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Wormhole teleportation and the SYK model

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High Energy Physics in the Quantum Era, KEK, 2-4 December 2024

Outline

- Wormholes
- The SYK model
- Wormhole teleportation in emergent space
- Long-range wormhole teleportation
- Exhibiting wormhole dynamics on real-world quantum systems
- Supersymmetric wormholes and emergent time

Quantum Communications Channels for Fundamental Physics (QCCFP) Collaboration:

Samantha Davis (Caltech)
Daniel Jafferis (Harvard)
Stavros Klaoudatos (Caltech)
David Kolchmeyer (MIT)
Nicolai Lauk (Caltech)
Joe Lykken (Fermilab)
Damian Musk (Caltech)
Hartmut Neven (Google Quantum AI)
Maria Spiropulu (Caltech)
Alex Zlokapa (MIT)

Papers:
https:// arXiv.org/abs/1911.07416
https://www.nature.com/articles/s41586-022-05424-3
https://arXiv.org/abs/2303.15423
https:// arXiv.org/abs/2405.07876
and to appear

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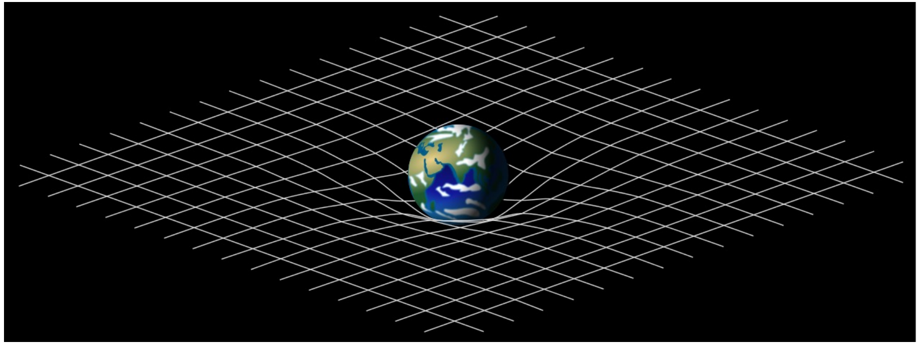
Lots of questions about quantum gravity

- What is the correct UV completion for gravity
- If you built a collider that could reach Planck-scale energies, what would you see
- What is the detailed mechanism by which evaporating black holes avoid information loss
- What was the Big Bang
- What was inflation
- What is dark energy

I will not address any of these questions in this talk

Questions about quantum gravity: this talk

- Is quantum gravity holographic?
- Is the fabric of spacetime explained by quantum entanglement?
- Can we recognize and study wormholes far away from the semi-classical regime?



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Quantum gravity: theoretical laboratories

- Pretty hopeless to examine deep questions about quantum gravity in flat space
- Instead, use backgrounds that have large, comprehensible, semi-classical features: e.g. black holes and wormholes
- Black holes and wormholes in AdS/CFT are even better, because you have a controlled UV completion and explicit holography
- To understand in detail the role of quantum entanglement, it is useful to look at the simplest case, quantum gravity in AdS_2 spacetime with coordinates x and t

Jackiw-Teitelboim 2-dimensional gravity

- In gravity with one space and one time dimension, there is no propagating graviton and the Einstein-Hilbert term is topological
- However, we can make a nontrivial dilaton-gravity theory called J-T gravity

$$S[g,\Phi] = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} d^2x\sqrt{-g}\,\Phi(R+2) + S_{bdry} + S_0\chi + S_m[g,\phi]$$

- Solving the equation of motion of the dilaton puts us in a (nearly) AdS_2 background
- Letting u denote proper time on the boundary, the 2d gravity theory is then the dynamics of t(u) and x(u)
- And x(u) is determined from t(u) from a renormalization condition

2-dimensional gravity in the Schwarzian approximation

- So 2D gravity here means the dynamics of the AdS boundary.
- The remnant of general coordinate invariance is that the effective action for t(u) should be SL(2,R) invariant; this is the Schwarzian action:

$$S_{\text{Sch}} = -C \int du \{t, u\} \qquad \{t, u\} = \frac{t'''}{t'} - \frac{3}{2}\left(\frac{t''}{t'}\right)^2$$

- This sounds pretty far away from quantum gravity in the real world
- But, in fact, J-T gravity describes physics near the horizon of 4-dimensional near-extremal charged black holes after a Kaluza-Klein reduction

for a review, see T. Mertens and G. Turiaci, arXiv:2210.10846

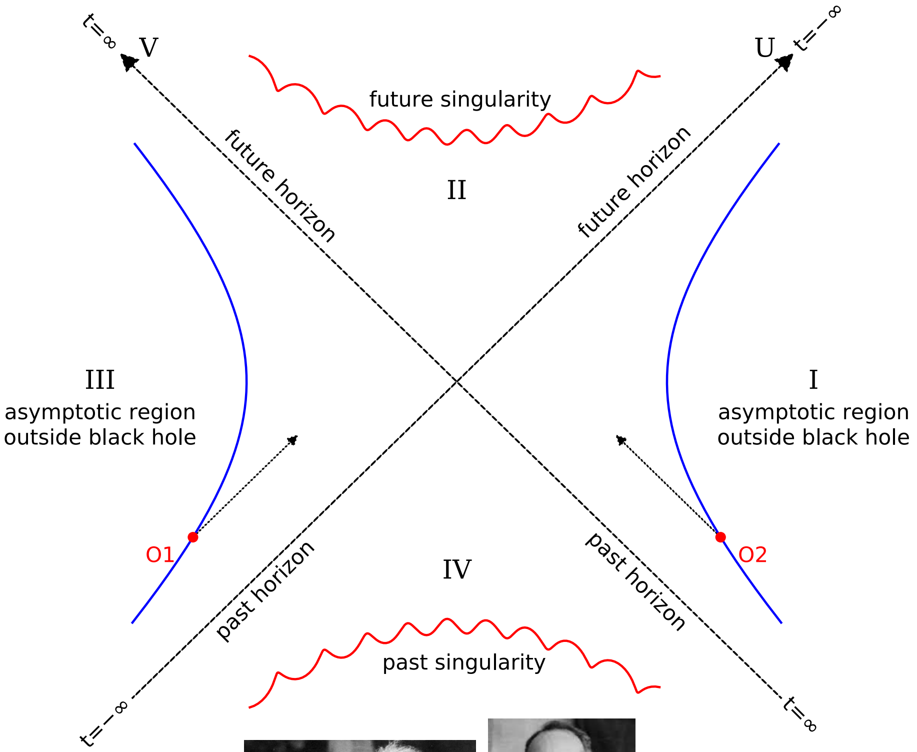
Einstein-Rosen bridge

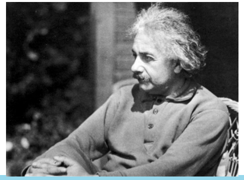
- Otherwise known as the Kruskal extension of the Schwarzschild metric for a static black hole
- Looks like two connected black holes
- This is the original example of a wormhole
- But this wormhole is non-traversable

A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935)

- Juan Maldacena later showed in the AdS/CFT context that you should think of this as two entangled black holes

J. Maldacena, hep-th/0106112





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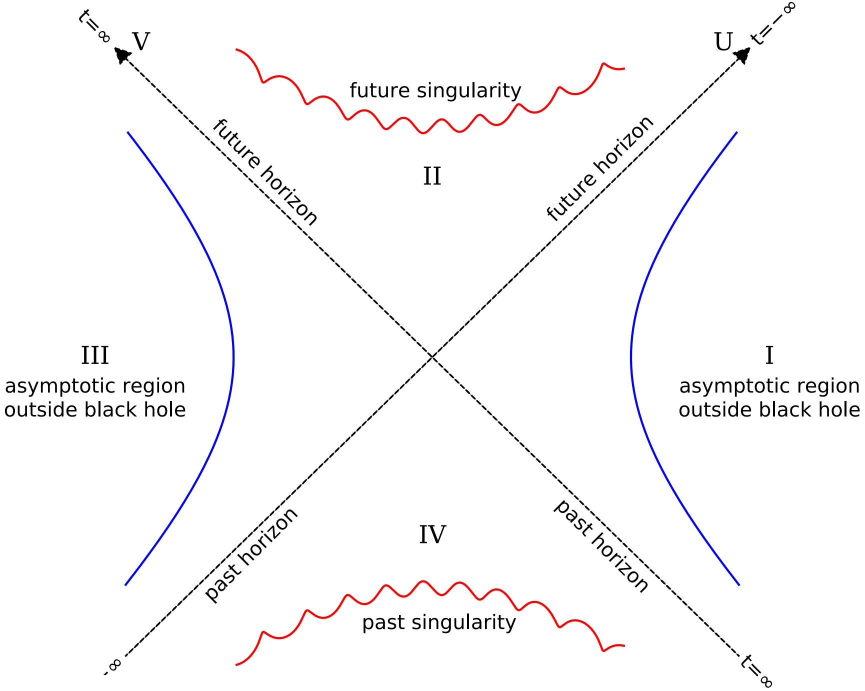
Einstein-Rosen bridge

- So this wormhole is non-traversable
- It has been known since the 1980s that to make such a wormhole traversable you would need to somehow inject **negative energy**



Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity

Michael S. Morris and Kip S. Thorne
Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125

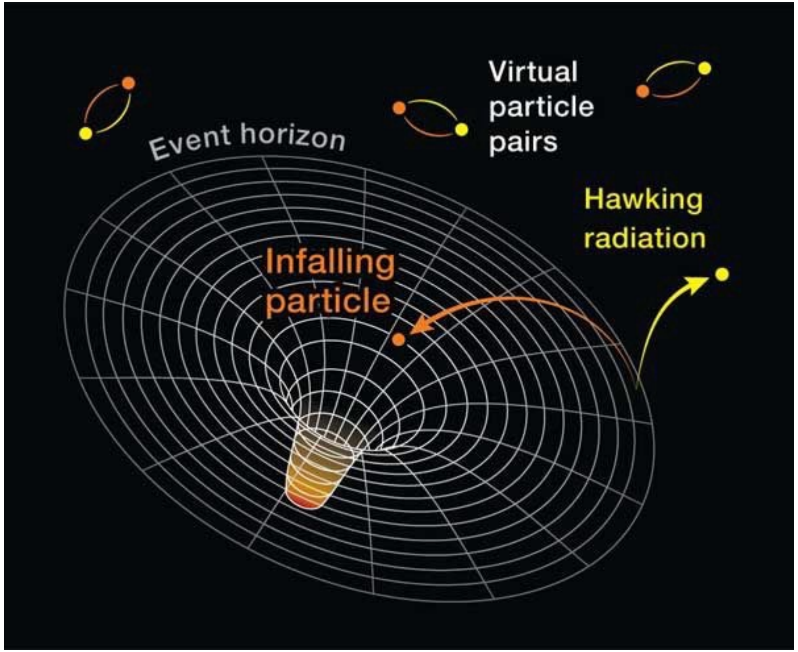
(Received 16 March 1987; accepted for publication 17 July 1987)

Hawking radiation and negative energy

S. Hawking, Commun.Math.Phys. 43 (1975) 199-220

- The statement that wormholes cannot be traversable unless you inject negative energy is closely related to Hawking radiation from black holes

- Both are effects of quantum gravity



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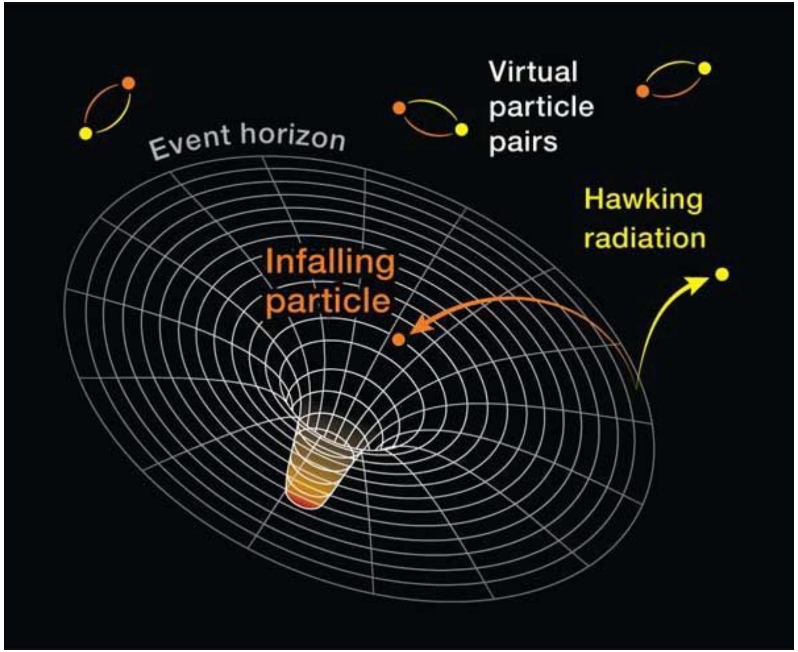
Hawking radiation and negative energy

S. Hawking, Commun.Math.Phys. 43 (1975) 199-220

- Hawking said in his original paper that a careful calculation of Hawking radiation should also show **negative energy moving into the black hole**

- This is because any positive energy/mass falling into a black hole makes the horizon expand, while **Hawking radiation must make the black hole horizon shrink**



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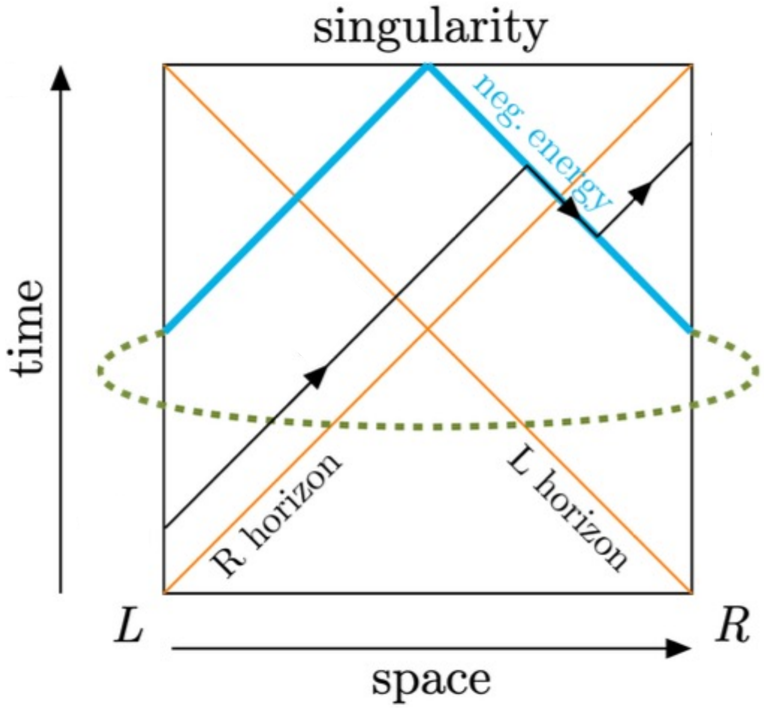
Traversable wormholes

P. Gao, D. Jafferis, A. Wall, arXiv:1608.05687

- In 2016 Daniel Jafferis et al showed how to make traversable wormholes
- This requires a quantum effect resulting from a direct interaction between the left and right entrances of the wormhole
- This quantum effect acts as a negative energy pulse
- Opens up a traversable path, but only in a finite time window



Bottom line; need a left-right coupling operator with a negative coupling constant: $H_L + H_R + \mu V$

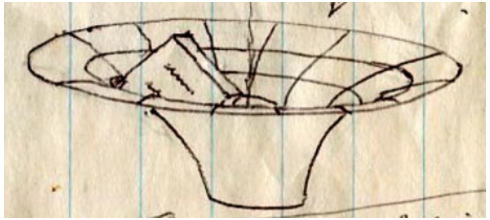
Traversable wormholes in 2D

This sounds exciting, but it is difficult to find a pair of black holes, let alone entangle them

But it turns out that you can also make a 1+1 dim. traversable wormhole (in a certain limit) by entangling two copies of a quantum mechanical system, the SYK model

In this case you are doing quantum gravity in (nearly) AdS_2

Or near the horizon of a 4-dimensional near-extremal charged black hole



SYK model

S. Sachdev and J.-w. Ye, Phys. Rev. Lett. 70 (1993) 3339

A. Kitaev, “A simple model of quantum holography.”
Talks at KITP, April 7 and May 27, 2015

- SYK is a quantum mechanical model of **N** interacting Majorana fermions ψ_j, interacting **q** at a time with random couplings scaled by an overall coupling **J**
- Hamiltonian is strongly coupled but nonlocal (no spatial dependence)

$$H_{\text{SYK}} = \sum_{1\le j_1<\ldots<j_q\le N} J_{j_1\ldots j_q}\,\psi^{j_1}\ldots\psi^{j_q}$$

random couplings sampled with
mean zero and variance = $J^2(q-1)!/N^{q-1}$

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SYK model scrambling dynamics

S. Sachdev and J.-w. Ye, Phys. Rev. Lett. 70 (1993) 3339

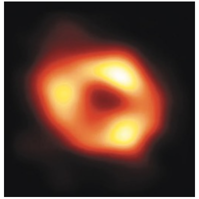
A. Kitaev, "A simple model of quantum holography." Talks at KITP, April 7 and May 27, 2015

- A single fermion ψ_j rapidly evolves over time into a linear combination of strings of many fermions
- This operator "size" growth is exponentially fast, parameterized by the Lyapunov exponent λ_L
- Quantum information encoded in a simple operator thus quickly becomes "scrambled", i.e. delocalized into multipartite entanglement

$$H_{\mathrm{SYK}} = \sum_{1 \leq j_1 < \ldots < j_q \leq N} J_{j_1 \ldots j_q} \psi^{j_1} \ldots \psi^{j_q}$$

random couplings sampled with mean zero and variance = $J^2(q-1)!/N^{q-1}$

SYK model and black holes



Fast scrambling of quantum information is a characteristic feature of black hole dynamics.

This is why, when Alice throws her diary into a black hole, Bob can decode its contents in the emitted Hawking radiation (after suitable unscrambling)

P. Hayden and J. Preskill, arXiv:0708.4025

- Maldacena, Shenker, and Stanford have argued that black holes saturate an upper bound on the Lyapunov exponent $\lambda_L \to 2\pi/\beta$

J. Maldacena, S. Shenker, D. Stanford, arXiv:1503.01409

J. Maldacena and D. Stanford, arXiv:1604.07818

- Maldacena and Stanford then showed that the SYK model in a certain limit also saturates this upper bound

- So the SYK model is similar to a black hole in terms of its quantum dynamics

SYK wormholes in emergent space

J. Maldacena, D. Stanford, and Z. Yang, arXiv:1704.05333

Maldacena, Stanford, and Yang showed that you can make a traversable wormhole from two entangled copies of the SYK model, in the limit $N \gg q \gg \beta J \gg 1$

- In this semi-classical limit, there is an ***emergent space*** dimension corresponding to motion through the wormhole
- The explicit left-right interaction comes from a sum of Majorana bilinears: $V = i \sum_{j=0}^{N-1} \psi_j^\ell \psi_j^r$
- We would like to study in detail how space and gravity **emerges from quantum entanglement dynamics in such a system**
- Notice that here Newton's constant is an adjustable parameter: $G_N \sim \frac{q\beta J}{N}$
- "Gravity" here means 1+1 dim Jackiw-Teitelboim gravity

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ER = EPR hypothesis

A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935)

A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935)

J. Maldacena and L. Susskind, arXiv:1306.0533

J. Maldacena, arXiv:hep-th/0106112

Maldacena and Susskind in 2013 argued that the 1935 ER paper on wormholes and the 1935 EPR paper on entanglement are actually talking about the same thing

- The Einstein-Rosen bridge occurs because the two black holes are entangled
- More generally, wormholes are a result of entanglement between certain kinds of quantum systems

The ER=EPR hypothesis asserts that traversable wormholes **have an equivalent physical description as some form of quantum teleportation**

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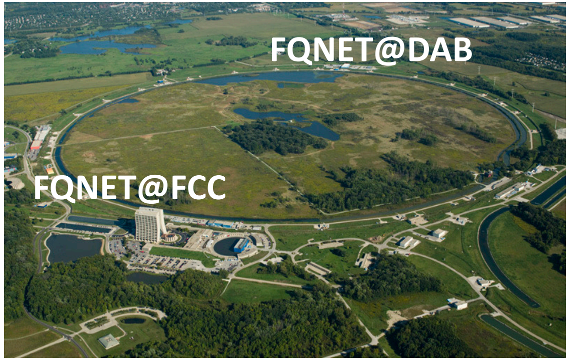
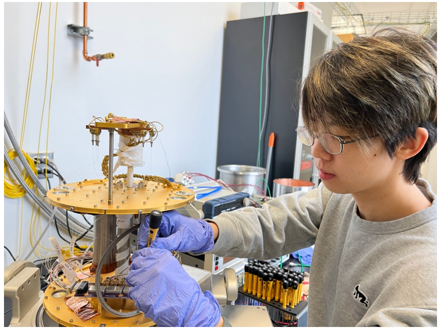
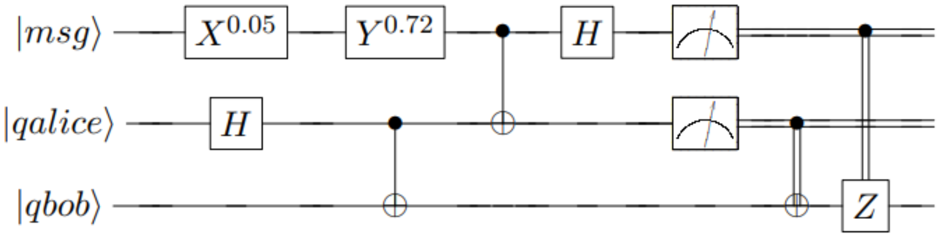
Quantum teleportation in the laboratory

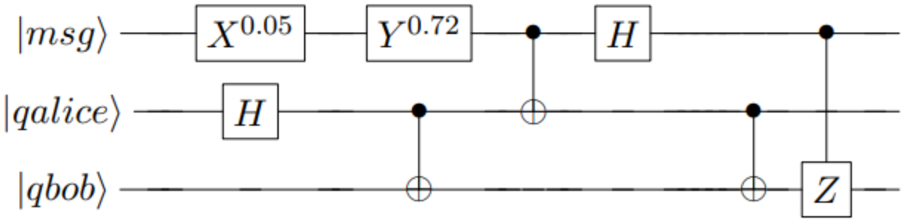
Quantum teleportation can be performed as a quantum circuit on a quantum processor







Example of a quantum circuit that performs (short-range) quantum teleportation



SYK wormhole teleportation

P. Gao and D. Jafferis, arXiv:1911.07416

Gao and Jafferis constructed an explicit protocol for performing quantum teleportation using this same setup of two entangled copies of SYK

- Their protocol is a new kind of many-body quantum teleportation between the two systems, where a message qubit gets swapped into one SYK system, then later appears in the other SYK system

They showed that in the limit $N \gg q \gg \beta J \gg 1$, for a particular (negative) value of μ, the teleportation has perfect fidelity

- In this semi-classical regime, **the teleportation has a physically equivalent holographic dual description**, where the quantum information of the message qubit propagates through a wormhole

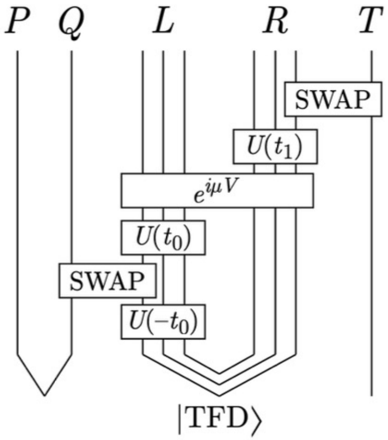
This is a concrete realization of the "ER = EPR" duality between teleportation and wormholes

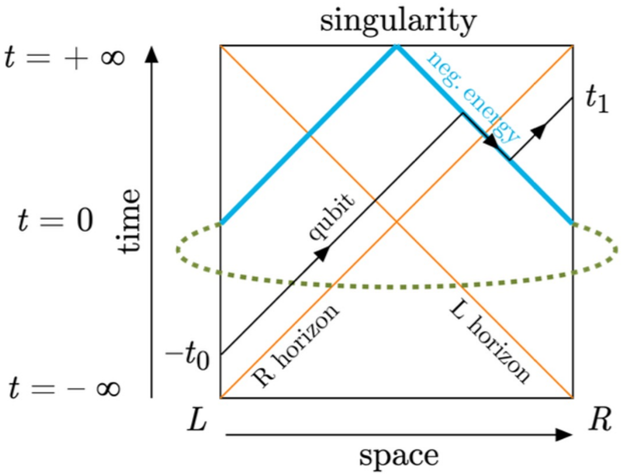
J. Maldacena and L. Susskind, arXiv:1306.0533

Wormhole teleportation

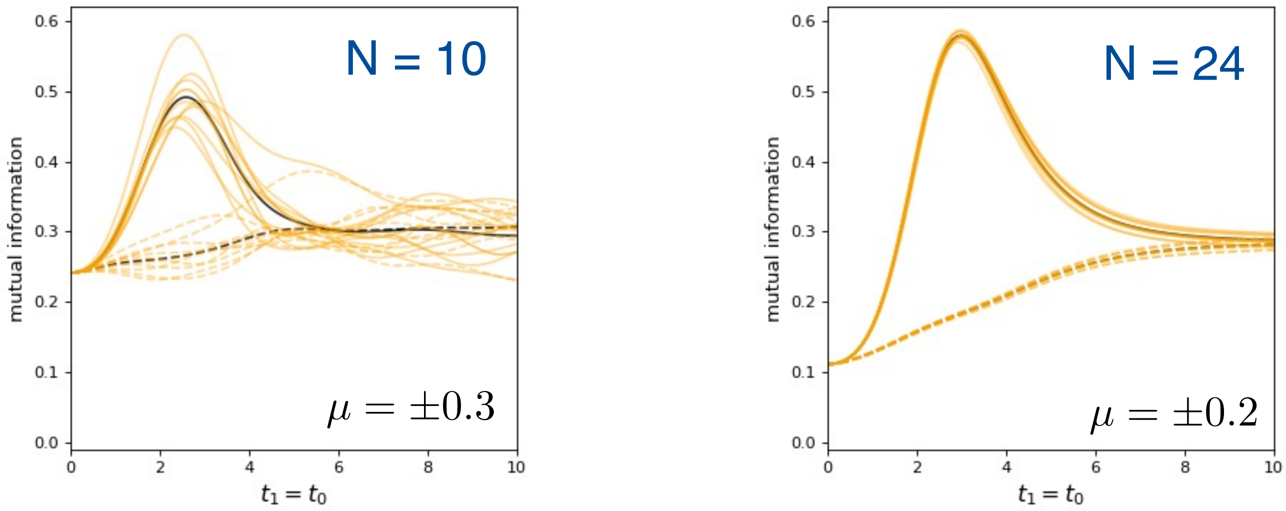
- To what extent can we exhibit and study any of these holographic features in a (classical) computer simulation of the wormhole teleportation protocol?

- Can we realize and study these holographic features on a quantum processor?





Wormhole teleportation at finite N



$$q = 4,\ J = 1,\ \beta = 4$$

Over some broad range of parameters, the mutual information between the message and readout qubits peaks at a characteristic time, but only for negative μ

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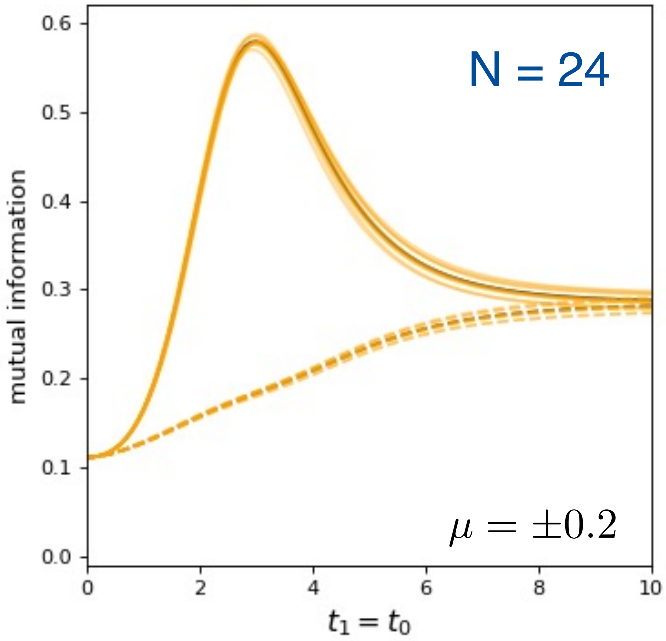
Wormhole teleportation at finite N

For finite N there are three distinct mechanisms transferring quantum information between the message and readout qubits:

- the holographic teleportation, occurring in a time window
- “teleportation by peak size”, which ramps up over time and dominates at late times
- direct transmission via left-right swapping, which occurs at t=0 but dies off rapidly



$q = 4, \ J = 1, \ \beta = 4$

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Size winding

A. Brown et. al, arXiv:1911.06314

S. Nezami et. al, arXiv:2102.01064

- A holographic duality implies that there are two physically equivalent descriptions, so there should be an explanation of wormhole teleportation ***that makes no reference to wormholes***
- This dual picture was discovered by the Stanford group and is called "teleportation by size winding"
- The key observation is that V in the holographic picture is a translation operator that, for the correct sign of μ, shifts a particle from inside the horizon to outside the horizon
- ***But V is also the size operator*** (up to an overall constant) for strings of Majorana fermions

$$V = i \sum_{j=0}^{N-1} \psi_j^{\ell} \psi_j^{r} \qquad \langle O_L \, V \, O_L \rangle = s - \frac{N}{2}$$

X-L Qi and A. Streicher, arXiv:1810.11958

This suggests that **the holographic picture should be equivalent to some coherent effect related to operator size**

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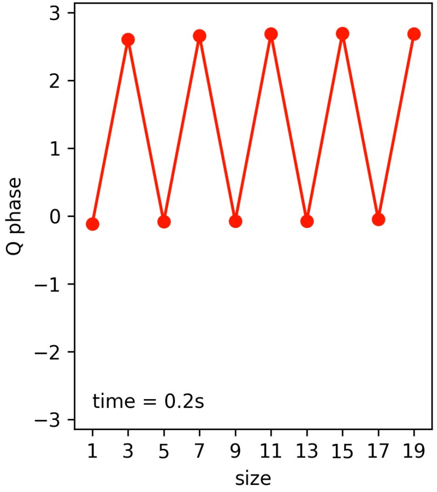
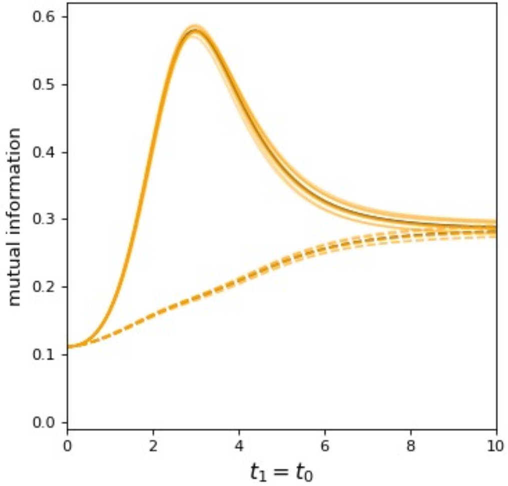
Size winding in action

- Start with the left-right system and a single left fermion inserted
- Write the full wavefunction as a complex vector in the “size basis” where V is diagonal $\psi_j^\ell(0)|\mathrm{tfd}\rangle$
- Define the $Q^\ell(s)$ as the sums of the squares of all the complex components corresponding to a given size

$$Q^\ell(s) = \sum_{|J|=s} \left(c_J^\ell(t)\right)^2$$



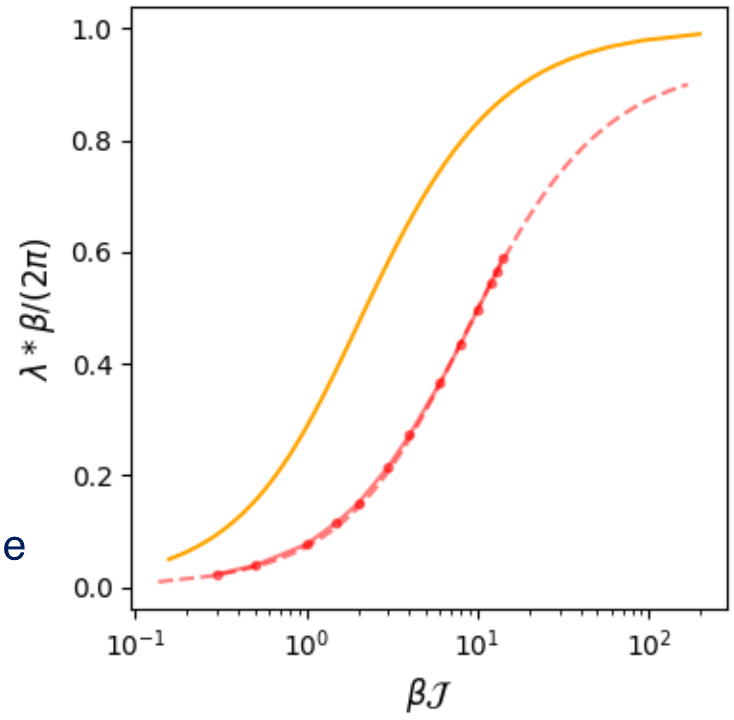


Time evolve with the SYK hamiltonian and plot the phases of the Q(s)’s as a function of the size s

This mysterious coherent effect of the random chaotic SYK dynamics is called size winding

Exhibiting other holographic properties: Lyapunov exponent

- As you saw, the slope of the size winding decreases rapidly as you approach the scrambling time
- We can fit this drop-off to an exponential and extract the Lyapunov exponent



J. Maldacena and D. Stanford, arXiv:1604.07818

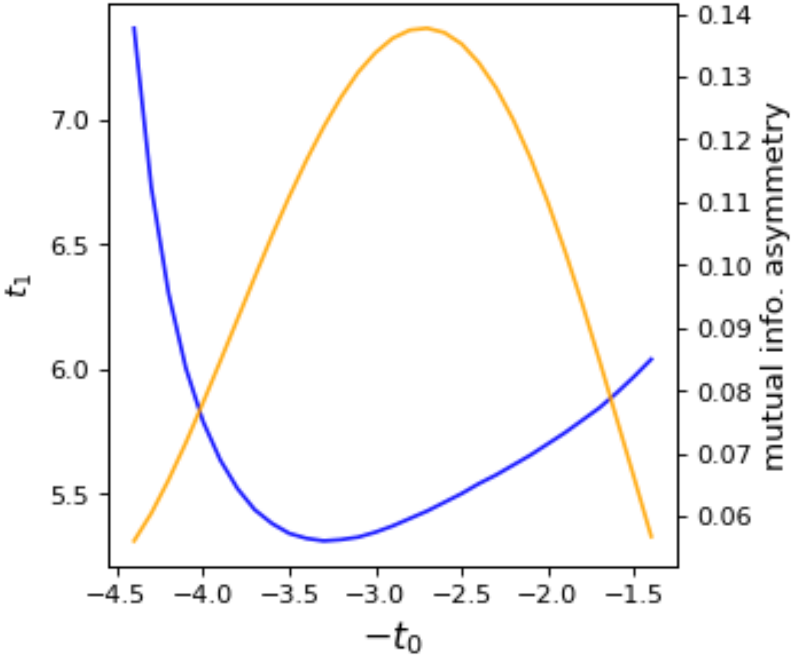
- Orange line is the large N, large q limit computed by the string theorists to show that SYK is like a black hole
- Red points are from our simulation with N=26, q=8

Exhibiting other holographic properties: causal time ordering

- Holographic teleportation predicts “first in, first out” time ordering for when the teleportation completes
- **This is the opposite of the many-body intuition**, realized by peak-size teleportation, that predicts “first in, last out”, i.e. if you scramble longer, then you need to also unscramble longer

- In our simulation we see both effects, since we have both kinds of teleportation happening
- The wormhole-like time ordering is enhanced at earlier times and larger β, as expected

$N = 26,\ q = 8,\ J = 1,\ \beta = 16$



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Long range wormhole teleportation?

- The wormhole teleportation protocol described so far requires a direct quantum interaction, using V, between the left and right systems
- In conventional Alice/Bob quantum teleportation, Alice instead uses a **classical channel** (e.g. a phone call) and instructs Bob to perform certain quantum gates on his system
- This classical channel allows teleportation to be long range (and thus “spooky”)
- It is easy to see that the operator V ***cannot*** be replaced by a classical channel between the left and right SYK systems

Long range wormhole teleportation

- To make wormhole teleportation long range, we replace the left-right operator V by a different operator V^b:

$$V^b = \sum_{j=0}^{N/2-1} \Gamma_j^{(2)\ell} \Gamma_j^{(2)r} \qquad \Gamma_j^{(2)\ell} = 2i\psi_{2j}^{\ell}\psi_{2j+1}^{\ell} \; ; \quad \Gamma_j^{(2)r} = 2i\psi_{2j}^{r}\psi_{2j+1}^{r}$$

- It is easy to see that now we can accomplish wormhole teleportation with a classical channel, where Alice measures the qubits of the left SYK system, and calls Bob to tell him to perform some simple gate operations of the right SYK system

- V^b ***is also a size operator***: it measures the number of Majoranas not counting any pair of Majoranas that live on the same qubit.
- Using this definition of size, size winding still works

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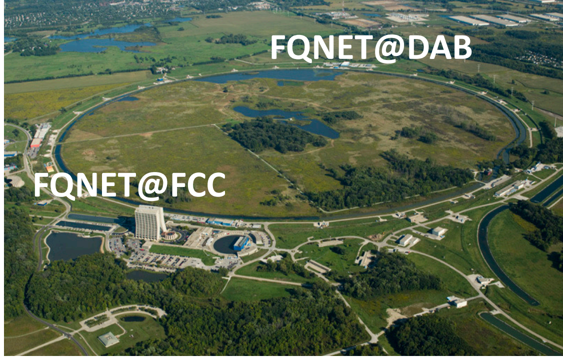
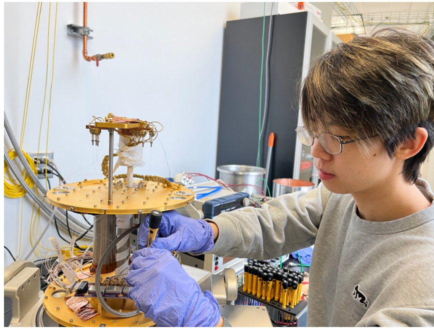
Wormhole teleportation on quantum processors





There are at least three strong motivations to realize this physics on quantum processors:

- Quantum teleportation on a quantum processor **is not a simulation** – you are actually teleporting quantum information from one place to another.
- With two copies of, say, the N=100 SYK model, you would have **a unique laboratory** to address questions about quantum gravity and holography
- Long-range wormhole teleportation may also lead to practical applications in quantum networks (error protection?)

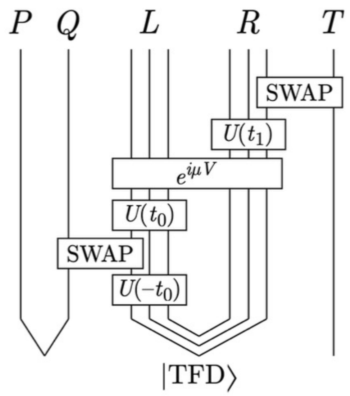
Wormhole teleportation on quantum processors

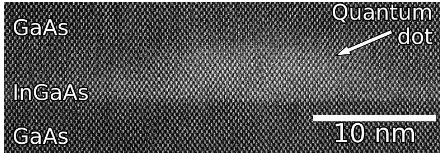
Ideally you want to realize the SYK hamiltonian on a quantum analog system, so that scrambling is the natural time evolution of the system

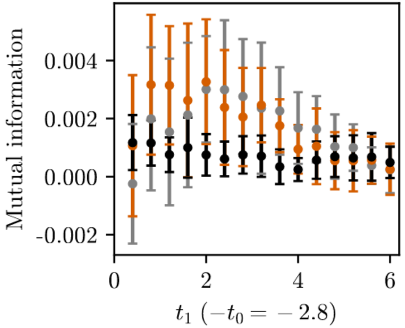
H. Shackleton et. al, arXiv:2309.05741; P. Uhrich et. al, arXiv:2303.11343

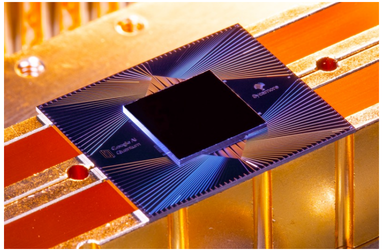
Or you can use a digital quantum processor, at the price of increased circuit complexity

In both cases what you run will be some variation, simplification, or sparsification of two copies of SYK, so you have the additional burden of showing that the holographic duality is still there









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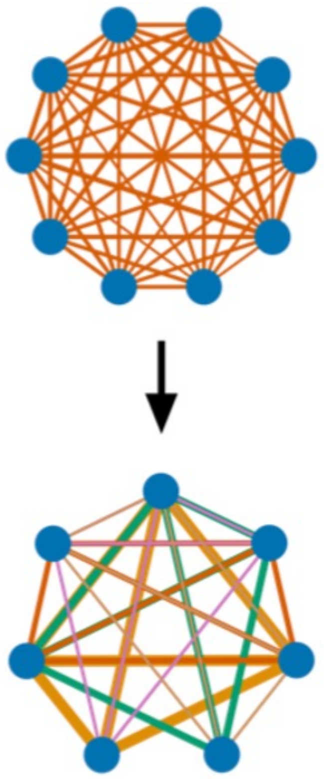
Wormhole teleportation on quantum processors

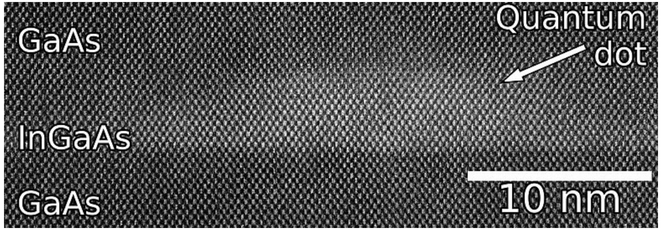
One approach that we and others have explored is **sparsification:**

- Since the SYK hamiltonian is nonlocal and the couplings are random, a typical term in the hamiltonian has no crucial importance
- This suggests that you should be able to sparsify the hamiltonian to O(N) terms and still preserve key dynamical features
- We developed a method to do this using machine learning tools (gradient descent), to “**learn**” sparse hamiltonians (430 terms -> 17 terms)



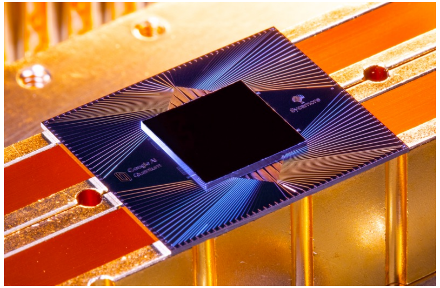
Wormhole teleportation on quantum processors: outlook

Analog SYK processors with quantum dots etc



- Close to realizing the SYK hamiltonian
- Entangling two copies a big challenge

Digital quantum computers



- Have performed wormhole teleportation for learned model
- Detailed study of wormhole properties will require error correction

Long-distance quantum teleportation networks

- Exist over fiber at scale of 60 km per link with pairwise entangled photons
- SYK-scale entanglement and processing a big challenge

Supersymmetric SYK and emergent time

- Combine 2N pairs of Majoranas to make N complex fermions
- Construct two supercharges with random coefficients

W. Fu, D. Gaiotto, J. Maldacena, S. Sachdev, arXiv:1610.08917

$$Q = \frac{i^{(\hat{q}-1)/2}}{\hat{q}!} \sum_{j_1, j_2, \ldots j_{\hat{q}}=0}^{N-1} C_{j_1, j_2, \ldots j_{\hat{q}}} \psi_{j_1} \psi_{j_2} \cdots \psi_{j_{\hat{q}}} \qquad \overline{Q} = \frac{i^{(\hat{q}-1)/2}}{\hat{q}!} \sum_{j_1, j_2, \ldots j_{\hat{q}}=0}^{N-1} \overline{C}_{j_1, j_2, \ldots j_{\hat{q}}} \bar{\psi}_{j_1} \bar{\psi}_{j_2} \cdots \bar{\psi}_{j_{\hat{q}}}$$

$$H = Q\overline{Q} + \overline{Q}Q$$

- This SUSY SYK model has a “BPS” subspace consisting of the zero modes of H
- For example, for N=16, $\hat{q} = 3$, nearly half of the states in the Hilbert space are zero modes
- Operators like. ψ_i and. $\psi_i \psi_j$ are BPS-like, since they anticommute/commute with Q
- Operators like. $\bar{\psi}_i \psi_j$ are not

SUSY SYK and emergent time

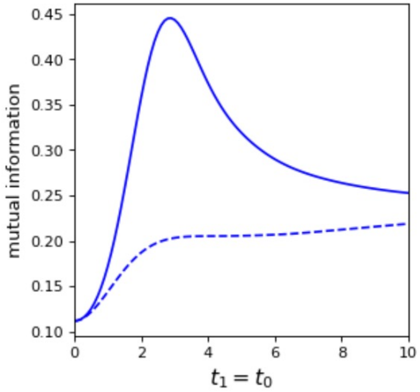
- Compute two-point functions of operators in the zero-mode subspace, and compare to the SUSY J-T gravity prediction
- For the BPS-like operators, **agrees with the gravity calculation within a few percent**
- In the holographic picture the boundary physics has no space and no time
- But bulk operators see both emergent space and **emergent time**

Operator	R-charge	Schwarzian prediction	Numerical answer (N=16)
ψ_i	0	0.111	0.110 ± 0.005
	$+1/3$	0.111	0.110 ± 0.005
$\psi_i\psi_j$	$+1/3$	0.0247	0.024 ± 0.003
$\bar{\psi}_i\psi_j$	$-1/3$	0.0282	0.027 ± 0.001
	0	0.0874	0.079 ± 0.001
	$+1/3$	0.0282	0.027 ± 0.001

H. Lin, J. Maldacena, L. Rozenberg, J. Shan, arXiv:2207.00407, arXiv:2207.00408

Operator	R-charge	Schwarzian prediction	N=16 $\hat{q}$=5 numerical
ψ_i	-1/5	0.0603	0.0592 ± 0.0004
	0	0.1404	0.1363 ± 0.0003
	+1/5	0.1404	0.1363 ± 0.0003
	+2/5	0.0603	0.0592 ± 0.0004
$\psi_i\psi_j$	0	0.0338	0.0343 ± 0.0004
	+1/5	0.0638	0.0638 ± 0.0004
	+2/5	0.0338	0.0343 ± 0.0004
$\bar{\psi}_i\psi_j$	-2/5	0.0149	0.0128 ± 0.0001
	-1/5	0.0681	0.0571 ± 0.0001
	0	0.0926	0.0770 ± 0.0001
	+1/5	0.0681	0.0571 ± 0.0001
	+2/5	0.0149	0.0128 ± 0.0001

Supersymmetric wormhole teleportation also works



Outlook

- We have a lot to learn from wormhole teleportation even in its simplest realizations with entangled copies of the SYK model
- It provides a concrete laboratory to investigate the relation between entanglement, chaotic dynamics, and some basic questions of quantum gravity
- It is exciting that wormhole teleportation with the SYK model is so close to teleportation experiments that we can perform in the laboratory
- We should also investigate variants of SYK (e.g. SUSY SYK, bosonic SYK, ...) for their holographic behaviors