Complex Langevin Lattice QCD for Color-Superconductivity at Extremely High Density

Kohtaroh Miura

KEK, Institute of Particle and Nuclear Studies, Theory Center

2024 December 11 at KEK-THEORY Workshop

Collaboration

Yuhma Asano	(University of Tsukuba)
Yuta Ito	(Tokuyama College)
Hideo Matsufuru	(KEK/SOKENDAI)
Kohtaroh Miura	(KEK-IPNS)
Yusuke Namekawa	(Hiroshima University)
Jun Nishimura	(KEK/SOKENDAI)
Asato Tsuchiya	(Shizuoka University)
Shoichiro Tsutsui	(RIKEN-iTHEMS)
Takeru Yokota	(RIKEN)

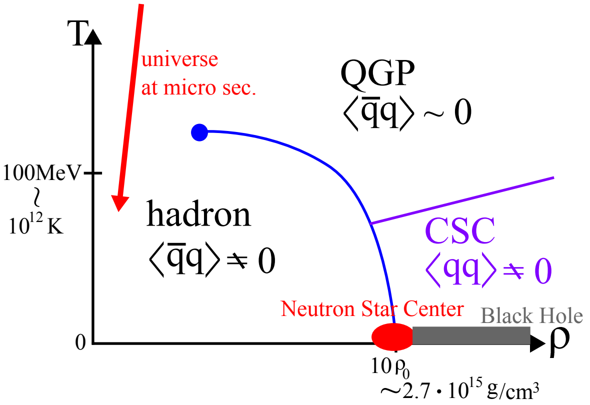
References

[1] T. Yokota, Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, A. Tsuchiya and S. Tsutsui, JHEP **06** (2023), 061 [arXiv:2302.11273 [hep-lat]].

[2] S. Tsutsui, Y. Asano, Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, A. Tsuchiya and T. Yokota, PoS **LATTICE2021**, 533 (2022) [arXiv: 2111.15095 [hep-lat]].

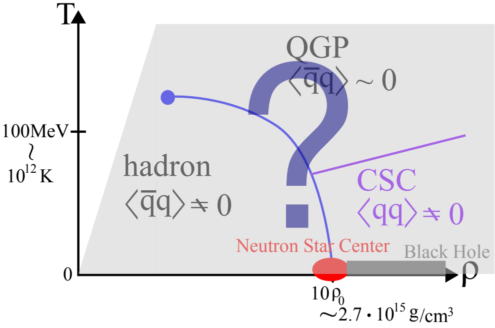
[3] Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, S. Shimasaki, A. Tsuchiya and S. Tsutsui, JHEP **10**, 144 (2020) [arXiv: 2007.08778 [hep-lat]].

QCD Phase Diagram



The QCD phase diagram is very interesting.

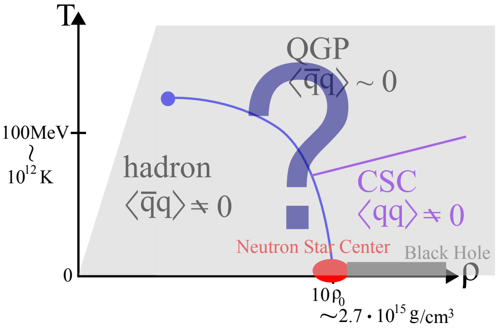
QCD Phase Diagram



High ρ: First principle calculations are very limited

due to Sign Problem in Monte Carlo based LQCDs.

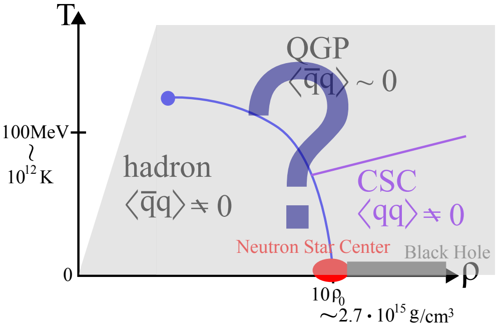
QCD Phase Diagram



High ρ: Colorsuperconductor (CSC, [4]- [7]) or Quarkyonic [9, 10]?

[4] Barrois (NPB'77), [5] Frautschi (SpringerUSA'80), [6] Bailin et al (NPB'81), [7] Alford et al (PLB'98), [9] McLerran et al (NPA'07), [10] Hidaka et al (NPA'08).

QCD Phase Diagram



High ρ: Attack the sign problem by
Complex Langevin based LQCD (CL-LQCD) for CSC.

c.f. Lefschetz Thimble, TRG, LYZ, Quantum Computer...

Small-Box QCD with Gap Equation

T. Yokota et al, JHEP'23. [1]:
Small-Box QCD (on Lattice) $L_s < \Lambda_{QCD}^{-1}$ ∼ One-Loop QCD Gap Eq.

Dyson eq. in Nambu basis

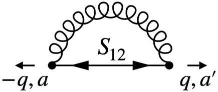
$$\mathbf{\Sigma} = \mathbf{S}^{-1} - \mathbf{S}_{\text{free}}^{-1}$$

Emerged by $\Sigma_{12(21)}$

$$\mathbf{S} = \begin{pmatrix} \langle \psi\bar{\psi} \rangle & \langle \psi\psi \rangle \\ \langle \bar{\psi}\bar{\psi} \rangle & \langle \bar{\psi}\psi \rangle \end{pmatrix}$$

Gap eq. in lowest order

$$\Sigma_{12(qaa')} =$$



q: momentum
a, a': internal d.o.f.

⇓ Linearlize

Critical point (Thouless criterion)

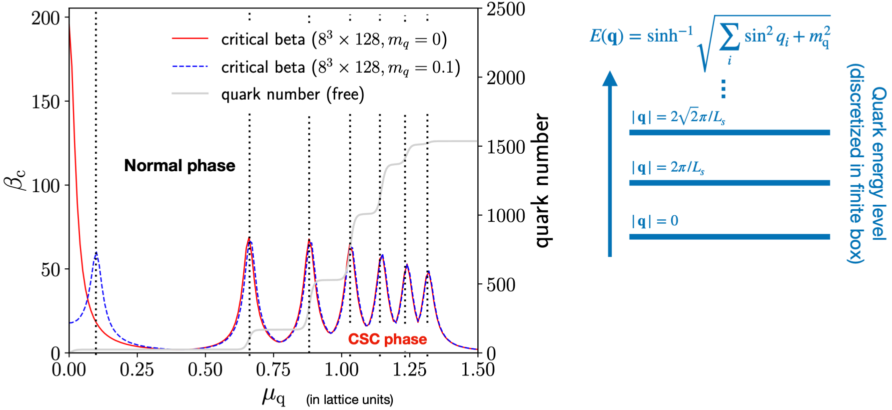
$$\frac{1}{V}\sum_{q'bb'} M_{(qaa')(q'bb')}\Sigma_{12(q'bb')} = \beta\Sigma_{12(qaa')}$$

$\beta = 2N_c/g^2$

- Max. eigenvalue of M
  = critical lattice coupling β_c = phase boundary of normal and CSC phases.
- No Gauge Fixing for the diquark Σ_{12} is required.

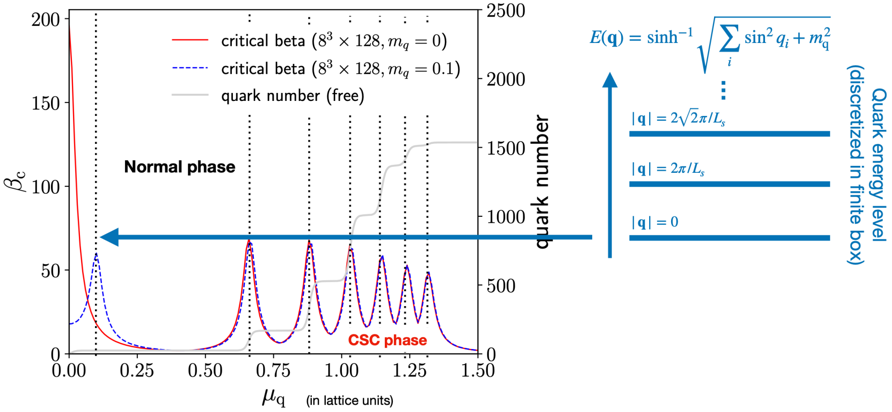
c.f. Small-Box QCD or QCD-like models at Continuum [11, 12, 13]: S. Hands et al, JHEP'12; JHEP'10; PLB'02.

Expected Phase Diagram in Small-Box LQCD



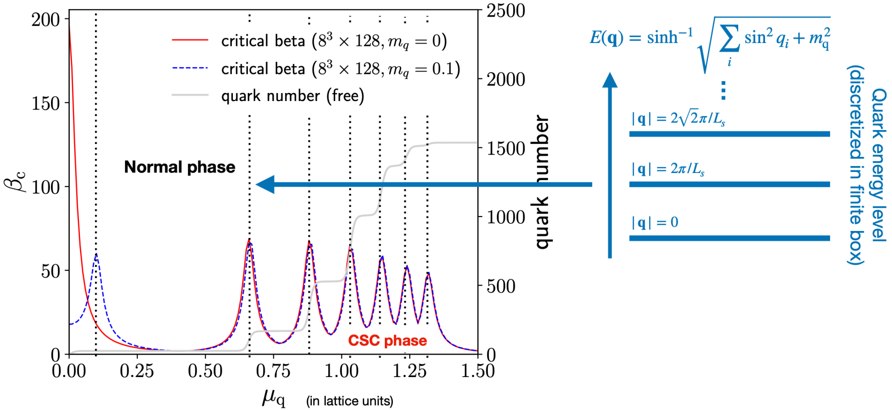
Slide borrowed from T. Yokota.

Expected Phase Diagram in Small-Box LQCD



Slide borrowed from T. Yokota.

Expected Phase Diagram in Small-Box LQCD



Slide borrowed from T. Yokota.

This Talk

Complex-Langevin Lattice QCD

- in a Small-Box,
- guided by One-Loop QCD Gap Equations,
- focusing on Colorsuperconductivity,
- in a gauge invariant way.

Table of Contents

Table of Contents

Complex Langevin in Lattice QCD

Complexification of Gluon Fields:

$$U_{\mu x} \in SU(3) \Longrightarrow \mathcal{U}_{\mu x}(\tau) \in SL(3,\mathbb{C}) \,. \qquad (1)$$

Parisi '83; Klauder '84;
Aarts, Seiler, Stamatescu '09;
Aarts, James,Seiler, Stamatescu '11;
Seiler, Sexty, Stamatescu '13; Sexty '14;
Fodor, Katz, Sexty, Torok '15;
Nishimura, Shimasaki '15;
Nagata, Nishimura, Shimasaki '15;
Sinclair, Kogut '16

Complex Langevin Equation:

$$\mathcal{U}_{\mu x}(\tau+\epsilon) = \exp\Big[i\lambda^a\big(-\epsilon\,\underbrace{v_{a\mu x}[\mathcal{U}(\tau)]}_{\text{drift term}} + \sqrt{\epsilon}\,\underbrace{\eta_{a\mu x}(\tau)}_{\text{gaussian noise}}\big)\Big]\mathcal{U}_{\mu x}(\tau) \,. \qquad (2)$$

c.f. For $z \in \mathbb{C}$, $\partial_\tau z(\tau) = \underbrace{-\partial_z S(z)}_{\text{drift term}} + \underbrace{\eta(\tau)}_{\text{gaussian noise}} \,.$

We expect:

$$\lim_{\tau\to\infty} \langle O[\mathcal{U}(\tau)]\rangle_\eta \overset{!}{=} \int DU\, P[U] O[U] \,. \qquad (3)$$

Complex Langevin in Lattice QCD

For a correct convergence of Complex-Langevin processes,
(So that associated Fokker-Planck Eq. achieves correct probability distribution,)

1. The Drift Term,

$$v_{a\mu x}[\mathcal{U}] = \lim_{\epsilon \to 0} \frac{S[e^{i\epsilon\lambda_a}\mathcal{U}_{\mu x}] - S[\mathcal{U}_{\mu x}]}{\epsilon} \,, \tag{4}$$

should be regular (no singular drift problem). To confirm, monitor that the histogram of

$$v_{max} = \left(\frac{1}{\mathrm{Vol}} \sum_{x,\mu} \sum_{i=1}^{N_c^2-1} |v_{a\mu x}|^2 \right)^{1/2} \,. \tag{5}$$

rapidly (exponentially) attenuates (Nagata et al, PRD'16).

2. The Unitarity Norm

$$\mathcal{N}_u = \frac{2}{\mathrm{Vol}} \sum_{x\nu} \mathrm{Tr}\left[\mathcal{U}_{x\mu}^\dagger \mathcal{U}_{x\mu} + \left(\mathcal{U}_{x\mu}^{-1}\right)^\dagger \mathcal{U}_{x\mu}^{-1} - 2 \times \mathbf{1}_3 \right] \,, \tag{6}$$

should stay small enough (no excursion of gluon fields into a deeply imaginary regime), which is achieved by a gauge cooling (Seiler et al, PLB'13).

Source-Term Method

- To extract a signal of diquark condensates on the lattice QCD, we introduce a source term

$$S_\phi = \sum_n m_\phi^2 \phi_n^{a,\dagger} \phi_{n,a} + S_J \;, \quad S_J = J \sum_n \left[\epsilon^{abc} (\chi_a \chi_b \phi_c)_n - h.c.\right] \; . \tag{7}$$

- The original staggered quark action $S_f = \bar{\chi} M \chi / 2$ is generalized as

$$S_f + S_J = \frac{1}{2} (\chi^T, \bar{\chi}) \tilde{M} \begin{pmatrix} \chi \\ \bar{\chi}^T \end{pmatrix} \;, \quad \tilde{M} = \begin{pmatrix} \Phi & -M^T \\ M & -\Phi^\dagger \end{pmatrix} \;, \quad \Phi_{nn'}^{ab} = 2J \epsilon^{abc} \phi_{n,c} \delta_{nn'} \; .$$

- Complex Langevin Eq. is characterized by $S_{eff} = S_G - (N_f/4) \mathrm{Log} Pf[\tilde{M}]$.

- One might think of measuring $\langle \bar{\phi} \phi \rangle$ which is proportional to our target $\langle (\epsilon \bar{\chi} \bar{\chi})(\epsilon \chi \chi) \rangle$. However this turns out to be too small: $\mathcal{O}(J^2)$.

- The relevant observable which we measure is the source-term itself

$$\frac{m_\phi^2}{2J} \cdot \langle \epsilon \chi \chi \phi + \epsilon \bar{\chi} \bar{\chi} \bar{\phi} \rangle \;, \tag{8}$$

with various J. This observable is equivalent to $\langle (\epsilon \bar{\chi} \bar{\chi})(\epsilon \chi \chi) \rangle$, which is not straightforward to calculate (Tsutsui et al, PoS LAT2021).

Table of Contents

Setup

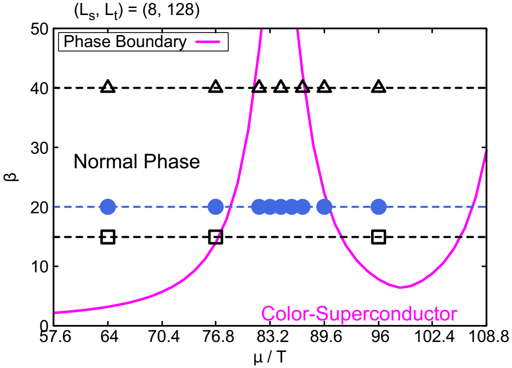
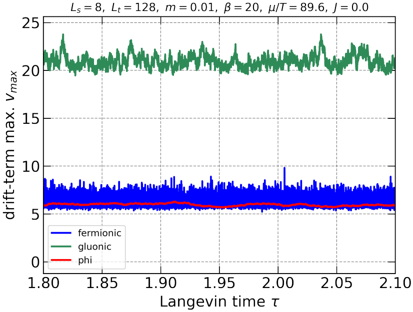
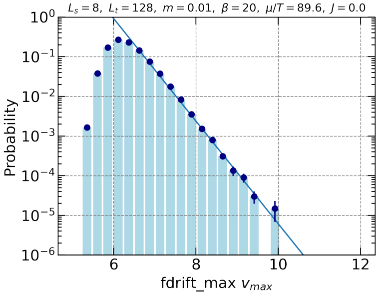


Figure:
Simulation points (blue circles) on the phase diagram predicted by the gap equation on the small-box LQCD.

- Small-box LQCD: $(L_s,\ L_t,\ m_{lat},\ \beta) = (8,\ 128,\ 0.01,\ 20.0)$, which realizes a cold ($L_t/L_s \gg 1$) and small-box LQCD $L_s a_{lat}(\beta) < \Lambda_{QCD}^{-1}$.
- At each simulation point, we investigate $\langle \epsilon\chi\chi\phi + h.c. \rangle$ for various J.

Example of Drift-Terms





- Left: An example of drift-terms Langevin trajectory.
- Right: A histogram for the fermion drift-term (blue trajectory in the left panel).
  - No Singular-Drift: The tail $v_{max} \geq 8$ is exponentially suppressed, $\propto e^{-3.0(2) \cdot v_{max}}$.
  - No-Excursion: $\mathcal{N}_U \sim \mathcal{O}(10^{-7})$.

Quark Number Density

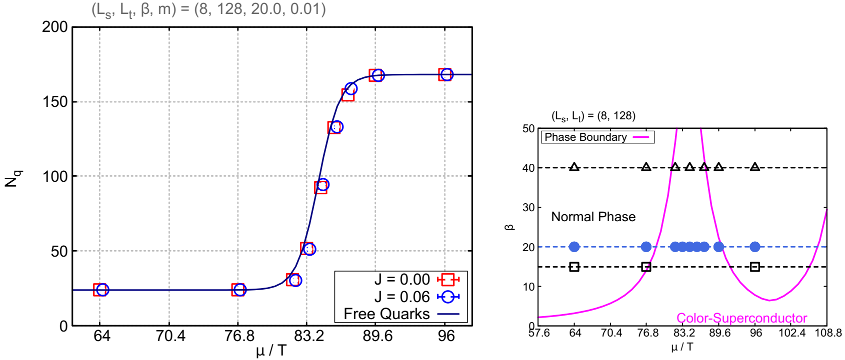
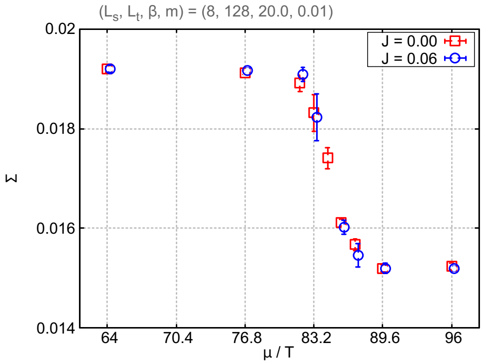


Figure: Quark number density vs. quark chemical potential (T units).

Stepwise Fermi-surface: $24 \cdot 1 \rightarrow 24 \cdot (1 + 6_{\vec{p}})$, where $24 = 4_{flv} \cdot 3_{col} \cdot 2_{spin}$.

Chiral Condensate



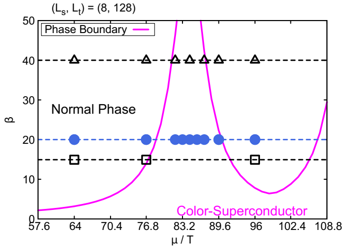
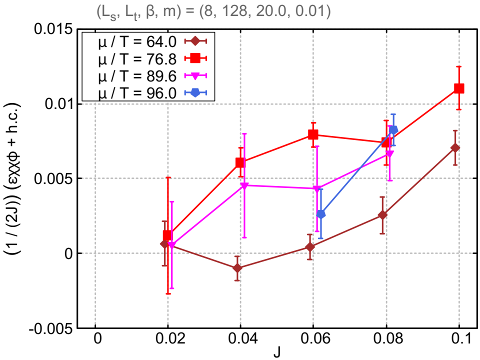


Figure: Chiral condensates vs. quark chemical potential (T units).

Chiral symmetry breaking tends to get restored with Fermi-surface formation.
(N.B. Polyakov loops stay zero-consistent.)

Diquark Source-Term: Summary



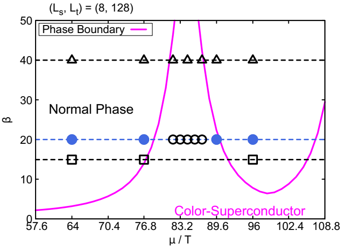
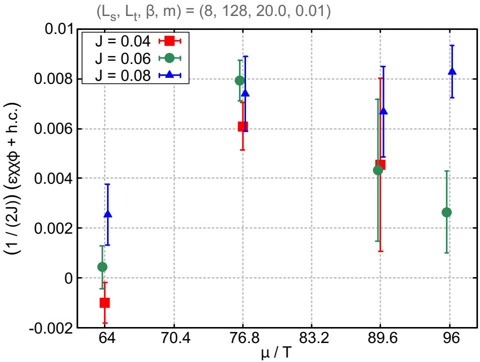


Figure: Comparison of Diquark Source Term vs. source-term coupling J.

Approaching the phase boundary from the normal phase to CSC, the response to J becomes more significant.

Diquark Source-Term: Summary



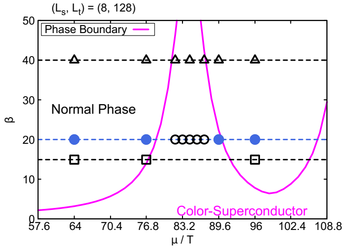


Figure: Comparison of Diquark Source Term vs quark chemical potential (T Units).

Approaching the phase boundary from the normal phase to CSC, the response to J becomes more significant.

Table of Contents

Summary and Future Perspective

- We have investigated a small-box lattice QCD at high density by using the complex Langevin method.
- We have invented the diquark source-term method and investigated the color-superconductivity in a gauge-invariant way.
- Approaching the phase boundary from the normal phase, a response to the source-term coupling J becomes more significant. This implies non-linear response associated with the CSC.
- A volume dependence of the above property is under investigation:
  Normal phase $\Longrightarrow$ Hadronic or Quarkyonic as $L_s a_{lat}(\beta)$ exceeds Λ_{QCD}^{-1}?
- Ultimately, we will take (1) a infinite volume limit and then (2) $J \to 0$ limit to see whether the diquark-source term remains finite (color-superconductivity emerges).

References

[1] T. Yokota, Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, A. Tsuchiya and S. Tsutsui, JHEP **06** (2023), 061 [arXiv:2302.11273 [hep-lat]].

[2] S. Tsutsui, Y. Asano, Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, A. Tsuchiya and T. Yokota, PoS **LATTICE2021**, 533 (2022) [arXiv: 2111.15095 [hep-lat]].

[3] Y. Ito, H. Matsufuru, Y. Namekawa, J. Nishimura, S. Shimasaki, A. Tsuchiya and S. Tsutsui, JHEP **10**, 144 (2020) [arXiv: 2007.08778 [hep-lat]].

[4] B.C. Barrois, Nucl. Phys. B **129** (1977) 390.

[5] S.C. Frautschi, Springer, U.S.A. (1980), p.19 [DOI:10.1007/978-1-4684-3602-0_2].

[6] D. Bailin and A. Love, Nucl. Phys. B **190** (1981) 175.

[7] M.G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422** (1998) 247 [hep-ph/9711395].

[8] M.G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Rev. Mod. Phys. **80** (2008) 1455 [arXiv:0709.4635].

References

[9] L. McLerran and R. D. Pisarski, Nucl. Phys. A **796** (2007), 83-100 [arXiv:0706.2191 [hep-ph]].

[10] Y. Hidaka, L. D. McLerran and R. D. Pisarski, Nucl. Phys. A **808** (2008), 117-123 [arXiv:0803.0279 [hep-ph]].

[11] S. Hands, T.J. Hollowood, and J.C. Myers, JHEP **12** (2010) 057 [arXiv:1010.0790].

[12] S. Hands, T.J. Hollowood, and J.C. Myers, JHEP **07** (2010) 086 [arXiv:1003.5813].

[13] S. Hands and D.N. Walters, Phys. Lett. B **548** (2002) 196 [hep-lat/0209140].

References

[14] G. Parisi, Phys. Lett. B **131** (1983) 393.

[15] J.R. Klauder, Phys. Rev. A **29** (1984) 2036.

[16] G. Aarts, E. Seiler, and I.-O. Stamatescu, Phys. Rev. D **81** (2010) 054508 [arXiv:0912.3360].

[17] G. Aarts, F.A. James, E. Seiler, and I.-O. Stamatescu, Eur. Phys. J. C **71** (2011) 1756 [arXiv:1101.3270].

[18] E. Seiler, D. Sexty, and I. O. Stamatescu, Phys. Lett. B **723**, 213-216 (2013) [arXiv:1211.3709 [hep-lat]].

[19] Z. Fodor, S.D. Katz, D. Sexty and C. Török, Phys. Rev. D **92** (2015) 094516 [arXiv:1508.05260].

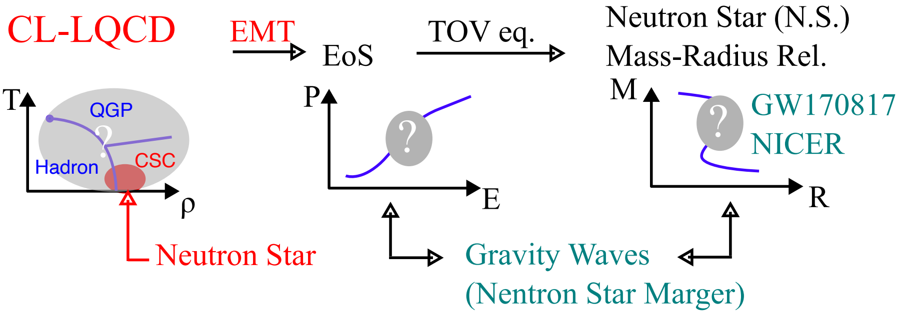
[20] K. Nagata, J. Nishimura and S. Shimasaki, PTEP **2016** (2016) no.1, 013B01 [arXiv:1508.02377 [hep-lat]].

[21] K. Nagata, J. Nishimura, and S. Shimasaki, Phys. Rev. D **94**, no.11, 114515 (2016) [arXiv:1606.07627 [hep-lat]].

[22] J.B. Kogut and D.K. Sinclair, Phys. Rev. D **100** (2019) 054512 [arXiv:1903.02622].

Table of Contents

From LQCD to Neutron Star Physics



Tolman–Oppenheimer–Volkoff (TOV) Equation

- Tolman–Oppenheimer–Volkoff (TOV) Equation:

$$\frac{dP(r)}{dr} = -G\frac{(M(r) + 4\pi r^3 P(r)/c^2)(E^2 + P^2)/c^2}{r^2(1 - 2GM/(rc^2))} , \quad \frac{dM(r)}{dr} = \frac{4\pi r^2 E}{c^2} . \tag{9}$$

- TOV Eq. is a static balance condition between neutron-star pressure and gravity in general relativity: $P(r + dr) - P(r) \sim GM(r)dr/r^2$.

- When the neutron-star EoS ($P = f(E)$) is combined with TOV Eq., $M - R$ relation is extracted.

Neutron Star Strcture

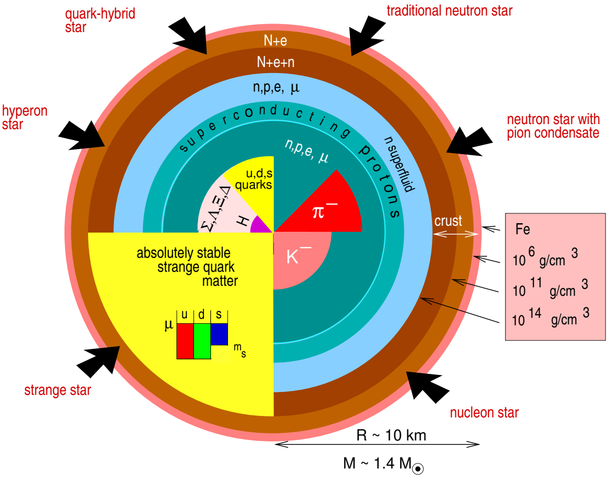


Fig: Neutron star inner structure. Quoted from [F. Weber, J. Phys. G: Nucl. Part. Phys. 27 (2001)]

Neutron Star Equation of State: Various Examples

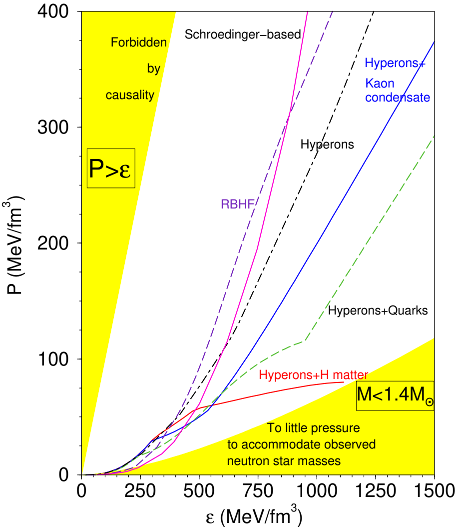


Fig: Various neutron star equation of state (EoS). Quoted from [F. Weber, J. Phys. G: Nucl. Part. Phys. 27 (2001)].

Neutron Star Mass: Observation

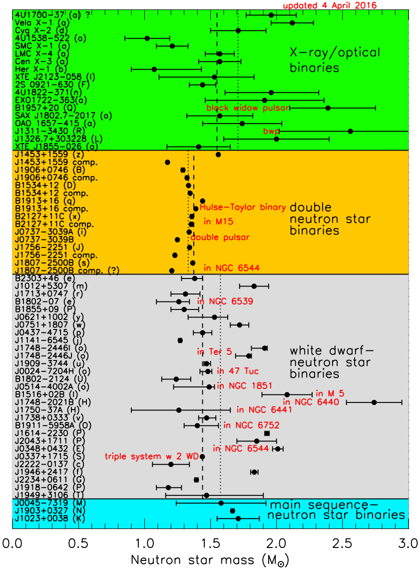


Fig: Summary of neutron star masses. The update of [PRL’05, J. Lattimer et al.]:

https://stellarcollapse.org/index.php/nsmasses.html

Neutron Star $M - R$ Relation

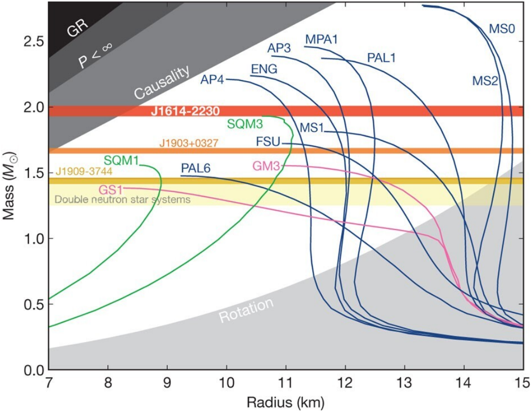


Fig: Various $M - R$ relations.
Quoted from [Nature'10, vol.467, P. Demorest et al.].