Bridging two semiclassical confinement mechanisms: monopole and center vortex

Yui Hayashi (YITP, Kyoto U.)

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Based on:
PRL **133**, 171902 (2024) [arXiv:2405.12402 [hep-th]] with Yuya Tanizaki (YITP)
also [arXiv:2410.21392 [hep-th]] with Tatsuhiro Misumi (Kindai U.) and Yuya Tanizaki (YITP)
(special thanks to Mithat Ünsal(NCSU))

Confinement mechanism(s)

Two promising scenarios for quark confinement: monopole and center vortex

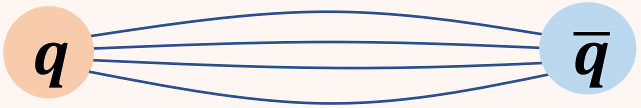
<u>Dual superconductor picture (monopole condensation)</u>

[Nambu '74, 't Hooft '75, Mandelstam '76,…]

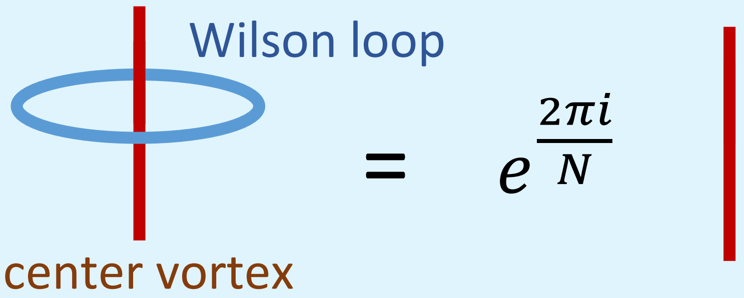
monopole condensation

⇒ dual Meissner effect

⇒ linear $q\bar{q}$-potential



<u>Center-vortex proliferation</u> ['t Hooft '78, …]



Center vortex: rotating Wilson loop by $e^{\frac{2\pi i}{N}}$.

Proliferation ⇒ $\langle W(C)\rangle \sim e^{-\sigma\,(\mathrm{Area})}$

cf.) restoration of $\mathbb{Z}_N^{[1]}$: proliferation of co-dim-2 defects

Connection between them? [Ambjørn-Giedt-Greensite '99, Engelhardt-Reinhardt '99, Cornwall '99,…]

"monopole as junction of center vortices"

Semiclassical approaches to confinement

Deformation with keeping confinement

SU(N) Yang-Mills theory
(strongly coupled, hard problem)

→

Deformed theory
(weakly coupled, easy problem)

↑

Solve this theory (semiclassically)
& study confinement/vacuum structure

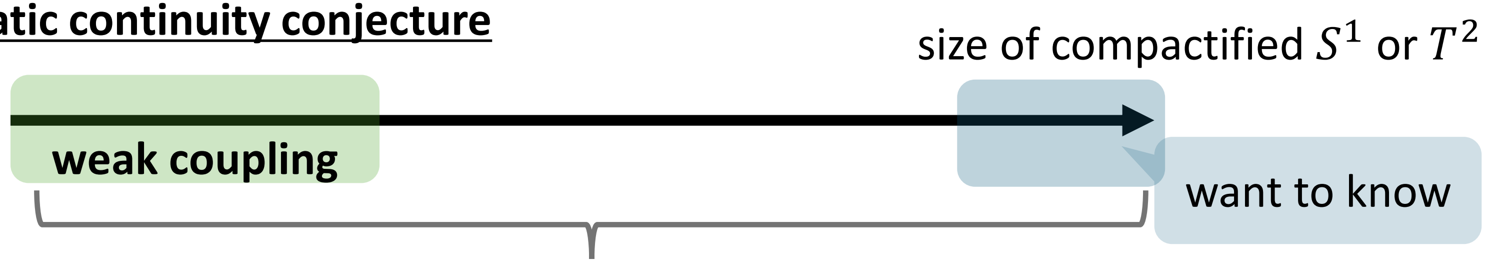
Semiclassical approaches to confinement

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement**.

compactification

center-stabilizing deformation
(to avoid deconfinement transition)

Ansatz: adiabatic continuity conjecture



“adiabatic continuity” (confinement phase, w/o transition)

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with “center-stabilizing deformation”

⇒ **3d $U(1)^{N-1}$ gauge theory**

+ monopole gas **(cf. [Polyakov ’77])**

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

Bridging two semiclassics

[YH, Tanizaki ‘24]

Question

Motto: deforming SU(N) YM to **weakly-coupled** one with **keeping confinement**.

<u>3d monopole semiclassics</u>

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+ monopole gas

⟷

<u>2d center vortex semiclassics</u>

[Tanizaki-Ünsal '22, …]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with 't Hooft flux

⇒ **confinement by 2d center-vortex gas**

Question: Relation between them?

How monopole transmutes to center vortex?

Interpolating setup

't Hooft flux

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{\left(\boldsymbol{S^1}\right)_3 \times \left(\boldsymbol{S^1}\right)_4}$

(L_4: always small)

center-stabilizing deformation

$L_3 \to \infty$

$L_3 \to L_4$

3d monopole semiclassics

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with center-stabilizing deformation

2d center vortex semiclassics

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

3d effective theory on $\mathbb{R}^2 \times (\boldsymbol{S^1})_3$

't Hooft flux

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times (\boldsymbol{S^1})_3 \times (\boldsymbol{S^1})_4$

(L_4: always small)

center-stabilizing deformation

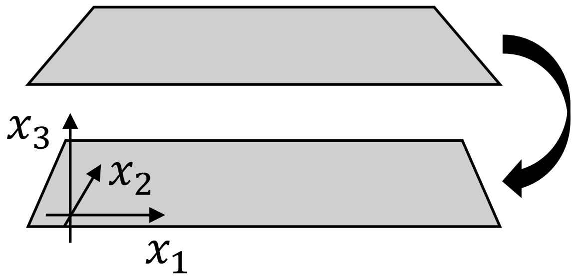
small L_4, adjoint higgsing by $P_4 \sim C$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (\boldsymbol{S^1})_3$
with "Weyl-permutation-twisted" boundary conditions

4d 't Hooft flux ($\mathbb{Z}_N^{[1]}$ background)
= 3d $\mathbb{Z}_N^{[0]}$-twisted boundary condition for $\boldsymbol{P_4}$
= Weyl Permutation (in terms of Cartan $U(1)^{N-1}$)



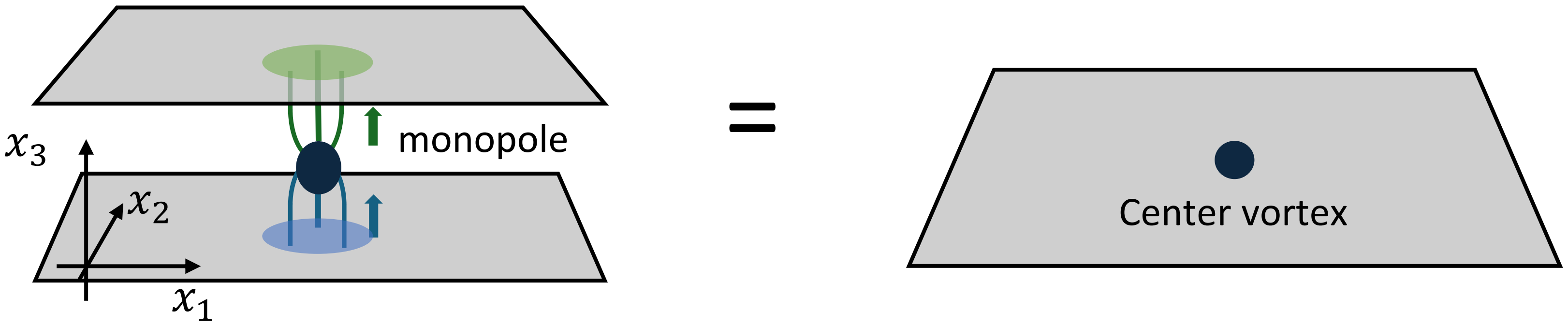
$$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$$

Claims:

setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times \underbrace{(S^1)_4}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

1. 3d effective theory on $\mathbb{R}^2 \times (S^1)_3$ ⇒ 2d center-vortex gas on $\mathbb{R}^2$

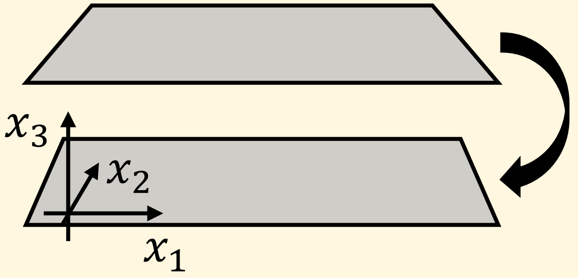
2. BPS/KK monopole in $\mathbb{R}^2 \times (S^1)_3$ (3d monopole-instanton)
⇒ center vortex on $\mathbb{R}^2$ (2d center-vortex-instanton)



(1) From 3d monopole gas to 2d center-vortex gas

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with "Weyl-permutation-twisted" boundary conditions

$$S_{3d}[\vec{\sigma}] = \int d^3x \left[\frac{\# g^2}{L_4} |d\vec{\sigma}|^2 - \# \, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$



$\vec{\sigma}(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

(dual photon: $d\vec{\sigma} = * \vec{f}$)

$\vec{\alpha}_1, \cdots, \vec{\alpha}_{N-1}$: simple roots (BPS monopoles)
$\vec{\alpha}_N$ $(= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$: affine root (KK monopole)

$L_3 \ll \Lambda^{-1}$**: restricted to** $\vec{\sigma} = S^{-1}\vec{\sigma}$

Zeromode: only N vacua
$\vec{\sigma} = \vec{\sigma}_k = \frac{2\pi k}{N}(\vec{\mu}_1 + \cdots + \vec{\mu}_{N-1})$
$k = 0, \cdots, N-1$

2d center-vortex gas

$$Z_{\mathbb{R}^2 \times (S^1)_3} = \int_{\substack{\vec{\sigma}(\boldsymbol{x}, x_3 + L_3) \\ = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)}} \mathcal{D}\vec{\sigma} \, \mathrm{e}^{-S_{3d}[\vec{\sigma}]} \approx \sum_{\substack{\vec{\sigma} = \vec{\sigma}_k \\ k \in \mathbb{Z}_N}} \mathrm{e}^{-S_{3d}[\vec{\sigma}]} = \sum_{k \in \mathbb{Z}_N} \mathrm{e}^{\# V_{2d} \mathrm{e}^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta + 2\pi k}{N}\right)} = Z_{\text{2d gas}}$$

✅ Understand (somewhat ad-hoc) 2d center-vortex semiclassics from 3d monopole semiclassics

(2) Microscopically: monopole in $\mathbb{R}^2 \times (S^1)_3$

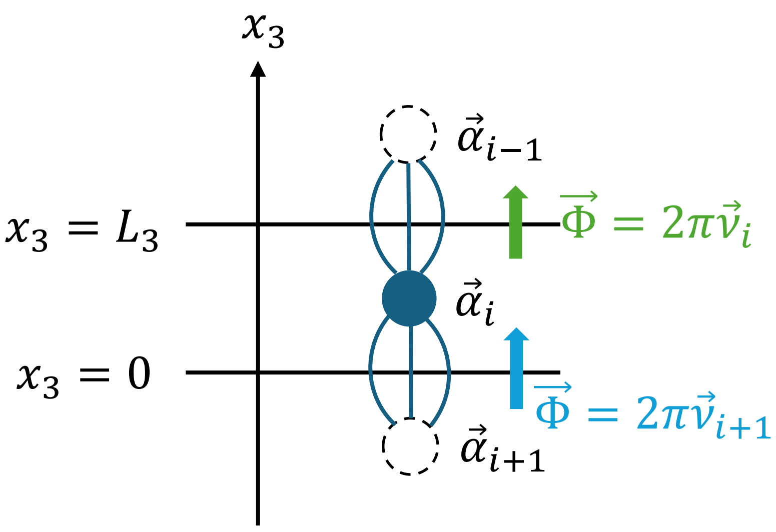
- BPS/KK monopole in 3d effective theory:

magnetic charge $\vec{\alpha}_i$ $\Rightarrow \nabla^2 \vec{\sigma} \sim 2\pi\vec{\alpha}_i \, \delta^{(3)}(x - x_*)$

boundary condition: $\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$\Rightarrow$ "mirror image": infinite chain of BPS/KK monopoles

$$\vec{\sigma} \sim \text{"} \sum_{n\in\mathbb{Z}} \frac{\vec{\alpha}_{i-n\,(\text{mod N})}}{|x - x_* - nL_3\,\hat{x}_3\,|} \text{"}$$



- A proper expression (with good convergence):

$$\vec{\sigma} \sim \sum_{k\in\mathbb{Z}} \left[\sum_{\ell\in\mathbb{Z}_N} \vec{v}_{i-\ell\,(\text{mod N})} \left\{ \frac{1}{|x - x_* - (Nk+\ell)L_3\,\hat{x}_3|} - \frac{1}{|x - x_* - (Nk+\ell+1)L_3\,\hat{x}_3|} \right\} \right]$$

$\vec{v}_i$: weight vector of defining representation
$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}$

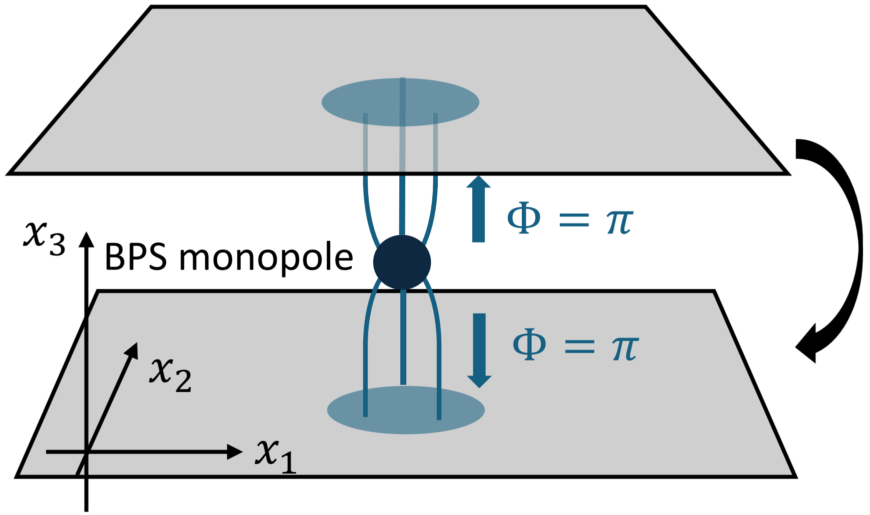
outgoing magnetic flux
$\vec{\Phi} = 2\pi\vec{v}_i$

incoming magnetic flux
$\vec{\Phi} = 2\pi\vec{v}_{i+1}$

Example: SU(2) case

Adjoint higgsing by $P_4 : SU(2) \to U(1)$
$\Rightarrow$ one compact scalar $\sigma \sim \sigma + 2\pi$

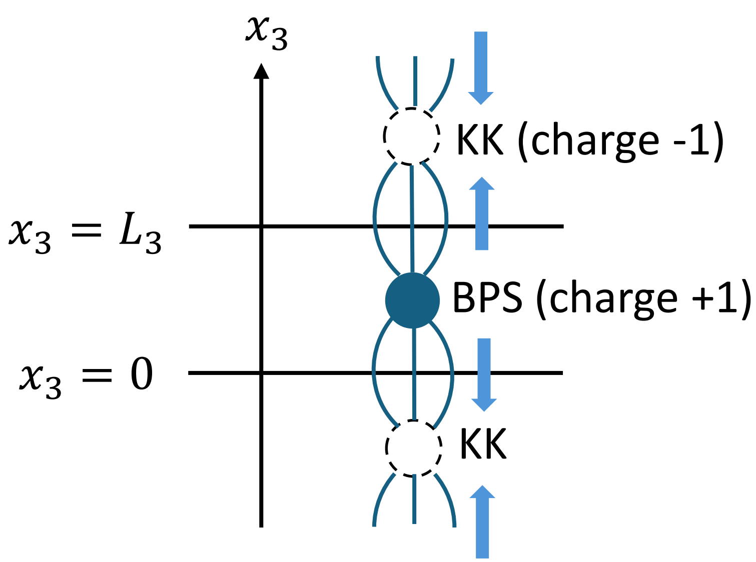
- One compact scalar $\sigma \sim \sigma + 2\pi$
- BPS monopole: magnetic charge +1, KK monopole: magnetic charge -1
- boundary condition (from ‘t Hooft twist): $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$



Weyl-permutation

$$\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma\,(\boldsymbol{x}, x_3)$$

“mirror image” solution:



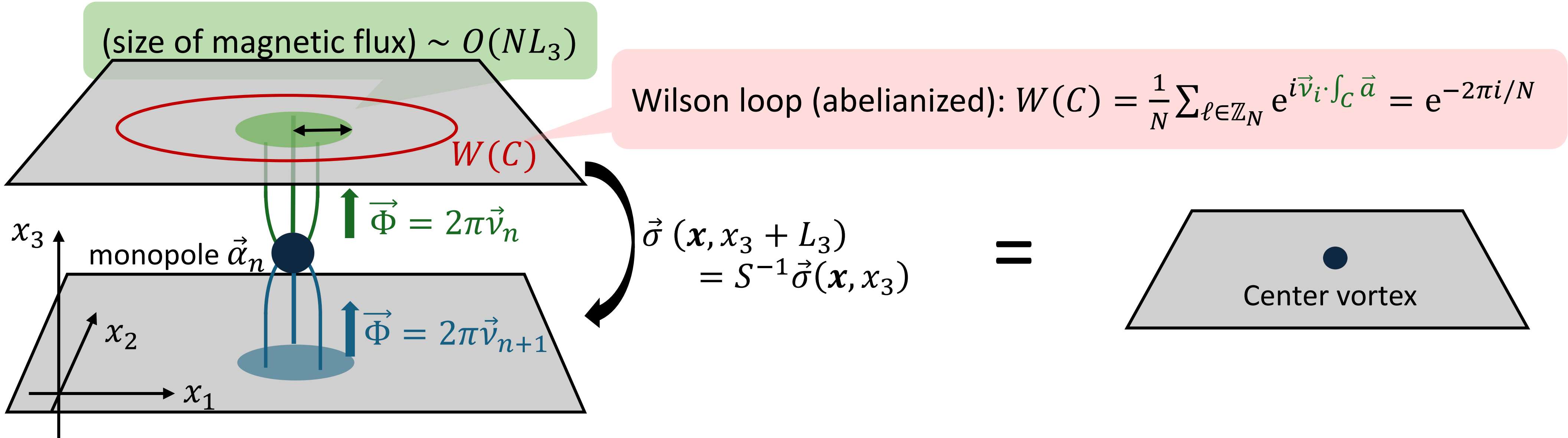
“Flux Fractionalization”:
1/N fractional magnetic flux, rotating the Wilson loop by a center element (-1)

3d BPS/KK monopoles become 2d center vortex

- The magnetic flux (of size $O(NL_3)$) is indeed center vortex:
  Wilson loop acquires $\mathrm{e}^{-2\pi i/N}$ phase.
- **3d BPS/KK monopole-instanton = 2d center-vortex-instanton**:

The 3d/2d semiclassical confinement mechanisms are essentially same!

- **"monopole as junction of center vortex" (realizing the old expectation!)**
- Species of monopole (BPS/KK) is included in extended moduli $x_3 \in [0, NL_3)$



Summary (of main topic)

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"

⇒ **confinement by 3d monopole gas**

2d center vortex semiclassics

[Tanizaki-Ünsal '22, …]

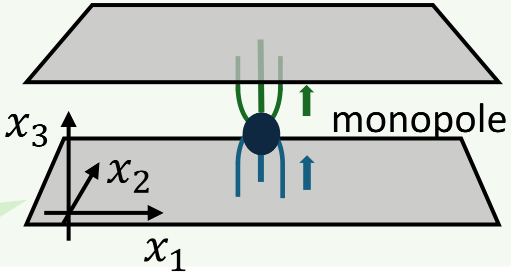
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with 't Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$

"monopole as junction of center vortices"



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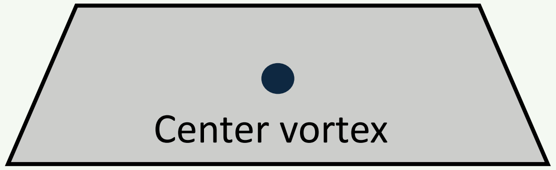
Center vortex in 2d



Bonus: 3d/2d continuity for SYM

[YH, Misumi, Tanizaki ‘24]

$\mathcal{N} = 1$ super-Yang-Mills theory

- **$\mathcal{N} = 1$ SYM theory = one-flavor (massless) adjoint QCD**

Field contents: SU(N) gluon a_μ+ adjoint Weyl fermion λ (“gluino”)

Well-known IR scenario: $(\mathbb{Z}_{2N})_{\mathbf{chiral}} \to \mathbb{Z}_2$ **SSB**

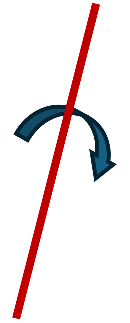
- 3d semiclassics is well developed [Davies-Hollowood-Khoze-Mattis **‘99**, ……]
- One can consider 2d semiclassics for SYM [Tanizaki-Ünsal ’22], but there were some unclear points.

Here, we focus:

- **One of the unclear points of 2d semiclassics:**
  2d Wilson loop shows the perimeter law. deconfinement? What happens?

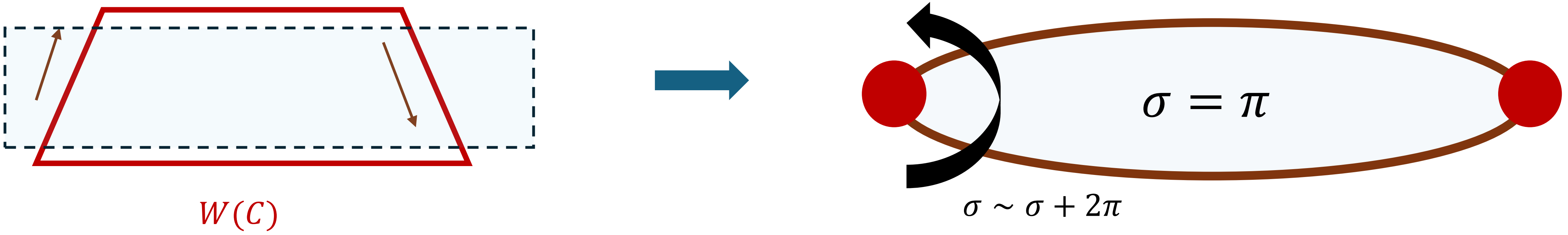
Let us observe what happens from the 3d perspective.

Wilson loop in 3d semiclassics



- Let us consider SU(2) $\mathcal{N} = 1$ SYM, for simplicity.
- **Wilson loop: a defect operator with nontrivial monodoromy $\boldsymbol{\sigma \sim \sigma + 2\pi}$**
- Monopole carries fermionic zeromodes; **Magnetic bion** (BPS-$\overline{\mathrm{KK}}$ molecule; magnetic charge 2/topological charge 0) **induces the bosonic potential:** $\sim \cos(2\sigma)$: **two minima**
- **Double string picture**:

The Wilson loop emits two kinks [Anber-Poppitz-Sulejmanpasic '15]



confining string = pair of two kinks

Wilson loop transmutes to domain wall

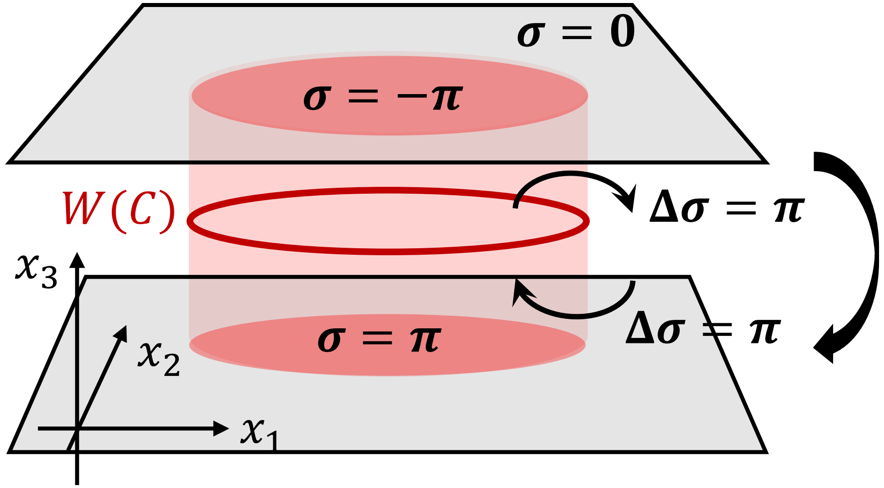
- We consider SU(2) $\mathcal{N}=1$ SYM.
- **Double string picture**: magnetic-bion potential $\sim \cos(2\sigma)$

The Wilson loop (defect $\sigma \sim \sigma + 2\pi$) emits two kinks ($\Delta\sigma = \pi$) [Anber-Poppitz-Sulejmanpasic '15]

- **Reduction from 3d to 2d**: consider $\mathbb{R}^2 \times S^1$ with the twisted boundary condition

For large Wilson loop $|C| \gg L_3$:

This is domain wall of $(\mathbb{Z}_{2N})_{\mathbf{chiral}}$!



$\sigma(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$

3d area law/2d perimeter law
(Area) = L_3 × (Perimeter)

Wilson loop becomes chiral DW ⇒ $(\mathbb{Z}_{2N})_{\mathbf{chiral}}$ SSB implies perimeter law.

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Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center vortex semiclassics

[Tanizaki-Ünsal ’22, …]

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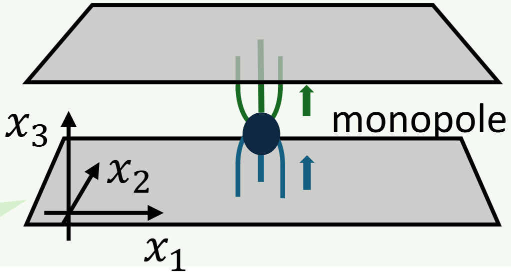
This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$

“monopole as junction of center vortices”



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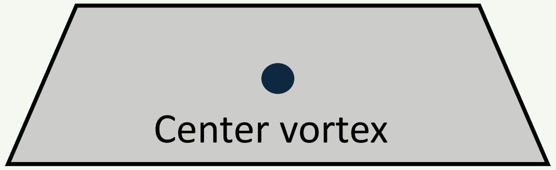
Center vortex in 2d



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3d Monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…] (cf. [Davies-Hollowood-Khoze-Mattis ’99,…])

- **SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with "center-stabilizing deformation"** [Ünsal-Yaffe ’08]:

$$S = S_{YM} + \int d^3x \sum_{n=1}^{[N/2]} a_n \,|\mathrm{tr}\,(P^n)|^2$$

Add a potential for Polyakov loop (by hand) to keep center symmetry

⇒ Center symmetry is kept for **small S^1** (, realizing weak-coupling confinement)

- **3d effective theory on $\mathbb{R}^3$**

The Polyakov loop behaves as an adjoint scalar field.

At the center symmetric vacuum, "$\langle P \rangle \sim C$" (up to gauge)

⇒ adjoint higgsing $SU(N) \to U(1)^{N-1}$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d effective theory = 3d $U(1)^{N-1}$ gauge theory + monopoles

- Polyakov confinement by dilute gas of monopoles (in 3d Abelian gauge theory) [Polyakov ’77]

Magnetic Debye screening ⇒ magnetic fluctuations enhanced ⇒ area law $\langle W(C) \rangle \sim e^{-\sigma\,(\mathrm{Area})}$

3d Monopole semiclassics (some details)

“compactness of adjoint higgs”
[Kraan-van Baal ‘98] [Lee-Lu ‘98]

- **N kinds of monopoles**: $Q_{top} = 1/N$ fractional instantons

(N-1) BPS monopoles (for $U(1)^{N-1}$) + KK monopole

Magnetic charge: $\vec{\alpha}_1$ $\vec{\alpha}_2$ ……. $\vec{\alpha}_{N-1}$ $\vec{\alpha}_N (= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$

$\vec{\alpha}_1, \cdots, \vec{\alpha}_{N-1}$: simple roots
$\vec{\alpha}_N \ (= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$: affine root

- **3d effective theory**

3d abelian duality: $U(1)^{N-1}$ gauge field→ $U(1)^{N-1}$-valued compact boson $\vec{\sigma}$ $(d\vec{\sigma} = * \vec{f})$

In terms of $\vec{\sigma}$ (dual photon/magnetic potential), the 3d effective theory is,

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \# \, e^{-\frac{8\pi^2}{N g^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

Monopole amplitude
$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{N g^2}} \, e^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\,\theta/N}$

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, ……] (cf. [Yamazaki-Yonekura ‘17])

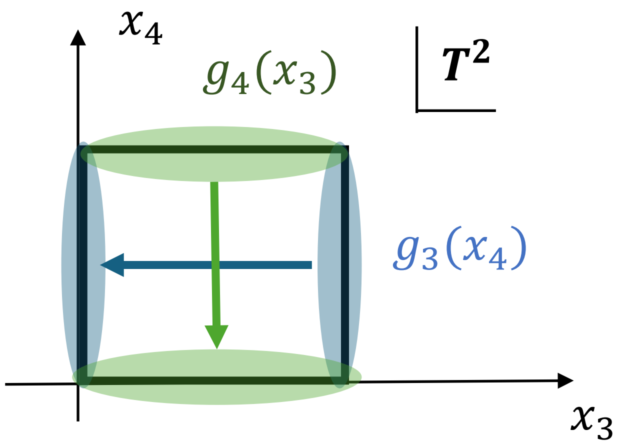
Setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t Hooft flux

- **‘t Hooft flux for T^2** (or $\mathbb{Z}_N^{[1]}$ background)

A unit ‘t Hooft flux ⇔ choose $g_3(0) g_4(L) g_3^\dagger(L) g_4^\dagger(0) = e^{\frac{2\pi \mathrm{i}}{N}}$.

($g_3(x_4), g_4(x_3)$: transition functions on T^2)

Up to gauge, we can take $g_3 = S,\ g_4 = C$ (shift and clock matrices of $SU(N)$).



$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - \mathrm{i}\, g_3^\dagger \mathrm{d} g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - \mathrm{i}\, g_4^\dagger \mathrm{d} g_4 \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

- **Consequences from ‘t Hooft-twisted compactification**
  - ✓**Center symmetry is kept at small T^2**

    Classically, $P_3 = S$ and $P_4 = C \Rightarrow \langle \mathrm{tr}\, P_3 \rangle = \langle \mathrm{tr}\, P_4 \rangle = 0$.
  - ✓**Perturbatively gapped gluons: O(1/NL) KK mass**
  - ✓**Numerical evidence for center vortex/fractional instantons** (as a local solution)

    [Gonzalez-Arroyo–Montero ’98, Montero ’99, ……].

exists locally,
(not globally if `regularity’ at infinity is imposed)

2d center-vortex semiclassics [Tanizaki-Ünsal '22]

- **Dilute gas of center vortices**

The center-vortex and anti-center-vortex vertices are:

$$[\mathcal{V}] = K e^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}, \qquad [\bar{\mathcal{V}}] = K e^{-\frac{8\pi^2}{Ng^2}-i\,\theta/N}$$

with a dimensionful constant K.

For calculating partition function, we compactify $\mathbb{R}^2$ without 't Hooft flux.
⇒ total topological charge is constrained $Q_{top} \in \mathbb{Z}$

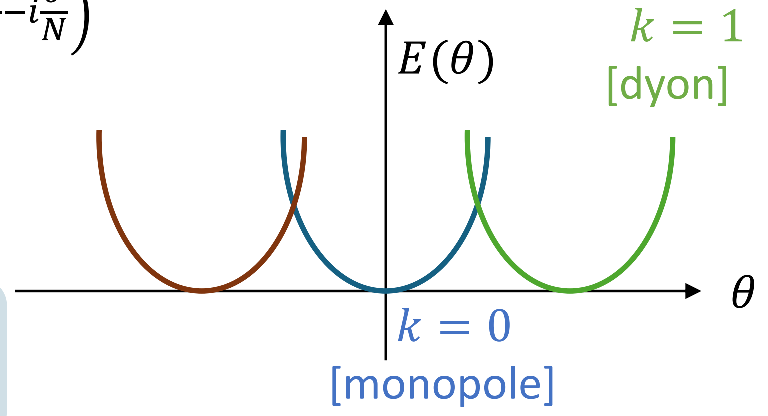
Then, the dilute gas approximation yields, (only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$Z_{2d} = \sum_{n,\bar{n}\geq 0} \frac{1}{n!\,\bar{n}!}\delta_{n-\bar{n}\in N\mathbb{Z}} \left(VKe^{-\frac{8\pi^2}{Ng^2}+i\frac{\theta}{N}}\right)^{n}\left(VKe^{-\frac{8\pi^2}{Ng^2}-i\frac{\theta}{N}}\right)^{\bar{n}}$$

$$= \sum_{k\in\mathbb{Z}_N} \exp\left[-V\left(-2Ke^{-\frac{8\pi^2}{Ng^2}}\cos\left(\frac{\theta-2\pi k}{N}\right)\right)\right]$$

N semiclassical vacua

Energy density of k-th vacuum
→multibranch structure!



✅ One can also derive area-law falloff of the Wilson loop from the dilute gas of center vortices.