Universal structure of Islands in evaporating black holes

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Based on [arXiv:2407.20921]

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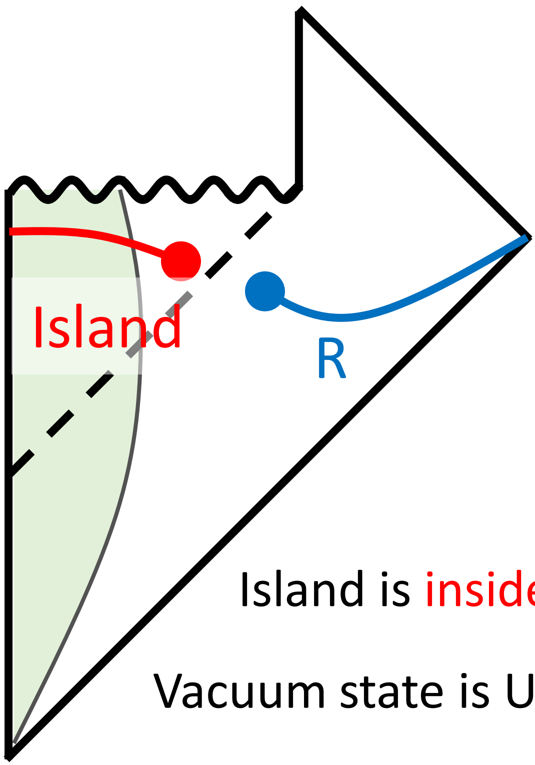
Position of islands does not depend on details of black holes

Hawking radiation in R has information of island

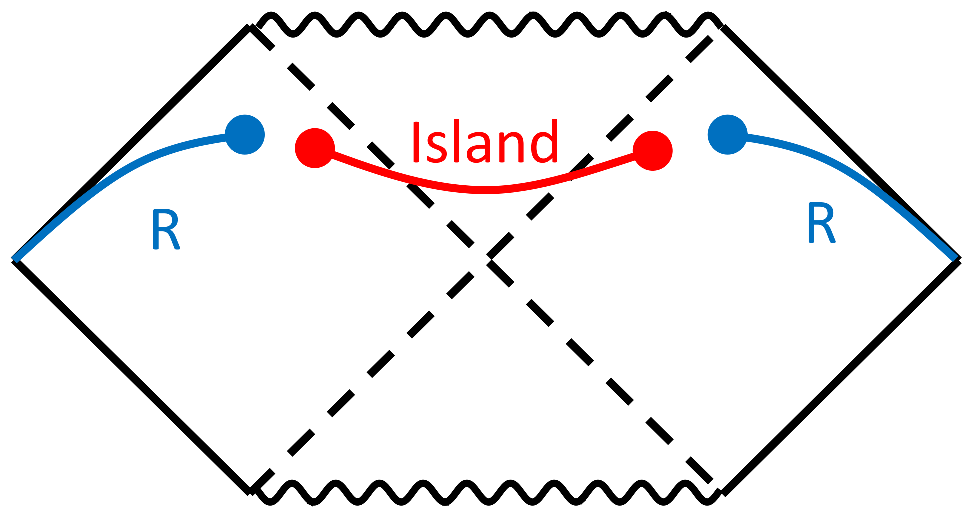
Evaporating black hole (1-sided)



Island is inside the horizon

Vacuum state is Unruh vacuum

Eternal black hole (2-sided)



Island extends outside the horizon

Vacuum state is Hartle-Hawking vacuum

This structure is determined by universal structure near the horizon

Plan of Talk

1. Entanglement entropy and islands
    - Islands appear as a consequence of replica trick
    - Gravity part: conical singularity at branch points
    - Matter part: correlation function of twist operators
2. Preferred coordinates of vacuum states
3. Islands in black hole spacetimes

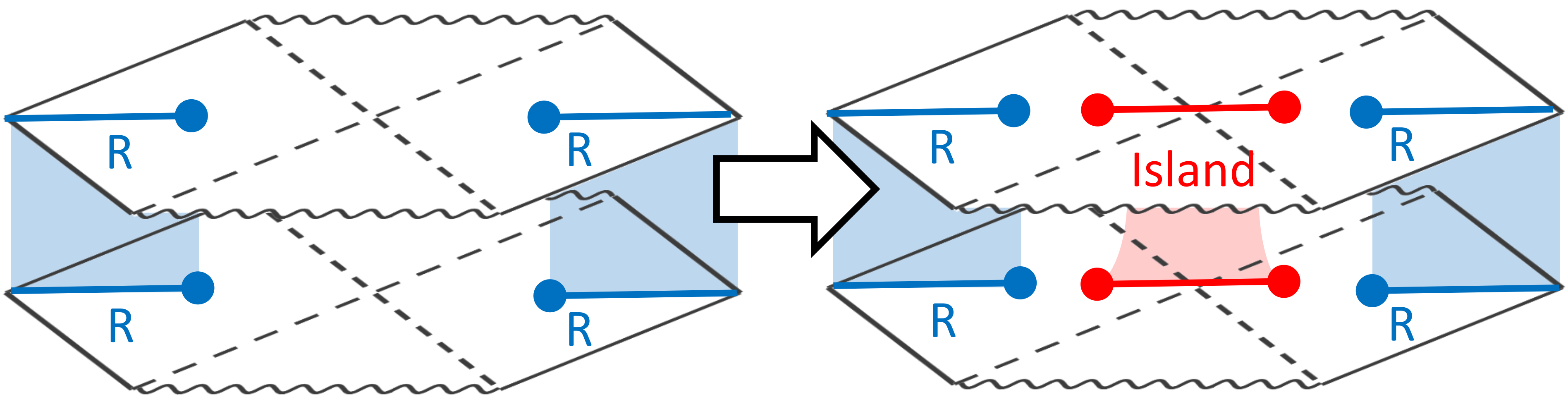
Island is wormhole between replica spacetimes

Entanglement entropy of Hawking radiation = Entanglement entropy of region R

Replica trick with gravity

→ Wormhole geometry between replicas is allowed = Island

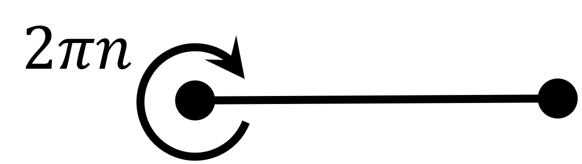
Entanglement entropy of R with gravity = Entanglement entropy of R∪I without gravity



Gravity part of entanglement entropy comes from conical singularity at branch point

Einstein-Hilbert action (+ UV divergent part of 1-loop effective action)

$$\log Z = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\,R + \cdots$$



n-sheeted spacetime has conical singularity at ends of branch cuts

$$R \propto (n-1)\delta(x - x_1) \quad \Rightarrow \quad \log Z_n = (n-1)\frac{\text{Area}}{4G_N} + \cdots$$

Entanglement entropy is area of branch point

$$S_{\text{grav}} = \frac{\text{Area}}{4G_N}$$

More generally, given by Wald entropy

$$S_{\text{grav}} = S_{\text{Wald}} \sim \int \frac{\partial \mathcal{L}}{\partial R_{trtr}}$$

Matter part of entanglement entropy is given by correlation function of twist operators

Non-local term can be calculated by using twist operators

$$Z_n = \left\langle e^{\varphi(x_1)} e^{-\varphi(y_1)} e^{\varphi(x_2)} e^{-\varphi(y_2)} \dots \right\rangle$$

e^{φ}: Twist operator $\quad \langle \varphi(x)\varphi(y) \rangle \sim \log|x-y|^2$

Divergent part should be regularized by cut-off in proper length

$$S_{\mathrm{matter}} \sim -\sum \log\left|x_i - x_j\right|^2 - \sum \log\left|y_i - y_j\right|^2 + \sum \log\left|x_i - y_j\right|^2$$

Divergence for $i = j$

$$-2\sum \rho - \sum \log {\epsilon_{\mathrm{prop}}}^2$$

$$\log \epsilon^2 = 2\rho + \log {\epsilon_{\mathrm{prop}}}^2$$

metric

$$ds^2 = -e^{2\rho} dU dV + r^2 d\Omega^2$$

Non-local term depends on vacuum state because correlation function depend on vacuum state

Plan of Talk

1. Entanglement entropy and islands

2. Preferred coordinates of vacuum states
   - Vacuum state is defined by using a (time) coordinates
   - Hawking radiation appears due to change of coordinates associated to vacuum state

3. Islands in black hole spacetimes

Vacuum state is defined by using a (time) coordinate

Creation and annihilation operators are defined by Fourier modes

$$\phi(x) = \int \frac{d\omega}{\sqrt{2\omega}} \left(a_\omega e^{-i\omega u} + a_\omega^\dagger e^{i\omega u} + b_\omega e^{-i\omega v} + b_\omega^\dagger e^{i\omega v} \right)$$

and vacuum state is defined by annihilation operators

$$a_\omega |0\rangle = 0 \qquad\qquad b_\omega |0\rangle = 0$$

Correlation function in this vacuum is exactly given by

$$\langle 0|\phi(x_1)\phi(x_2)|0\rangle = \frac{1}{4\pi} \log |(v_1 - v_2)(u_1 - u_2)|$$

Vacuum state is defined by using a (time) coordinate

We can do the same in another coordinate $U = U(u), V = V(v)$

$$\phi(x) = \int \frac{d\omega}{\sqrt{2\omega}} \left(A_\omega e^{-i\omega U} + A_\omega^\dagger e^{i\omega U} + B_\omega e^{-i\omega V} + B_\omega^\dagger e^{i\omega V} \right)$$

Vacuum state associated with this expansion is different from $|0\rangle$

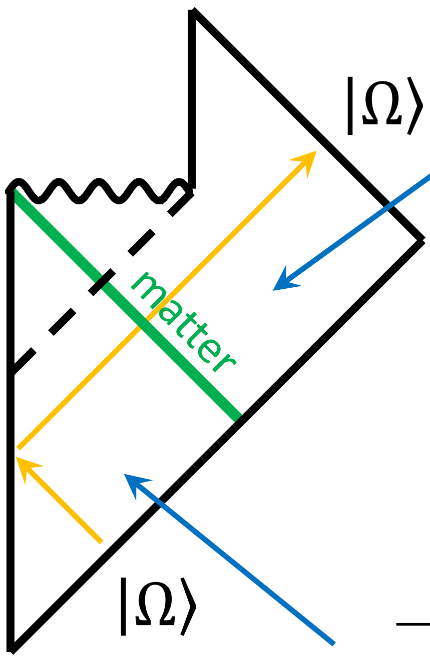
$$A_\omega |\Omega\rangle = 0 \ , \quad B_\omega |\Omega\rangle = 0 \qquad \text{but} \qquad a_\omega |\Omega\rangle \neq 0 \ , \ \ b_\omega |\Omega\rangle \neq 0$$

Correlation function in $|\Omega\rangle$ is different from that in $|0\rangle$

$$\langle \Omega | \phi(x_1) \phi(x_2) | \Omega \rangle = \frac{1}{4\pi} \log |(V_1 - V_2)(U_1 - U_2)|$$

$$\neq \langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle = \frac{1}{4\pi} \log |(v_1 - v_2)(u_1 - u_2)|$$

Hawking radiation appears due to change of coordinates associated to vacuum state



After gravitational collapse

$$ds^2 \simeq -du dv + r^2 d\Omega^2$$

State: $|\Omega\rangle$ Vacuum: $|0\rangle$ $(a_\omega , a_\omega^\dagger)$

Particle number $\langle\Omega|a_\omega^\dagger a_\omega|\Omega\rangle = \dfrac{1}{e^{2\pi\omega/\kappa} - 1}$

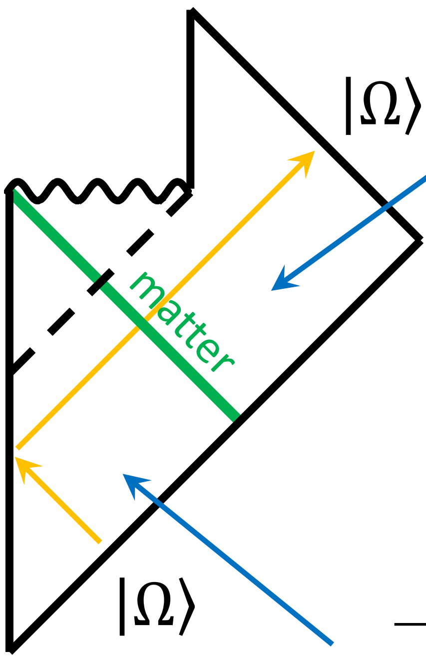
Hawking radiation in Planck distribution

Before gravitational collapse $ds^2 = -dU dv + r^2 d\Omega^2$

State: $|\Omega\rangle$ Vacuum: $|\Omega\rangle$ $(A_\omega , A_\omega^\dagger)$ $U = -e^{-\kappa u}$

Particle number $\langle\Omega|A_\omega^\dagger A_\omega|\Omega\rangle = 0$

Hawking radiation appears due to change of coordinates associated to vacuum state



After gravitational collapse

$$\langle\Omega|T_{uu}|\Omega\rangle = \lim_{u'\to u}\langle\Omega|{:}\,\partial_u\phi(u)\partial_{u'}\phi(u'){:}\,|\Omega\rangle$$

$$= \lim_{u'\to u}\partial_u\partial_{u'}\left(\log|U(u)-U(u')| \underbrace{-\log|u-u'|}_{\text{Normal ordering}}\right)$$

$$= \{U,u\} \sim \kappa^2 \quad \Rightarrow \quad \left\langle T_{uu}^{(4D)}\right\rangle \sim \kappa^2 \sim T_H^2$$

Stefan-Boltzmann law

Before gravitational collapse

$$\langle\Omega|T_{uu}|\Omega\rangle = \lim_{u'\to u}\langle\Omega|{:}\,\partial_u\phi(u)\partial_{u'}\phi(u'){:}\,|\Omega\rangle$$

$$= \lim_{u'\to u}\partial_u\partial_{u'}\left(\log|U-U'| \underbrace{-\log|U-U'|}_{\text{Normal ordering}}\right) = 0$$

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1. Entanglement entropy and islands

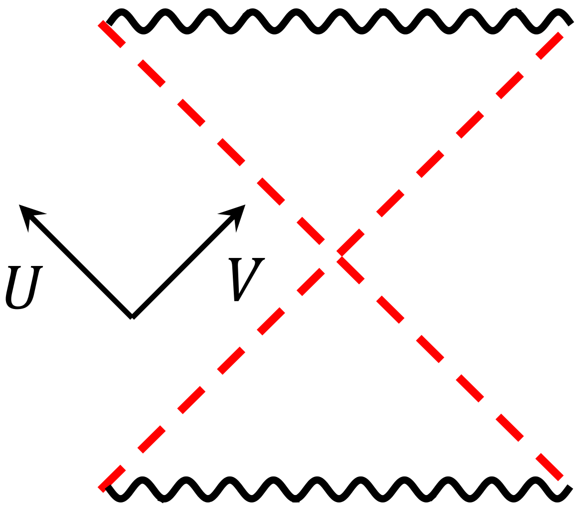
2. Preferred coordinates of vacuum states

3. Islands in black hole spacetimes

    - Near horizon geometry of stationary black holes
    - Island extends outside the horizon for stationary black holes
    - Vacuum state is Unruh vacuum for evaporating black holes
    - Island is inside the horizon for evaporating black holes

Near horizon geometry of stationary black holes

Stationary black holes have the bifurcate horizon



Near horizon metric

$$ds^2 \simeq -dUdV + r^2 d\Omega^2$$

Stationary

⇨ Killing vector

$$\xi = \partial_v \text{ (on horizon)}$$

$$\xi = \kappa V \partial_V - \kappa U \partial_U$$

$$(V = e^{\kappa v})$$

Hartle-Hawking vacuum ⇨ Kruskal coordinates

$$ds^2 = -e^{2\rho} dUdV + r^2 d\Omega^2$$

$$\rho \simeq 0 + \underline{\rho_1 UV}$$

$$r \simeq r_h - \underline{r_1 UV}$$

Correction

Island extends outside the horizon of stationary black holes

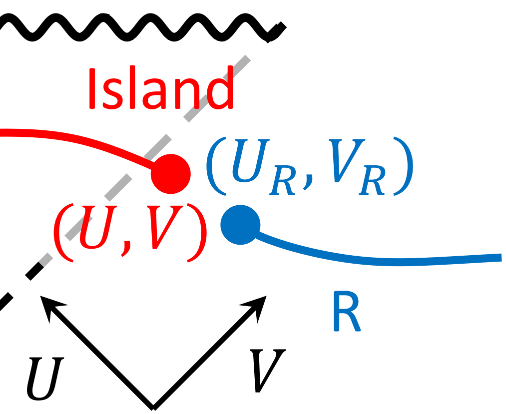
Entanglement entropy

$$S = S_{\text{grav}} + S_{\text{matter}}$$

$$S_{\text{grav}} = A - BUV \qquad S_{\text{matter}} = \frac{c}{6}\sum \log|(U - U_R)(V - V_R)|$$

Position of island is fixed so that entanglement entropy is extremized

$$\partial_U S = 0 \qquad \partial_V S = 0$$



$$0 = \partial_V S = -BU + \frac{c}{6}\frac{1}{(V - V_R)} + \cdots$$

$$\Rightarrow \quad U = \frac{c}{B(V - V_R)} < 0$$

Island extends to outside the horizon

Vacuum state is Unruh vacuum for evaporating black holes

Unruh vacuum ⇨ Eddington-Finkelstein coordinates

$$ds^2 = -e^{2\rho} dU dv + r^2 d\Omega^2$$

$$\rho \simeq \frac{1}{2}\kappa(v)\, v + \rho_1(v) U e^{\kappa v} \qquad r \simeq r_h(v) - r_1(v) U e^{\kappa v}$$

Time evolution ⇨ v-dependence $\kappa(v)$, $r_h(v)$

Time evolution of gravitational entropy

$$\delta E = \frac{\kappa}{2\pi} \delta S_{\mathrm{BH}}$$

$$\delta E = -T_{vv} = -\frac{c\kappa^2}{48\pi}$$

⇨

$$\frac{dA}{dv} = \frac{dS_{\mathrm{BH}}}{dv} = -\frac{c\kappa}{24}$$

Island is inside the horizon of evaporating black holes

Entanglement entropy

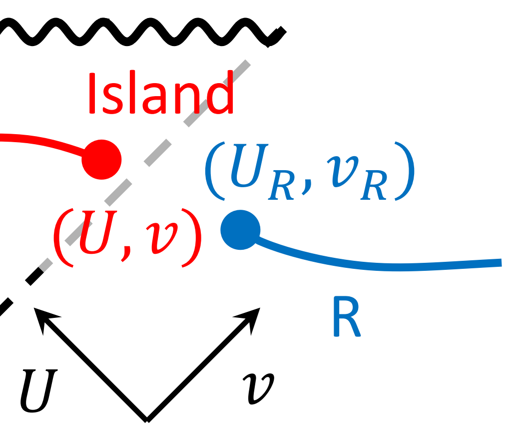
$$S \simeq A(v_0) - \frac{c\kappa v}{24} - BUe^{\kappa v} + \frac{c}{6}\log|(U - U_R)| + \frac{c\kappa v}{12}$$

$$= A(v_0) - BUe^{\kappa v} + \frac{c}{6}\log|(U - U_R)| + \frac{c\kappa v}{24}$$

Endpoints of island is quantum extremal surface

$$0 = \partial_v S = -BUe^{\kappa v} + \frac{c\kappa}{24}$$

$$\Rightarrow \quad U = \frac{c\kappa}{24B} > 0$$



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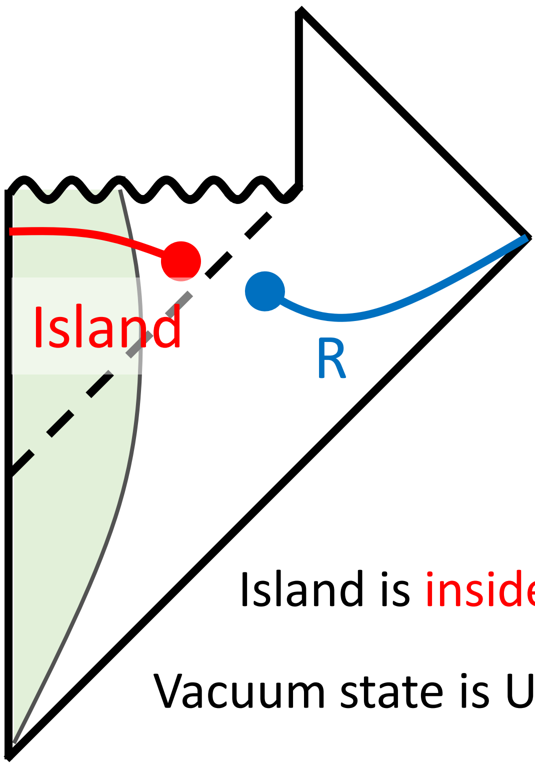

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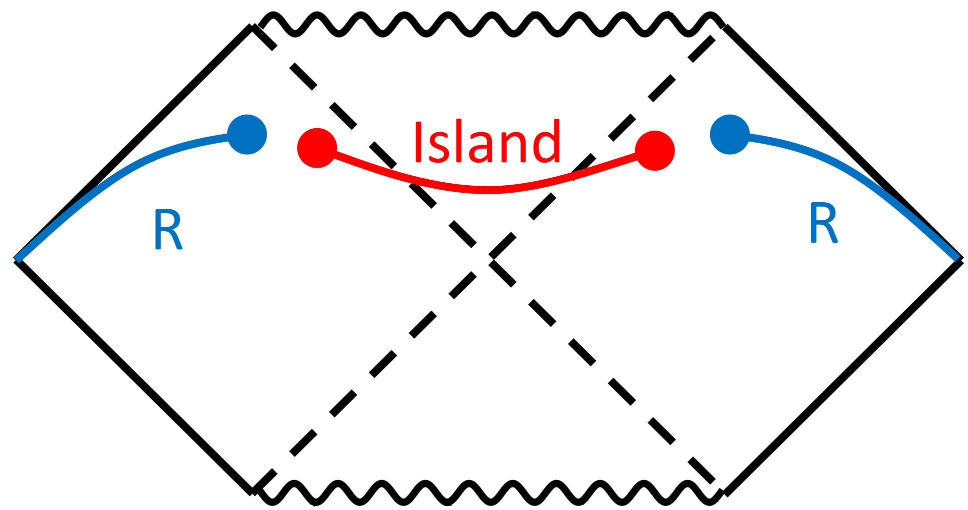

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Thank you