A scaling relation in large-N gauge theories at finite temperature

KEK Theory workshop 2024

Dec. 11th, 2024

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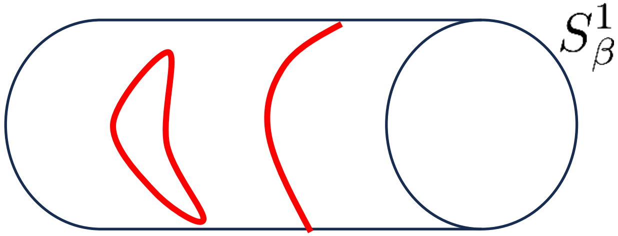
Based on collaboration
with Takehiro Azuma (*Setsunan University*)

2406.10672 (PTEP 2024)

Introduction

d-dim $\mathrm{U}(N)$ Yang-Mills theory (+ adjoint matters) (d≧1) $N \to \infty$

Finite temperature



Find a scaling relation with respect to temperature in loop equation.

⇒

- Strong constraints on the Polyakov loop effective potential
  (≒ Gauge theory version of the Landau–Ginzburg theory)
- Strong constraints on various observables.

✓ Non-perturbative relation independent of the details of the models.

✓ It is applicable not only to the thermal S^1_β but also to any S^1.

1. Loop equation at finite temperature (review)

2. Scaling relation in loop equation

3. The Polyakov loop effective potential at large-N

4. Summary

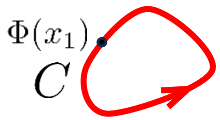
1. Loop equation at finite temperature

$\mathrm{U}(N)$ d-dim YM theory (d ≧ 1)
(+ adjoint matters $\Phi(x)$)

★ Building blocks of gauge theory

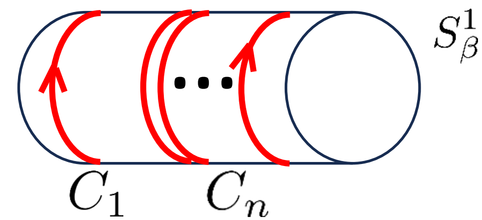
→ Loop operator for contour $C := W(C)$

$$\frac{1}{N}\mathrm{Tr}\left[\mathrm{P}\exp\left(i\int_C A_\mu dx^\mu\right)\right], \quad \frac{1}{N}\mathrm{Tr}\left[\mathrm{P}\exp\left(i\int_C A_\mu dx^\mu\right)\Phi(x_1)\right], \quad \cdots$$



$$:= W(C_0)$$

★ Winding loop operators at finite temperature



C_n : Contour winding n times

$$:= W(C_n)$$

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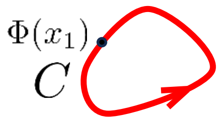

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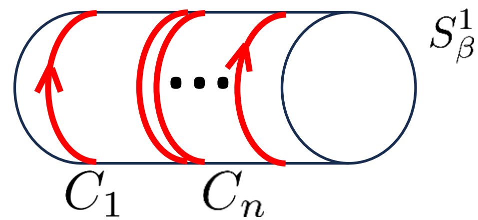

C_n : Contour winding n times

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★ Loop equation (Schwinger-Dyson eq.): non-perturbative correlation for $W(C)$

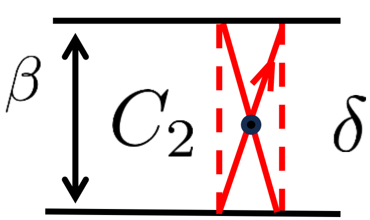
Makeenko-Migdal (1979), Eguchi-Kawai (1982), Gocksch-Neri (1983)

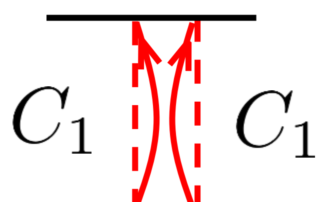
Roughly, the following expression:

$$\langle \delta W(C_m)\rangle = \sum_k \langle W(C_{m-k})\rangle\langle W(C_k)\rangle + O(1/N^2)$$

δ ≒ A local insertion of an Euler-Lagrange eq..

ex) m=2





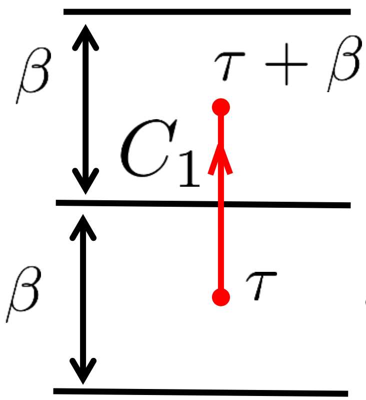
Features

- Conservation of the winding number.
- No explicit temperature dependence. (δ is independent of temperature).
- Large-N factorization.

1. Loop equation at finite temperature

★ Building blocks of gauge theory

→ Temperature ← non-zero $\langle W(C_n)\rangle$

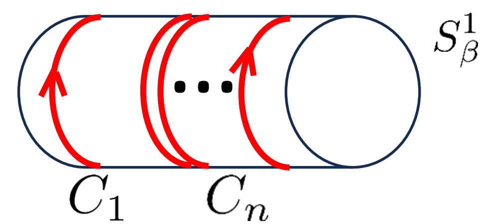


$W(C_1)$ is an "open" Wilson line.
→ This can be non-zero due to the periodicity $A_\mu(\tau+\beta)=A_\mu(\tau)$.

$\langle W(C_1)\rangle \neq 0$ → Temperature dependence in the loop equations.

$\mathrm{U}(N)$ d-dim YM theory (d≧1)
(+ adjoint matters $\Phi(x)$)

★Winding loop operators at finite temperature



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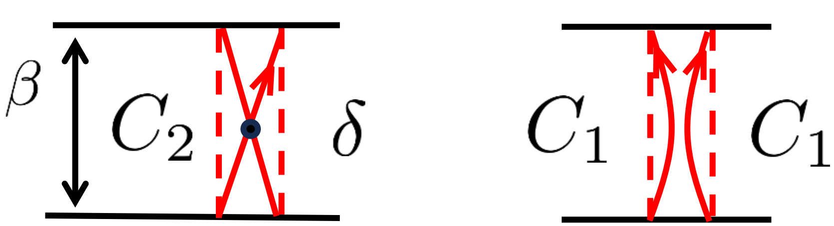

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2. Scaling relation in loop equation

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★ Loop equation

$$\langle \delta W(C_m) \rangle = \sum_k \langle W(C_{m-k}) \rangle \langle W(C_k) \rangle + O(1/N^2)$$

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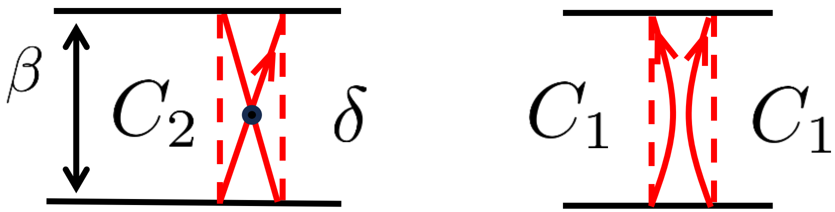

Features

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Scaling relation: For any natural number m, we can show

Loop equation at β with the substitution $\begin{cases} \langle W(C_{nm}) \rangle \to \langle W(C_n) \rangle \\ \langle W(C_n) \rangle = 0 \quad (n \notin m\mathbf{Z}) \end{cases}$

= Loop equation at $m\beta$ $\qquad (N \to \infty)$

This is for bosonic systems. Need a modification for fermions, which I skip in this talk.

2. Scaling relation in loop equation

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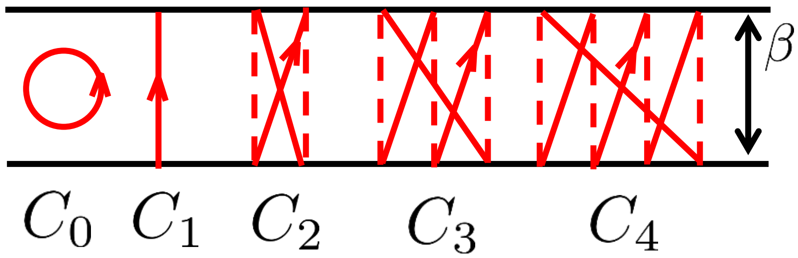
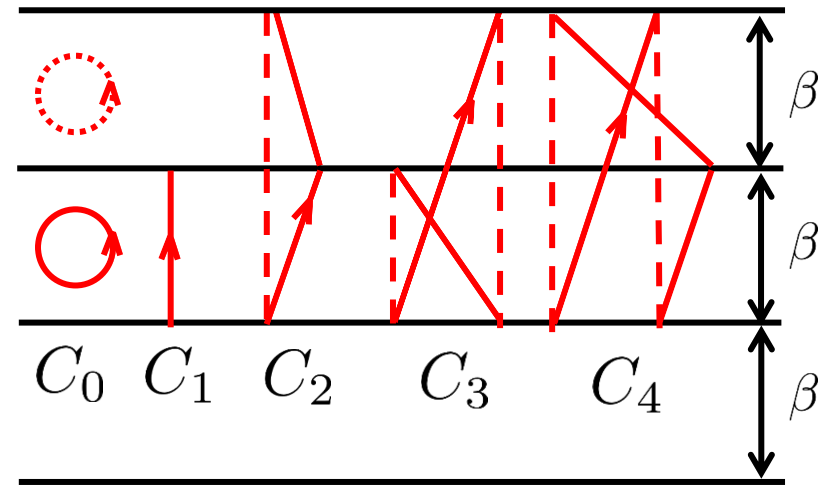
ex) m=2 case

Mirror images



$0 \leq \tau < \beta$

$=$



$-\infty < \tau < \infty$

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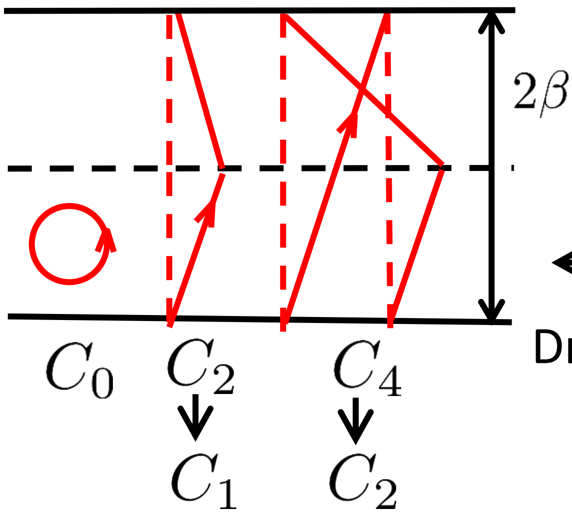
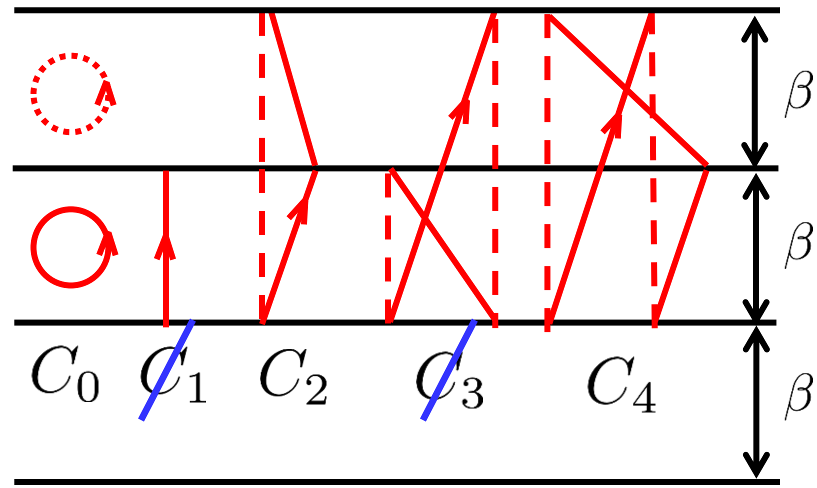


Only the loops at 2β survive.

Dropping the odd winding loops

Mirror images



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3. The Polyakov loop effective potential at large-N

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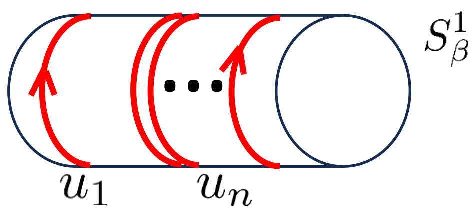
(Review) The low effective theory of finite temperature gauge theories

Order parameter:

$$u_n = \frac{1}{N}\mathrm{Tr}\left(\mathrm{P}e^{i\int_0^{n\beta} A_\tau d\tau}\right)$$

Polyakov loop operators winding the temporal circle n times.



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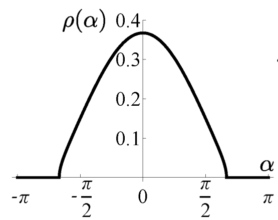

← Eigenvalue distribution of A_τ

All $\{u_n\}$ are required to represent the phases.

The effective potential (LG theory): $V(\beta, \{u_n\})$

Constraint: Winding number conservation $u_n u_m u_l \to n + m + l = 0$

$$\longrightarrow \quad V(\beta, \{u_n\}) = \epsilon(\beta) + \sum_{n=1}^{\infty} a_n(\beta) u_n u_{-n} + \sum_{n=1}^{\infty} b_n(\beta) u_n^2 u_{-n}^2 + \cdots .$$

The coefficients $\epsilon(\beta),\ a_n(\beta),\ b_n(\beta), \cdots$ are determined depending on the details of the models.

$N \to \infty$ $\quad \epsilon(\beta) = \epsilon$ Gocksch-Neri (1983)

Infinitely many constraints are imposed from our scaling relation!

3. The Polyakov loop effective potential at large-N

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Scaling relation

Loop equation at β with the substitution $\begin{cases} \langle W(C_{nm})\rangle \to \langle W(C_n)\rangle \\ \langle W(C_n)\rangle = 0 \quad (n \notin m\mathbf{Z}) \end{cases}$

= Loop equation at $m\beta$ $\quad (N \to \infty)$

→ $V(\beta, \{u_n\})$ has to satisfy the following relation for any *m*:

$$V(\beta, \{u_n\})|_{u_n=0\ (n\notin m\mathbf{Z}),\ u_{mn}\to u_n} = V(m\beta, \{u_n\})$$

→ $$V(\beta, \{u_n\}) = \epsilon(\beta) + \sum_{n=1}^{\infty} a_n(\beta) u_n u_{-n} + \sum_{n=1}^{\infty} b_n(\beta) u_n^2 u_{-n}^2 + \cdots .$$

$$\begin{cases} \epsilon(\beta) = \epsilon \\ a_n(\beta) = a_1(n\beta) \\ b_n(\beta) = b_1(n\beta) \\ \cdots \end{cases}$$

All the coefficients are strongly constrained.

$$V(\beta, \{u_n\}) = \epsilon + \sum_{n=1}^{\infty} a_1(n\beta) u_n u_{-n} + \sum_{n=1}^{\infty} b_1(n\beta) u_n^2 u_{-n}^2 + \cdots .$$

3. The Polyakov loop effective potential at large-N

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★ Effective potential → physical quantities

Ex) u_n in the confinement phase: $u_n = \frac{1}{N} \mathrm{Tr}\left(\mathrm{P} e^{i\int_0^{n\beta} A_\tau d\tau} \right)$

$$\langle |u_1(T)| \rangle = \frac{1}{\sqrt{n}} \langle |u_n(nT)| \rangle$$

→ Any adjoint large-N gauge theories satisfy this relation if it is confined.

Note) $\{u_n\}$ are zero in the confinement phase, but their ratios are finite.

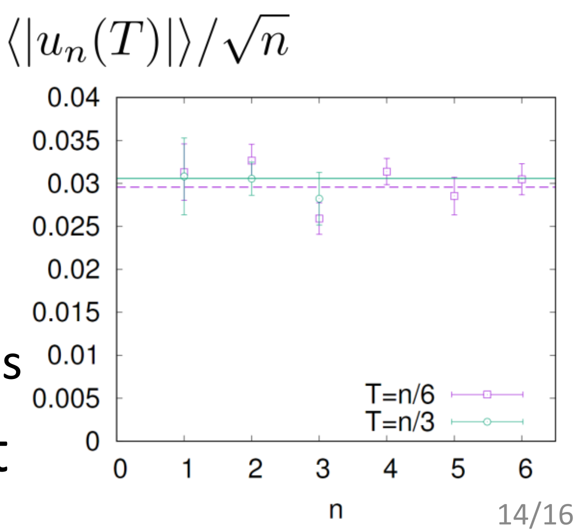
Ex) Bosonic BFSS matrix model (Monte-Carlo)

$$S = \int_0^\beta dt \mathrm{Tr} \left\{ \sum_{I=1}^{D} \frac{1}{2} (D_t X^I)^2 - \sum_{I,J=1}^{D} \frac{g^2}{4} [X^I, X^J]^2 \right\}$$

D=3, N=30

Line: scaling relation
Dots: numerical results

→ Good agreement



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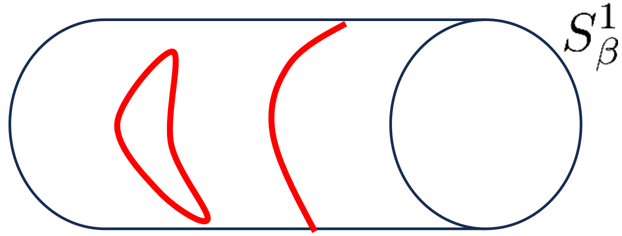
Finite temperature



→ Scaling relation in the loop equations.

✓ Non-perturbative relation independent of the details of the models.

✓ It is applicable not only to the thermal S^1_β but also to any S^1.

⇒
- Strong constraints on the Polyakov loop effective potential
- Strong constraints on various observables.

Future directions

- Application to various adjoint theories.
- Gauge/gravity correspondence → scaling relation in string theory?