Complex Langevin simulation of the IKKT matrix model with or without “gauge-fixing” the Lorentz symmetry

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0. Introduction

Ishibashi-Kawai-Kitazawa-Tsuchiya('97)

IKKT matrix model：nonperturbative formalism of the superstring theory

$$S_{\rm b} = -\frac{N}{4}{\rm tr}\left([{\rm A}^\mu, {\rm A}^\nu][{\rm A}_\mu, {\rm A}_\nu]\right)$$

$$S_{\rm f} = -\frac{N}{2}{\rm tr}\left(\bar{\Psi}\Gamma^\mu[{\rm A}_\mu, \Psi]\right)$$

N×N Hermitian matrices

$A_\mu(\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha(\alpha = 1, \cdots, 16)$ Majorana Weyl spinor

SO(9,1) Lorentz symmetry

Space-time emerges from D.O.F of matrices.

($\mathcal{N} = 2$ SUSY includes the translation $A_\mu \to A_\mu + \alpha_\mu \mathbf{1}$)

To study the emergence of space-time, we need to take large N.

Definition the IKKT matrix model Talk by W. Piensuk

- “Gauge-unfixed” model (with cutoff)

  Model defined without gauge fixing Lorentz symmetry

  The artifact of cutoff remains.

- “Gauge-fixed” model

  Model defined with gauge fixing Lorentz symmetry from the beginning.

Plan of talk

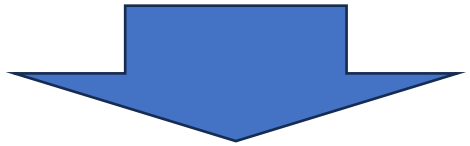
0. Introduction
1. Brief Review of the emergence of space-time
2. How to perform the numerical simulation
3. Results for the gauge-unfixed model
4. Results for the gauge-fixed model
5. Summary and Discussion

1. Brief review of the emergence of space-time

Eq. of motion : $[A^\nu, [A_\nu, A_\mu]] = 0$

Appendix A of H. C. Steinacker, JHEP, 02, 033, arXiv:1709.10480 [hep-th]

Classical solutions are exhausted by the diagonal ones $[A_\mu, A_\nu] = 0$



We add a Lorentz invariant mass term to the action.

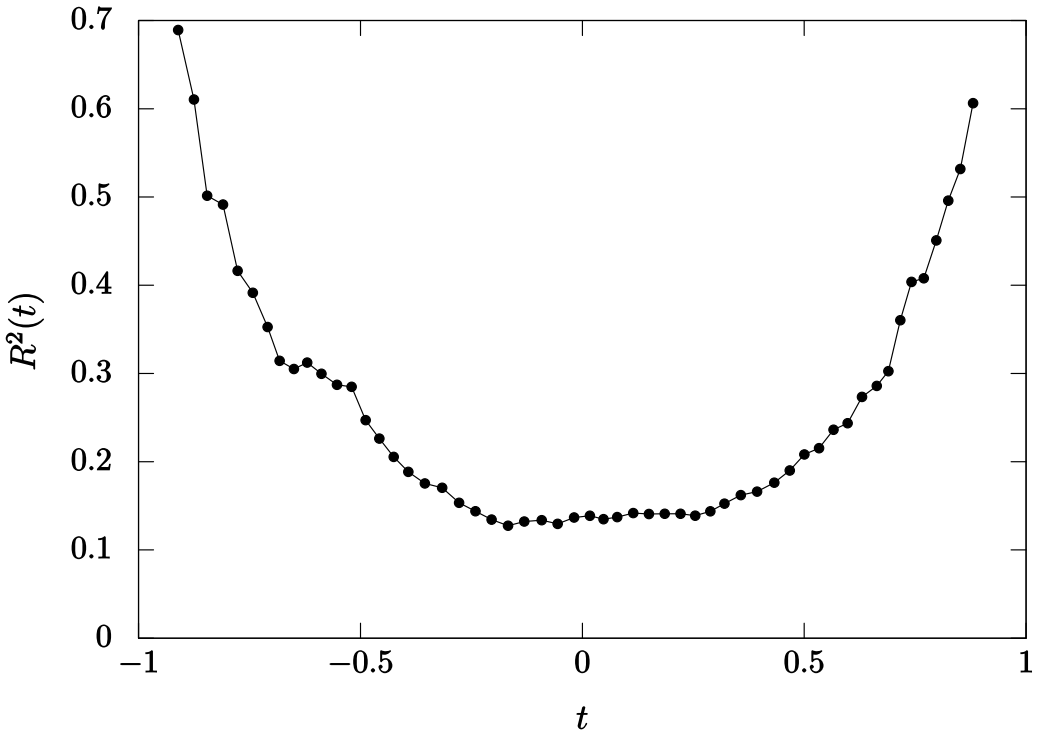
$$S_\gamma = -\frac{1}{2} N\gamma \mathrm{Tr}\,(A_\mu)^2 = \frac{1}{2} N\gamma \left\{ \mathrm{Tr}\,(A_0)^2 - \mathrm{Tr}\,(A_i)^2 \right\}$$

Expanding behavior of classical solutions

Eq. of motion : $[A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$

Hatakeyama-Matsumoto-Nishimura
-Tsuchiya-Yosprakob
PTEP 2020 (2020) 4, 043B10

Typical Hermitian solutions show expanding behavior for $\gamma > 0$.
(but not for $\gamma < 0$)

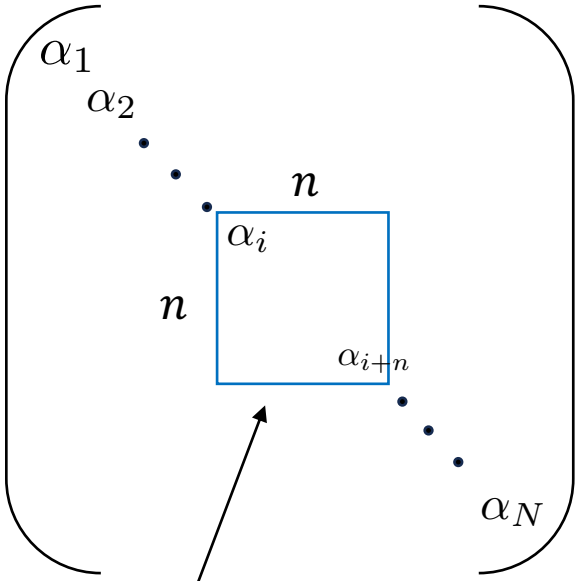


However, dimension of space-time is determined by quantum effect.

Diagonalize the A_0 using SU(N) symmetry

$$A_0 = \mathrm{diag}(\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_N), \quad (\alpha_1 < \alpha_2 < \cdots < \alpha_N)$$

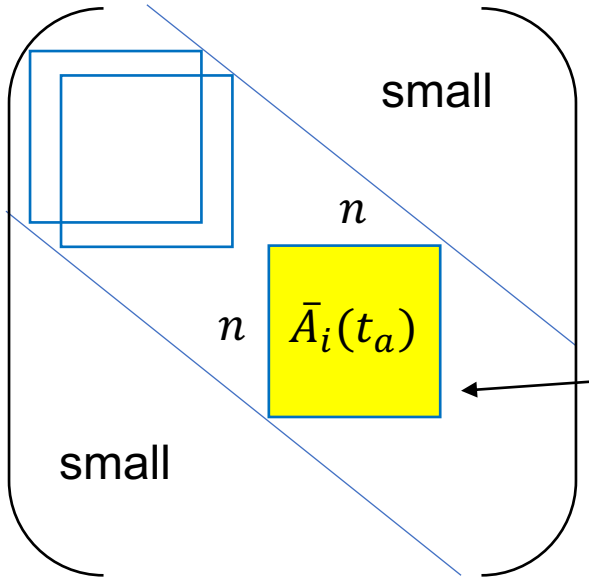
$A_0 =$



$$\bar{\alpha}_i = \frac{1}{n}\sum_{\nu=1}^{n} \langle \alpha_{i+\nu} \rangle$$

$$t_a = \sum_{i=1}^{a} |\bar{\alpha}_i - \bar{\alpha}_{i-1}|$$

$A_i =$



space at time t_a

$$R^2(t) = \left\langle \frac{1}{n} \mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle$$:Expansion of space

2. How to perform the simulation

Complex Langevin method Parisi('83), Klauder('84)

The effective action

$$S_{\text{eff}} = -iN\left(\frac{1}{2}\text{tr}[A_0, A_i]^2 - \frac{1}{4}\text{tr}[A_i, A_j]^2\right) - \frac{i}{2}N\gamma(\text{tr}A_0^2 - \text{tr}A_i^2)$$

$$- \log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a$$

τ_a: auxiliary variables that impose $\alpha_1 < \alpha_2 < \cdots < \alpha_N$ automatically $\alpha_i = \sum_{k=1}^{i-1} e^{\tau_k}$ $(i = 2,3,\cdots N)$

Complex Langevin equation

$$\frac{d\tau_a}{dt} = -\frac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a(t) \ ,$$

$$\frac{d(A_i)_{ab}}{dt} = -\frac{\partial S_{\text{eff}}}{\partial (A_i)_{ba}} + (\eta_i)_{ab}(t)$$

τ_a: complex variables

A_i: complex matrices

η_a, η_i: Gaussian noise

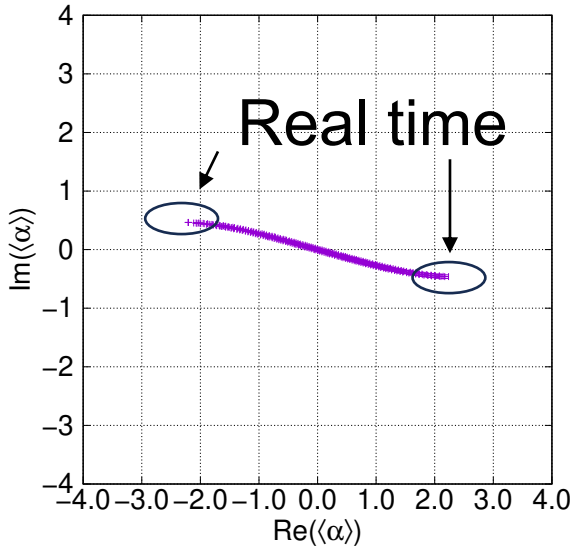
3. Results for the gauge-unfixed model

Result for bosonic model

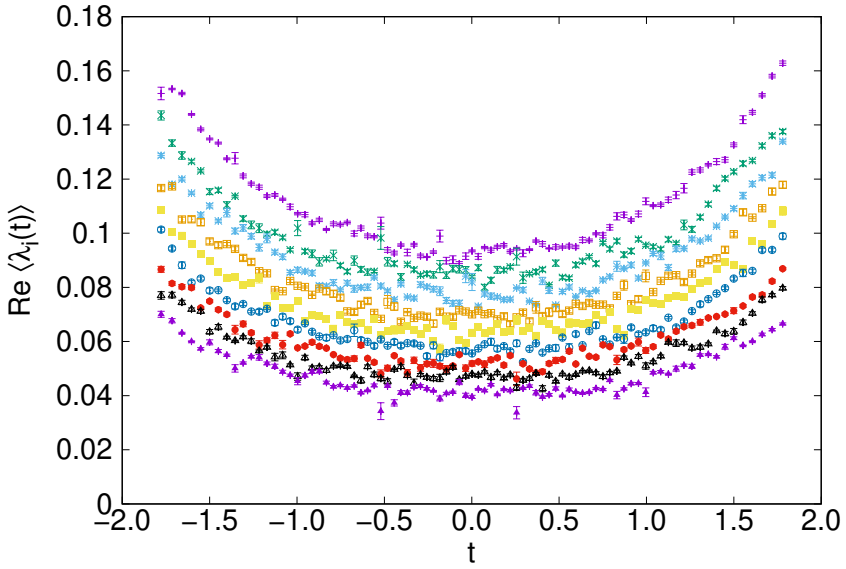
Result for bosonic model $N = 96, \gamma = 4$

Hirasawa-Anagnostopoulos-Azuma-Hatakeyama-Nishimura-Papadoudis-Tsuchiya, (2024)
arXiv : 2407.02491 [hep-th]

Eigenvalues of α_i



$\lambda_i(t)$: Eigenvalues of $T_{ij}(t) = \left\langle \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t)) \right\rangle$



Spatial symmetry $SO(9)$ is not broken spontaneously.

When the eigenvalues of the Dirac operator close to zero, the complex Langevin simulation does not work (singular drift problem).

Nishimura-Shimasaki(2015)

To avoid this problem, we add the mass term to fermionic action.

$$S_{m_{\mathrm{f}}} = iNm_{\mathrm{f}}\mathrm{Tr}\left[\bar{\Psi}_\alpha \left(\Gamma_7\Gamma_8^\dagger\Gamma_9\right)_{\alpha\beta}\Psi_\beta\right] \quad \begin{cases} m_{\mathrm{f}} \to \infty & : \text{bosonic model} \\ m_{\mathrm{f}} \to 0 & : \text{original model} \end{cases}$$

Complex Langevin method works only for $m_{\mathrm{f}} \geq 3.5$.
But the results for $m_{\mathrm{f}} = 3.5$ are still qualitatively same as bosonic model.

Mimicking the effect of SUSY

To mimic the effect of SUSY without decreasing m_{f}, we modify the Lorentz invariant mass term in order to suppress the bosonic fluctuation. G. Bonelli, JHEP 08 (2002) 922

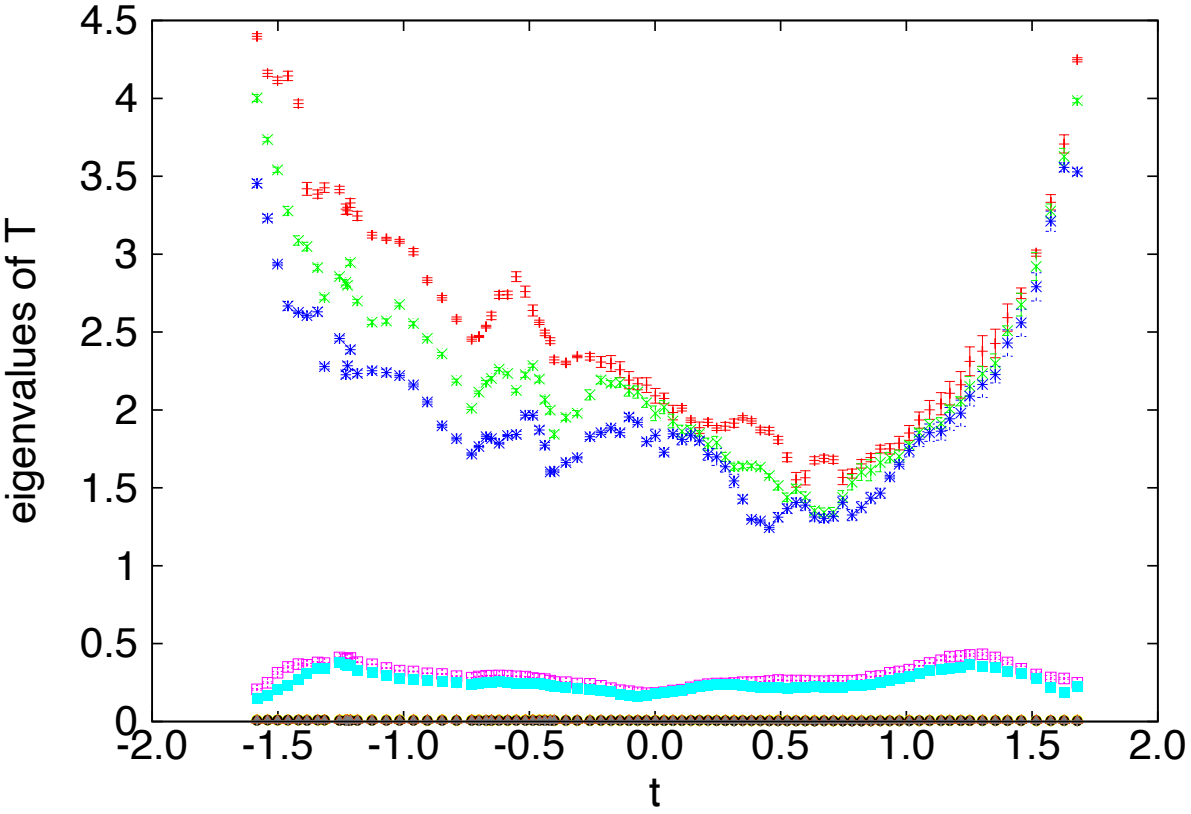
$$S_\gamma = \frac{1}{2} N\gamma \left\{ \mathrm{Tr}\,(A_0)^2 - \sum_{i=1}^{\tilde{d}} \mathrm{Tr}\,(A_i)^2 - \xi \sum_{j=\tilde{d}+1}^{9} \mathrm{Tr}\,(A_j)^2 \right\}, \quad (\xi > 1)$$

$(9 - \tilde{d})$ fluctuation of bosonic matrices is suppressed.

Lorentz symmetry is broken explicitly $SO(9,1) \to SO(\tilde{d},1)$.

Result for the SUSY case (1)

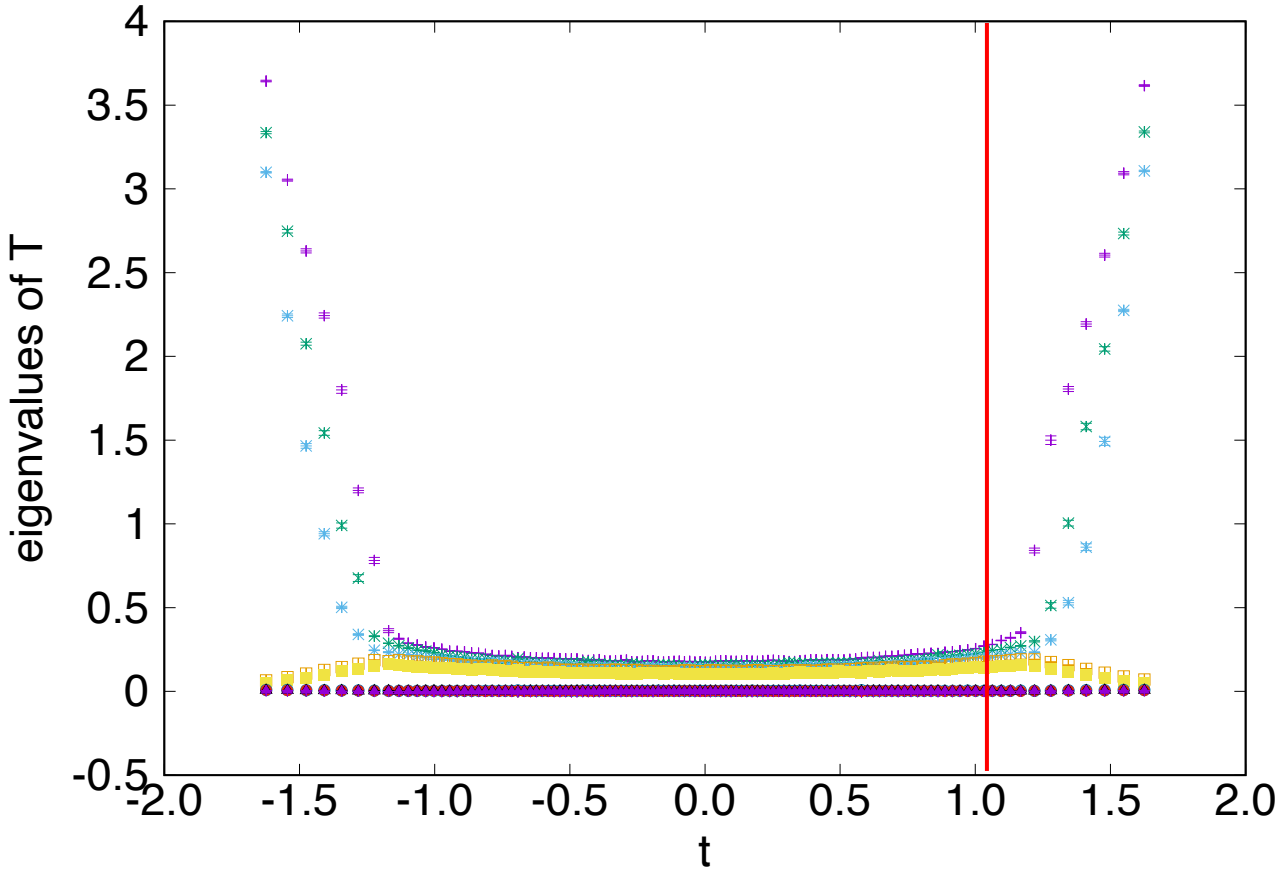
$$N = 96, \gamma = 4, m_{\mathrm{f}} = 3.5, \tilde{d} = 5, \xi = 16$$



SSB from $SO(5)$ to $SO(3)$ occurs at early time.

Result for the SUSY case (2)

$$N = 128, \gamma = 4, m_{\mathrm{f}} = 5.0, \tilde{d} = 5, \xi = 10$$



$SO(\tilde{d})$ symmetry is preserved at early time and broken to $SO(3)$ at some point in time.

Expanding (3+1) D space-time appears at late time.

4. Results for the gauge-fixed model

Gauge unfixed model has artifact of Lorentz breaking cutoff.
Gauge fixed model respects Lorenz symmetry completely.
It remains to be seen whether results are the same.

“Gauge-fixed” model Talk by W. Piensuk
: Model defined by “gauge fixing” Lorentz symmetry from the beginning

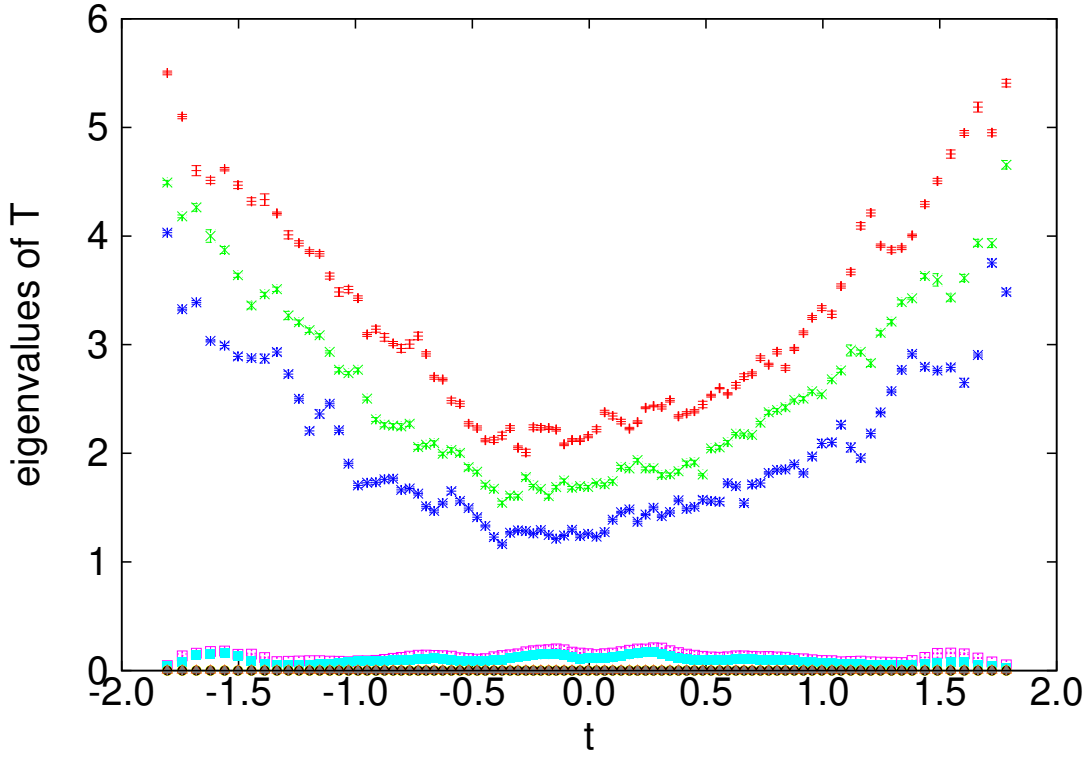
$$Z_{\mathrm{g.f}} = \int dA e^{iS[A]} \mathrm{Pf}\mathcal{M}(A)\Delta_{\mathrm{FP}}[A] \prod_{i=1}^{d} \delta(\mathrm{tr}(A_0 A_i)).$$

FP determinant is given by

$$\Delta_{\mathrm{FP}}[A] = \det\Omega,\ \Omega_{ij} = \mathrm{tr}(A_0)^2\delta_{ij} + \mathrm{tr}(A_i A_j).$$

Result for the SUSY case

$$N = 96, \gamma = 4, m_{\mathrm{f}} = 3.5, \tilde{d} = 5, \xi = 16$$



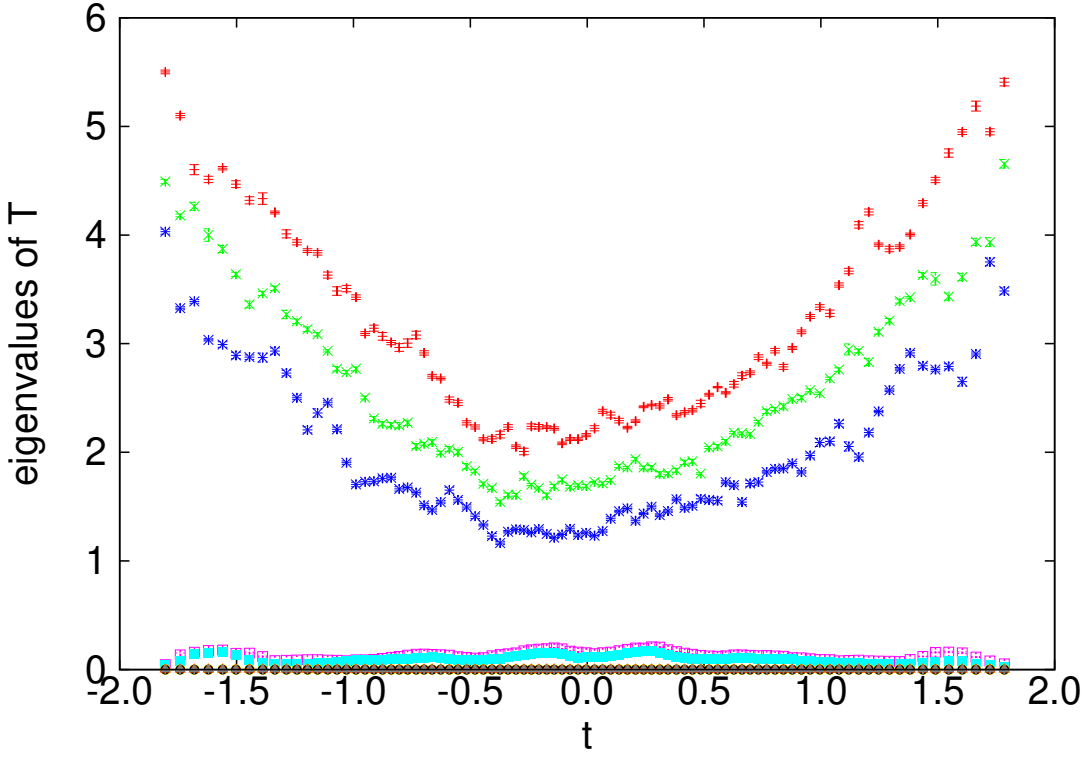
SSB from $SO(5)$ to $SO(3)$ occurs at early time.

Qualitatively same as Gauge-unfixed model

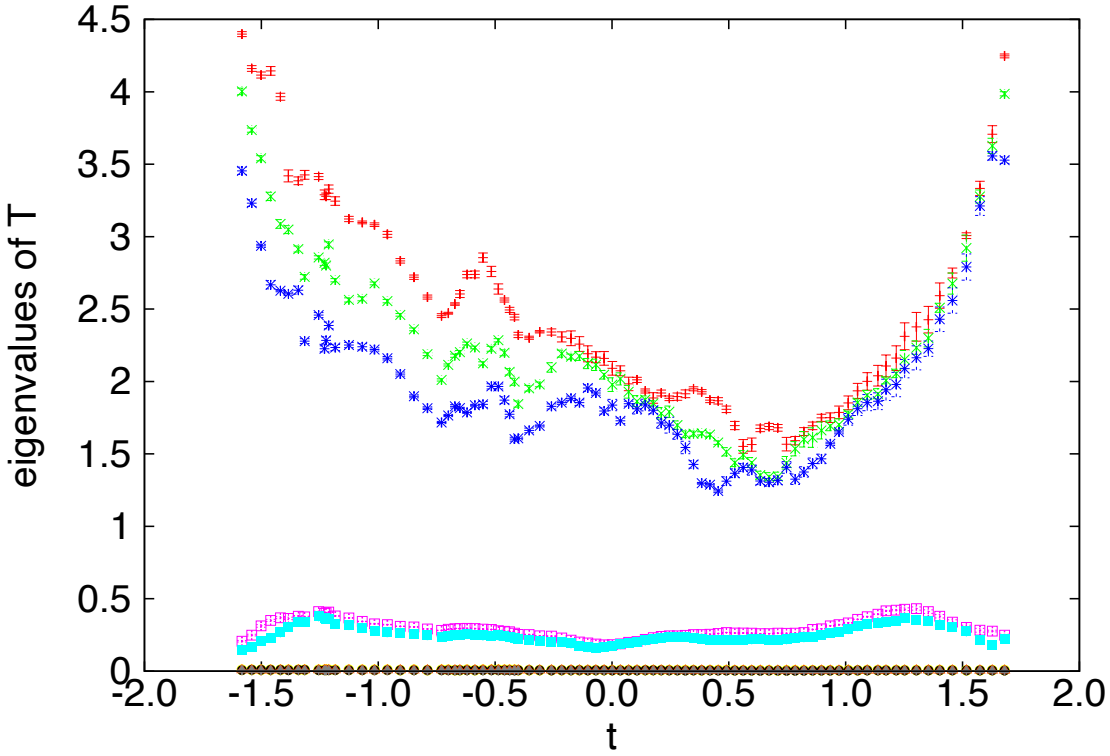
Result for gauge-fixed model (1)

$$N = 96, \gamma = 4, m_{\mathrm{f}} = 3.5, \tilde{d} = 5, \xi = 16$$

gauge-fixed model



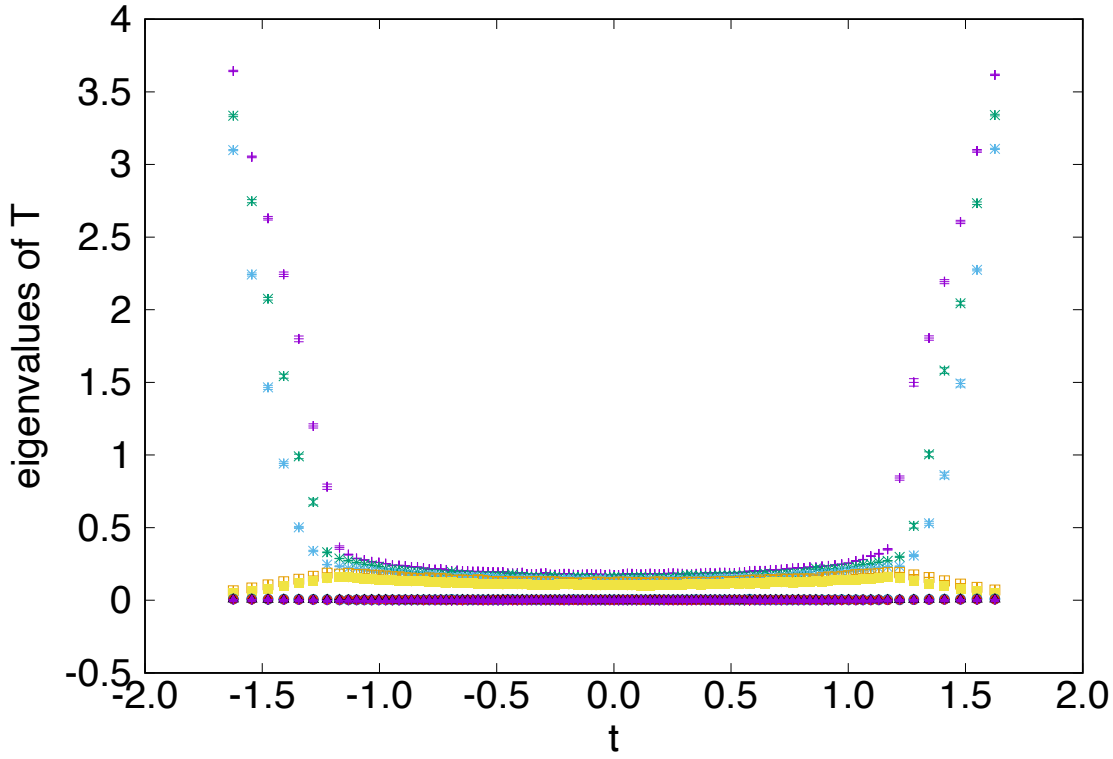
gauge-unfixed model



Result for gauge-fixed model (2)

$$N = 128, \gamma = 4, m_{\mathrm{f}} = 5.0, \tilde{d} = 5, \xi = 10$$

gauge-unfixed model



Numerical calculation for the gauge-fixed model is in progress.

Is (3+1) D expanding space-time maintained in the gauge-fixed model ?

3. Summary and Discussion

Summary and Discussion (1)

- IKKT matrix model is promising candidate for the nonperturbative formalism of superstring theory.
- We have studied the model with or without gauge fixing Lorentz symmetry.
- Gauge unfixed model (including artifacts of Lorentz symmetry breaking)
  - Bosonic model : No SSB of spatial symmetry $SO(9)$
  - Model includes SUSY : (3+1) D expanding space-time was observed.

Summary and Discussion (2)

- Gauge fixed model (respecting Lorentz symmetry)
  - Model includes SUSY

    : Qualitatively same result with gauge-unfixed model is obtained at N=96.

- Numerical calculation for the gauge-fixed model with SUSY at N=128.

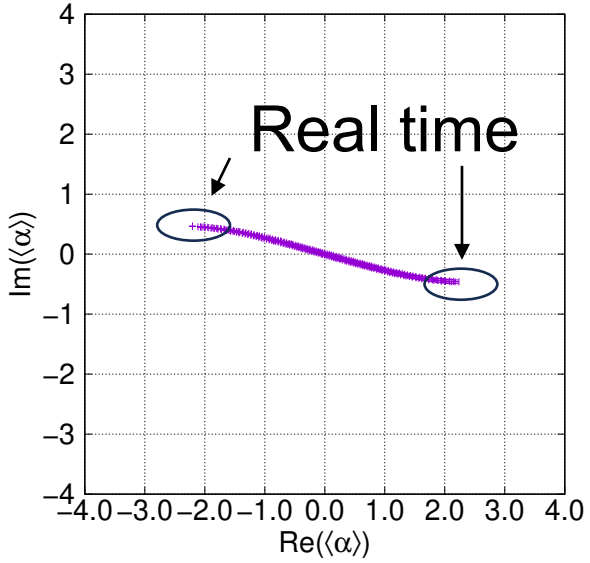
  : Whether (3+1) expanding behavior can be observed

- Can we reduce m_{f} while maintaining 3-dimensional expanding behavior ?

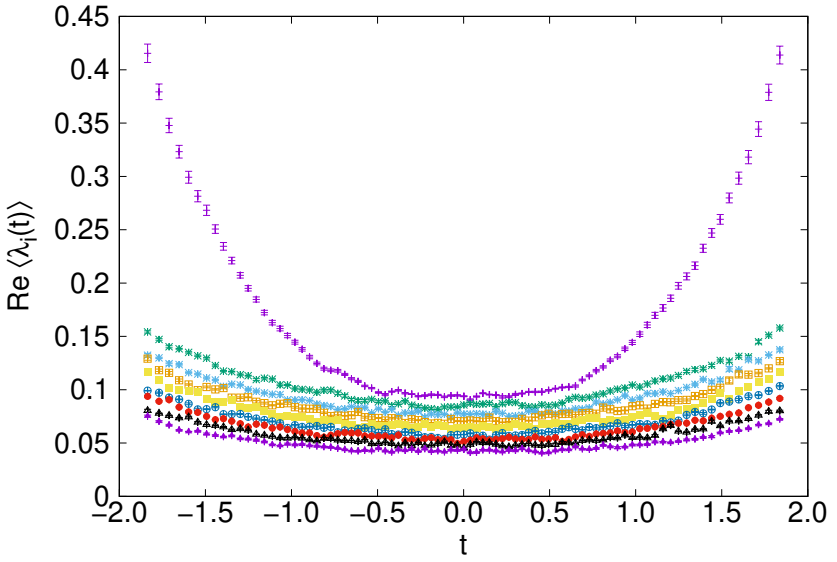
Backup Slides

Previous study for gauge-unfixed model $N = 96, \gamma = 4$

Eigenvalues of α_i



$\lambda_i(t)$: Eigenvalues of $T_{ij}(t)$



$$T_{ij}(t) = \left\langle \frac{1}{n} \mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t)) \right\rangle$$

Expanding
1+1 dimension
space-time

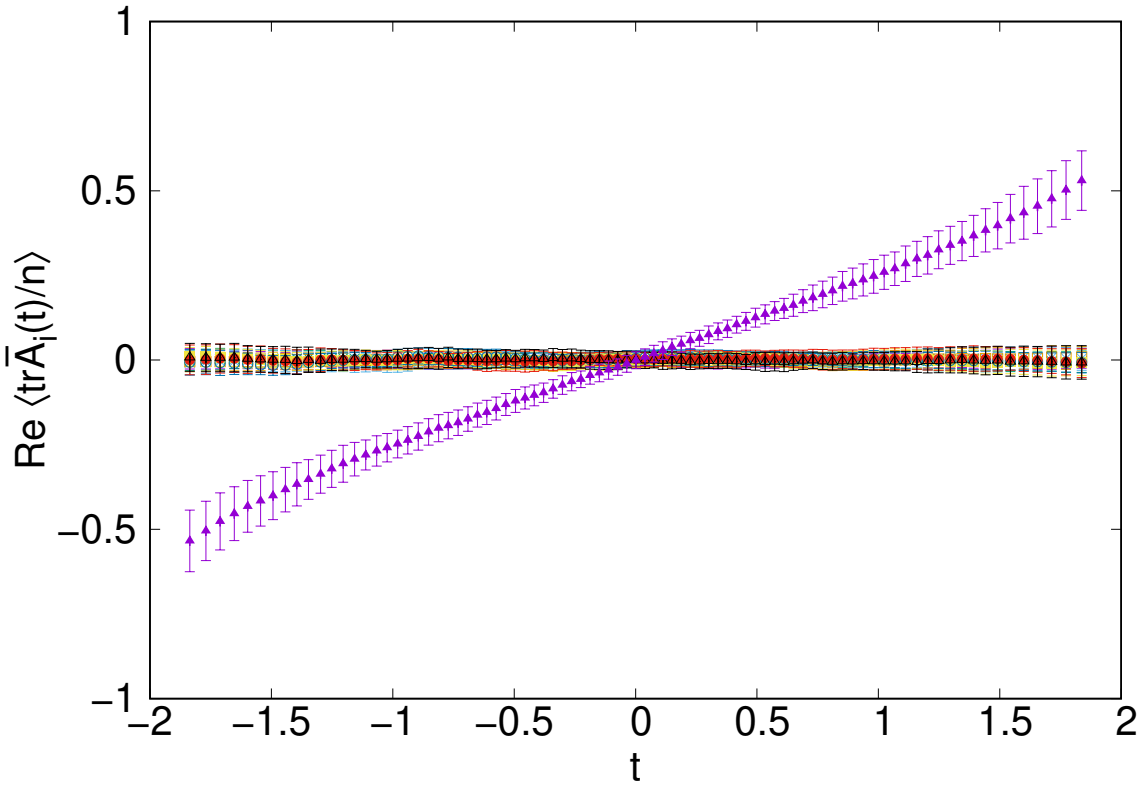
However, in fact, this result is due to the Lorentz boost during the simulation.

The effect of the Lorentz boost

Trace of each spatial block matrix : $\bar{A}_i(t)$

One of these grows linearly in time.

Configurations are Lorentz boosted.

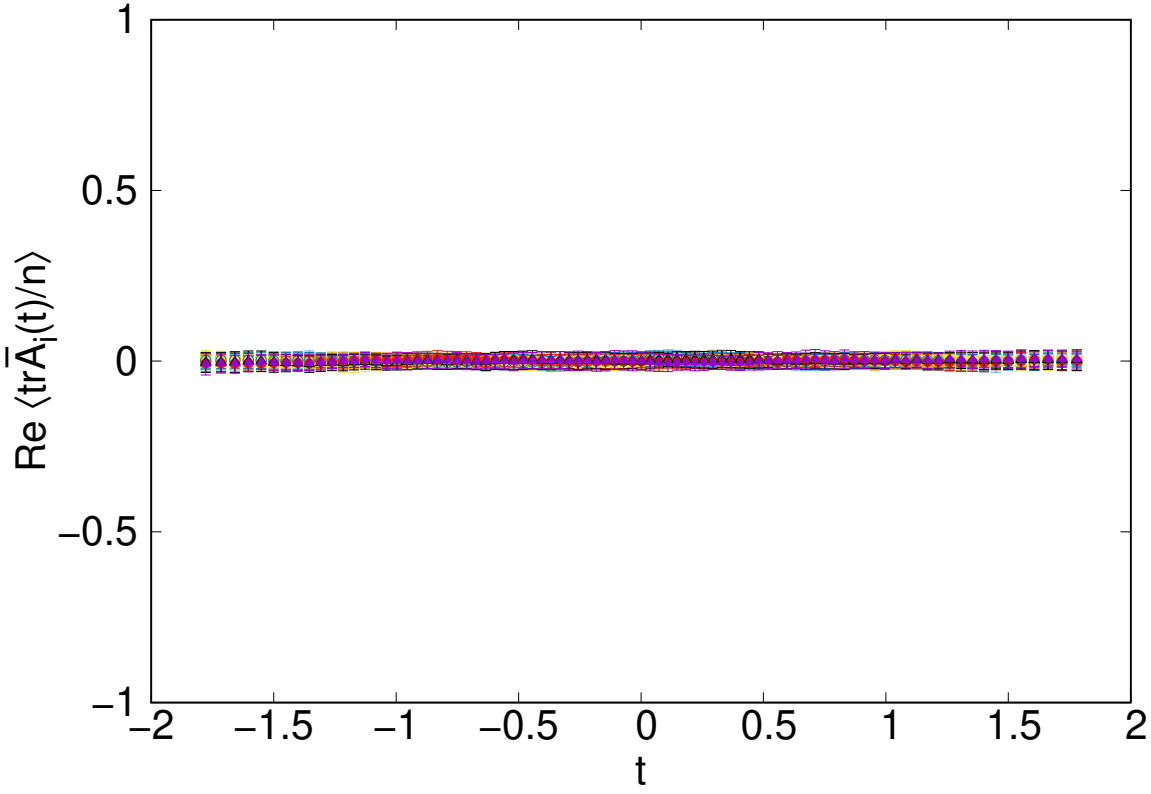


Choose a Lorentz frame by minimizing the quantity

$$\mathcal{T} = \mathrm{Tr}\left(A_0^\dagger A_0\right).$$

This can be achieved by performing the (1+1)-dimensional Lorentz transformation.

$$\begin{pmatrix} A_0' \\ A_i' \end{pmatrix} = \begin{pmatrix} \cosh\sigma & \sinh\sigma \\ \sinh\sigma & \cosh\sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix}$$



After the Lorentz transformation, the linear growth has disappeared.