Investigating 9d/8d non-supersymmetric branes and theories from supersymmetric heterotic strings

Joint work with Yuta Hamada [arXiv:2409.04770, hep-th]

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1.Introduction and Summary

Introduction: Standard Model and Superstring theories

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- Superstring theories are expected to derive this model from fundamental principles, eventually.

Introduction: Standard Model and Superstring theories

- The Standard Model is highly reliable, but there is no explanation why the nature choose this model.
- Superstring theories are expected to derive this model from fundamental principles, eventually.
- For this reason, vacua of superstring theory should be widely explored, and indeed have been investigated extensively. But many unexplored areas still remain, especially in the absence of spacetime supersymmetry (SUSY).
- Certain types of ~~SUSY~~ superstring theories can be constructed from SUSY ones!

(Non-)Supersymmetric Heterotic Strings and Our Results

Backgrounds:

- 10d SUSY and ~~SUSY~~ Het are already identified[Gross et al, 1985, Dixon-Harvey,1986]. Examples:

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$$9d\text{: SUSY } A_{2n-1} \times 2E_{9-n}.$$

Here, we denote $E_1 = A_1,\ E_3 = A_2 \times A_1,\ E_4 = A_4,\ E_5 = D_5.$

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Our work:

We applied this orbifolding method to 9d theories and obtained the following results:

$$9d\text{: SUSY } A_{2n-1} \times 2E_{9-n} \overset{\text{orbifolding}}{\rightarrow} \text{~~SUSY~~ } C_n \times E_{9-n}, 1 \leq n \leq 9, n \neq 7$$

2.Supersymmetric Heterotic Strings

Superstring Theories

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Let me review the basics of SUSY heterotic strings.

SUSY Heterotic String Theories

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$(10-d)$-dimensional heterotic strings are consist of three contents:

- Coordinate bosons:$(X^0_{L,R}, \cdots, X^{9-d}_{L,R}) : \Sigma \to \mathbb{R}^{1,9-d}$,
- Worldsheet spinors:$(\psi^0_R, \cdots, \psi^9_R)$,
- Internal bosons : $(X^{10-d}_L, \cdots, X^{25}_L; X^{10-d}_R, \cdots, X^9_R) : \Sigma \to T^{16+d,d}$,

where $T^{16+d,d} = \mathbb{R}^{16+d,d}/\Gamma^*_{16+d,d}$ is a $(16+d,d)$-dimensional torus, and $\Gamma^*_{16+d,d}$ is a $(16+d,d)$ lattice. The theory allows for a choice of lattice.

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Is any lattice $\Gamma^*_{16+d,d}$ allowed in heterotic strings? No.

Hilbert Space and Massless Spectrum

The Hilbert space (before GSO projection) of this $(10-d)$-dimensional theory consists of three parts:

$$\mathcal{H}^{8-d,8-d}_{\text{Boson}} \text{ :the bosonic Fock space generated by } \left(\alpha^i_*, \tilde{\alpha}^i_*\right)^{2\leq i\leq 9-d},$$

$$\mathcal{H}^{0,8}_{\text{Fermion}} \text{ :the fermionic Fock space generated by } \left(\tilde{\psi}^i_*\right)^{2\leq i\leq 9},$$

$$\begin{aligned} \mathcal{H}^{16+d,d}_{\text{Internal}} &= \mathcal{H}_{\text{Oscillator}} \otimes \mathcal{H}_{\text{Momentum}} \\ &= \mathcal{H}^{16+d,d}_{\text{Boson}} \otimes \left(\bigoplus_{p\in\Gamma} \mathbb{C}\,|p\rangle\right). \end{aligned} \tag{1}$$

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The total Hilbert space is

$$\mathcal{H} = \mathcal{H}^{8-d,8-d}_{\mathrm{Boson}} \otimes \mathcal{H}^{0,8}_{\mathrm{Fermion}} \otimes \mathcal{H}^{16+d,d}_{\mathrm{Internal}}. \tag{2}$$

The 10d massless spectrum consists of supergravity/vector multiplets:

(Dilaton, B field, Metric ; Dilatino, Gravitino) +(Gauge fields ; Gaugino)

Torus Partition Function and Modular Invariance

Let τ be a complex moduli of a torus, $q = \exp 2\pi i\tau$, $\eta(\tau)$ be the Dedekind eta function, and $\theta_{1,2,3,4}(\tau)$ are the theta functions. The torus partition function is

$$Z(\tau,\bar{\tau}) = \frac{1}{(\mathrm{Im}\,\tau)^{\frac{8-d}{2}}(\eta\bar{\eta})^{8-d}} \cdot \frac{1}{2\bar{\eta}^4}\left(\bar{\theta}_3^4 - \bar{\theta}_4^4 - \bar{\theta}_2^4\right) \cdot \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(p_L,p_R)\in\Gamma_{16+d,d}} q^{\frac{1}{2}p_L^2}\bar{q}^{\frac{1}{2}p_R^2} \tag{3}$$

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It's classicaly known that[Schellekens-Warner,1986], for the anomaly cancellation on the target spacetime theory, $Z(\tau,\bar{\tau})$ should be modular invariant:

$$\begin{aligned} Z(\tau+1,\bar{\tau}+1) =& Z(\tau,\bar{\tau}), \\ Z\left(-\frac{1}{\tau},-\frac{1}{\bar{\tau}}\right) =& Z(\tau,\bar{\tau}) \end{aligned} \tag{4}$$

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The first and the second factor are modular invariant, then the last factor should also be.

Lattices should be Even Self-Dual

The modular transformation properties of the previous factor are given as follows:

$$\begin{aligned} Z_\Gamma(\tau) :=& \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(p_L,p_R)\in\Gamma} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2}, \\ T : Z_\Gamma(\tau+1) =& \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(p_L,p_R)\in\Gamma} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2} e^{\pi i (p_L^2 - p_R^2)}, \\ S : Z_\Gamma\left(-\frac{1}{\tau}\right) =& \frac{1}{\eta^{16+d}\bar{\eta}^d} \sum_{(p_L,p_R)\in\Gamma^*} q^{\frac{1}{2}p_L^2} \bar{q}^{\frac{1}{2}p_R^2}. \end{aligned} \tag{5}$$

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Therefore, for Z_Γ to be modular invariant, $(16+d, d)$ lattice Γ should satisfy the following conditions(called *even* and *self-dual*):

$$\begin{aligned} \forall (p_L, p_R) \in \Gamma,\ \ p_L^2 - p_R^2 \in 2\mathbb{Z}, \\ \Gamma^* =\Gamma. \end{aligned} \tag{6}$$

Even Self-Dual Lattices

Thus, our story came down to a purely mathematical discussion:

How many $(16+d,d)$ even self-dual lattices are there? It's already known!

$$\begin{aligned} &(16,0) : E_8 \times E_8 \text{ root lattice, } \mathrm{Spin}(32)/\mathbb{Z}_2 \text{ lattice} \\ &(17,1) : \text{unique, but many different expressions} \\ &(18,2) : \text{unique, but many different expressions} \\ &\quad \cdots \end{aligned} \tag{7}$$

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In 10 dimensions, there are two gauge symmetries: $E_8\times E_8$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$.

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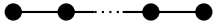
More presicely, if the root lattice $\Gamma_{\mathfrak{g}}$ of a Lie algebra $\mathfrak{g}$ with rank $16+d$, can be embedded in $\Gamma_{16+d,d}$, then $\mathfrak{g}$ is a gauge symmetry of $(10-d)$-dimensional heterotic strings.

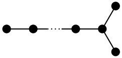
Dynkin Diagram

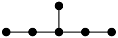
Gauge symmetries are described by Dynkin diagrams graphically:

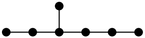
$A_n(\mathrm{SU}(n+1))$ 

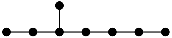
$B_n(\mathrm{SO}(2n+1))$ 

$C_n(\mathrm{Sp}(2n))$ 

$D_n(\mathrm{SO}(2n))$ 

E_6 

E_7 

E_8 

The number of nodes is rank of the algebra.

It's natural to denote $E_5 := D_5,\ E_4 := A_4,\ E_3 := A_2 \times A_1,\ E_1 := A_1.$

9d SUSY Heterotic Strings and Extended Dynkin Diagram

- Recently, the list of maximal gauge symmetries of SUSY(16 supercharges) heterotic strings on T^d are identified[Freitas et al, 2020]. There are 44 maximal gauge symmetries in 9d SUSY heterotic strings.

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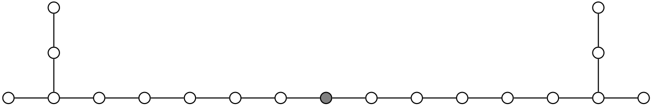
- There is an interesting game to obtain them. You start from the following diagram:



This diagram, with 19 nodes, is not a Dynkin diagram, but rather called an *extented Dynkin diagram*, which appears in the context of affine Lie algebra.

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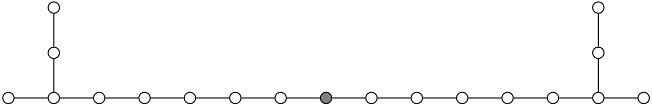

  This diagram, with 19 nodes, is not a Dynkin diagram, but rather called an *extented Dynkin diagram*, which appears in the context of affine Lie algebra.
- First you remove one node from each side of the gray node. If the remained diagram is a Dynkin diagram, then it is a gauge symmetry of 9d SUSY theories; if not, it is not.

Table of Gauge Symmetries in 9d SUSY Heterotic Strings

$2E_8 \times A_1$	$2E_7 \times A_3$	$2E_6 \times A_5$	$2D_5 \times A_7$
$2A_4 \times A_9$	$2A_2 \times 2A_1 \times A_{11}$	$2A_1 \times A_{15}$	A_{17}
$E_8 \times A_5 \times A_4$	$E_8 \times E_7 \times A_2$	$E_7 \times E_6 \times A_4$	$E_7 \times D_{10}$
$E_7 \times D_5 \times A_5$	$E_7 \times A_{10}$	$E_7 \times A_9 \times A_1$	$E_7 \times A_7 \times A_2 \times A_1$
$E_7 \times A_6 \times A_4$	$E_8 \times E_6 \times A_3$	$E_6 \times D_{11}$	$E_6 \times D_5 \times A_6$
$E_6 \times A_{11}$	$E_6 \times A_{10} \times A_1$	$E_6 \times A_8 \times A_2 \times A_1$	$E_6 \times A_7 \times A_4$
D_{17}	$D_{16} \times A_1$	$D_{14} \times A_2 \times A_1$	$D_{13} \times A_4$
$D_{12} \times D_5$	$D_5 \times A_{12}$	$D_5 \times A_{11} \times A_1$	$D_5 \times A_9 \times A_2 \times A_1$
$D_5 \times A_8 \times A_4$	$E_8 \times D_9$	$E_8 \times A_8 \times A_1$	$A_{16} \times A_1$
$E_8 \times A_6 \times A_2 \times A_1$	$A_{14} \times A_2 \times A_1$	$A_{13} \times A_4$	$A_{13} \times A_2 \times 2A_1$
$A_{12} \times A_4 \times A_1$	$E_8 \times A_9$	$A_{10} \times A_4 \times A_2 \times A_1$	$E_8 \times D_5 \times A_4$

All of them have rank 17. Red ones will be used in the later.

Now, it's times to see non-supersymmetric heterotic strings!

3.Non-Supersymmetric Heterotic Strings

Heterotic Strings Without Spacetimes Supersymmetry

- Since SUSY is not observed yet in this real world, investigation of ~~SUSY~~ superstring theories is needed.
- In the case of 10d heterotic strings, not only SUSY theories(such as $E_8 \times E_8$ and $SO(32)$), but also ~~SUSY~~ theories are already known.

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- There are six theories with rank 16, and only one theory with rank 8: ~~SUSY~~ E_8 theory.
- All of these 10d ~~SUSY~~ theories can be constructed from 10d SUSY theories, by the method called *asymmetric orbifolding*.

Roughly speaking, our work is an application of this method to dimensions lower than 10.

10d Asymmetric Orbifolding and the ~~SUSY~~ E_8 theory.

The 10d $E_8 \times E_8$ theory have two $\mathbb{Z}_2$ symmetries:

- The spacetime(not worldsheet) fermion number $(-1)^{F_{\text{Spacetime}}}$. It acts on $\mathcal{H}^{0,8}_{\text{Fermion}}$.

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- Exchanging symmetry g: $E_8 \overset{g}{\leftrightarrow} E_8$. It acts on $\mathcal{H}^{16+d,d}_{\text{Internal}}$:

$$g \left| \left(x_1^{(E_8)}, x_2^{(E_8)} \right) \right\rangle = \left| \left(x_2^{(E_8)}, x_1^{(E_8)} \right) \right\rangle \tag{8}$$

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10d Asymmetric Orbifolding is a certain projection with these symmetries:

$$\begin{aligned} &\frac{1+(-1)^F}{2}\mathcal{H} \\ \to &\frac{1+g(-1)^{F_{\text{Spacetime}}}}{2}\frac{1+(-1)^F}{2}\mathcal{H}, \end{aligned} \tag{9}$$

where F is the worldsheet fermion number. Only $g(-1)^{F_{\text{Spacetime}}} = 1$ states survive.

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What does occur?

Asymmetric Orbifolding

Remind of the massless spectrum in the 10d $E_8 \times E_8'$ heterotic strings:

(Dilaton, B field, Metric ; Dilatino, Gravitino) $+$(Gauge fields ; Gaugino)

Asymmetric Orbifolding

Remind of the massless spectrum in the 10d $E_8 \times E_8'$ heterotic strings:

(Dilaton, B field, Metric ; Dilatino, Gravitino) $+$(Gauge fields ; Gaugino)

- effect of $(-1)^{F_{\text{Spacetime}}}$: dilatino and gravitino ($g = 1$) are projected out:

$$\frac{1 + g(-1)^{F_{\text{Spacetime}}}}{2} \left|\text{Dilatino,Gravitino}\right\rangle = \frac{1 + 1 \cdot (-1)}{2} \left|\text{D,G}\right\rangle = 0, \tag{10}$$

- effect of g : gauge symmetry reduced from $E_8 \times E_8$ to E_8:

$$g\left(\left|A^{\mu}_{E_8}\right\rangle \pm \left|A^{\mu}_{E_8'}\right\rangle\right) = \pm\left(\left|A^{\mu}_{E_8}\right\rangle \pm \left|A^{\mu}_{E_8'}\right\rangle\right). \tag{11}$$

Thus, we obtained the 10d ~~SUSY~~ E_8 theory!

Asymmetric Orbifolding

Remind of the massless spectrum in the 10d $E_8 \times E_8'$ heterotic strings:

(Dilaton, B field, Metric ; Dilatino, Gravitino) +(Gauge fields ; Gaugino)

- effect of $(-1)^{F_{\text{Spacetime}}}$: dilatino and gravitino ($g = 1$) are projected out:

$$\frac{1 + g(-1)^{F_{\text{Spacetime}}}}{2} \left|\text{Dilatino,Gravitino}\right\rangle = \frac{1 + 1 \cdot (-1)}{2} \left|\text{D,G}\right\rangle = 0, \tag{10}$$

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Thus, we obtained the 10d ~~SUSY~~ E_8 theory!

Then, it's quite natural to attempt this process to dimensions lower than 10. Let me explain additional $\mathbb{Z}_2$ symmetry in lower dimensions.

Folding of Dynkin Diagram

- A_{2n-1} diagram possess $\mathbb{Z}_2$ symmetry, which flips the left and right of the diagram.
- By taking the quotient of the diagram with this symmetry, it leads to C_n type diagram: 

  This process is called folding of Dynkin diagram.

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- All of non simply-laced Lie algebras(B_n, C_n, F_4, G_2) can be obtained by the folding of simply-laced Lie algebras($A_n, D_n, E_{6,7,8}$):

$$A_{2n-1} \to C_n, \quad A_{2n} \to B_n, \quad D_n \to B_{n-1}, \quad D_4 \to G_2, \quad E_6 \to F_4, \tag{12}$$

- In 9 dimensions, only $A_{2n-1} \to C_n$ appear. (In dimensions lower than 9, these other folding would appear).

(Repost)Table of gauge symmetries in 9d SUSY heterotic strings

$2E_8 \times A_1$	$2E_7 \times A_3$	$2E_6 \times A_5$	$2D_5 \times A_7$
$2A_4 \times A_9$	$2A_2 \times 2A_1 \times A_{11}$	$2A_1 \times A_{15}$	A_{17}
$E_8 \times A_5 \times A_4$	$E_8 \times E_7 \times A_2$	$E_7 \times E_6 \times A_4$	$E_7 \times D_{10}$
$E_7 \times D_5 \times A_5$	$E_7 \times A_{10}$	$E_7 \times A_9 \times A_1$	$E_7 \times A_7 \times A_2 \times A_1$
$E_7 \times A_6 \times A_4$	$E_8 \times E_6 \times A_3$	$E_6 \times D_{11}$	$E_6 \times D_5 \times A_6$
$E_6 \times A_{11}$	$E_6 \times A_{10} \times A_1$	$E_6 \times A_8 \times A_2 \times A_1$	$E_6 \times A_7 \times A_4$
D_{17}	$D_{16} \times A_1$	$D_{14} \times A_2 \times A_1$	$D_{13} \times A_4$
$D_{12} \times D_5$	$D_5 \times A_{12}$	$D_5 \times A_{11} \times A_1$	$D_5 \times A_9 \times A_2 \times A_1$
$D_5 \times A_8 \times A_4$	$E_8 \times D_9$	$E_8 \times A_8 \times A_1$	$A_{16} \times A_1$
$E_8 \times A_6 \times A_2 \times A_1$	$A_{14} \times A_2 \times A_1$	$A_{13} \times A_4$	$A_{13} \times A_2 \times 2A_1$
$A_{12} \times A_4 \times A_1$	$E_8 \times A_9$	$A_{10} \times A_4 \times A_2 \times A_1$	$E_8 \times D_5 \times A_4$

For the modular invariance, only reductions that decrease the rank by 8 are allowed. Then, only these red ones ($2E_{9-n} \times A_{2n-1}, n \neq 7$) are allowed to apply this method.

They possesses two $\mathbb{Z}_2$ symmetries:

g: exchanging two E_{9-n} 's.

g_{folding}: flipping A_{2n-1}.

9d Asymmetric Orbifolding

9d Asymmetric Orbifolding is a certain projection with these symmetries:

$$\frac{1+(-1)^F}{2}\mathcal{H} \rightarrow \frac{1+g(-1)^{F_{\text{Spacetime}}}g_{\text{folding}}}{2}\frac{1+(-1)^F}{2}\mathcal{H}, \quad (13)$$

where F is the worldsheet fermion number.

- the effect of $(-1)^{F_{\text{Spacetime}}}$: dilatino and gravitino are projected out
- the effect of g : gauge symmetry is reduced from $2E_{9-n}$ to E_{9-n}.
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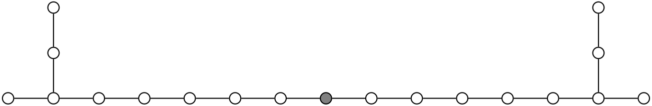
remark: We should add new sector, called *twisted sector* to make partition function modular invariant. New massless particles come from this sector.

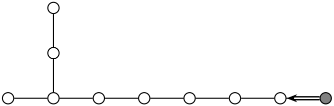
Results(9d ~~SUSY~~ Gauge Symmetries and Additional Massless Spectrum)

9d **SUSY** theory	9d ~~**SUSY**~~ theory	**Twisted sector**	
		$\bar{O}_8$	$\bar{C}_8$
A_{17}	C_9	$\mathbf{1}$	$\mathbf{152}$
$A_{15} \times 2A_1$	$C_8 \times A_1$	$\mathbf{1}$	$(\mathbf{1},\mathbf{3}) \oplus (\mathbf{16},\mathbf{2}) \oplus (\mathbf{119},\mathbf{1})$
$A_{11} \times 2A_2 \times 2A_1$	$C_6 \times A_2 \times A_1$	$\mathbf{1}$	$(\mathbf{65},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3})$
$A_9 \times 2A_4$	$C_5 \times A_4$	$\mathbf{1}$	$(\mathbf{44},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24})$
$A_7 \times 2D_5$	$C_4 \times D_5$	$\mathbf{1}$	$(\mathbf{27},\mathbf{1}) \oplus (\mathbf{8},\mathbf{10}) \oplus (\mathbf{1},\mathbf{45})$
$A_5 \times 2E_6$	$C_3 \times E_6$	$\mathbf{1}$	$(\mathbf{14},\mathbf{1}) \oplus (\mathbf{1},\mathbf{78})$
$A_3 \times 2E_7$	$C_2 \times E_7$	$\mathbf{1}$	$(\mathbf{5},\mathbf{1}) \oplus (\mathbf{1},\mathbf{133})$
$A_1 \times 2E_8$	$C_1 \times E_8$	$\mathbf{1}$	$(\mathbf{1},\mathbf{248})$

$\bar{O}_8, \bar{C}_8$ are the scalar and conjugate spinor representation of $\mathrm{Spin}(7)$, respectively. All of them contain tachyon.

Extended Dynkin diagrams for 9d ~~SUSY~~ can be obtained by the folding of the one for 9d SUSY:


,



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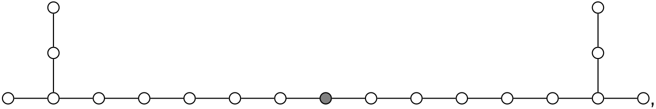

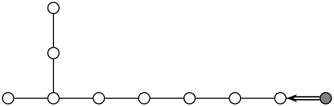

The rules of the game are same as before: First you remove one node to the left of the gray node. If the remained diagram is a Dynkin diagram, then it is a gauge symmetry of 9d $\cancel{\text{SUSY}}$ theories; if not, it does not.

Future Directions

- Theories we obtained from 9d rank 17 SUSY theories could also be obtained by the compactification of the 10d ~~SUSY~~ E_8 theory:

SUSY ~~SUSY~~

$$
\begin{array}{ccc}
\text{10d } E_8 \times E_8 & \xrightarrow{\text{orbifolding}} & \text{10d } E_8 \\
\text{on } S^1 \downarrow & & \downarrow \text{on } S^1 \\
\text{9d rank17} & \xrightarrow[\text{orbifolding}]{} & \text{9d rank9}
\end{array}
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- Assuming the completeness conjecture, which states that there exist physical states for all possible charges, $\mathbb{Z}_2$ symmetries we used predict new branes with $\mathbb{Z}_2$ charges in 9d SUSY heterotic strings. What are the properties of them?

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- M-theoretic interpretation? I assume that M-theory compactified on the Möbius strip with certain Pin^+ structure would know something.

Thank you for listening!