de Sitter from M-theory

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Difficulties with de Sitter in string theory:

- Supersymmetry algebra cannot be realized in dS
- Various No-Go theorems (e.g. Maldacena–Nuñez (2000)) forbid $\mathrm{Mink}_d/\mathrm{dS}_d$ vacua only using classical ingredients

  Can be evaded by including higher-curvature effects, localized sources (e.g. O-planes), quantum effects (Casimir energy), ...

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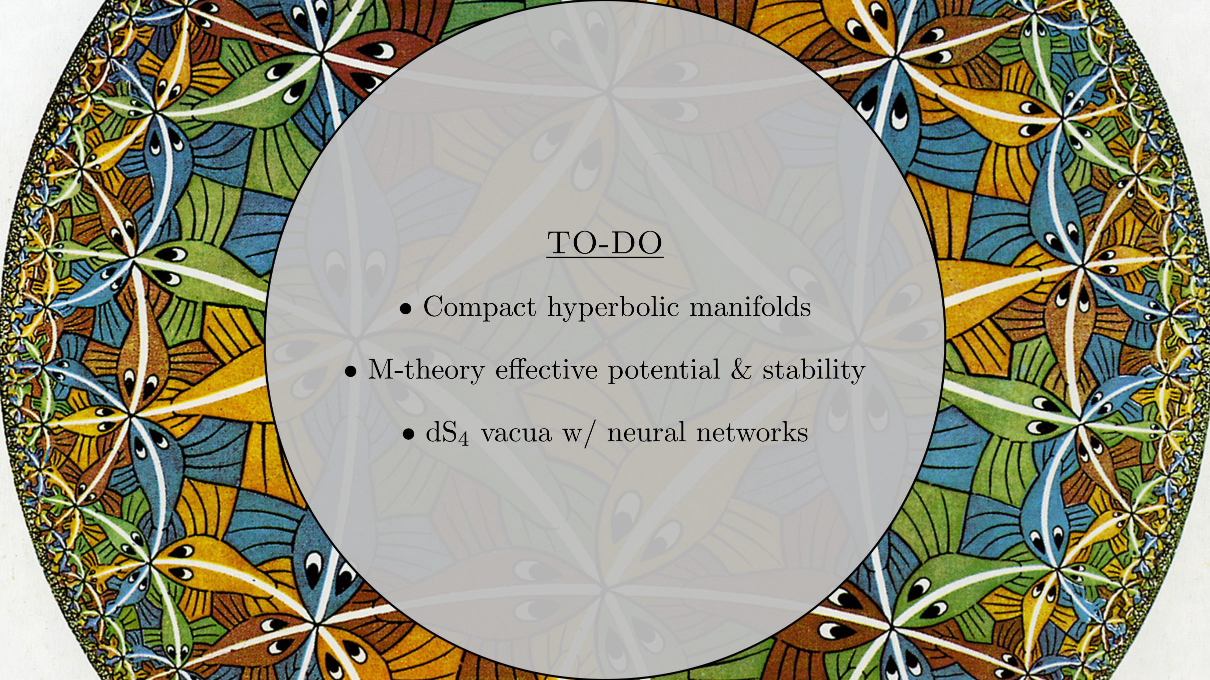
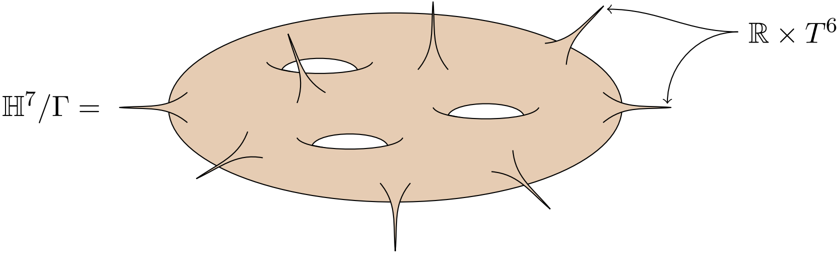
Today: explicit construction realizing this proposal



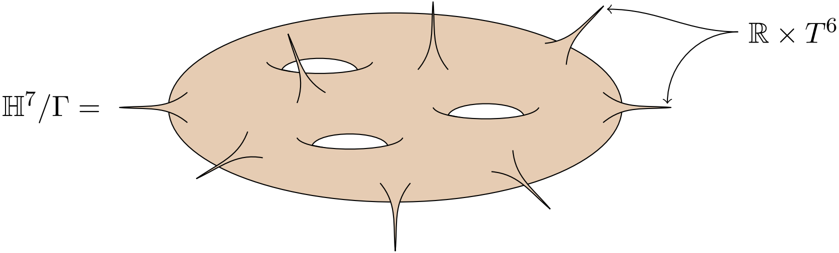
TO-DO

- Compact hyperbolic manifolds
- M-theory effective potential & stability
- dS_4 vacua w/ neural networks

Compact Hyperbolic Manifolds

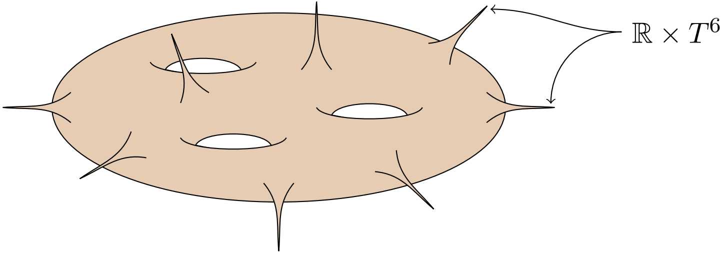


Requirements for X:



Requirements for X:

- Large volume

$\mathbb{H}^7/\Gamma =$ 

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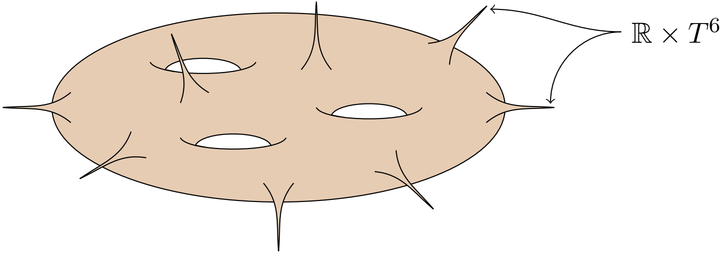
$\mathbb{H}^7/\Gamma =$ 

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- Highly symmetric (for numerics to be practical)

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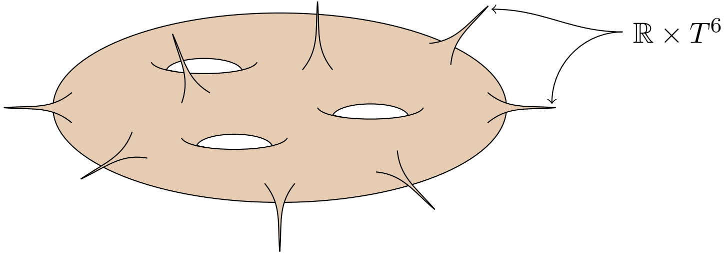

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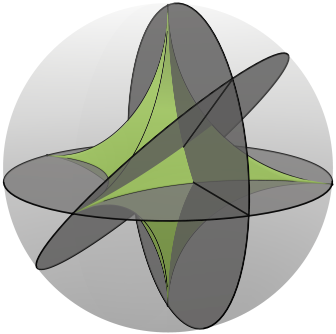
right-angled polytope P $\longrightarrow$	cusped manifold X_{cusp} $\longrightarrow$	compact manifold X
P^{E_7}: $V = 702,\ F = 56$	X^7_{cusp}: $8,257,536$ cusps	X^7: vol $\approx 2 \times 10^9$
P^{B_3}: $V = 6,\ F = 8$	X^3_{cusp}: 6 cusps	X^3: vol ≈ 7.33

$P^{B_3} \subset \mathbb{H}^3$ (ideal octahedron)



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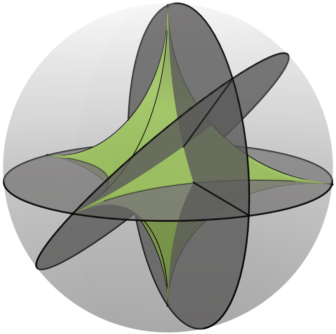
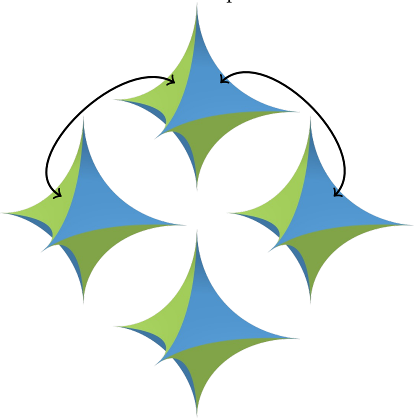


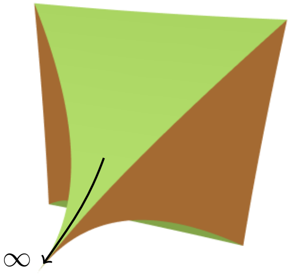
$\Longrightarrow$

X^3_{cusp}



glue together 2^c copies of P

Cusp in X^3_{cusp}



$$\mathrm{d}s^2_{\mathrm{cusp}} = \frac{\mathrm{d}z^2}{z^2} + z^2 \mathrm{d}s^2_{T^2}$$

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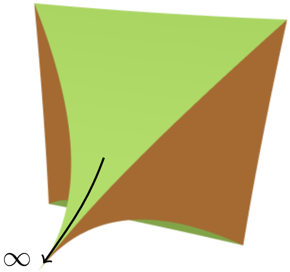
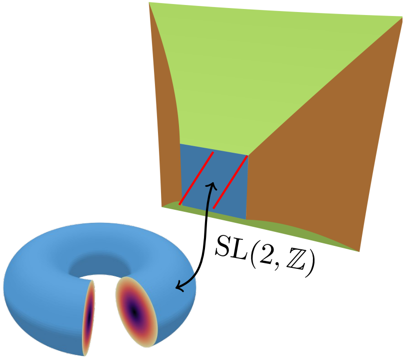


$$ds^2_{\text{cusp}} = \frac{dz^2}{z^2} + z^2 ds^2_{T^2}$$

$\Longrightarrow$

Filled cusp in X^3



$$ds^2 = \frac{dz^2}{z^2 - z_{\text{h}}^2} + \left[(z^2 - z_{\text{h}}^2)d\theta_1^2 + z^2 ds^2_{T^1}\right]/\mathbb{Z}^2$$

size z_{h} of Casimir cycles
controlled by winding numbers

Effective Potential

Low-energy dynamics of M-theory:

$$S_{11} = \int_{11} \left(R^{(11)} - \frac{1}{2}|G_4|^2 + \frac{1}{6} C_3 \wedge G_4 \wedge G_4 + \dots \right) \to \int_{11} \left(R^{(11)} - \frac{1}{2}|G_7|^2 + \dots \right)$$

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Consider warped products $M^{1,3}_{\text{symm}} \times X^7$:

$$\mathrm{d}s^2_{11} = e^{2A(y)} \mathrm{d}s^2_{4,\text{symm}} + g^{(7)}_{ij}(y)\, \mathrm{d}y^i \mathrm{d}y^j$$

$$g^{(7)} = e^{2B(y)} (\underbrace{\gamma}_{\substack{\text{unit} \\ \text{curvature}}} + \underbrace{h}_{|h| \ll 1})$$

$$G_7 = N_7\, e^{-2A} \mathrm{vol}_X$$

Effective potential [Douglas – '09]

$$V_{\text{eff}}^{(4)} = \int_7 e^{4A} \left(-R^{(7)} - 42(\partial A)^2 + 2|G_7|^2 - \frac{1}{2} {T_{\text{Cas}}}^i{}_i \right) + \frac{\lambda}{2} \left(\frac{1}{\ell_4^2} - \int_7 e^{2A} \right)$$

Importantly, this only depends on the 7d data.
The Casimir energy ${T_{\text{Cas}}}^i{}_i \sim \rho(L_{\text{cyc}}) \sim -\frac{1}{L_{\text{cyc}}^{11}}$ is localized to cusp-like regions of X^7

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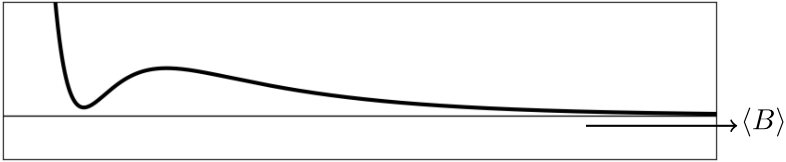
Crude picture of volume stabilization:

$$R^{(7)},\ (\partial A)^2 \sim e^{-2B} \qquad |G_7|^2 \sim e^{-14B} \qquad T_{\text{Cas}} \sim e^{-11B}$$



Procedure:

- Solve $\delta V_{\text{eff}}/\delta A = 0$ for A in terms of other fields:

$$-12\nabla^2(e^{2A}) + Ue^{2A} = -\lambda\,, \qquad U = 2R^{(7)} - 4|G_7|^2 + {T_{\text{Cas}}}^i{}_i$$

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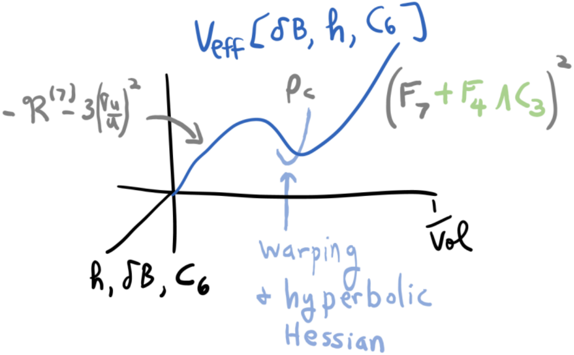
Free parameters:

- X^7_{cusp} (built from P^{E_7}) determines number of cusp-like regions, volume, etc.
- Winding numbers determine filling geometry and size of minimal cycles
- 7-flux N_7, helping to stabilize overall volume

Stability:

- Crude picture for volume stabilization
- Mostow–Prasad rigidity theorem: fluctuations h are massive
- Exponential warping avoids naïve instability in cusp-like regions

[De Luca, Silverstein, Torroba – 21]



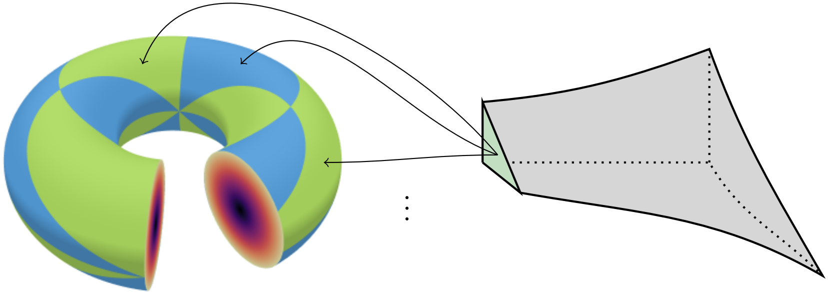
from [2104.13380]

dS_4 vacua w/ NNs

Leverage X^7's high degree of symmetry to solve on domain D which is $\approx 10^{14}$ times smaller

[GL – 25xx.xxxx]

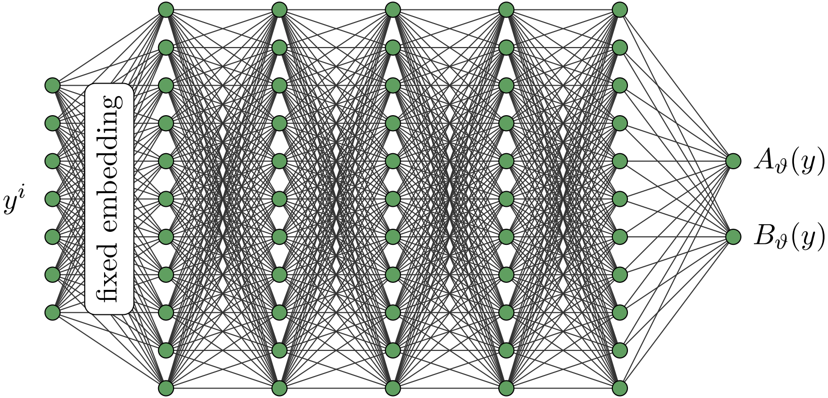
3d analogue:

$D =$ 

Cost to pay is the complicated shape of D and variety of boundary conditions

Coupled PDEs in 7d domain with variety of boundary conditions: use NN parametrization

[GL – 25xx.xxxx]



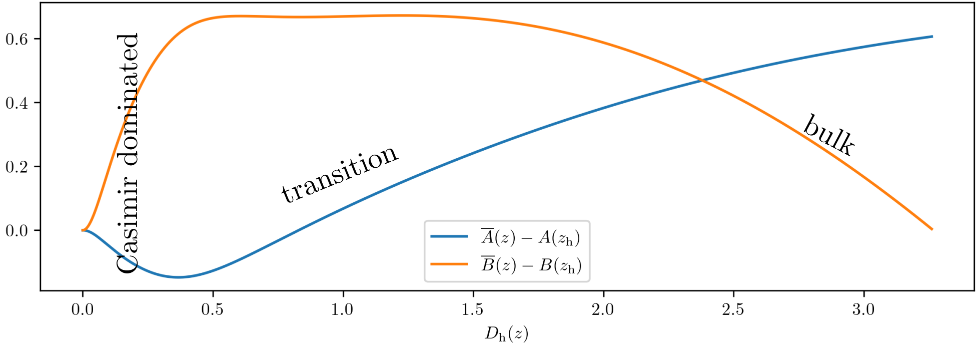
First hidden layer is untrainable and ensures all boundary conditions are satisfied (e.g. $y^i \mapsto \sin y^i, \cos y^i$)

During training, minimize residual loss:

$$\mathcal{L} = \sum_{n=1}^{N} \left\| \mathrm{PDE}[A_\vartheta, B_\vartheta](y_n) \right\|^2$$

[GL – 25xx.xxxx]

$N_7 = 200$, $w = (1,1,1,2,2,2)$, averaging over transverse torus:



Summary

- Hyperbolic compactification of M-theory with competition between curvature, flux and Casimir terms
- X^7 generically has many cusp-like regions supporting Casimir energy
- Searching for vacua . . .
  - Use highly-symmetric X^7 to reduce to solving on much smaller domain
  - NN parametrization for high-dimensional PDEs and satisfying complicated boundary conditions
  - Can understand behavior of solution in different regions (e.g. with Schrödinger equation intuition)
  - In progress: scan over parameters to find vacua under most control (tunably small curvatures and large volumes) & confirm stability numerically (stay tuned)