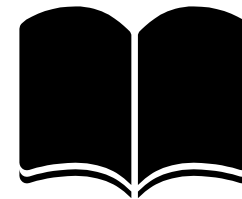


[YN, T. Matsuura, and M. Koashi, 2210.06661 (2022)]

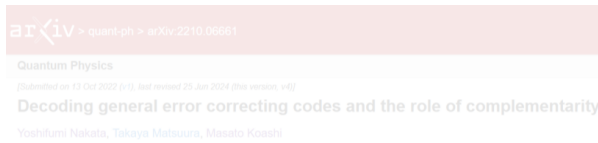


[T. Utsumi & YN, 2405.06051 (2024)]

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1. **Introduction: Quantum Error Correction & physics**
2. Decoding the Hayden-Preskill protocol
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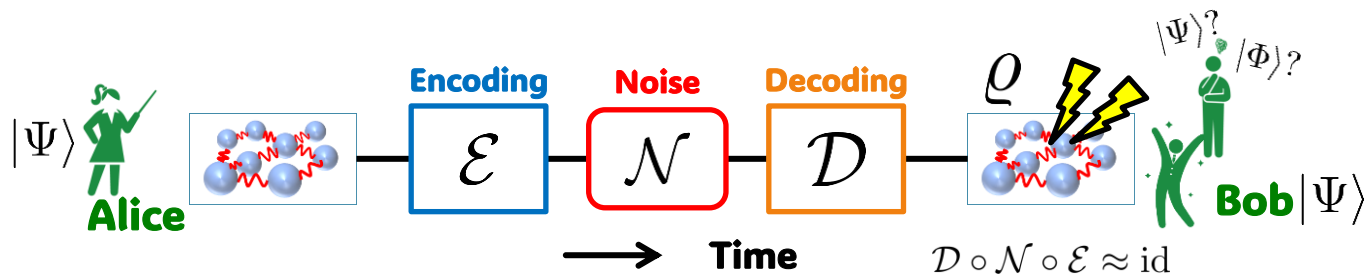
1.

Introduction

- 1. The ABC's of Quantum Error Correction (QEC)**
- 2. Why should we, physicists, care about QEC?**

Introduction - Quantum Error Correction -

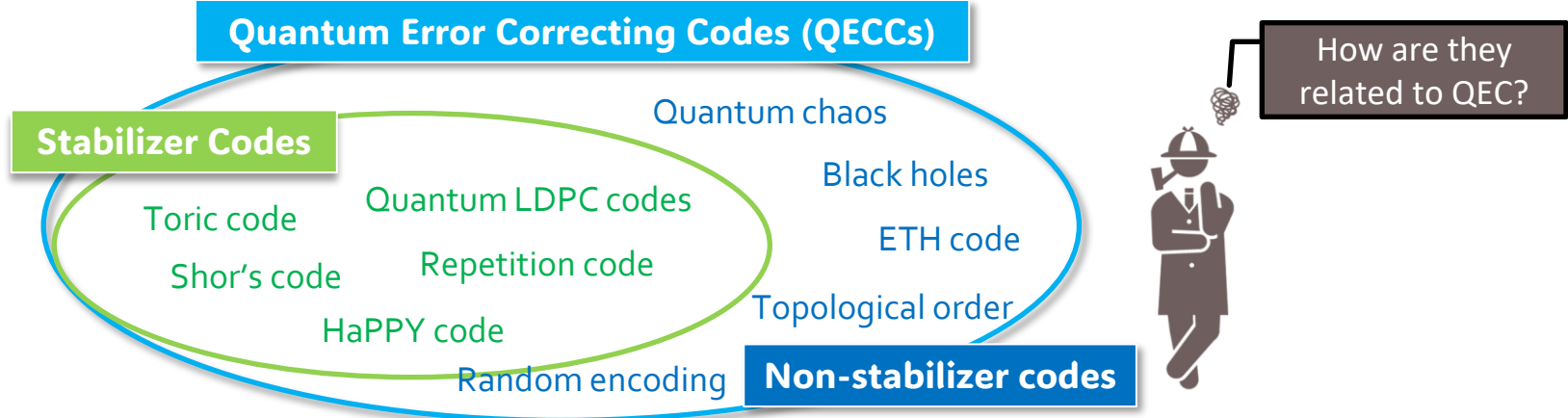
- **Quantum Error Correction (QEC)** is a method to effectively cancel noise in a quantum system.
 - QEC is a key to achieve **a large-scale quantum information processing**.
 - Growing interest in theoretical physics.
 - **Quantum gravity, quantum chaos, and new quantum phases** related to QEC.



Quantum Error Correcting Code (QECC) = $(\mathcal{E}, \mathcal{D})$

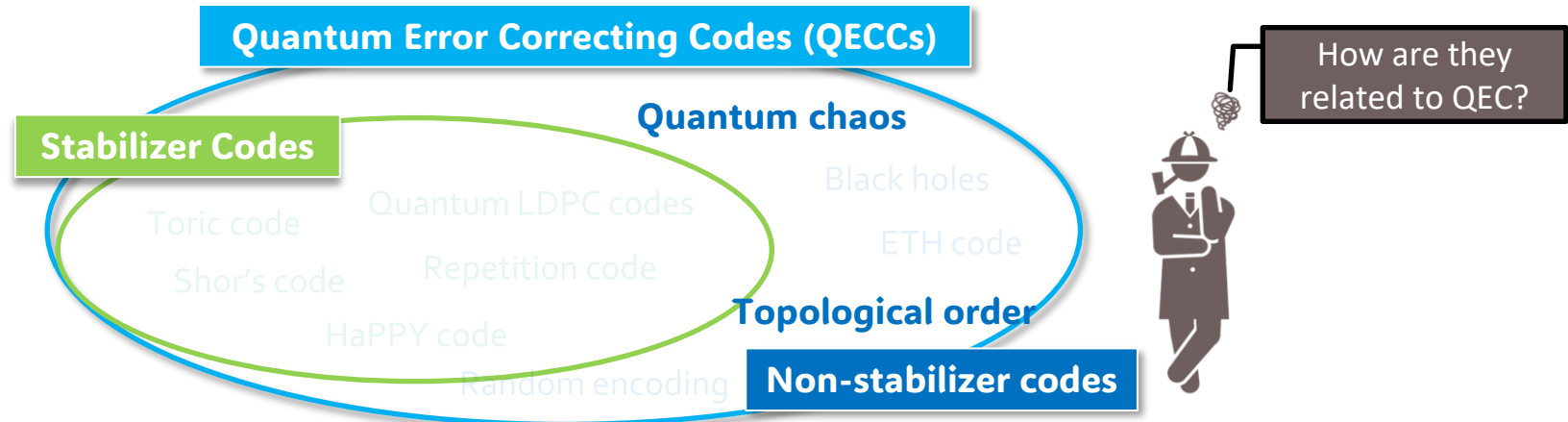
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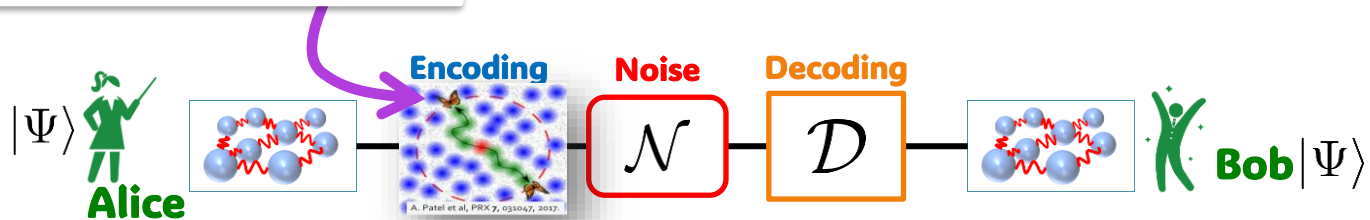
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Introduction - QEC and Quantum Chaos -

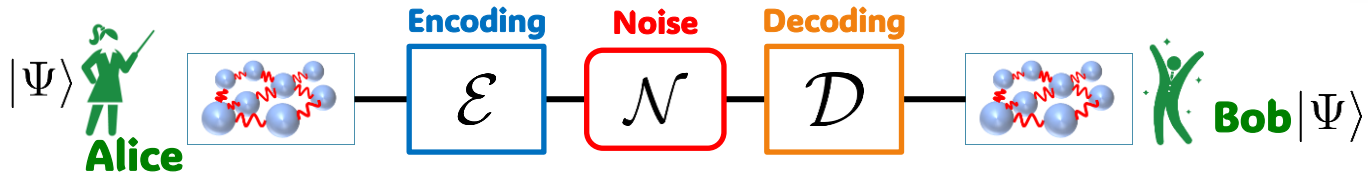
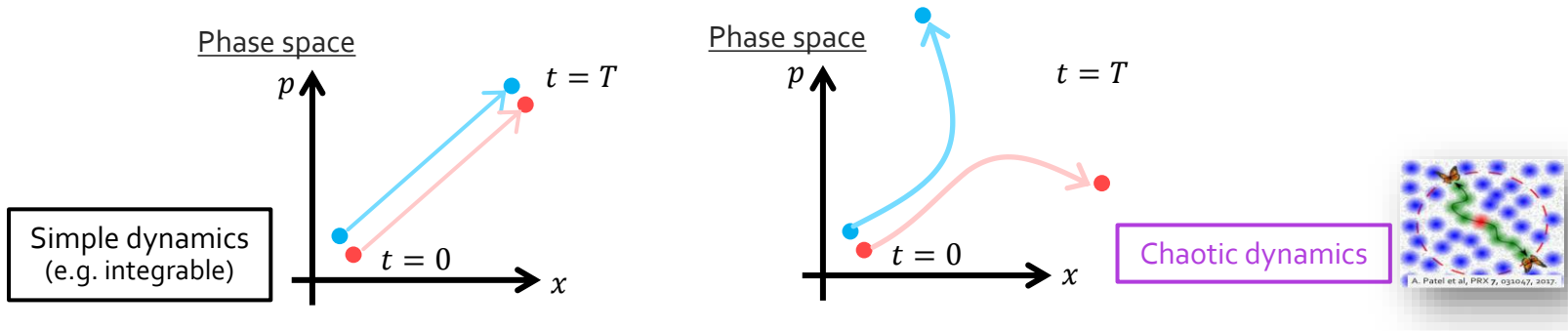
- The idea is to use **chaotic dynamics** for **encoding** quantum states.

Use quantum chaotic dynamics!



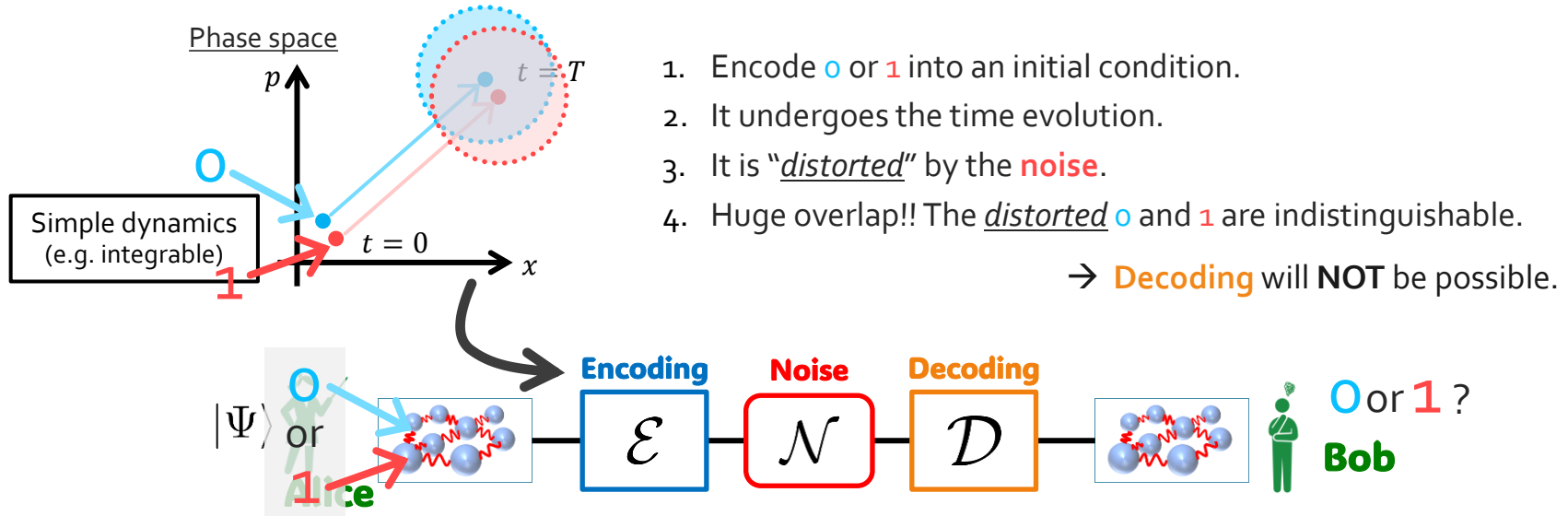
Introduction - QEC and Quantum Chaos -

- The idea is to use **chaotic dynamics** for **encoding** quantum states.
- **Chaotic dynamics** is sensitive to initial conditions (at least **classically**).



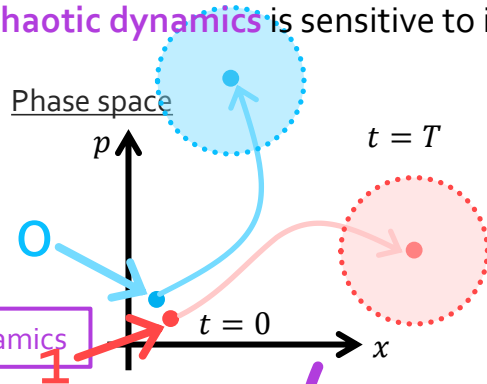
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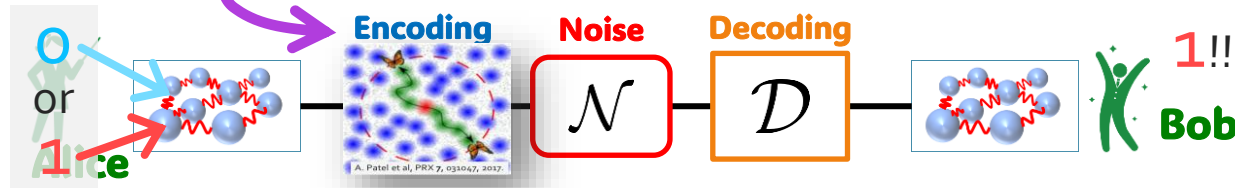
Introduction - QEC and Quantum Chaos -

- ❑ The idea is to use **chaotic dynamics** for **encoding** quantum states.
- ❑ **Chaotic dynamics** is sensitive to initial conditions (at least **classically**).



1. Encode **0** or **1** into an initial condition.
2. It undergoes the **chaotic** time evolution.
3. It is "distorted" by the **noise**.
4. No overlap!! The distorted **0** and **1** are distinguishable.

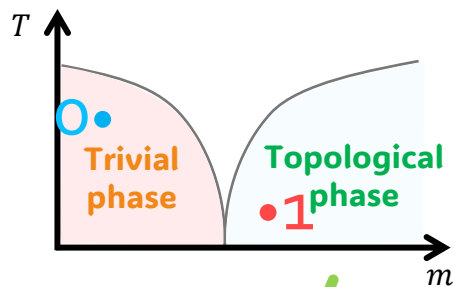
→ **Decoding** should be possible.



Introduction - QEC and Topological Order -

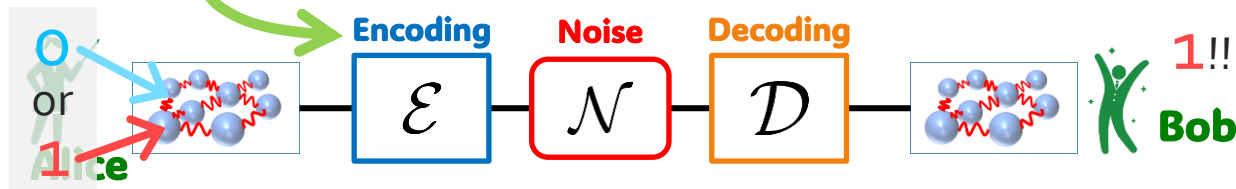
- Similarly, we can use a **topological order** for **encoding** quantum states.

Phase diagram



1. Encode **0** into a state in a **topological phase** and **1** into a state in a **trivial phase**.
2. **No local noise** can change the phase.

→ The information **0** and **1** is protected by the **topological order**.
→ **Decoding** should be possible.



Introduction - QEC in Physics -

- ❑ **Chaotic dynamics** is sensitive to initial conditions (at least **classically**).
- ❑ **Topological order** is not broken by local operations.

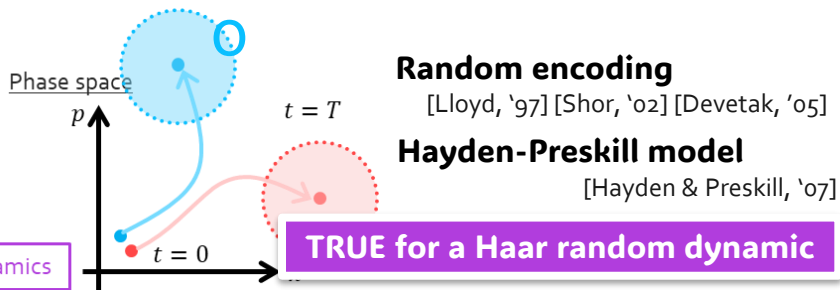
- ❑ **Caution!**

1. **QUANTUM** case is not so trivial due to, e.g., **coherence** ($\alpha|0\rangle + \beta|1\rangle$), not **0** or **1**.
2. Unitary dynamics does **NOT** change the distance.

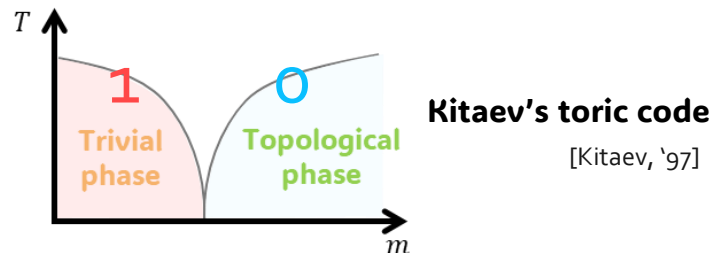


Physics intuition is useful for constructing QECCs!

By sharpening the intuition, they are turned out to be useful for QEC with suitable settings.

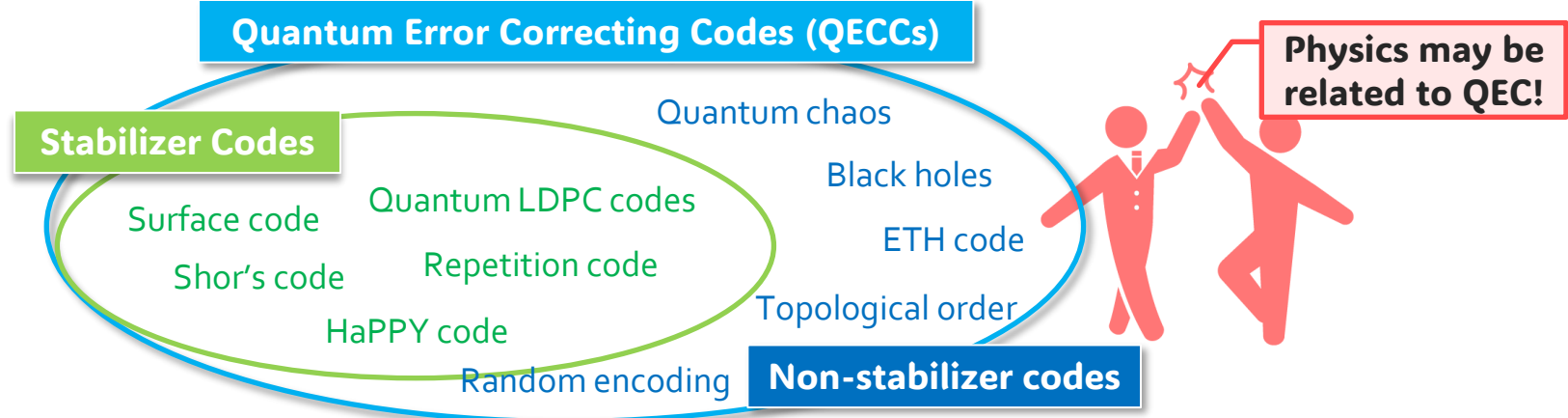


Phase diagram



Introduction - Quantum Error Correction -

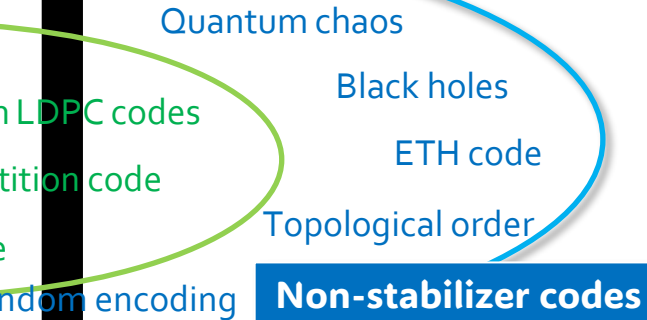
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Correcting Codes (QECCs)



In this talk, we will focus on **non-stabilizer codes**.



Features of non-stabilizer codes

- 😊 Many **physically-relevant QECCs**.
- 😊 They sometimes have **better code performance** than stabilizer ones.
- 😞 **Not as simple as stabilizer codes.**
 - In particular, **decoding** is highly non-trivial.

Introduction - Decoding non-stabilizer codes -

- Recently, I have been working on the decoding problem of non-stabilizer codes.

arXiv > quant-ph > arXiv:2210.06661

Quantum Physics

[Submitted on 13 Oct 2022 (v1), last revised 25 Jun 2024 (this version, v4)]

Decoding general error correcting codes and the role of complementarity

Yoshifumi Nakata, Takaya Matsuura, Masato Koashi

- Based on the complementarity principle.
- Construct a decoder from two “classical decoders”.

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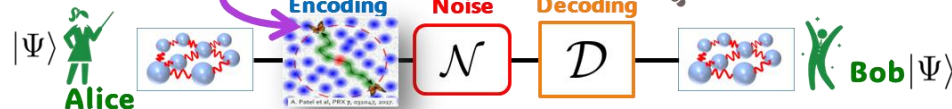
Explicit decoders using fixed-point amplitude amplification based on QSVT

Takeru Utsumi, Yoshifumi Nakata

- Clever use of a quantum algorithm (QSVT).
- Generalization of the Yoshida-Kitaev decoder.



Use, e.g., quantum chaotic dynamics!



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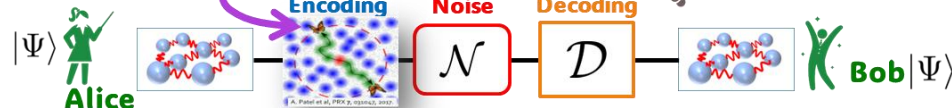
Quantum Physics

We will focus on this construction and demonstrate it in a toy model of QEC, that is, the Hayden-Preskill model.

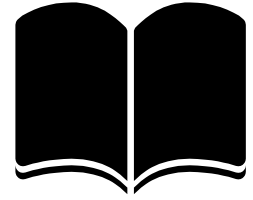
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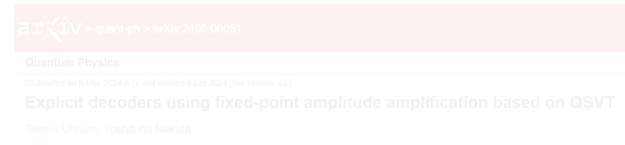
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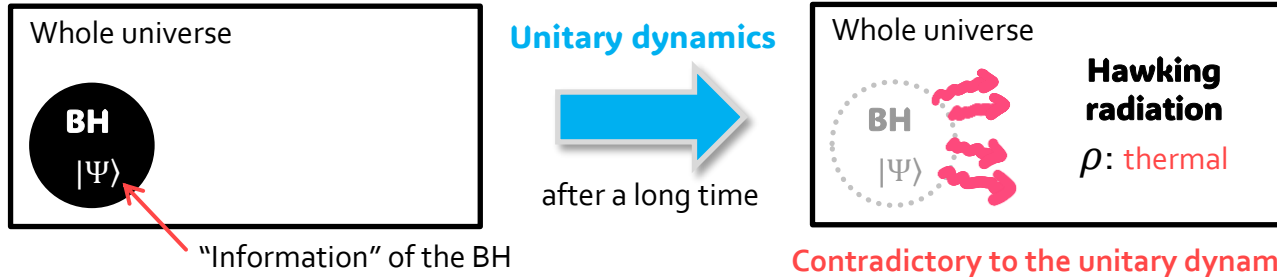


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Decoding the Hayden-Preskill

□ What is the **Hayden-Preskill model**?

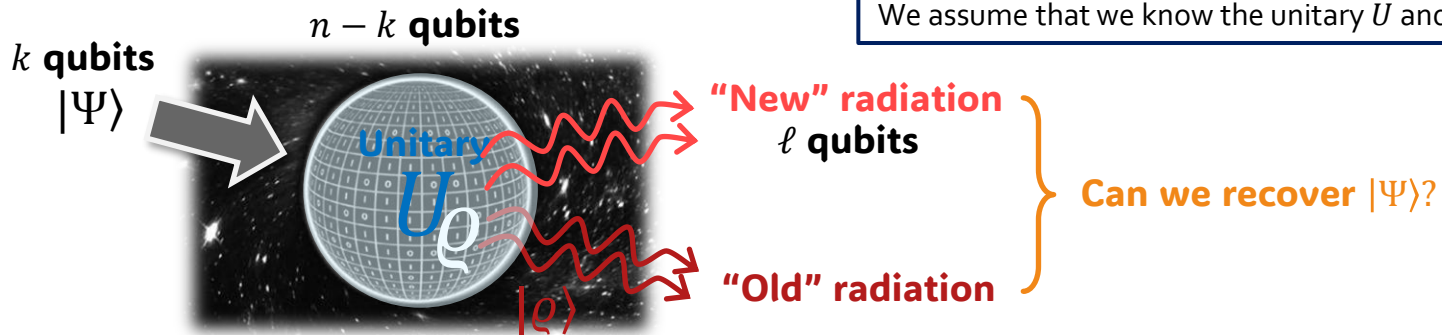
- A (oversimplified) toy model of the **black hole information paradox**



- If we believe **unitarity**, the "information" of the BH should be recoverable from the radiation.
- **Motivation of the Hayden-Preskill: how much radiation is needed for the recovery?**

Decoding the Hayden-Preskill

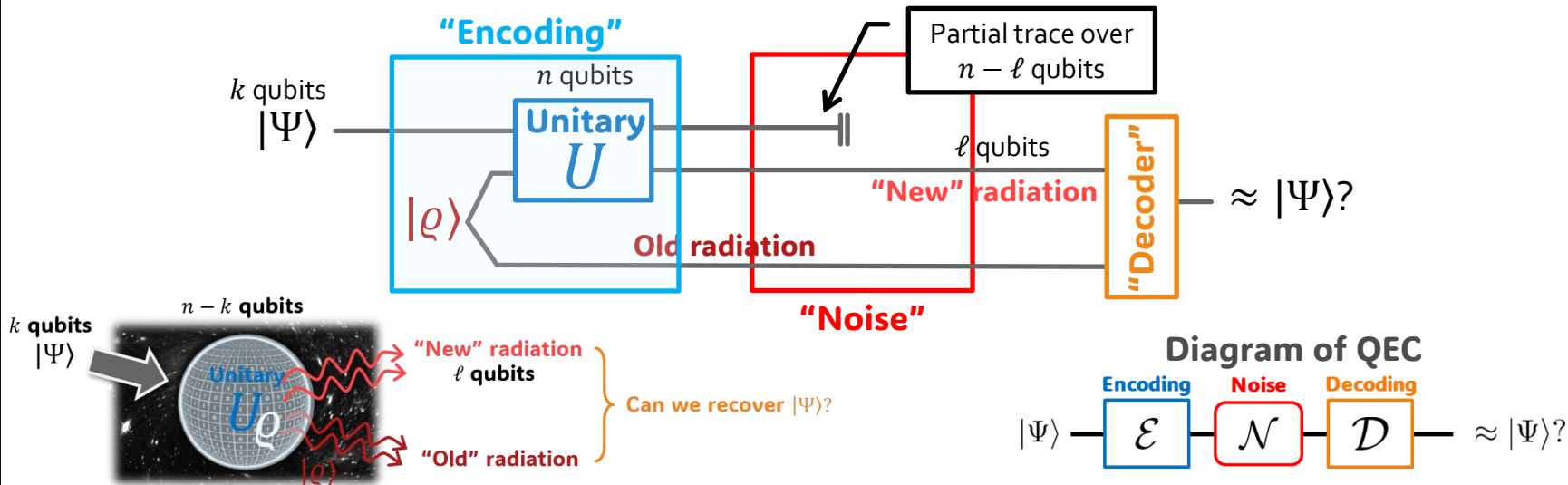
- What EXACTLY is the Hayden-Preskill model?
 - A (oversimplified) toy model of the black hole information paradox



- If $\ell = n$, $|\Psi\rangle$ is recovered by U^\dagger .
- How large should ℓ be for the recovery of the k -qubit state $|\Psi\rangle$ to be possible?

Decoding the Hayden-Preskill

- What EXACTLY is the **Hayden-Preskill model**?
 - A (oversimplified) toy model of the **black hole information paradox**
 - The Hayden-Preskill model is also a simple toy model of **QEC**.



Decoding the Hayden-Preskill

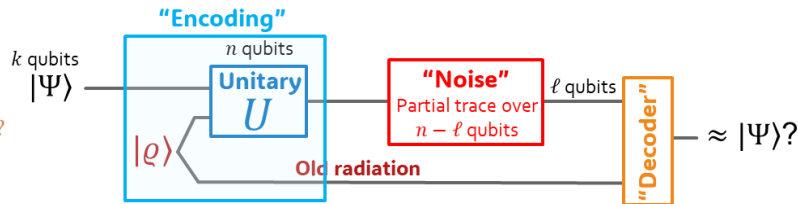
□ What EXACTLY is the **Hayden-Preskill model**?

- A (oversimplified) toy model of the **black hole information paradox**
- The Hayden-Preskill model is also a simple toy model of QEC.

1. **Encoding** = "BH unitary dynamics"
2. **Noise** = the partial trace over $n - \ell$ qubits

□ Hayden & Preskill used a standard **QEC** technique, and showed the following.

If U is Haar random \Rightarrow the recovery error $\Delta \leq 2^{\frac{\ell_{th} - \ell}{2}}$ where $\ell_{th} = (n + k - H_2(\rho))/2$.



Decoding the Hayden-Preskill

- Hayden & Preskill used a standard QEC technique, and showed the following.

If U is Haar random \Rightarrow the recovery error $\Delta \leq 2^{\frac{\ell_{th}-\ell}{2}}$ where $\ell_{th} = (n + k - H_2(\rho))/2$.

- Dep. on the initial entropy of the BH, ℓ_{th} ranges from k (for max entropy) to $(n + k)/2$ (for zero entropy).
- Unfortunately, the proof does not EXPLICITLY provide a decoder.

How can we explicitly “decode” the Hayden-Preskill protocol?



Decoding the Hayden-Preskill

How can we explicitly “decode” the Hayden-Preskill protocol?

1. **The Petz recovery map** [Barnum & Knill, JMP '02]

Good: it works for *any noise*.

Bad: *inefficient*, and too complicated to improve the construction.

2. **Yoshida-Kitaev decoder** [Yoshida & Kitaev, '17]

Good: **Clear Q. circuit** construction.

Bad: *inefficient*, and *works only for decoding the Hayden-Preskill protocol*.



Decoding the Hayden-Preskill

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3. **Classical-to-Quantum decoder** [YN, Matsuura & Koashi, '22]

Good: Reduces the problem to a **CLASSICAL** one (= hopefully easy to improve), and works for **any noise**.

Bad: **inefficient**.



Decoding the Hayden-Preskill

How can we explicitly “decode” the Hayden-Preskill protocol?

- The Petz(-like) recovery map** [Barnum & Knill, JMP '02] [Utsumi & Nakata, '24]
Good: it works for *any noise*.
Bad: slightly improved, but *inefficient*.
- (generalized) Yoshida-Kitaev decoder** [Yoshida & Kitaev, '17] [Utsumi & Nakata, '24]
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Decoding the Hayden-Preskill

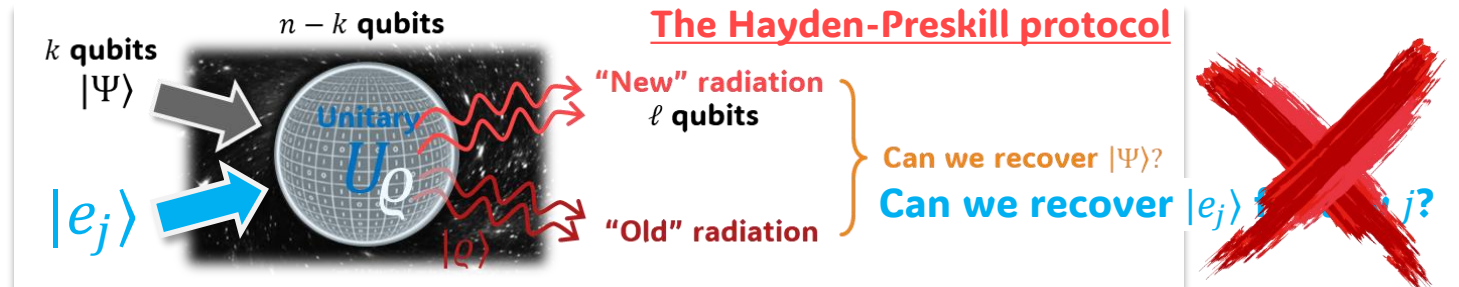
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Good: Reduces the problem to a **CLASSICAL** one (= hopefully easy to improve), and works for *any noise*.
Bad: *inefficient*.

The decoder works for any noise, but we'll focus on the HP protocol.

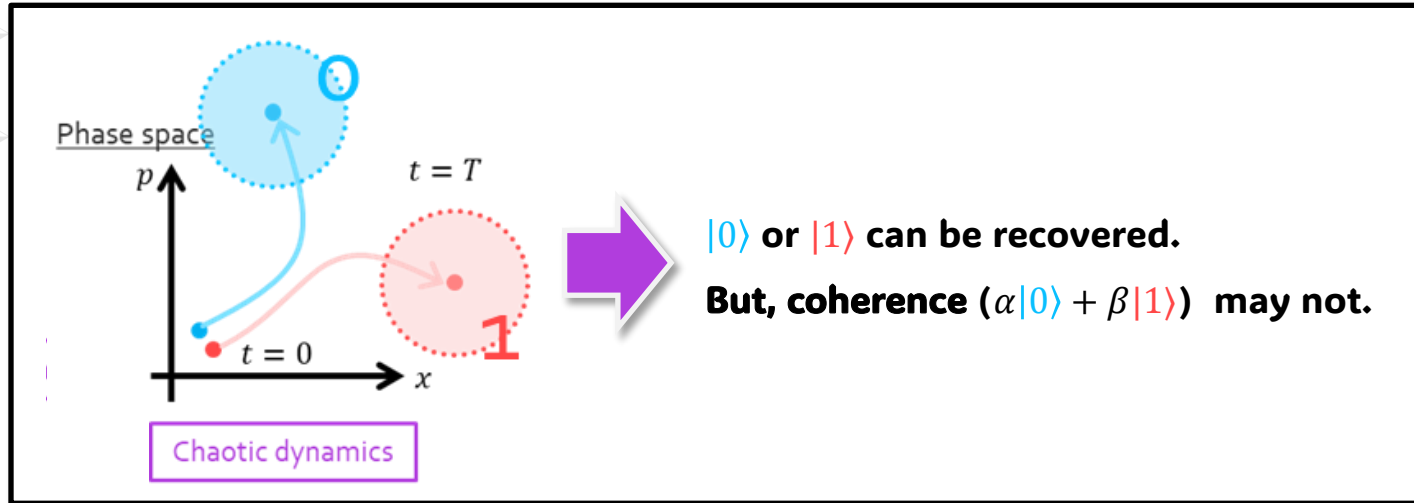
Classical-to-Quantum decoder

- Goal: to construct a **decoding quantum circuit** for general **QECCs**, incl. the Hayden-Preskill.
 - **A difficulty**: need to consider **ALL** state $|\Psi\rangle \in \mathcal{H}^A$.
 - $|\Psi\rangle = c_1|e_1\rangle + c_2|e_2\rangle + \dots + c_{2^k}|e_{2^k}\rangle$, where $\{|e_j\rangle\}_j$ is a basis.
 - Enough to consider **basis states** $\{|e_j\rangle\}_j$? **Answer: No, it's not enough.**



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$|e_j\rangle$

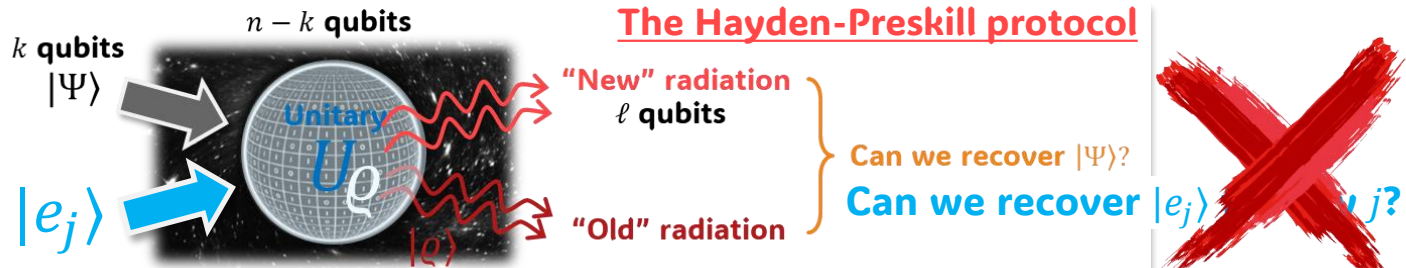
"Old" radiation

Can we recover $|e_j\rangle$ for any j ?

Classical-to-Quantum decoder

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 - Enough to consider **basis states** $\{|e_j\rangle\}_j$? **Answer: No, it's not enough.**
 - Recovery of a basis $\not\Rightarrow$ superposition (coherence) maintained.

Use the **complementarity principle** of quantum theory!

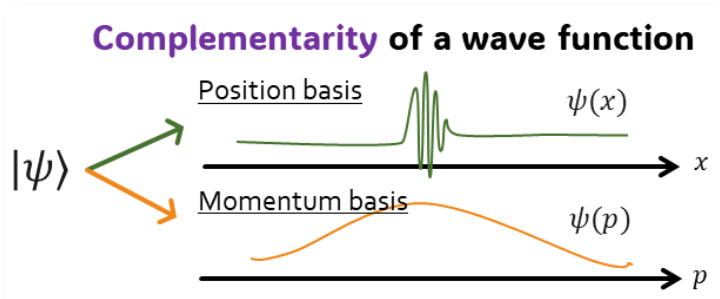


Classical-to-Quantum decoder

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Use the **complementarity principle of quantum theory!**

- If **one observable is definite**, its **complementary counterpart is indefinite**. Hence, **two complementary observables** are necessary to **FULLY describe** a quantum system.
- If we can decode "**a pair of complementary bases**", we may decode **any state** $|\Psi\rangle \in \mathcal{H}^A$!



A pair of complementary bases

Eigenbasis of the **Pauli Z** = $\{|0\rangle, |1\rangle\}$

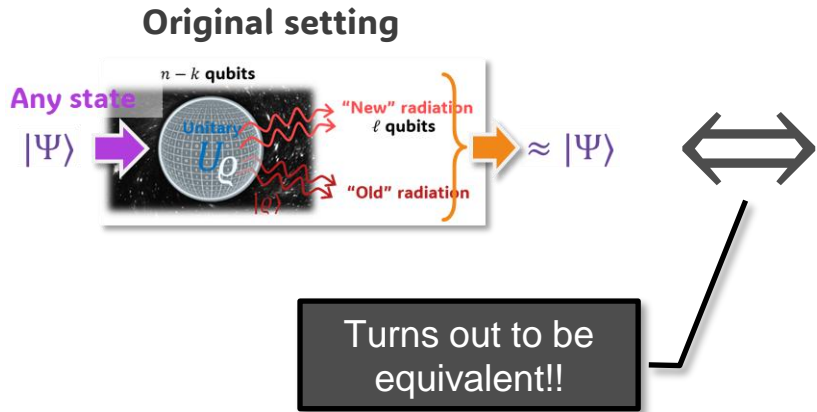
Eigenbasis of the **Pauli X** = $\{|+\rangle, |-\rangle\}$



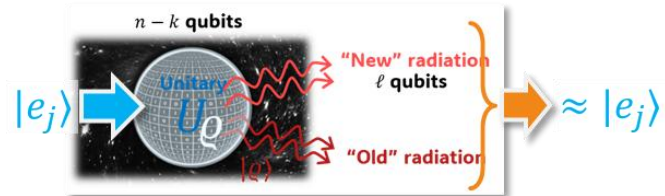
$$|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$$

Classical-to-Quantum decoder

- Goal: to construct a **decoding quantum circuit** for general **QECCs**, incl. the Hayden-Preskill.
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A basis $E = \{|e_j\rangle^A\}_{j=1,\dots,2^k}$



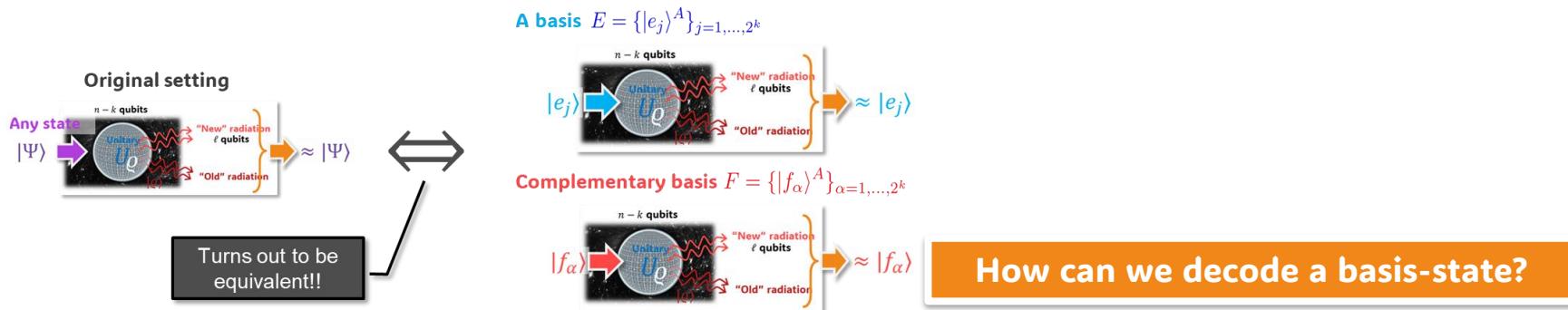
Complementary basis $F = \{|f_\alpha\rangle^A\}_{\alpha=1,\dots,2^k}$



How can we decode a basis-state?

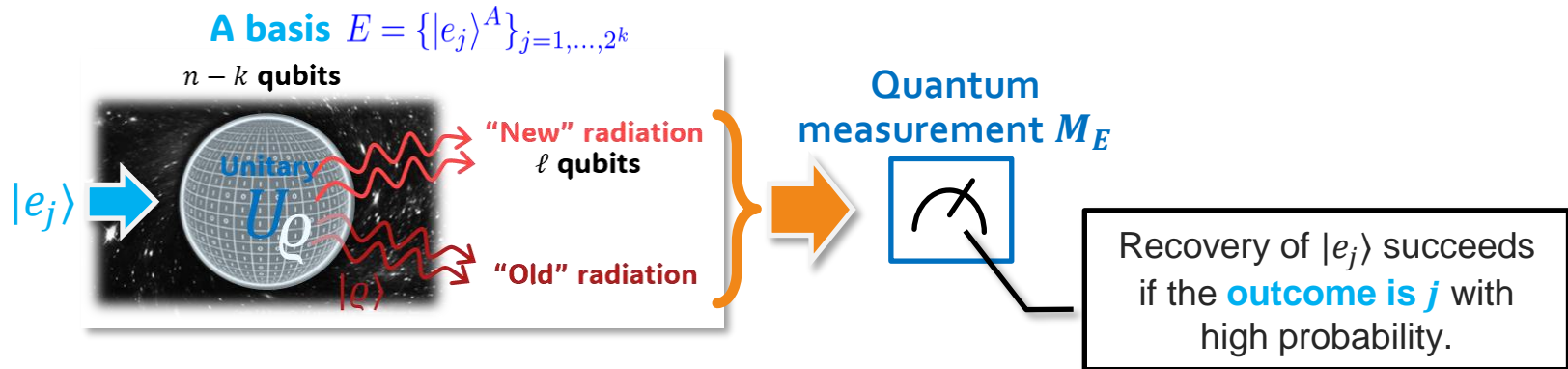
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 - The input is one of the 2^k basis states $\{|e_j\rangle\}_{j=1,\dots,2^k}$. Enough to identify **the labelling** $j \in \{1, \dots, 2^k\}$.



Classical-to-Quantum decoder

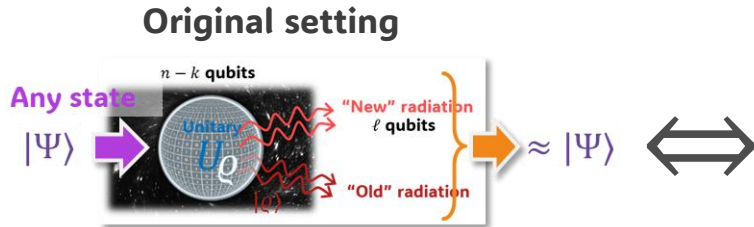
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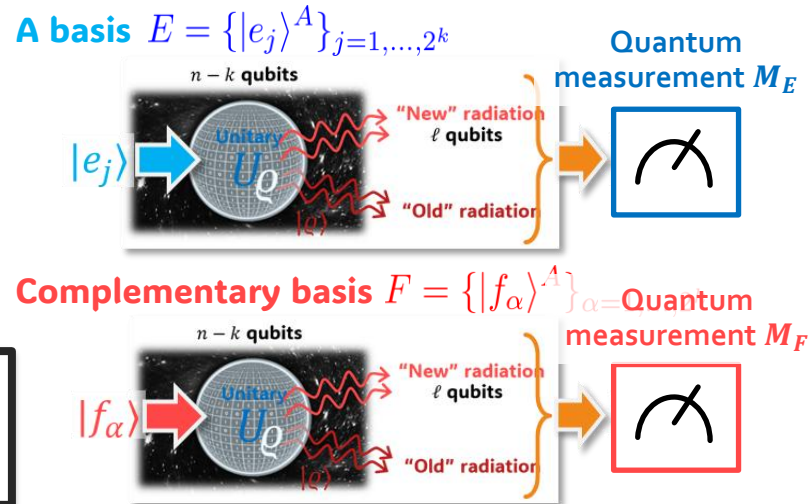
How can we decode a basis-state?

Classical-to-Quantum decoder

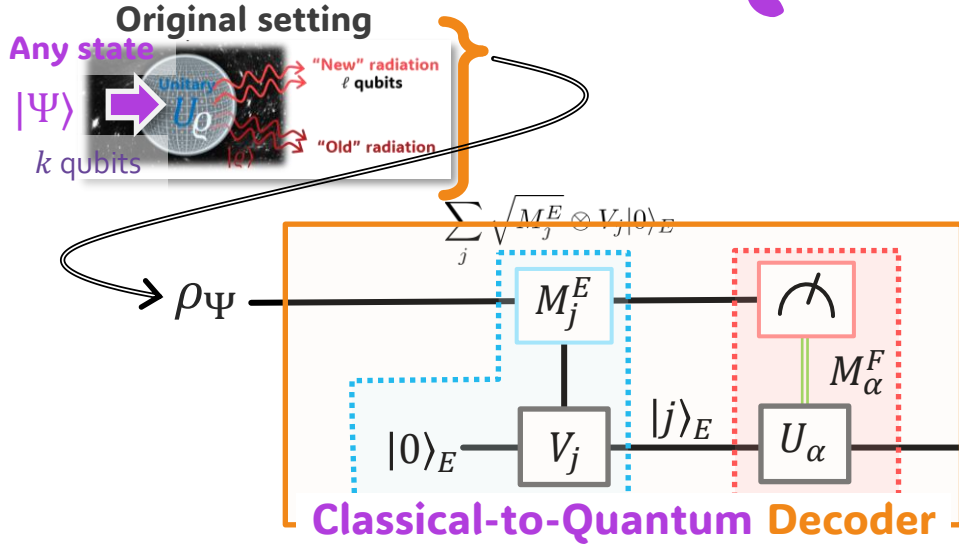
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It should be possible to construct a **decoder** from **two quantum measurements**.



Classical-to-Quantum decoder

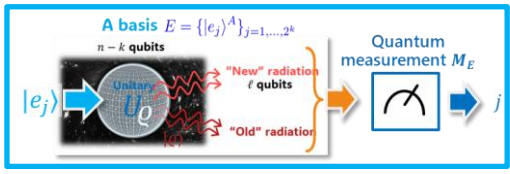


Theorem

The recovery error Δ of the **Classical-to-Quantum decoder** satisfies

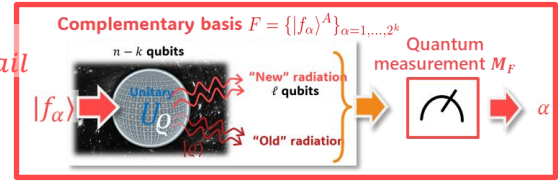
$$\Delta \leq \sqrt{P_{\text{fail}}^E (2 - P_{\text{fail}}^E)} + \sqrt{P_{\text{fail}}^F}$$

$\Delta \approx |\Psi\rangle?$



Failure probability P_{fail}^F

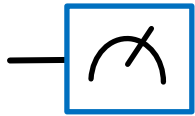
$M_F = \{M_\alpha^F\}_\alpha$



Classical-to-Quantum decoder

- Based on the **complementarity** idea, a **decoder** can be constructed.
 - If **two good Q measurements** are given, recovering the complementary bases, we can EXPLICITLY construct a **good decoder**.
 - Standard approach to find such good Q measurements → our **decoder** is **near optimal**.
 - ➔ Unfortunately, **no efficient construction** is known..., need an improvement.
- This decoder works for ANY **encoding** and **noise**.

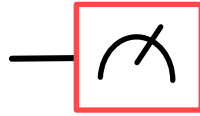
Quantum measurement M_E
for a basis E.



Failure probability P_{fail}^E



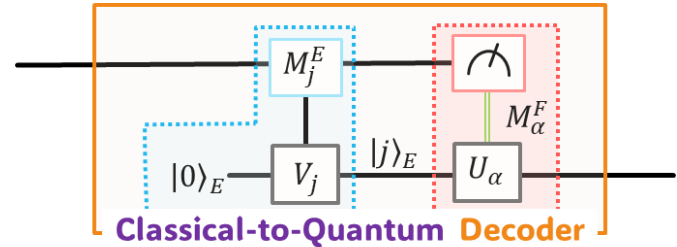
Quantum measurement M_F
for a complementary basis F.



Failure probability P_{fail}^F



$$\Delta \leq \sqrt{P_{fail}^E (2 - P_{fail}^E)} + \sqrt{P_{fail}^F}$$



Classical-to-Quantum decoder

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- This decoder works for ANY **encoding** and **noise**.



Classical-to-Quantum decoder

□ Based on the complementarity idea, a decoder can be constructed.

■ The recovery error of the decoder is given by the failure probabilities of the two Q measurements

1. **The Petz(-like) recovery map** [Barnum & Knill, JMP '02] [Utsumi & Nakata, '24]

Good: it works for *any noise*.

Bad: *inefficient*.



Could be improved, but needs full Q analysis.

2. **(generalized) Yoshida-Kitaev decoder** [Yoshida & Kitaev, '17] [Utsumi & Nakata, '24]

Good: **Clear Q. circuit** construction and works for *any noise*.

Bad: *inefficient*.



Can **NOT** be improved.

PROVEN

3. **Classical-to-Quantum decoder** [YN, Matsuura & Koashi, '22]

Good: From **CLASSICAL** ones, and works for *any noise*.

Bad: *inefficient*.



Could be improved, and **Classical analysis** suffices.

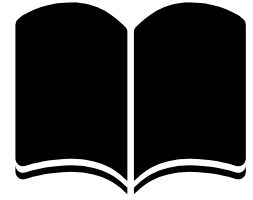
HaPPY code

Random encoding

Non-stabilizer codes

Can we find EFFICIENT measurements?

Outline



1. Introduction: Quantum Error Correction & physics
2. Decoding the Hayden-Preskill model
3. Conclusion



[YN, T. Matsuura, and M. Koashi, 2210.06661 (2022)]

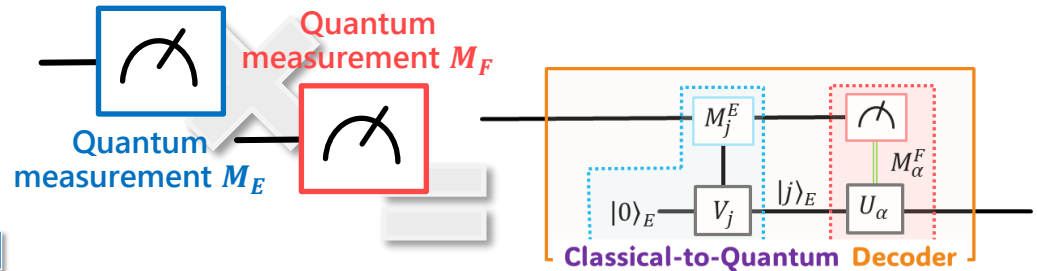
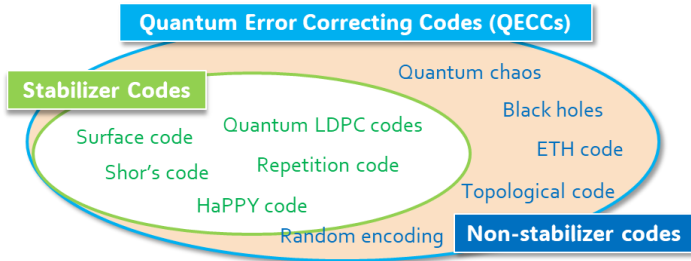


[T. Utsumi & YN, 2405.06051 (2024)]

Conclusion

- ❑ There are many **QECCs** that are related to **complex physics phenomena**.
 - They are usually **non-stabilizer codes**, so **decoding** is highly non-trivial.
- ❑ The **complementarity principle** helps the **decoding problem**
 - From **two Q measurements** for a pair of **complementary bases**, a **decoder** can be explicitly constructed.

∃ Many topics in Quantum Information that we can explore based on the physics intuition.





Advertisement

A book about Quantum Information is now available!

- Axiomatic summary of quantum mechanics.
- All basic notion in QI.
- Quantum Error Correction in detail.
- Canonical Typicality.
- Hayden-Preskill protocol.
- Haar random calculus. etc...

Will be useful both for students & experts.

Thank you for listening!

[YN, T. Matsuura, and M. Koashi, 2210.06661 (2022)]



Special thanks to

- Human pictgram 2.0
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- "Quantum Butterfly Effect in Weakly Interacting Diffusive Metals" by A. A. Patel, D. Chowdhury, S. Sachdev, and B. Swingle, PRX 7, 031047 (2007) for the figure of chaos.