Species Scale in One-loop Correction

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Ongoing work with Y. Hamada, S. Iso, and M. Miyaji

Introduction

Why Scale?

- Scales play a crucial role in the description of physics. In fact, many phenomenological puzzles are tied to the behavior of scales and their hierarchies.

- In a low-energy effective field theory (EFT) including gravity, we typically encounter two different scales:

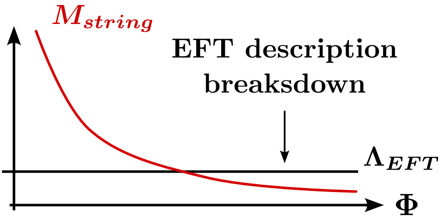
$$\Lambda_{\text{EFT}} \ll M_{\text{planck}}.$$

In this regime, quantum gravity effects are expected to be negligible in the EFT description.

Scale of Quantum Gravity

- **According to string theory, the EFT picture is valid only if:**

$$\Lambda_{\text{EFT}} < M_{\text{string}} \sim M_{\text{planck}}\, e^{-\Phi}, \quad \text{dilaton} : \Phi \to +\infty.$$



This provides a nontrivial constraint on EFT.

- **This viewpoint underlies the swampland program.**

Theme of the Swampland Program

The Swampland program organizes our understanding of quantum gravity as learned from string theory.

Importantly, the conjectures obtained from string theory can be understood as aspects of the quantum nature of gravity itself, independent of string theory.

$$\Lambda_{\text{EFT}} < M_{\text{string}} \sim M_{\text{planck}} e^{-\Phi}.$$

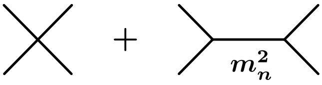
Could this relation hold generically without relying on string theory?

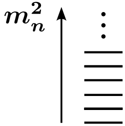
Can Quantum Gravity Tell Us Anything About the Scale?

Infinitely Many Particles

A perturbative quantum gravity should involve an infinite tower of states. [Arkani-Hamed and Huang[2], 2020]

Tree-level UV completion for gravity:





Tower of states

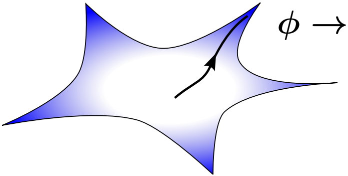
Unitarity at $s = E \to \infty$ is achieved by softening:

$$\frac{1}{s} \quad \longrightarrow \quad \frac{1}{s \cdot \Pi_{n\geq 1}(1 - s/m_n^2)}$$

Infinitely Many Particles in String Theory

In all string theory examples [Ooguri and Vafa, 2006 +]

$$m_n \sim \exp(-\alpha\phi) \quad \text{with} \quad \alpha \gtrsim \mathcal{O}(1)$$



Tower of light states is either [Lee, Lerche and Weigand, 2019]

- **Kaluza-Klein (KK) tower:** $m_n = n/R$
- **String tower:** $m_n = \sqrt{n} M_s$

Species bound

The species bound connects the infinite tower of states (species) to the scale of quantum gravity.

Naively, one might say this is given by M_{planck}.

However, with N_{species} light states (either KK or string), gravity becomes strongly coupled at a lower scale [Dvali, 2007]:

$$\Lambda_{\text{EFT}} < \Lambda_{\text{species}} = \frac{M_{\text{planck}}}{\sqrt{N_{\text{species}}}}.$$

Rationale for the Species Bound

Renormalization of G_N by the species [Dvali, 2007]

Perturbative description breaks down at $p^2 \sim \Lambda_{\text{species}}$:

$$\frac{1}{p^2} \sim \frac{1}{p^2} \cdot \frac{1}{M_{\text{planck}}^2} < T(p)T(-p) > \cdot \frac{1}{p^2} \sim \frac{N_{\text{species}}}{M_{\text{planck}}^2}$$

$$\Longrightarrow \quad \Lambda_{\text{species}} = \frac{M_{\text{planck}}}{\sqrt{N_{\text{species}}}}$$

Issues with the Justification

- **The number of light states is truncated by hand.**
- **N_{species} appears to be a free parameter.**

Question: What determines the number of species N_{species}?

One-loop correction from infinite Kaluza-Klein particles

Set up & Methods

4d Einstein gravity + 1d Kaluza-Klein tower

Radion

$$S = \frac{M_{\text{planck},4}^2}{2} \int R - (\nabla \phi)^2 - \sum_{n \geq 0} \left(|\nabla \psi_n|^2 + m_{KK,n}^2(\phi) |\psi_n|^2 \right)$$

Kaluza-Klein scalar

The heat kernel method

One-loop correction from KK modes with $n \geq 1$

$$S_{\text{1-loop}} = -\int \sqrt{-g} \sum_{n \geq 1} \int_{\Lambda_{QG}^{-2}(\phi_0)}^{\infty} \frac{du}{u} \, \text{tr} \, e^{-u(-\nabla^2 + m_{KK,n}^2(\phi))}$$

Cutoff of quantum gravity

Finite 'number' from infinite particles

Consider the large mass $m_{KK,n}(\phi_0)$ expansion:

$$S^k_{\text{1-loop}} = -\int \sqrt{-g}\, O(R^k) \int_{\Lambda_{QG}^{-2}(\phi_0)}^{\infty} du\, u^{k-3} \sum_{n\geq 1}^{\infty} e^{-u m^2_{KK,n}(\phi_0)}$$

Set $N := \Lambda_{QG}/m_{KK}$. Then, a built-in cutoff emerges:

$$\sum_{n\geq 1}^{\infty} e^{-(n/N)^2} \to \text{Finite.}$$

Thus, $2k$-dimensional correction reads

$$S^k_{\text{1-loop}} \sim -\int \sqrt{-g} \left(\frac{1}{m_{KK}^{2k-4}} + \frac{N}{\Lambda_{QG}^{2k-4}} \right) O(R^k).$$

Species Scale from Number Proportional Term

One-loop corrected action is

$$S \sim -\int \sqrt{-g}\left(M_{\text{planck},4}^2 R + N R^2 + \cdots\right) \quad \textbf{with} \quad N = \frac{\Lambda_{QG}}{m_{KK}}$$

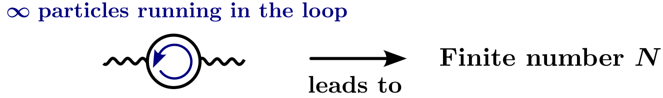
For the perturbative expansion to be valid up to $E_* = \Lambda_{QG}$,

$$M_{\text{planck},4}^2 E_*^2 \gtrsim N E_*^4 \quad \Longrightarrow \quad \Lambda_{QG} \lesssim M_{\text{planck},5} = \Lambda_{\text{species}}$$

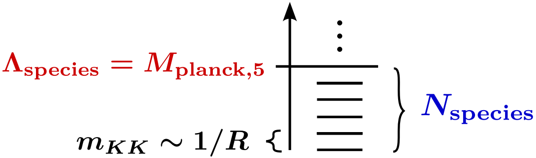
This shows that the species scale indeed serves as a scale of quantum gravity.

Number of Species in KK Tower

Number N appears naturally in the EFT calculation:



$$N = N_{\text{species}} = \frac{\Lambda_{\text{species}}}{m_{KK}} \quad \text{when} \quad \Lambda_{QG} = \Lambda_{\text{species}}$$



Question: N_{species} **counts the number of states below** Λ_{species}**?**

One-loop Correction from Infinite String States

One-loop Correction Formula

In contrast to the KK tower, a string tower exhibits

- A different mass spectrum: $m_n = \sqrt{n} M_s$
- Degeneracy at each level: $d_n \sim e^{c\sqrt{n}}, \quad c \sim \mathcal{O}(1)$

Assuming all string states are scalar, $2k$-dimensional one-loop correction reads

$$S^k_{\text{1-loop}} \sim - \int \sqrt{-g} \, \frac{\exp\left(\frac{c^2}{4}\left(\frac{\Lambda_{QG}}{M_S}\right)^2\right)}{\Lambda_{QG}^{2k-d}} \mathcal{O}(R^k).$$

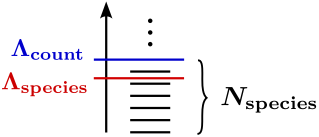
Number of Species in the String Tower

Defining N as

$$N = \exp\left(\frac{c^2}{4}\left(\frac{\Lambda_{QG}}{M_S}\right)^2\right) \quad \Longrightarrow \quad \Lambda_{\text{species}} \simeq M_s\sqrt{\log(g_s^{-2})}$$

This matches with the scale [Mende and Ooguri, 89], where string tree-level approximation becomes invalid.

However, this conflicts with the naive counting:



$$\Lambda_{\text{species}} < \Lambda_{\text{count}} \simeq M_s \log(g_s^{-2})$$

Conclusion

Conclusion

- The species bound connects the infinite tower of light states to the scale of quantum gravity.
- A bottom-up approach confirms that the species scale serves as the cutoff of quantum gravity.
- In the case of the string tower, the species scale derived from field theory aligns with results from the string S-matrix.