K_{27} as a Symmetry of Closed Strings and Branes

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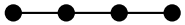
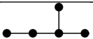
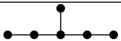
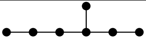
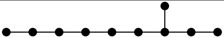
Dimensional reduction of 11D SUGRA on a torus down to $11 - n$ dimensions gives Kaluza-Klein scalars generating a coset space possessing the following 'exceptional symmetries'

Dimension	Exceptional Symmetry Group	Coset Space
10 (IIA)	$\mathrm{O}(1,1)$	-
10 (IIB)	$\mathrm{SL}(2)$ $\bullet$	$\mathrm{SL}(2)/\mathrm{SO}(2)$
9	$\mathrm{GL}(2)$ $\bullet_R$	$\mathrm{GL}(2)/\mathrm{SO}(2)$
8	$E_3 \sim A_2 \times A_1$ 	$\mathrm{SL}(3) \times \mathrm{SL}(2)/\mathrm{SO}(3) \times \mathrm{SO}(2)$
7	$E_4 \sim A_4$ 	$\mathrm{SL}(5)/\mathrm{SO}(5)$
6	$E_5 \sim D_5$ 	$\mathrm{SO}(5,5)/\mathrm{SO}(5) \times \mathrm{SO}(5)$
5	E_6 	$E_6/\mathrm{Sp}(8)$
4	E_7 	$E_7/\mathrm{SU}(8)$
3	E_8 	$E_8/\mathrm{SO}(16)$
2	E_9 	$E_9/I_c(E_9)$
1	E_{10} 	$E_{10}/I_c(E_{10})$

Introduction: The E_{11} Program

In 2001 it was conjectured [1] that the initial 11D theory possesses E_{11}, with Dynkin diagram

$$\begin{array}{ccccccccccccccccccc} & & & & & & & & & & & & & & \bullet & & 11 & & \\ & & & & & & & & & & & & & & | & & & & \\ \underset{1}{\bullet} & - & \underset{2}{\bullet} & - & \underset{3}{\bullet} & - & \underset{4}{\bullet} & - & \underset{5}{\bullet} & - & \underset{6}{\bullet} & - & \underset{7}{\bullet} & - & \underset{8}{\bullet} & - & \underset{9}{\bullet} & - & \underset{10}{\bullet} \end{array} \tag{1}$$

as an exceptional symmetry group, explaining the above table.

In 2015 it was shown [2] that the low energy effective action of the bosonic sector of M-theory arises from the non-linear realization of

$$E_{11} \otimes_s l_1 / I_c(E_{11}). \tag{2}$$

Here l_1 is the vector representation of E_{11}, and $I_c(E_{11})$ is a generalization of the coset subgroups in the table.

This is a generalization of the vielbein formulation of general relativity, which is based on $\mathrm{GL}(11) \otimes_s l_1 / \mathrm{SO}(1,10)$.

The Kac-Moody Algebra K_{27}

In 2001 it was also conjectured [1] that the Kac-Moody algebra K_{27}

$$\begin{array}{ccccccccccccccccc} & & & & & & \bullet\ 26 & & & & & & & & \bullet\ 27 & & \\ & & & & & & | & & & & & & & & | & & \\ \underset{1}{\bullet} & - & \underset{2}{\bullet} & - & \underset{3}{\bullet} & - & \underset{4}{\bullet} & - & \underset{5}{\bullet} & - & \cdots & - & \underset{23}{\bullet} & - & \underset{24}{\bullet} & - & \underset{25}{\bullet} \end{array} \qquad (3)$$

is a symmetry of the 26D closed bosonic string.

In this talk we will show [3] that the low energy effective action of the 26D closed bosonic string arises from the non-linear realization of

$$K_{27} \otimes_s l_1 / I_c(K_{27}) \qquad (4)$$

The Kac-Moody Algebra K_{27}

K_{27} possesses generators R^α given by ($a = 1, ..., 26$)

$$K^a{}_b \ , \ R \ ; \ R^{a_1 a_2} \ , \ R^{a_1..a_{22}} \ ; \ R_{a_1 a_2} \ , \ R_{a_1..a_{22}} \ ; \\ R^{a_1..a_{24}} \ , \ R^{a_1..a_{23},b} \ , \ R^{a_1..a_{25},b_1..b_{19}} \ ; \ \dots \tag{5}$$

These generators satisfy an algebra of the form

$$[K^a{}_b, R] = 0 \ , \ [K^a{}_b, R^{c_1 c_2}] = 2\delta^{[c_1}{}_b R^{|a|c_2]} \ , \ [R^{a_1 a_2}, R^{b_1 b_2}] = 0 \ , \\ [R^{a_1 a_2}, R^{b_1..b_{22}}] = R^{a_1 a_2 b_1..b_{22}} + R^{b_1..b_{22}[a_1,a_2]} \ , \ \dots \tag{6}$$

We associate fields A_α to the K_{27} generators of equation (5)

- $h_a{}^b$ — Graviton
- ϕ — Dilaton
- $A_{a_1 a_2}$ — Kalb-Ramond
- $A_{a_1..a_{22}}$ — Dual Kalb-Ramond
- $A_{a_1..a_{24}}$ — Dual Scalar
- $h_{a_1..a_{23},b}$ — Dual Graviton

Involution Invariant Subalgebra of K_{27}

We now define the subalgebra $I_c(K_{27})$.

The Serre relations are preserved by the involution I_c

$$I_c(E_A) = \eta_{AB} F_B \quad , \quad I_c(F_A) = \eta_{AB} E_B \quad , \quad I_c(H_A) = \eta_{AB} H_B \quad , \tag{7}$$

$$I_c(AB) = I_c(A) I_c(B) \ . \tag{8}$$

We can define involution invariant combinations, which generate $I_c(K_{27})$.

The $I_c(K_{27})$ subalgebra is explicitly generated by

$$J_{a_1 a_2} = \eta_{a_1 e} K^e{}_{a_2} - \eta_{a_2 e} K^e{}_{a_1} \ ; \tag{9}$$

$$\begin{aligned} S_{a_1 a_2} &= R^{b_1 b_2} \eta_{b_1 a_1} \eta_{b_2 a_2} - R_{a_1 a_2} \ , \\ S_{a_1..a_{22}} &= R^{b_1..b_{22}} \eta_{b_1 a_1} .. \eta_{b_{22} a_{22}} - R_{b_1..b_{22}} \ ; \\ S_{a_1..a_{24}} &= R^{b_1..b_{24}} \eta_{b_1 a_1} .. \eta_{b_{24} a_{24}} + R_{b_1..b_{24}} \ , \ \dots \end{aligned} \tag{10}$$

Vector Representation of K_{27}

The vector representation of K_{27} is denoted l_1 and possesses generators l_A given by

$$P_a \;;\; Q^a \;,\; Z^{a_1..a_{21}} \;;\; Z^{a_1..a_{23}}_{\{1\}} \;,\; Z^{a_1..a_{23}}_{\{2\}} \;,$$
$$Z^{a_1..a_{22},b} \;,\; Z^{a_1..a_{24},b_1..b_{19}} \;,\; Z^{a_1..a_{25},b_1..b_{18}} \;;\; \ldots \tag{11}$$

The higher coordinates represent charges of higher branes in the theory.

They generate a '**generalized space-time**' with coordinates x^A given by

$$x^a \;,\; y_a \;;\; x_{a_1..a_{21}} \;,\; .... \tag{12}$$

The x^a, y_a are the D_{26} coordinates of *Double Field Theory* (DFT).

Vector Representation of K_{27}

The $R^{a_1..a_{22}}$ sends P_a into $Z^{a_1..a_{21}}$ via $[R^{a_1..a_{21}}, P_b] = 21\delta_b^{[a_1} Z^{a_2..a_{22}]}$, and Q^a into the $Z^{a_1..a_{23}}$ under a similar relation. This implies that we will have to introduce branes into our eventual closed string theory.

It is thought [2] that the higher coordinates may reflect the breakdown of space-time near a black hole, giving an approximate description of more fundamental degrees of freedom.

"Space-time is doomed" in the E_{11} program, we must extend the usual x^μ into a 'generalized space-time' x^Π.

Nonlinear Realisation of $K_{27} \otimes_s l_1 / l_c(K_{27})$

The non-linear realisation of $K_{27} \otimes_s l_1 / l_c(K_{27})$ begins from a **group element**

$$g = g_{l_1} g_{K_{27}} \quad ,$$
$$g_l = \exp(x^a P_a + y_a Q^a + \ldots) \quad , \tag{13}$$
$$g_{K_{27}} = \Pi_\alpha \exp(A_\alpha R^\alpha) = .. \exp(A_{a_1 a_2} R^{a_1 a_2}) \exp(h_a{}^b K^a{}_b).$$

We then compute the **Maurer-Cartan form**

$$\Omega \equiv g^{-1} dg = dx^\Pi E_\Pi{}^A (l_A + G_{A,\alpha} R^\alpha) \tag{14}$$

- $E_\Pi{}^A$ is the **generalized vielbein** of a **generalized geometry** on the generalized space-time.
- G_α are **generalized covariant derivatives** of the goldstone fields A_α.

Note the $G_\alpha = dx^\Pi E_\Pi{}^A G_{A,\alpha}$ involve derivatives ∂_A w.r.t. the generalized coordinates x^A.

The **generalized vielbein** is given explicitly by

$$E_\Pi{}^A = (\det e)^{-\frac{1}{2}} \begin{bmatrix} e_\mu{}^a & -2e^{-\phi} A_{\mu a} & -22 e^{\phi} A_{\mu a_1..a_{21}} & \dots \\ 0 & e^{-\phi} e_a^\mu & 0 & \dots \\ 0 & 0 & e^{\phi} e_{a_1..a_{21}}^{\mu_1..\mu_{21}} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad (15)$$

The covariant derivatives G_α are given explicitly by $(e_a{}^b \equiv (e^h)_a{}^b)$

$$G_a{}^b = (e^{-1} de)_a{}^b \quad , \quad G = d\phi \quad ;$$

$$G_{a_1 a_2} = e^{-\phi} e_{\mu_1}{}^{a_1} e_{a_2}{}^{\mu_2} dA_{\mu_1 \mu_2} \quad ,$$

$$G_{a_1..a_{22}} = e^{+\phi} e_{a_1}{}^{\mu_1} \dots e_{a_{22}}{}^{\mu_{22}} dA_{\mu_1..\mu_{22}} \quad ;$$

$$G_{a_1..a_{24}} = e_{a_1}{}^{\mu_1} \dots e_{a_{24}}{}^{\mu_{24}} (dA_{\mu_1..\mu_{24}} - dA_{[\mu_1..\mu_{22}} A_{\mu_{23}\mu_{24}]}) \qquad (16),$$

$$G_{a_1.a_{23},b} = e_{a_1}{}^{\mu_1} . e_{a_{23}}{}^{\mu_{23}} e_b{}^\nu (dh_{\mu_1.\mu_{23},\nu} - dA_{[\mu_1.\mu_{22}} A_{\mu_{23}]\nu} + dA_{[\mu_1..a_{22}} A_{\mu_{23}\nu]}) ,$$

$$\vdots$$

Nonlinear Realisation of $K_{27} \otimes_s l_1 / I_c(K_{27})$

The group element of $K_{27} \otimes_s l_1 / I_c(K_{27})$ satisfies $g \equiv g' = gh$ for $h \in I_c(K_{27})$.

Under the local $h \in I_c(K_{27})$ transformations $g \rightarrow gh$, the covariant derivatives transform covariantly since

$$\Omega \quad \rightarrow \quad \Omega' = h^{-1}(\Omega)h + h^{-1}dh \tag{17}$$

In other words, the covariant derivatives of (16) transform under (17) into linear combinations of one another.

The set of all covariant derivatives form a trivial irrep of $K_{27} \otimes_s l_1 / I_c(K_{27})$.

Nonlinear Realisation of $K_{27} \otimes_s l_1 / I_c(K_{27})$

Setting $h = I - \Lambda^{a_1 a_2} S_{a_1 a_2} - \Lambda^{a_1..a_{22}} S_{a_1..a_{22}} + \ldots$ they transform as

$$
\begin{aligned}
\delta G_a{}^b &= 4\Lambda^{eb}\overline{G}_{ea} - \tfrac{1}{6}\Lambda^{e_1 e_2}\overline{G}_{e_1 e_2}\delta^b{}_a + (22)^2 21!\Lambda^{e_1..e_{21} b} G_{e_1..e_{21} a} \\
&\quad - \tfrac{11}{12} 22! \Lambda^{e_1..e_{22}} G_{e_1..e_{22}} \delta^b{}_a \ , \\
\delta G &= \tfrac{1}{3}\Lambda^{e_1 e_2}\overline{G}_{e_1 e_2} - \tfrac{22!}{6}\Lambda^{e_1..e_{22}} G_{e_1..e_{22}} \ ; \qquad (18) \\
\delta \overline{G}_{a_1 a_2} &= 2 \cdot 2\Lambda_{e[a_1} G_{(a_2]}{}^{e)} - 2\Lambda_{a_1 a_2} G + \tfrac{24!}{12}\Lambda^{e_1.e_{22}} G_{e_1.e_{22} a_1 a_2} \ + .. \ , \\
\delta G_{a_1.a_{22}} &= 2 \cdot 22\Lambda_{e[a_1.a_{21}} G_{(a_{22}]}{}^{e)} + 2\Lambda_{a_1.a_{22}} G - 4 \cdot 23\Lambda^{e_1 e_2} G_{e_1 e_2 a_1.a_{22}} + .., \\
\delta G_{a_1.a_{24}} &= \Lambda_{[a_1 a_2} G_{a_3.a_{24}]} - \Lambda_{[a_1.a_{22}} G_{a_{23} a_{24}]} + .. \ , \\
\delta G_{a_1.a_{23},b} &= G_{[a_1.a_{22}}\Lambda_{a_{23}]b} - G_{[a_1.a_{22}}\Lambda_{a_{23} b]} - \Lambda_{[a_1.a_{22}} G_{a_{23}]b} + \Lambda_{[a_1.a_{22}} G_{a_{23} b]}.
\end{aligned}
$$

The transformations in equation (17) are on the differential forms as a whole.

In $\Omega = dx^\Pi E_\Pi{}^A(l_A + G_{A,\alpha} R^\alpha)$, the A index will change under $\Omega' = h^{-1}\Omega h + h^{-1} dh$ and must be compensated via the A in $E_\Pi{}^A$.

This means that the A in ∂_A varies, so derivatives rotate into one another.

To level one we have derivatives $\partial_a, \hat{\partial}^a, \hat{\partial}^{a_1..a_{21}}$, and the Cartan form coefficients $G_{A,\alpha}$ transform under (24) in the A index as

$$\delta G_{a,\alpha} = -2\Lambda_{ae}\hat{G}^{e,}{}_\alpha - 22\Lambda_{ae_1..e_{21}} G^{e_1..e_{21},}{}_\alpha \ , \qquad (19)$$
$$\delta \hat{G}^{a,}{}_\alpha = -10\Lambda^{ae} G_{e,\alpha} \quad , \quad \delta G^{a_1..a_{21},}{}_\alpha = -11 \cdot 21! \Lambda^{a_1..a_{21}e} G_{e,\alpha} \ .$$

First Order Duality Relations

We now construct a non-trivial irrep of $K_{27} \otimes_s l_1 / I_c(K_{27})$.

Seek linear combinations of covariant derivatives that transform covariantly into one another under $I_c(K_{27})$.

These are **first order duality relations**

$$
\begin{aligned}
D_a &\equiv G_a + \frac{1}{12}\varepsilon_a{}^{c_1..c_{25}} G_{c_1,c_2..c_{25}} \\
\overline{D}_{a_1 a_2 a_3} &\equiv G_{[a_1,a_2 a_3]} + \frac{1}{6}\varepsilon_{a_1 a_2 a_3}{}^{c_1..c_{23}} G_{c_1,c_2..c_{23}} \\
D_{a,b_1 b_2} &\equiv (\det e)^{1/2}\omega_{a,b_1 b_2} - \varepsilon_{b_1 b_2}{}^{c_1..c_{24}} G_{c_1,c_2..c_{24},a}
\end{aligned}
\tag{20}
$$

- D_a is the dilaton duality,
- $\overline{D}_{a_1 a_2 a_3}$ is the Kalb-Ramond duality,
- $D_{a,b_1 b_2}$ is the gravity dual-gravity duality.

We are finding a formulation of the 26D Closed Bosonic String in terms of First Order duality relations.

First Order Duality Relations

However, the derivatives in say

$$\overline{D}_{a_1 a_2 a_3} \equiv G_{[a_1,a_2 a_3]} + \frac{1}{6}\varepsilon_{a_1 a_2 a_3}{}^{c_1..c_{23}} G_{c_1,c_2..c_{23}}$$

also vary under $\Omega' = h^{-1}\Omega h + h^{-1}dh$

In order to construct a consistent irrep, we must generalize (20) to include higher level derivative contributions.

First Order Duality Relations

We must generalize $\overline{D}_{a_1 a_2 a_3}$ to a duality involving level one derivative contributions

$$\overline{\mathcal{D}}_{a_1 a_2 a_3} \equiv \overline{\mathcal{G}}_{[a_1 a_2 a_3]} + \tfrac{1}{6}\varepsilon_{a_1 a_2 a_3}{}^{e_1..e_{23}}\mathcal{G}_{e_1 e_2..e_{23}} \quad , \tag{21}$$

where now

$$\overline{\mathcal{G}}_{[a_1 a_2 a_3]} \equiv \overline{G}_{[a_1, a_2 a_3]} + \frac{1}{5}\hat{G}_{[a_1, a_2 a_3]} + \frac{4 \cdot 22 \cdot 23}{3}\hat{G}^{e_1..e_{21},}{}_{e_1..e_{21} a_1 a_2 a_3}$$
$$+ 2 \cdot 22 \cdot 23\, \hat{G}^{e_1..e_{21},}{}_{e_1..e_{21}[a_1 a_2, a_3]} \tag{22}$$

and

$$\mathcal{G}_{[a_1 a_2..a_{23}]} \equiv G_{[a_1, a_2...a_{23}]} + (2/21!)\hat{G}_{[a_1...a_{21}, a_{22} a_{23}]} - (4/5)\hat{G}^{c,}{}_{c a_1..a_{23}}$$
$$+ (2/5)\hat{G}^{c,}{}_{a_1..a_{23}, c} \tag{23}$$

Similarly

$$D_a \equiv G_a + \tfrac{1}{12}\varepsilon_a{}^{c_1..c_{25}} G_{c_1,c_2..c_{25}} \quad ,$$

generalies into

$$\mathcal{D}_a \equiv \mathcal{G}_a + e_1 \varepsilon_a{}^{b_1..b_{25}} G_{b_1,b_2..b_{25}} \tag{24}$$

where we have

$$\mathcal{G}_a \equiv G_a + \frac{1}{15}\hat{\hat{G}}{}^{e,}{}_{ea} - \frac{22}{3}\hat{G}^{e_2..e_{22},}{}_{e_2..e_{22}a} \tag{25}$$

D_{a,b_1b_2} gets a similar modification into $\mathcal{D}_{a,b_1b_2}$ [3].

We call the $\mathcal{D}_{a,\alpha}$ analog of $D_{a,\alpha}$ the l_1-extension of $D_{a,\alpha}$.

First Order Duality Relations

Under the transformations of equation (18) and (19), the l_1-extended duality relations get sent into

$$\begin{aligned} \delta\mathcal{D}_a &= \tfrac{1}{2}\Lambda^{e_1 e_2}\overline{D}_{a e_1 e_2} + \dots \\ \delta\overline{\mathcal{D}}_{a_1 a_2 a_3} &= -2D_{[a}\Lambda_{b_1 b_2]} + 2\Lambda^{c}{}_{[a_1}D_{|c|,a_2 a_3]} + \dots \\ \delta\mathcal{D}_{a,b_1 b_2} &= -\tfrac{11}{12}\varepsilon_{b_1 b_2}{}^{c_1..c_{24}}\Lambda_{a c_1..c_{21}}\overline{D}_{c_{22}c_{23}c_{24}} \\ &\quad + \tfrac{11}{8}\varepsilon_{b_1 b_2}{}^{c_1..c_{24}}\Lambda_{c_1..c_{22}}\overline{D}_{c_{23}c_{24}a} + \dots \end{aligned} \tag{26}$$

The first order duality relations along with their l_1-extensions and higher level contributions form an irreducible representation of $K_{27} \otimes_s l_1 / I_c(K_{27})$.

We can thus consistently set all duality relations in the representation, and their l_1-extensions, to zero, and indeed interpret them as duality relations.

Second-Order Equations of Motion

We can use the first-order duality relations in equation (20) to form **second-order equations of motion** (eom).

We do this by projecting out one of the two fields using derivatives:

$$E = \partial_\mu[(\det e)^{1/2} D^\mu]$$
$$E^{\nu_1\nu_2} = \partial_\mu[(\det e)^{\frac{1}{2}} \overline{D}^{\mu,\nu_1\nu_2}] \qquad (27)$$

We can then form l_1-extended versions of (27) and vary these under (19) and (20).

For example, E gets l_1-extended into

$$\mathcal{E} = \{(\det e)^{\frac{1}{2}} e_a{}^{\mu}\partial_{\mu} G^a - G_{c,a}{}^c G^a + \frac{1}{2} G_{a,c}{}^c G^a\}$$
$$+\frac{2}{3}\frac{1}{10}\partial_{\mu}\{(\det e)^{\frac{1}{2}} \overline{\hat{G}}^{\tau_2,}{}_{\tau_2}{}^{\mu}\} - \frac{2}{10}\overline{G}^{[\mu,\tau_1\tau_2]}\hat{G}_{\tau_2,(\mu\tau_1)}$$
$$-\frac{4}{10}\overline{\hat{G}}^{c,}{}_{,cb}G^b - 22\frac{22!}{6\cdot 11\cdot 21!}\partial_{\mu}\{\hat{G}^{\tau_1,\tau_2..\tau_{22}\mu}\Lambda_{\tau_1..\tau_{22}}(\det e)^{\frac{1}{2}}\}$$
$$-\frac{21!21}{11\cdot 21!}\hat{G}^{e_2..e_{22},}{}_{e_2..e_{22}b}G^b \tag{28}$$
$$-\frac{22}{11\cdot 21!}\varepsilon^{a_1..a_{26}}\overline{G}_{[a_1,a_2a_3]}\hat{G}_{a_5..a_{24},[a_{25}a_{26}]}(\frac{1}{6\cdot 6e_2})$$
$$-e_1\varepsilon^{e_1..e_{26}}G_{e_1,e_2e_3}G_{e_4,e_5..e_{26}}$$

Second-Order Equations of Motion

The result of varying $\mathcal{E}$ and $\mathcal{E}^{\nu_1\nu_2}$ is that we find the second order eom

$$E = \partial_\nu[(\det e)^{\frac{1}{2}} G^\nu] + \tfrac{1}{2}\overline{G}_{[c_1,c_2c_2]}\overline{G}^{[c_1,c_2c_3]} = 0$$

$$E^{\nu_1\nu_2} \equiv \partial_\mu[(\det e)^{\frac{1}{2}}\overline{G}^{[\mu,\nu_1\nu_2]}] - G_\mu\overline{G}^{[\mu,\nu_1\nu_2]} = 0 \qquad (29)$$

$$(\det e)\tilde{E}_a{}^b \equiv (\det e)R_a{}^b - 9\overline{G}_{[a,e_1e_2]}\overline{G}^{[b,e_1e_2]} + \tfrac{1}{4}\delta_a{}^b\overline{G}_{[e_1,e_2e_3]}\overline{G}^{[e_1,e_2e_3]}$$
$$-6G_aG^b = 0$$

These are the familiar second order equations of motion of the closed bosonic string in 26 dimensions, coming from the action

$$S = \int d^{26}x \det e\{R - \tfrac{1}{6}(\partial^\mu\phi)(\partial_\mu\phi) - \tfrac{1}{3}e^{-\frac{2}{6}\phi}F_{\mu\nu\rho}F^{\mu\nu\rho}\}$$
$$= \int d^{26}x\{(\det e)R - \tfrac{1}{6}G^aG_a - 3G_{[a,bc]}G^{[a,bc]}\} \qquad (30)$$

Second-Order Equations of Motion

The eom of the closed bosonic string arise from K_{27} symmetry being realized non-linearly.

Taken all together the eom form an irreducible representation of $K_{27} \otimes_s l_1 / I_c(K_{27})$.

Conclusion

- The Kac-Moody algebra K_{27} is a symmetry of the theory closed bosonic strings with branes.
- Contains $\mathrm{O}(D, D)$ DFT for $D = 26$: ignoring node 26 gives $\mathrm{O}(26, 26)$, x^a, y_a DFT coordinates.
- Brane contributions are required in our theory of closed strings.
- Truncating the theory at any stage and throwing away higher fields or higher coordinates will cause everything to fail, there cannot be any ‘consistent truncations’.

Future Questions

- Compute Brane dynamics: Q^a represents the charge of a string. $Z^{a_1..a_{21}}$ is the charge of a 21-brane.
- Deeper relationship to E_{11} and other string theories.
- Kac-Moody Algebra interpretation of all 5 string theories at low energies? Have M-theory, IIA, IIB. Heterotic?
- Generalized Symmetries in K_{27}?

References

[1] P. West, E_{11} *and m-theory*, Class. Quant. Grav. 18, (2001) 4443, hep-th/ 0104081.

[2] Tumanov and P. West, *E11 must be a symmetry of strings and branes*, arXiv:1512.01644.

[3] K. Glennon and P. West, K_{27} *as a symmetry of closed bosonic strings and branes*, hep-th/2409.08649.