Quantum Black Brane in the 4D Standard Model

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Short summary

The Swampland cobordism conjecture predicts various new defects in theories.

This can be applied to the SM, which is in string landscape.

Specifically, a string-like defect is predicted in SM

Our work is . . .

Constructing this string-like defect in SM numerically as a Black String solution !

The horizon of this black string is supported by Casimir energy, so this is intrinsically quantum object.

Cobordism conjeture

Cobordant theories

- M, N : k-dim compact manifolds. The statement "compactified theory of QG on M and N are cobordant ($M \sim N$)" means there is a finite energy domain wall between these compactified theories.

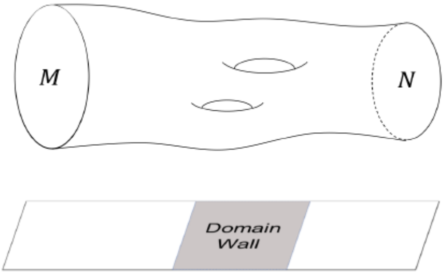


Figure from [J. McNamara and C. Vafa '19]

The cobordism conjecture

What [J. McNamara and C. Vafa '19] states is . . .

all compactified theories on k-dim manifolds are cobordant.

Actually, the existance of non-cobordant compactified theories means that there is a global symmetry in QG.

We do not know complete QG theory, so our possible approach is to confirm this statement in our simple theory models (string theory or SM).

The cobordism conjecture for SM

When we compactify SM on S^1, there are two non-cobordant theories : each of them corresponding to different choice of Fermions' spin structure(periodic or anti-periodic boundary condition when going around S^1).

$\Longleftrightarrow$ the global $\mathbb{Z}_2$ symmetry in SM : sign flipping of fermion fields.

We have to compensate a defect which break this symmetry : $\mathbb{Z}_2$ charged string-like defect !

The horizon supported by Casimir energy

How is a black string horizon supported ?

It is known that only spherical topology is allowed as horizon topology in 4D theory at least classically.

Then, how we can construct a black string solution as vacuum solution . . . ?

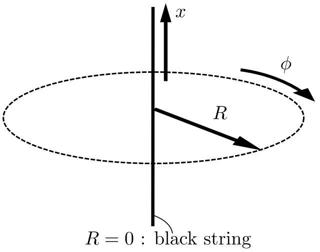
Actually, this theorem is valid under null energy condition.

The quantum collection for vacuum, Casimir energy, can violate the null energy condition !

Casimir energy

Casimir energy associated the S_1 surrounding the string is below.

$$V_{\text{Casimir}} = -\sum_{\text{particle}} (-1)^{2s_p} n_p \frac{m_p^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\theta_p)}{(2\pi R m_p n)^2} K_2(2\pi R m_p n)$$



θ_p : corresponds to the boundary condition of the particle when going around the string.

Recall that fermions have periodic boundary condition $(\theta_p = \pi/2)$ instead of anti-periodic, because this BS is $\mathbb{Z}_2$ charged

What we have to do

$$V_{\text{Casimir}} = -\sum_{\text{particle}} (-1)^{2s_p} n_p \frac{m_p^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi n\theta_p)}{(2\pi R m_p n)^2} K_2(2\pi R m_p n)$$

In our low energy case, only light particles are dominant

$\longrightarrow$ graviton, photon, neutrino

We assume neutrinos are Majorana and its mass spectrum is Normal hierarchy with lightest mass 0.

What we have to do is ...

Solving Einstein eq. with Casimir energy-momentum tensor with appropriate metric ansatz and boundarty conditions.

Metric ansatz and boundary conditions

Extremal ansatz

The simplest string-like metric ansatz is

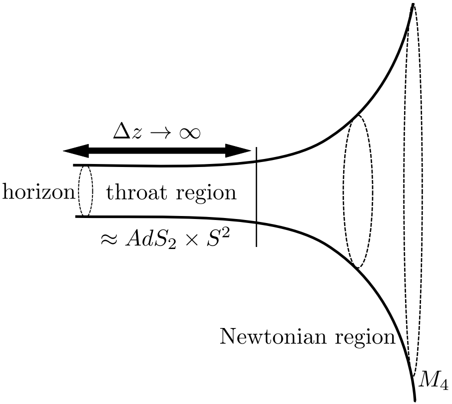
$$ds^2 = A^2(z)(-dt^2 + dx^2) + dz^2 + R^2(z)d\phi^2$$

z is a parameter which parametrize the radial direction (proper distance) and R is now a function of z.

Actually, the good candidate for such solution was suggested by Arkani-Hamed et. al. in 2007 (but they didn't construct it specifically). They asserted that the extremal black string interpolate 4D vacua.

Reissner-Nordstrom BH interpolating 4D vacua (review)

Recall the extremal Reissner-Nordstrom BH.



This BH interpolates between $AdS_2 \times S^2$ and M_4.

This $AdS_2 \times S^2$ is vacuum solution of 2-dim KK compactified Einstein-Maxwell theroy, where its moduli is stabilized by electric flux.

The black string interpolating 4D vacua (review)

They expected the same thing is true for $\mathbb{Z}_2$ charged black string.

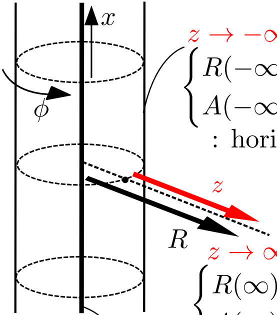
Actually, 4D SM has S^1 compactified vacuum solution, whose moduli (radion) is stabilized by the Casimir energy of light SM particles, with periodic boundary conditions for fermions.

In our assumption (neutrino mass spectrum) ...

the compactified vacuum solution is $AdS_3 \times S^1$, whose S^1 radius R_0 is neutrino scale.

Boundary conditions for extremal BS

$$ds^2 = A^2(z)(-dt^2 + dx^2) + dz^2 + R^2(z)d\phi^2$$



$z \to -\infty$

$$\begin{cases} R(-\infty) = R_0 \\ A(-\infty) \sim e^{z/l_{\mathrm{AdS}}} \to 0 \end{cases}$$

: horizon, $AdS_3 \times S^1$

$z \to \infty$

$$\begin{cases} R(\infty) \propto z \to \infty \\ A(\infty) = \mathrm{const.} \end{cases}$$

: infinite distance, M_4

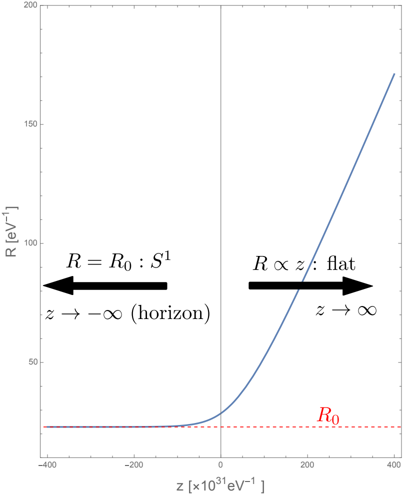
$R = 0$: conical singularity

So we adopt this boundary conditions.

At infinite distance, the Casimir energy is suppressed, so we expect flat metric.

Numerical Results

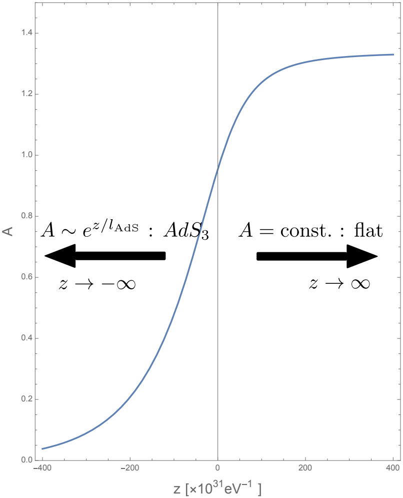
Extremal solution : R



As $z \to -\infty$, approaching to the stabilized constant value R_0 (neutrino scale) of radion in 3D effective theory.

As $z \to \infty$, $R \sim az$ with $a \sim 4.3 \times 10^{-34} \sim m_\nu / M_P$.
This means that the deficit angle θ going around the string is $\theta \sim \mathcal{O}(1)$

Extremal solution : A



The solution A also satisfies expected boundary conditions.

As $z \to -\infty$, $A(z)$ goes to 0 ($\Rightarrow$ coordinate singularity, horizon) exponentially with

$l_{\mathrm{AdS}} \sim 1.2 \times 10^{33} [\mathrm{eV}]$

Conclusion

Conclusion

- The cobordism conjecture predicts a $\mathbb{Z}_2$ charged string-like object in 4D SM.
- We could actually construct this object as the “quantum” black string in SM.

Some comments for the non-extremal case (1)

- The natural expansion of metric ansatz is

$$ds^2 = -A^2(z)dt^2 + B^2(z)dx^2 + dz^2 + R^2(z)d\phi^2$$

- With this ansatz, there are non-flat vacuum (Casimir is not considered) solutions [Bronnikov et. al. '19].
  $\longrightarrow$ the solutions can approach these vaccum solutions asymptotically.

Some comments for the non-extremal case (2)

- The horizon is at finite z. Because of shifting z symmetry of Einstein eq., we can always put the horizon at $z = 0$. Boundary conditions at horizon obtained from the condition that $z = 0$ is coordinate singularity but geometrical scalars are not diverge there.
- For Einstein eq., the physical integral constant is only $R(0)$, which is mass parameter (horizon size). When $R(0) \to R_0$, the solutions should approach the extremal solution.