TRG study of 3D Yang-Mills theories with the reduced tensor network formulation

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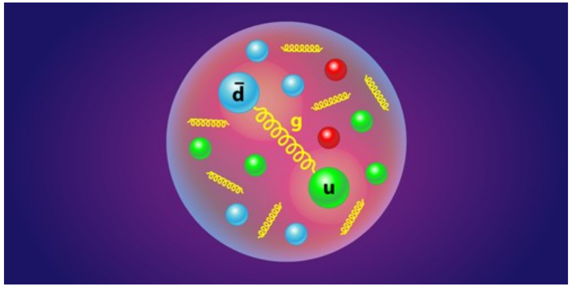
Based on ***PTEP 2024 (2024) 7, 073B05*** and ***2406.16763***

@ KEK Theory workshop

December 11th, 2024

Lattice QCD

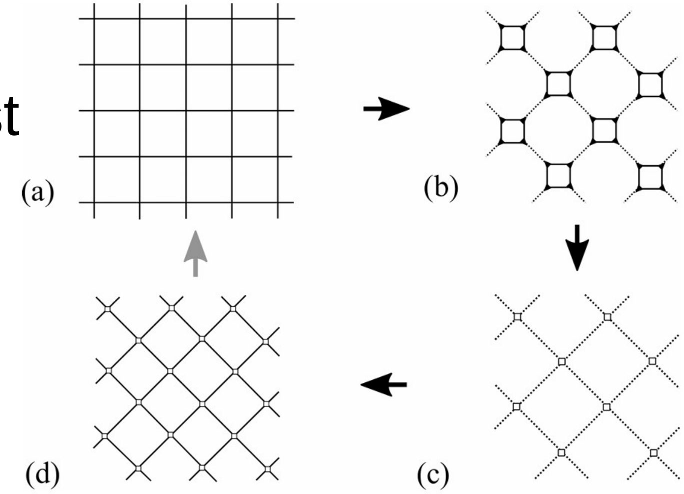
- non-perturbative formulation for quantum chromodynamics
- 4D Euclidean SU(3) gauge theory with Nf=2,3
    - Higher dimensions, Non-abelian, Multiple flavors
- MC computation suffers from the sign problem at finite θ and finite density
- MC computation also suffers from the topology freezing problem toward continuum limit



So far, we do not have a universal method that avoids all the problems...

Tensor renormalization group (TRG)

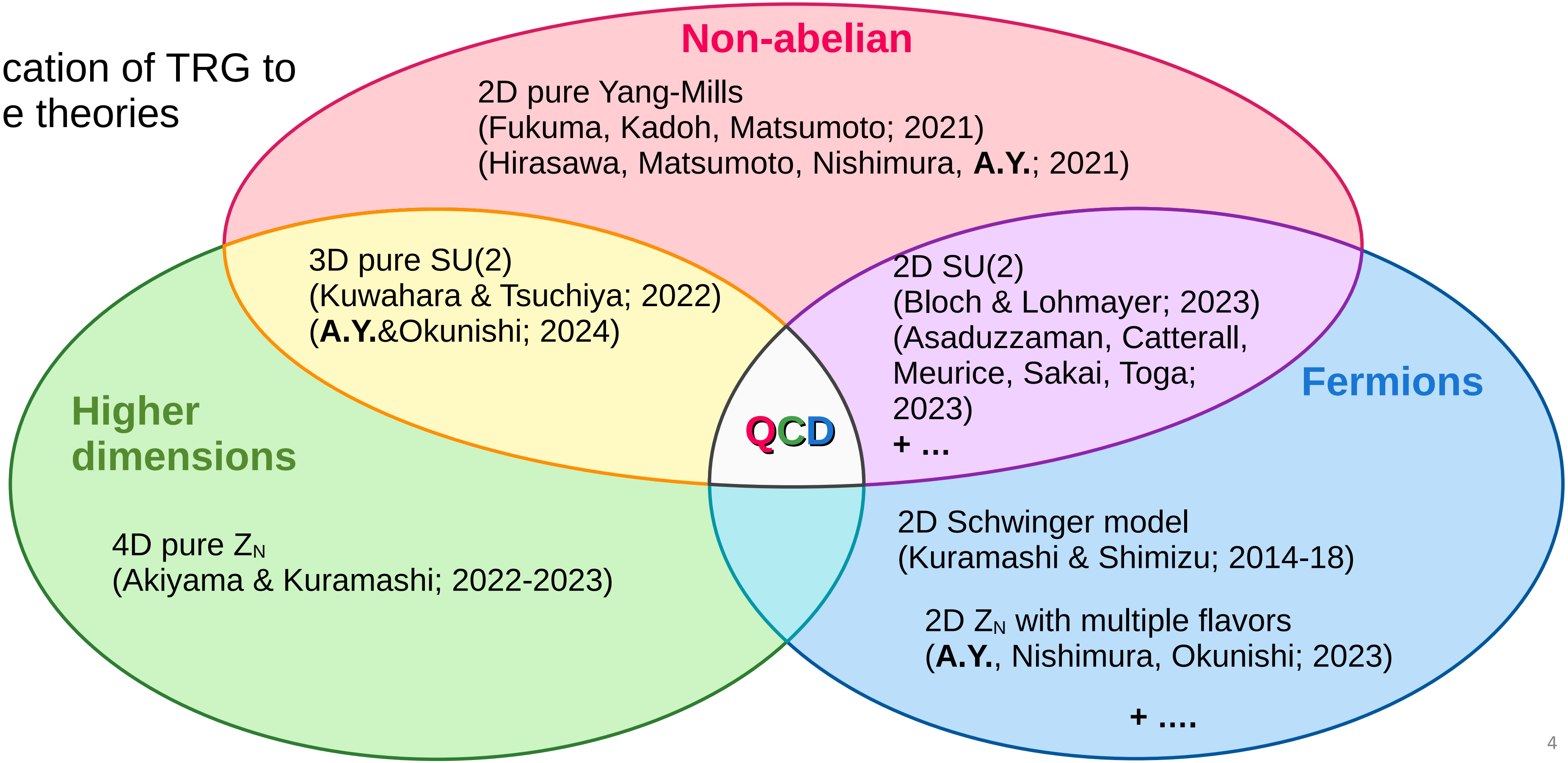
- An alternative to Monte Carlo methods based on coarse graining
- No sampling = No sign problem
- Can access large volumes with log cost
- Can handle fermion/Grassmann numbers directly; Grassmann TRG



[Figures from Okunishi-Nishino-Ueda; 2022]

Progress toward TRG study of Lattice QCD

Application of TRG to gauge theories



Progress toward TRG study of Lattice QCD

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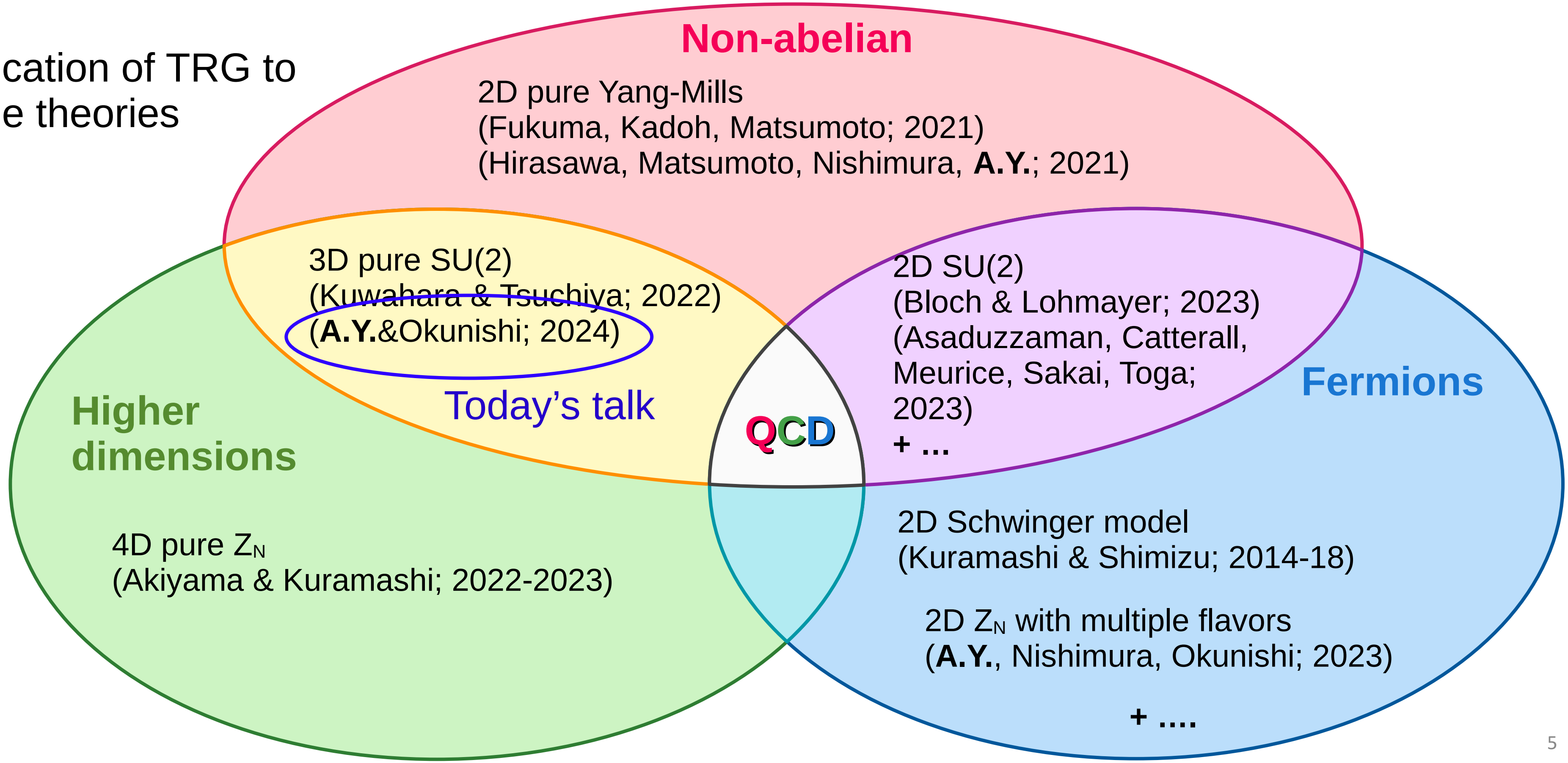
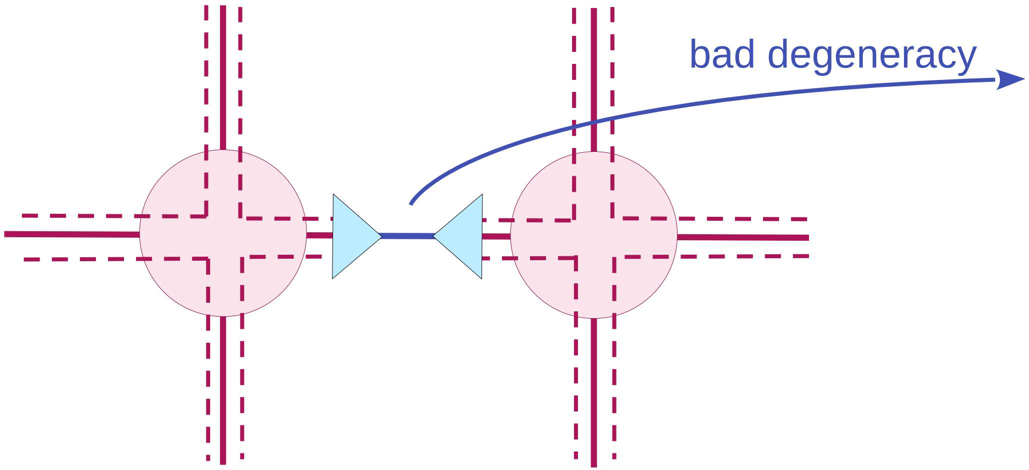


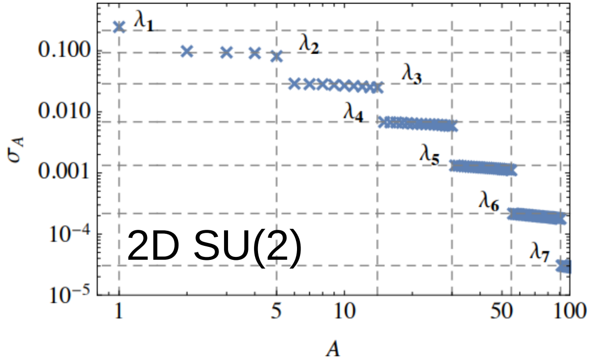
Outline

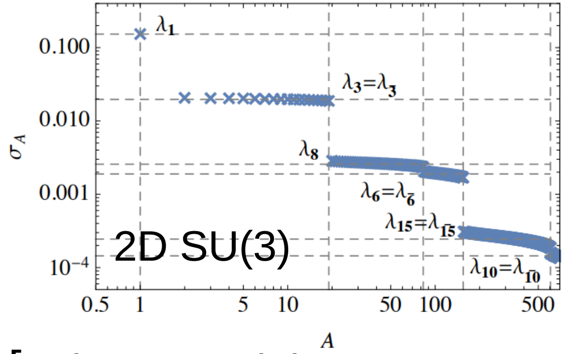
- Introduction
- Degeneracy in the tensor network
- Construction of the ‘armillary sphere’
- Result: 3D SU(2) & SU(3) thoery
- Summary

Why is non-abelian tensor network difficult?

Internal symmetry in SU(N) is a redundancy in the tensor network that cannot be truncate by an SVD







The entanglement structure is nonlocal...

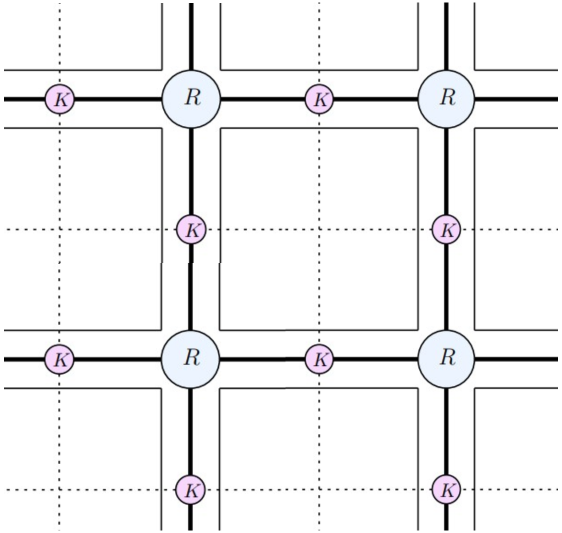
Figures from [Fukuma-Kadoh-Matsumoto; 2021]

Why is non-abelian tensor network difficult?

- Lesson from 1+1D: the (matrix) index loops can be traced out if we use character expansion

[Hirasawa, Matsumoto, Nishimura, **A.Y.**; 2021]

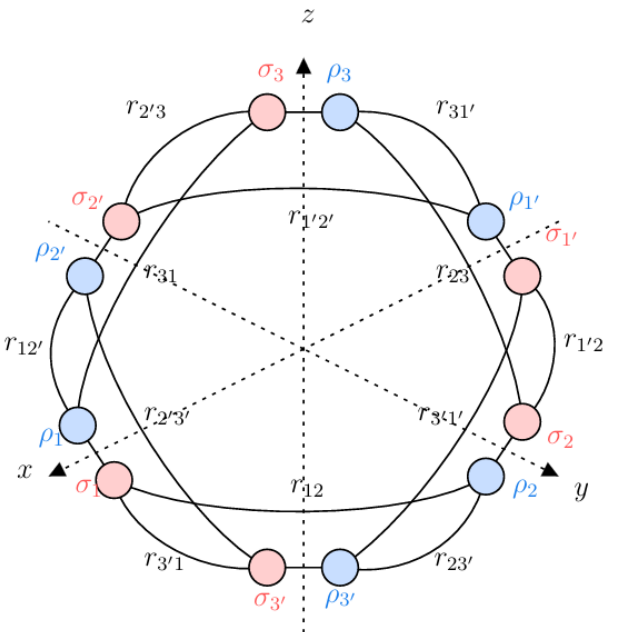
- Degeneracy is completely eliminated

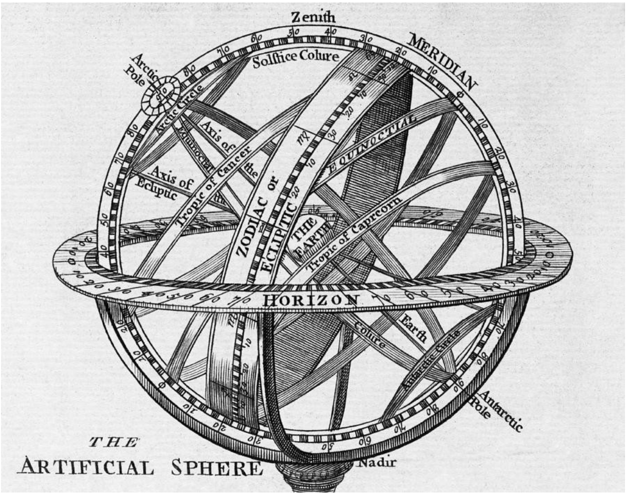


Question: Can we do the same thing in any dimensions?

The armillary sphere

Yes! There is a similar closed network in any dimension
Which we call the armillary sphere





armillary sphere
= intersecting circles

This was first noticed by [Oeckl & Pfeiffer;2001] in the context of the spin foam model.

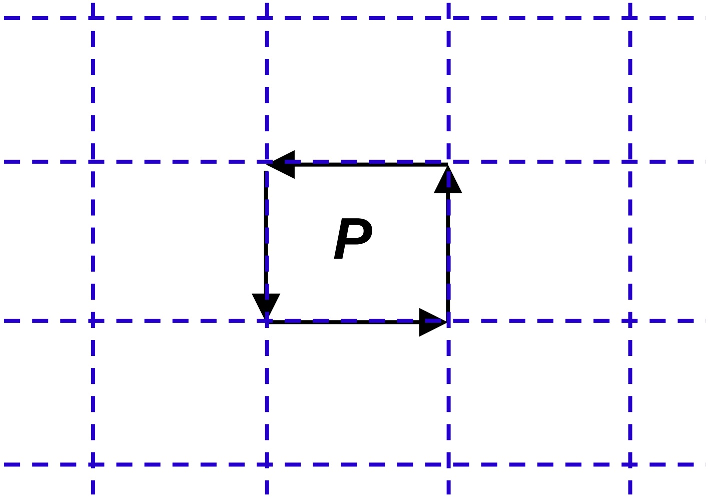
Yang-Mills theory

$$Z = \int DU\, e^{-S[U]};$$

$$S[U] = -\beta \sum_{n \in \Lambda_d} \sum_{1 \le \mu < \nu \le d} \Re \operatorname{tr} P_{n,\mu\nu}$$

$$P_{n,\mu\nu} \equiv U_{n,\mu} U_{n+\hat{\mu},\nu} U^\dagger_{n+\hat{\nu},\mu} U^\dagger_{n,\nu}$$

We consider the **plaquette action** as the lattice discretization for the YM theory.

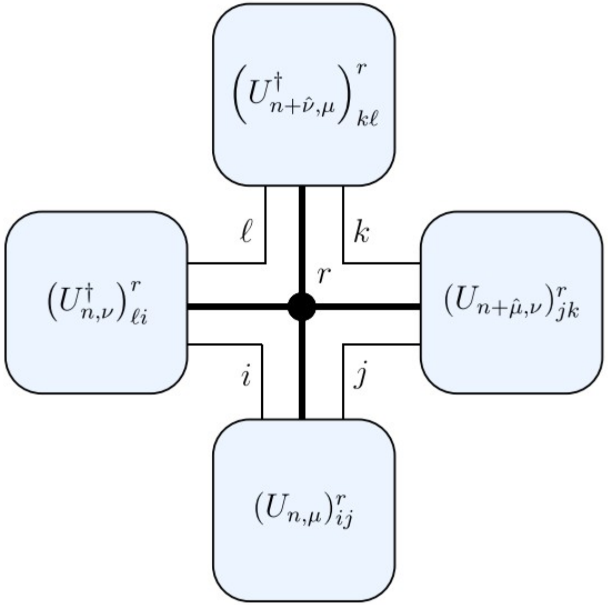


The armillary sphere

Step 1: perform character expansion on the Boltzmann weight

$$e^{\beta \Re \operatorname{tr} P_{n,\mu\nu}} = \sum_r f_r \operatorname{tr}_r(U_{n,\mu} U_{n+\hat{\mu},\nu} U^\dagger_{n+\hat{\nu},\mu} U^\dagger_{n,\nu})$$
$$= \sum_r f_r \sum_{i,j,k,l} (U_{n,\mu})^r_{ij} (U_{n+\hat{\mu},\nu})^r_{jk} (U^\dagger_{n+\hat{\nu},\mu})^r_{kl} (U^\dagger_{n,\nu})^r_{li}$$

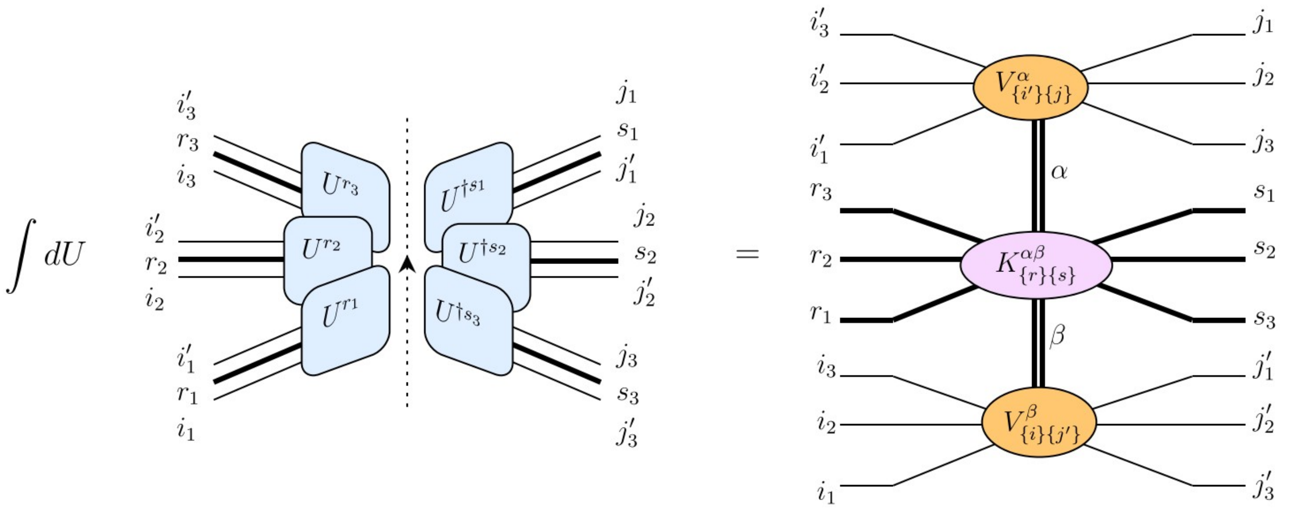
$\operatorname{tr}_r P_{n,\mu\nu} =$



The armillary sphere

Step 2: perform group integral on each link variable

$$\int dU_{n,\mu} (U_{n,\mu})^{r_1}_{i_1 i'_1} \cdots (U_{n,\mu})^{r_{d-1}}_{i_{d-1} i'_{d-1}} (U^\dagger_{n,\mu})^{s_1}_{j_1 j'_1} \cdots (U^\dagger_{n,\mu})^{s_{d-1}}_{j_{d-1} j'_{d-1}} = \sum_{\hat r \in D_{\otimes r}} \sum_{\hat s \in D_{\otimes s}} \sum_{\hat\imath,\hat\jmath} \frac{1}{\dim(\hat r)} C^{\hat r \hat\imath}_{\{r\}\{i\}} C^{\hat r \hat\jmath}_{\{r\}\{i'\}} C^{\hat s \hat\jmath}_{\{s\}\{j\}} C^{\hat s \hat\imath}_{\{s\}\{j'\}} \delta(\hat r \simeq \hat s)$$



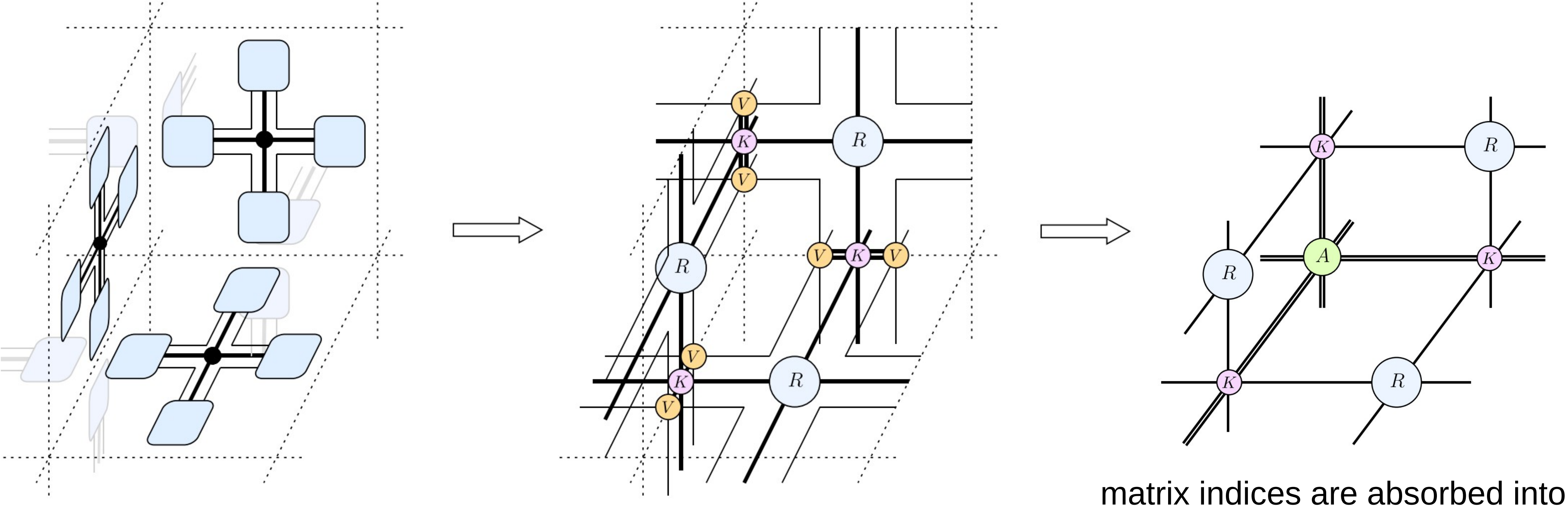
$$V^{\alpha}_{\{i\}\{j'\}} = \delta(\hat r \simeq \hat s) \sum_{\hat\imath} C^{\hat r \hat\imath}_{\{r\}\{i\}} C^{\hat s \hat\imath}_{\{s\}\{j'\}},$$

$$K^{\alpha\beta}_{\{r\}\{s\}} = \frac{1}{\dim(\hat r)} \delta_{\hat r \hat r'} \delta_{\hat s \hat s'} \delta(\hat r \in D_{\otimes r}) \delta(\hat s \in D_{\otimes s});$$

Note: matrix indices (thin lines) are neatly separated into two layers

The armillary sphere

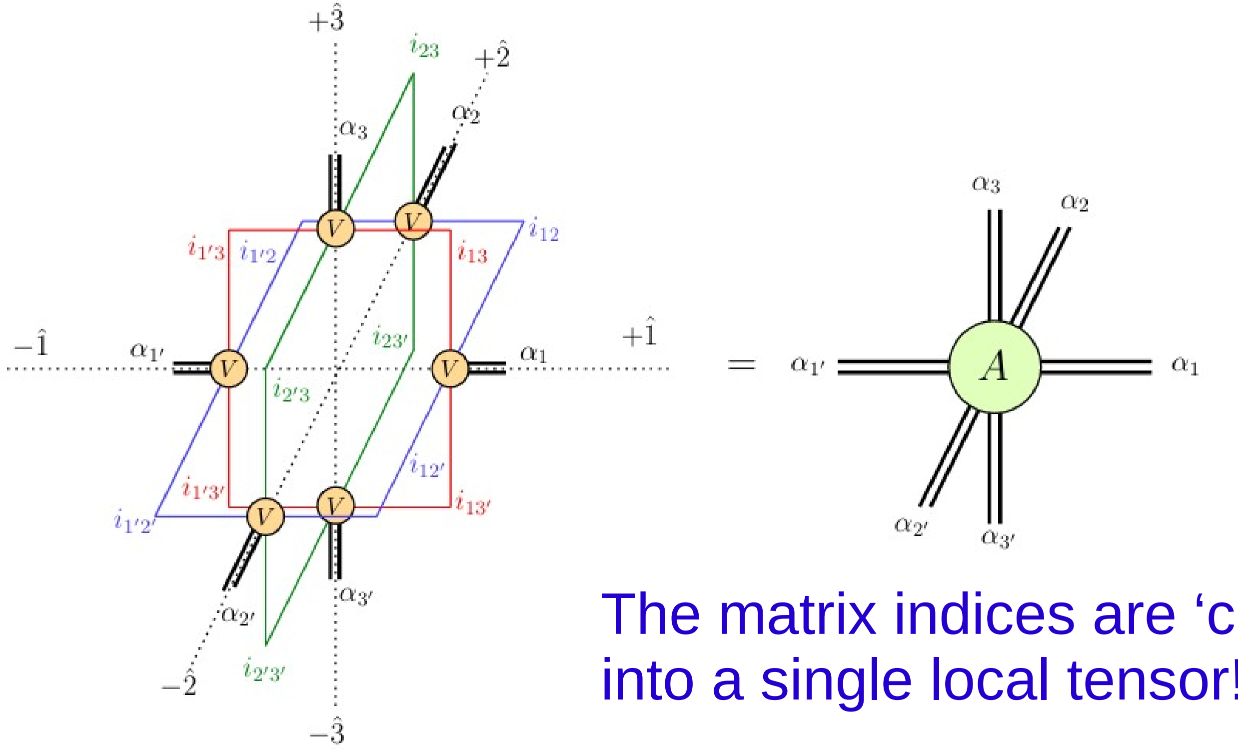
Step 3: Contract the matrix indices



matrix indices are absorbed into the armillary tensor ‘A’

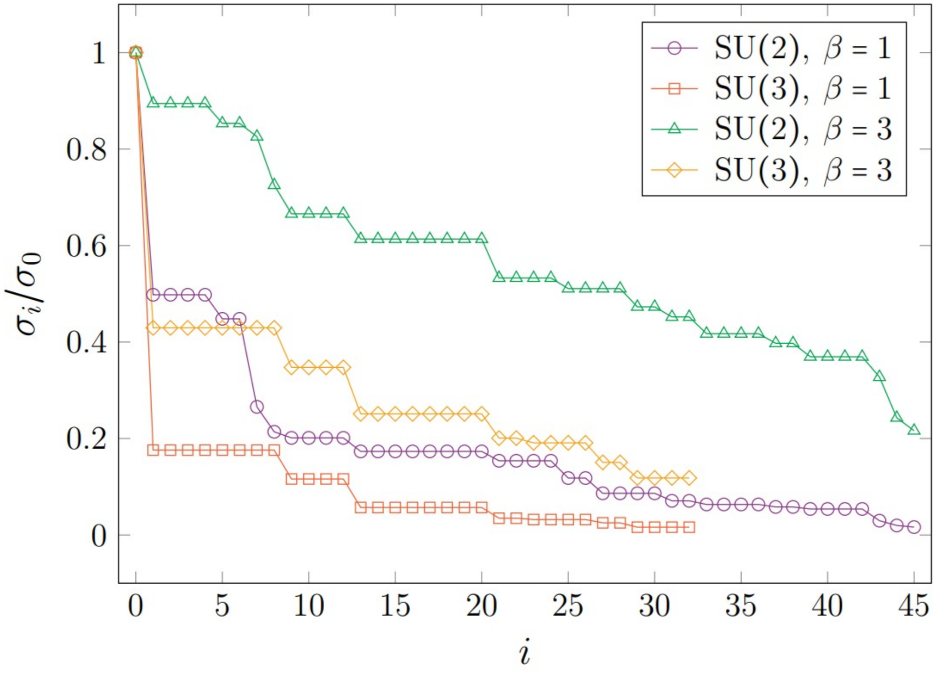
The armillary sphere

Step 3: Contract the matrix indices



The matrix indices are ‘collapsed’ into a single local tensor!

Result: singular value spectrum



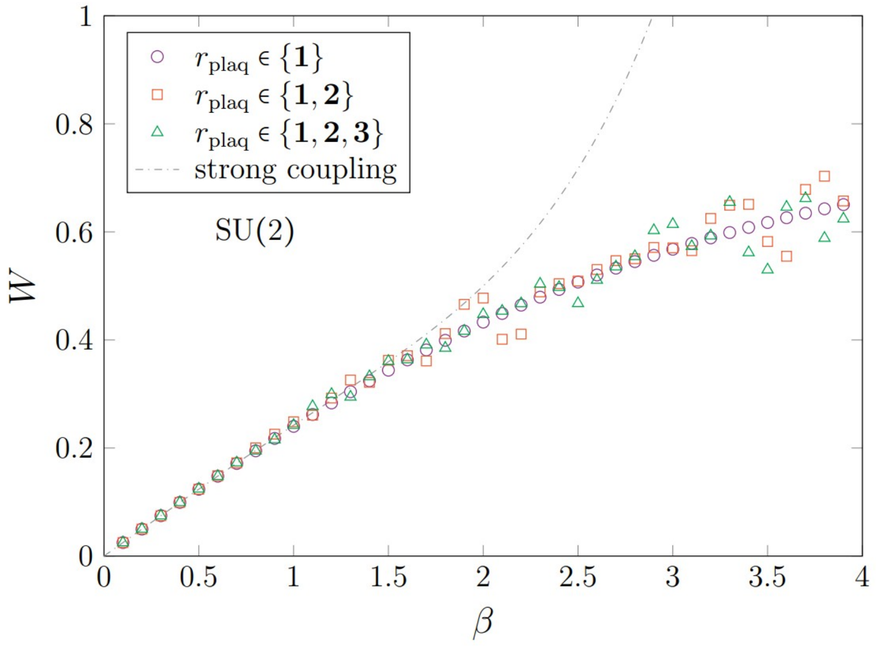
Singular value spectrum of the initial tensor do not have large degeneracy

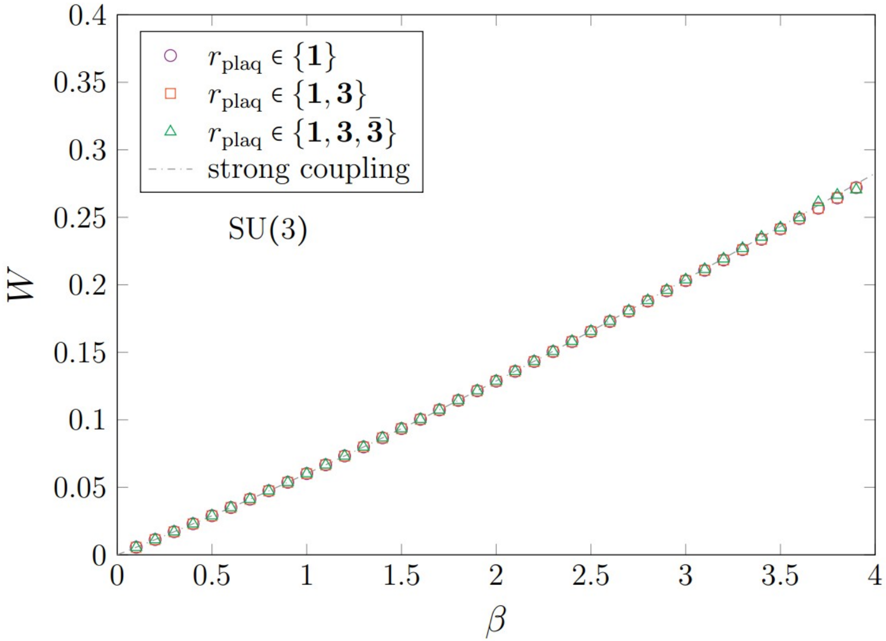
Result: average plaquette @ zero temperature

pure 2+1D SU(2) and SU(3) gauge theory

ATRG; $V = 16^3$; $D_{cut} = 16$

Average plaquette – consistent with strong coupling expansion

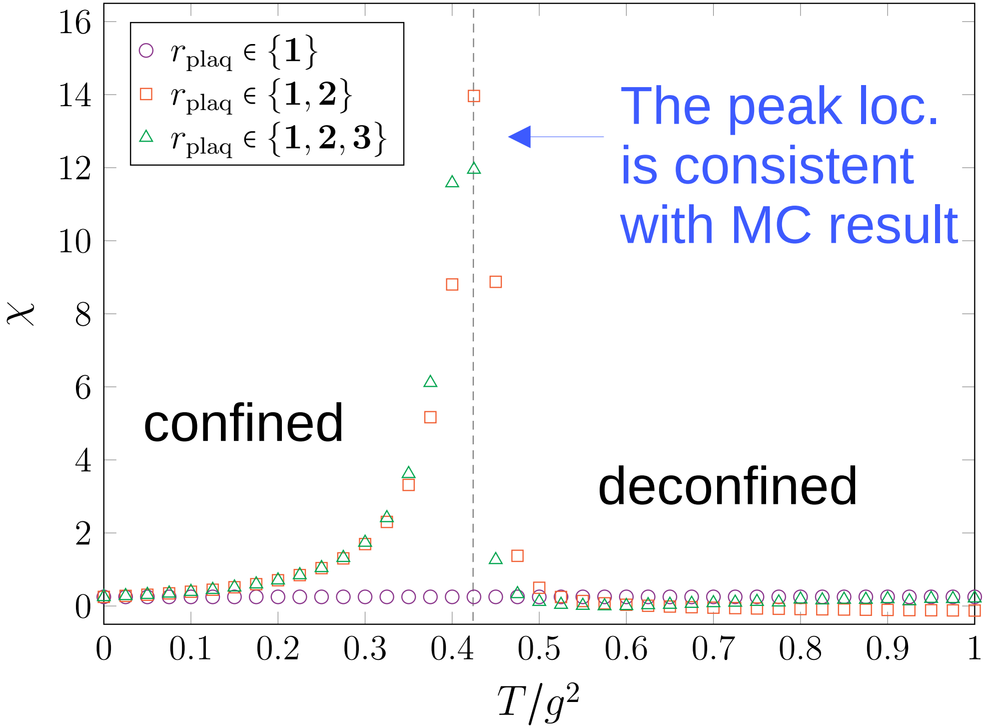


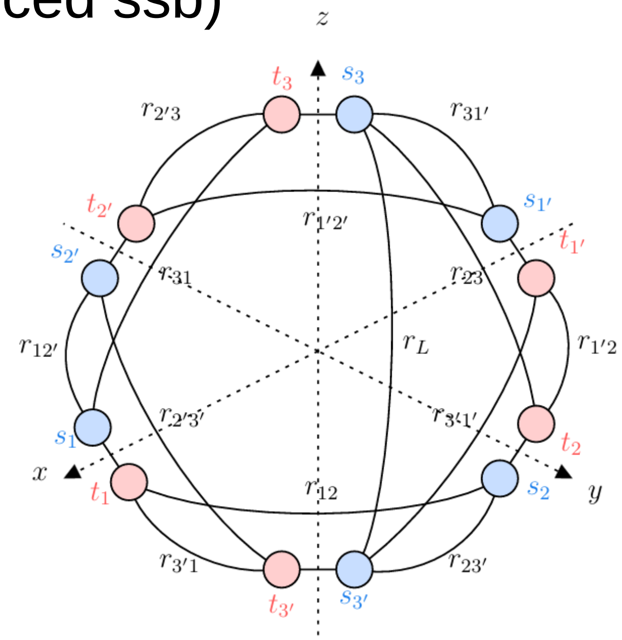


Result: deconfinement @ finite temperature

TRG; V = 1×1024^2; $D_{cut} = 64$; SU(2)

Polyakov loop susceptibility (with induced ssb)



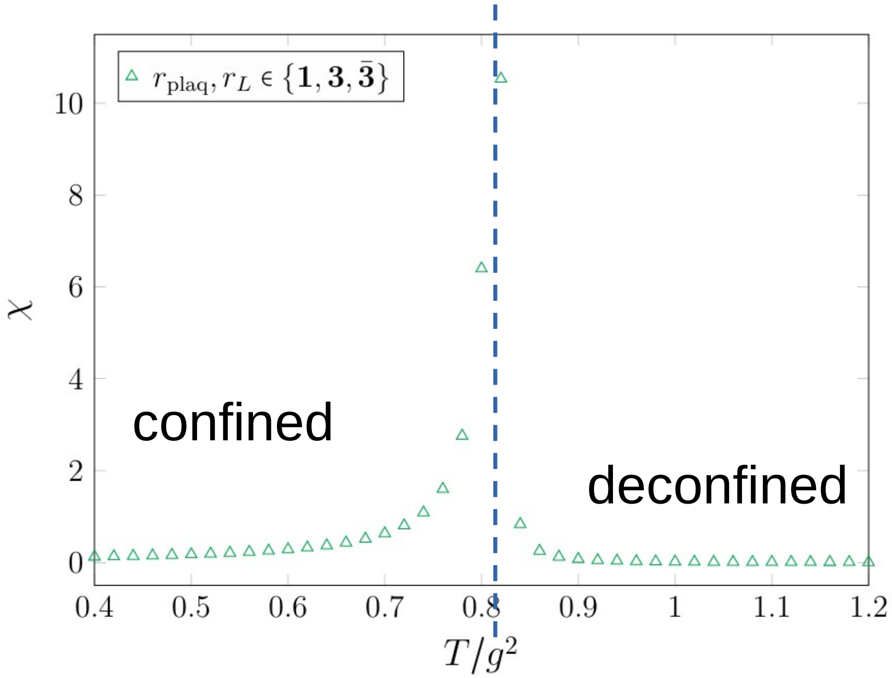


Armillary sphere with polyakov loop

Result: deconfinement @ finite temperature

TRG; $V = 1\text{x}1024^2$; $D_{cut} = 64$; **SU(3)**

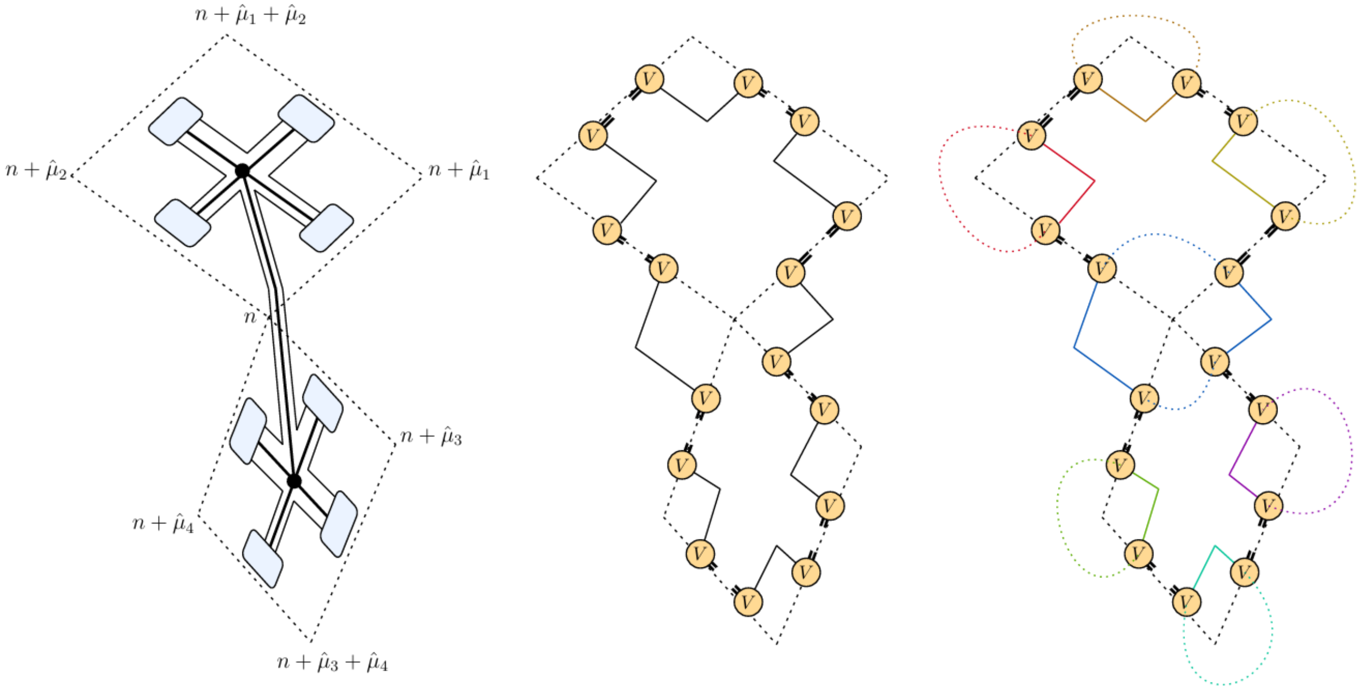
Polyakov loop susceptibility (with induced ssb)



to do: identify the order of the deconfinement transition (coming soon)

Possible generalizations

This idea could be generalized to any pure gauge theory (including the theta term)



The 4D "clover" topological charge

Summary

- Performing the TRG study for YM theory is difficult due to the non-local entanglement structure—large degeneracy in the tensor network
- This ‘armillary sphere’ structure can be contracted analytically
- The reduced tensor network no longer has the degeneracy.
- The 3D SU(2) and SU(3) calculations are demonstrated. The deconfinement transition is observed.

Future prospect

- Can we reduce the tensor network without character expansion? (Some variation of Gilt-TNR?) [Hauru,Delcamp,Mizera; 2017]
- Reduced TN with matter fields
- More in-depth analysis (any physical meaning? gauge fixing?)
- 4D gauge theory + theta term
- Etc.