Defining IKKT matrix model by gauge fixing Lorentz symmetry

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Outline

- Brief review of IKKT matrix model
- Artifacts from Lorentz-breaking regularization
- Defining Lorentzian IKKT model
- Summary

Definition of IKKT matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya [hep-th/9612115]

- The IKKT model:

Non-perturbative formulation of superstring theory

$$Z = \int \mathrm{d}A\mathrm{d}\psi\, e^{iS} \quad \text{where} \quad S = S_{\mathrm{b}} + S_{\mathrm{f}}$$

Bosonic: $$S_{\mathrm{b}} = -\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu][A^\mu, A^\nu]$$

Fermionic: $$S_{\mathrm{f}} = -\frac{N}{2}\mathrm{tr}\left(\overline{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right)$$

$A_\mu\ (\mu = 0, 1, \ldots, 9)$ $\psi_\alpha\ (\alpha = 1, 2, \ldots, 16)$ $N \times N$ Hermitian matrices

- Shift symmetry $A_\mu \to A_\mu + \alpha_\mu \mathbf{1}$ = Translation in SUSY

→ Eigenvalues of A_μ = spacetime coordinates

Spacetime emerges dynamically

The mass term as IR regulator

Pfaffian: real polynomial in A_μ

- Lorentzian model is not absolutely convergent $$Z = \int \mathrm{d}A\, e^{iS_b} \mathrm{Pf}\,\mathcal{M}(A)$$

- To regularize: Adding the Lorentz-invariant mass term as an IR regulator

Hatakeyama et al. [2201.13200]

$$S = S_b + S_m \qquad S_m = \frac{1}{2} N\gamma \left[e^{i\epsilon} \mathrm{Tr}(A_0)^2 - e^{-i\epsilon} \mathrm{Tr}(A_i)^2 \right]$$

ε : convergence factor

Bosonic model with mass term

- Classical equation of motion: $A_\mu \to \hat{A}_\mu = A_\mu / \sqrt{|\gamma|}$

$$Z = \int \mathrm{d}A\, e^{i\gamma^2 S[\hat{A}]} \qquad \left(\gamma^2 \leftrightarrow \frac{1}{\hbar} \right)$$ "classical limit" $\longrightarrow$ $$[A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$$

Complete set of classical solutions

- D-dimensional bosonic model at N=2

At $N \to \infty$, non-trivial solutions $A_\mu \neq 0$ with expanding behaviour appears only at $\gamma > 0$. Steinecker [1709.10480]

Non-trivial solutions $A_\mu \neq 0$

$\gamma > 0$	$\gamma < 0$
Trivial: $A_\mu = 0$	Trivial: $A_\mu = 0$
Pauli: $A_\mu = \sqrt{\frac{\gamma}{2}}\sigma_\mu \quad \mu = 1,2,3$; $\left(\text{diagonal subgroup of } \mathrm{SO}(3) \times \mathrm{SU}(2)\right) \times \mathrm{SO}(D-4,1)$; 3 extended spatial directions	
squashed Pauli: $A_i = \sqrt{\gamma}\sigma_i \quad i = 1,2$; $\left(\text{diagonal subgroup of } \mathrm{SO}(2) \times \mathrm{U}(1)\right) \times \mathrm{SO}(D-3,1)$; 2 extended spatial directions	

Divergent partition functions

- Non-trivial solutions break Lorentz symmetry ⇒ Flat directions
- Consequences of non-compact flat directions

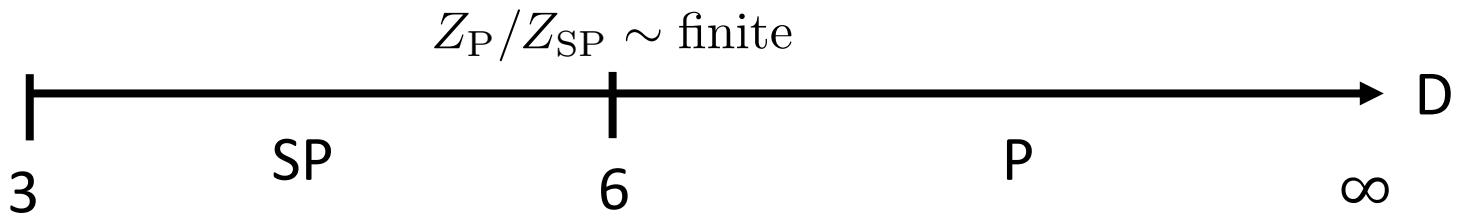
1. Divergent partition functions Use method similar to Hotta-Nishimura-Tsuchiya [hep-th/9811220]

Solutions	Trivial	Pauli (P)	squashed Pauli (SP)
Z	finite	$\varepsilon^{-(\frac{3D}{2}-6)}$	$\varepsilon^{-(D-3)}$

"The most diverging partition function = The most dominant configuration"

Prediction:

$\varepsilon \to 0$



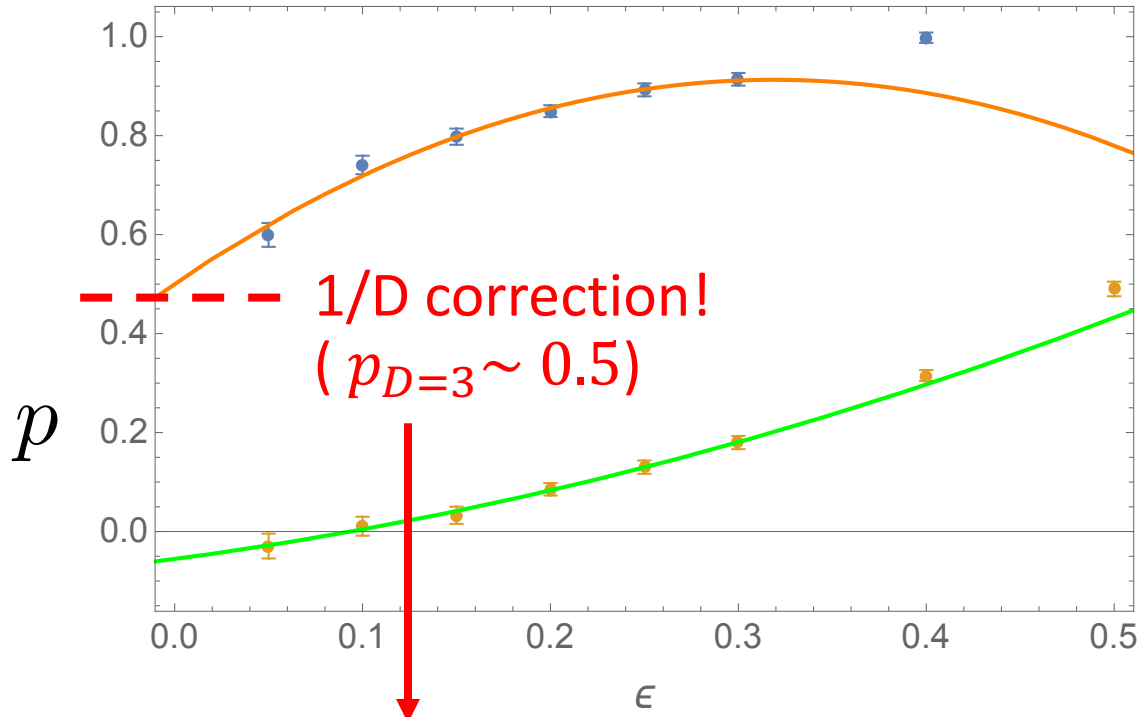
Numerical confirmation of divergence

$$S_m \sim e^{i\varepsilon}\mathrm{tr}(A_0^2) - e^{-i\varepsilon}\mathrm{tr}(A_i^2)$$
$$= \mathrm{tr}(A_0^2) - \mathrm{tr}(A_i^2) + i\varepsilon\mathrm{tr}(A_0^2) + i\varepsilon\mathrm{tr}(A_i^2)$$

$$Z \propto e^{iS_m}$$

$$\langle \mathrm{tr}(A_0^2) \rangle \sim \frac{d}{d\varepsilon} \ln Z \xrightarrow{Z \sim \varepsilon^{-p}} \frac{p}{\varepsilon}$$

SP at D=3 Prediction: p=0



- Power of divergence for SP:

$$p = D - 3 + \frac{b}{D} \quad b\sim 1.5$$

Numerical confirmation of transition

$T_{ij} = \mathrm{tr}(A_i A_j)$ # of large eigenvalues λ_i = spacetime dimensionality

Characteristic polynomial

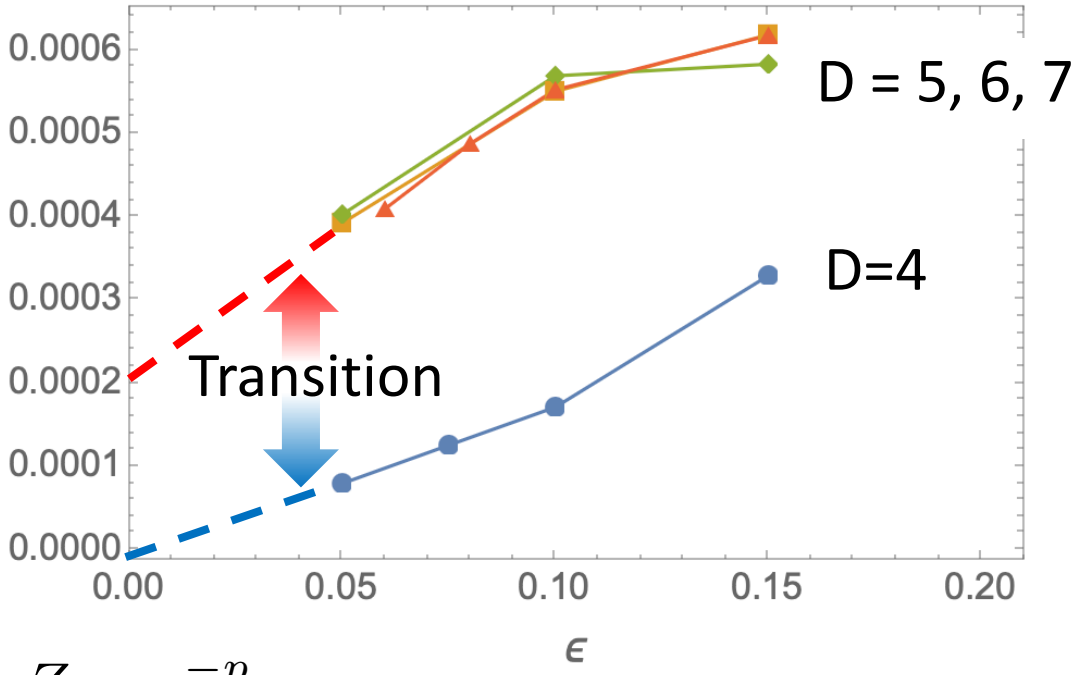
$$a_0\lambda^{D-1} + a_1\lambda^{D-2} + a_2\lambda^{D-3} + a_3\lambda^{D-4} + \cdots + a_{D-2}\lambda + a_{D-1} = 0$$

$$a_1 \sim \sum_i \lambda_i \qquad a_3 \sim \sum_{k>j>i} \lambda_i\lambda_j\lambda_k$$

P: 3D space = 3 large λ_i $\longrightarrow$ $a_3 \neq 0$

SP: 2D space = 2 large λ_i $\longrightarrow$ $a_3 = 0$

Order parameter $\left|\left\langle a_3/a_1^3 \right\rangle\right|^2$

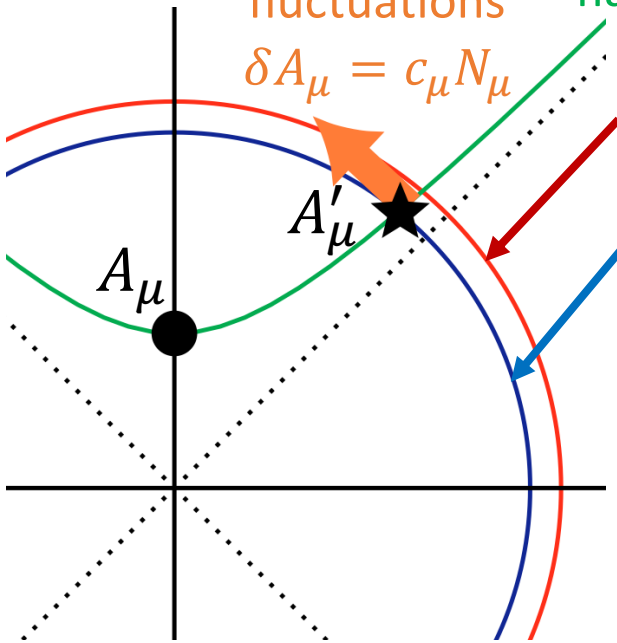


$Z \sim \varepsilon^{-p}$

- Transition between D=4 and D=5 $\Rightarrow$ Prediction for P: $p = \frac{3D}{2} - 6 + \frac{a}{D} \quad 4 < a < 5.5$

Classicalization

- Hessian controls fluctuations around saddles Asano-Nishimura-WP-Yamamori [2404.14045]



$$\left.\begin{aligned} &\mathrm{tr}(A'_0+\delta A'_0)^2 + \mathrm{tr}(A'_i+\delta A'_i)^2 = \Lambda + \delta\Lambda \\ &\mathrm{tr}(A'^2_0) + \mathrm{tr}(A'^2_i) = \Lambda \end{aligned}\right\} \quad \mathrm{tr}(A'_0\delta A'_0) + \mathrm{tr}(A'_i\delta A'_i) = 0$$

"constraints on fluctuations"

$$\delta S = \delta A_\mu H_{\mu\nu}(A)\delta A_\nu = \delta A'_\mu H_{\mu\nu}(A')\,\delta A'_\nu$$

$\delta S(A_\mu)$ is finite → Find eigenvalues of $H_{\mu\nu}(A')$

Solutions	Eigenvalues of $H_{\mu\nu}(A'_\mu)$		check $\frac{\#\text{ of finite eigenvalues}}{D}$
	Divergent	Finite	
P	4	2	classicalizes at $D \to \infty$
SP	4	$D-3$	classicalizes only at $D = 3$

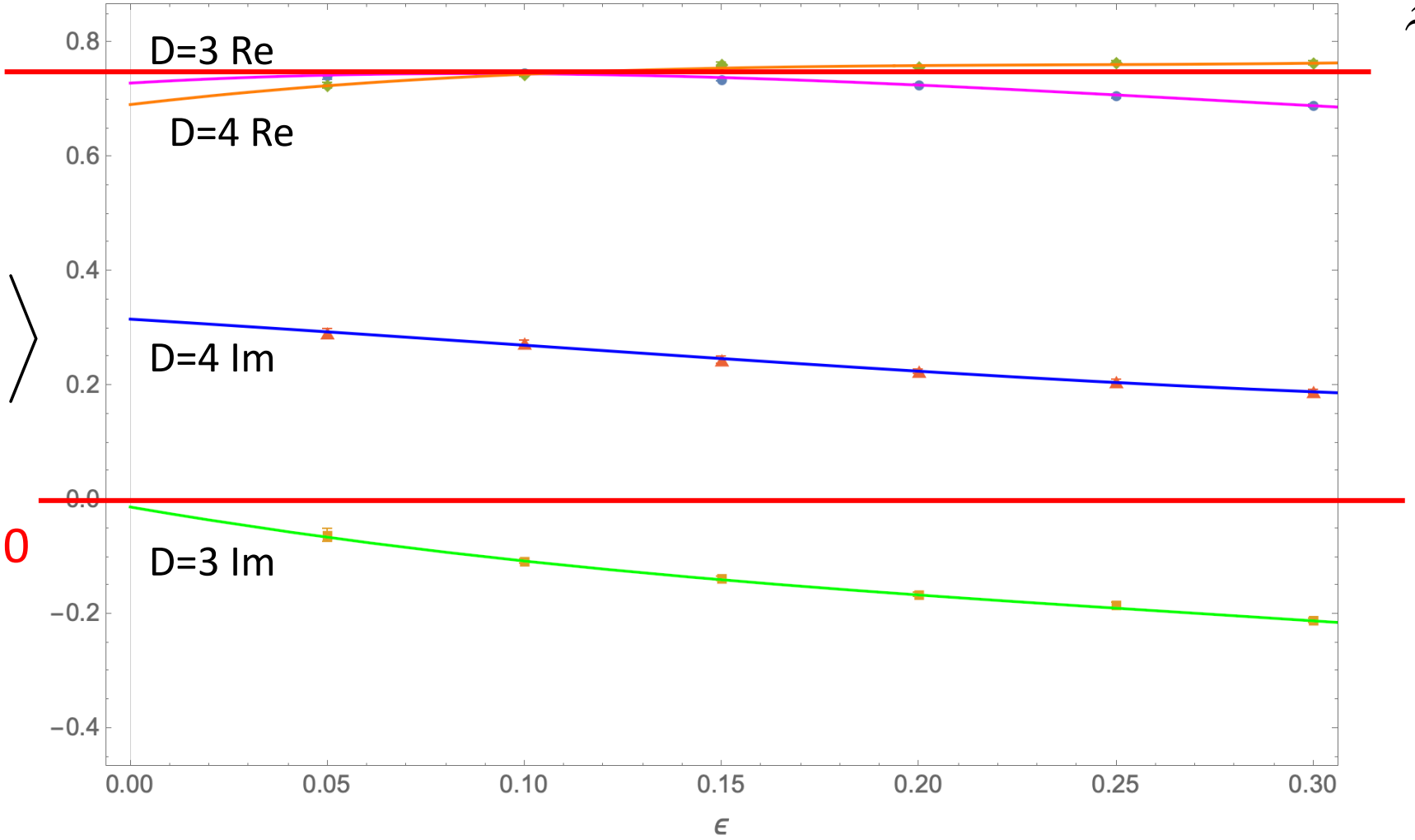
Numerical confirmation of classicalization (SP)

classical prediction

$\tilde{\gamma}/2 = 0.75$

$\left\langle \frac{1}{N}\mathrm{tr}(A^2) \right\rangle$

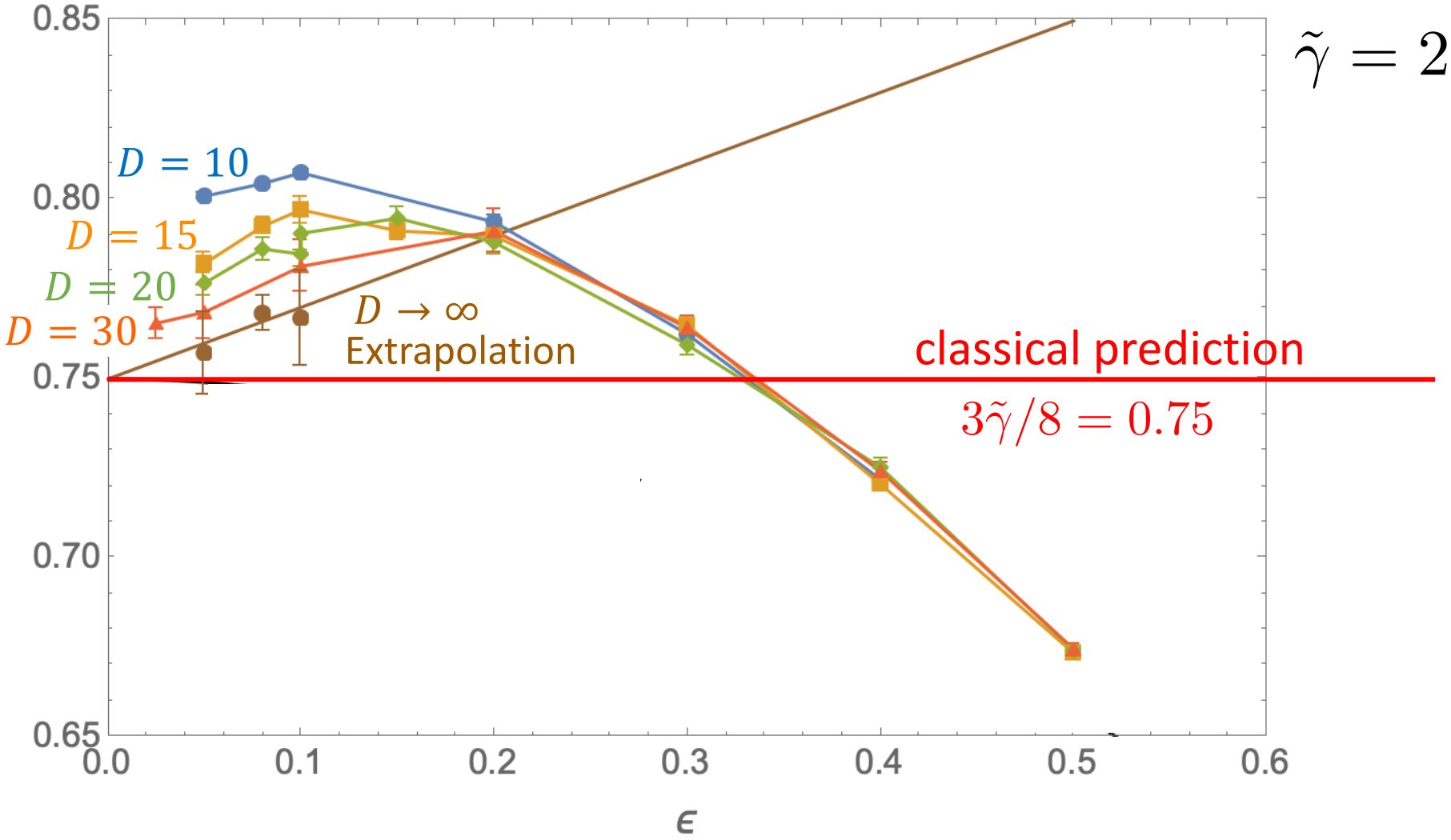
classical prediction = 0



$\tilde{\gamma} = 1.5$

Numerical confirmation of classicalization (P)

$$\mathrm{Re}\left\langle \frac{1}{N}\mathrm{tr}(A^2)\right\rangle$$



Gauge fixing Lorentz symmetry

- Divergent partition function & classicalization are artifacts of Lorentz-breaking cutoff
- We gauge-fix Lorentz symmetry by minimizing $\mathrm{tr}(A_0^2)$

Simulation evidence for the need of gauge fixing: Hirasawa et al. [2407.03491]

N. Yamamori's talk

$$Z_{\mathrm{g.f.}} = \int dA\, e^{iS[A]}\, \mathrm{Pf}\mathcal{M}(A)\, \Delta_{\mathrm{FP}}[A] \prod_{i=1}^{d} \delta(\mathrm{tr}(A_0 A_i))$$

$$\Delta_{\mathrm{FP}}[A] = \det\left(\mathrm{tr}(A_0^2)\mathbf{1} + \mathrm{tr}(A_i A_j)\right)$$

- Classical solutions are modified by FP determinant A. Tripathi's talk
- Numerical simulations has been done

Lefschetz thimble: A. Tripathi's talk

CLM: N. Yamamori's talk

Summary

- We show the artifacts of introducing Lorentz-breaking cutoff
    1. Partition functions around non-trivial solutions are divergent
    2. Only classical contribution of Lorentz-invariant observables survives and discuss the mechanism at certain dimensions (D=3 for SP and $D \to \infty$ for P)

- We propose a new definition of IKKT model by gauge fixing Lorentz symmetry

- Prospects:
    - Larger N
    - Include fermionic term

    } Discuss dimensionality and expansion behaviour

Back up

Classical solutions with expanding behaviour

Bosonic model with mass term

- Classical equation of motion: $A_\mu \to \hat{A}_\mu = A_\mu / \sqrt{|\gamma|}$

$$Z = \int \mathrm{d}A\, e^{i\gamma^2 S[\hat{A}]} \qquad \left(\gamma^2 \leftrightarrow \frac{1}{\hbar}\right) \text{“classical limit”} \longrightarrow [A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$$

- There are non-trivial solutions $A_\mu \neq 0$ with expanding behaviour at $\gamma > 0$.
  No such solutions at $\gamma < 0$ Steinecker [1709.10480]

- 3D space are expected with fermions.

  Configurations A_μ with $A_1 = A_2 \neq 0$
  and $A_{3-10} = 0$ gives Pf(M) = 0. $\Rightarrow$ suppressed

  Krauth-Nicolai-Nishimura [hep-th/9803117]

  Nishimura-Vernizzi [hep-th/0003223]