Entanglement-enhanced AC magnetometry in the presence of Markovian noises

Yuichiro Matsuzaki, Chuo University

T. Sichanugrist, H. Fukuda, T. Moroi, K. Nakayama, S. Chigusa, N. Mizuochi, M. Hazumi, and **Y. M.** arXiv:2410.21699 (2024)

Collaborators

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- Takeo Moroi (Tokyo University)
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- So Chigusa (Berkley National University)
- Mizuochi Norikazu (Kyoto University)
- Hazumi Masao (KEK)

Main point

- To measure DC magnetic field with qubits, we cannot improve the sensitivity by using a cat (GHZ) state under the effect of high-frequency parallel noise

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

- To measure AC magnetic field with qubit, we can improve

the sensitivity by using a cat state under the effect of high-frequency parallel noise for some cases.

T. Sichanugrist, H. Fukuda, T. Moroi, K. Nakayama, S. Chigusa, N. Mizuochi, M. Hazumi, and **Y. M.**
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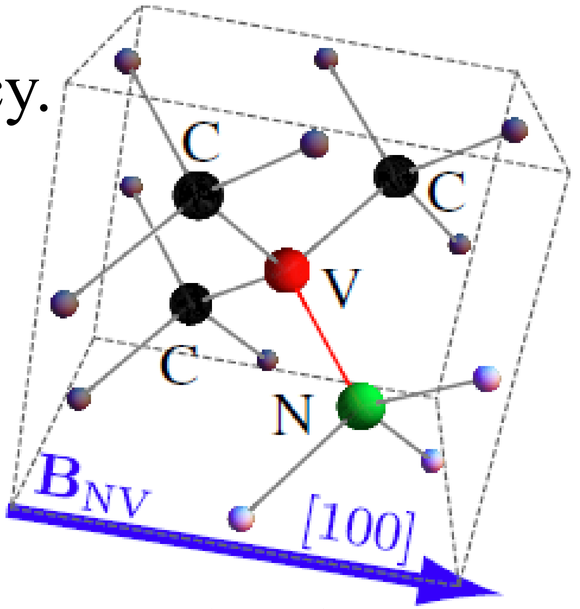
Nitrogen vacancy centers in diamond

NV center is a nearest-neighbor pair of a nitrogen atom and a vacancy.
→Electron spins are trapped, which we can use as a qubit
→ Candidate to realize quantum enhanced magnetic field sensing



S. Saito, et al. *PRL* 111.10 (2013): 107008.

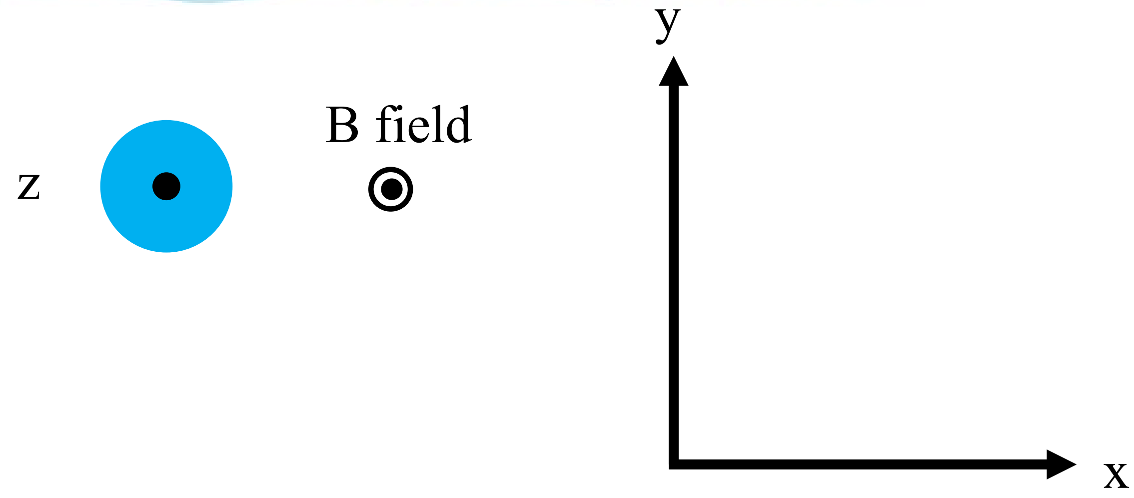
Advantage

- Long coherence time (a few milli seconds at room temperature)
- Spin polarization is possible by illuminating optical laser
- Spin readout can be performed via photoluminescence

C. Degen et al., Reviews of modern physics 89.3 (2017): 035002.

Hamiltonian for an electron spin

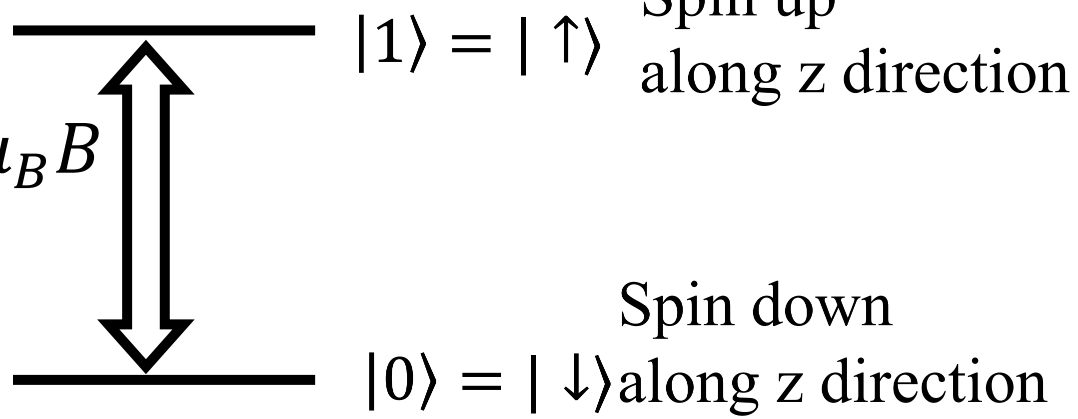
$$H = \frac{1}{2}\omega\hat{\sigma}_z$$



$$\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Zeeman splitting

$$\omega = g\mu_B B$$



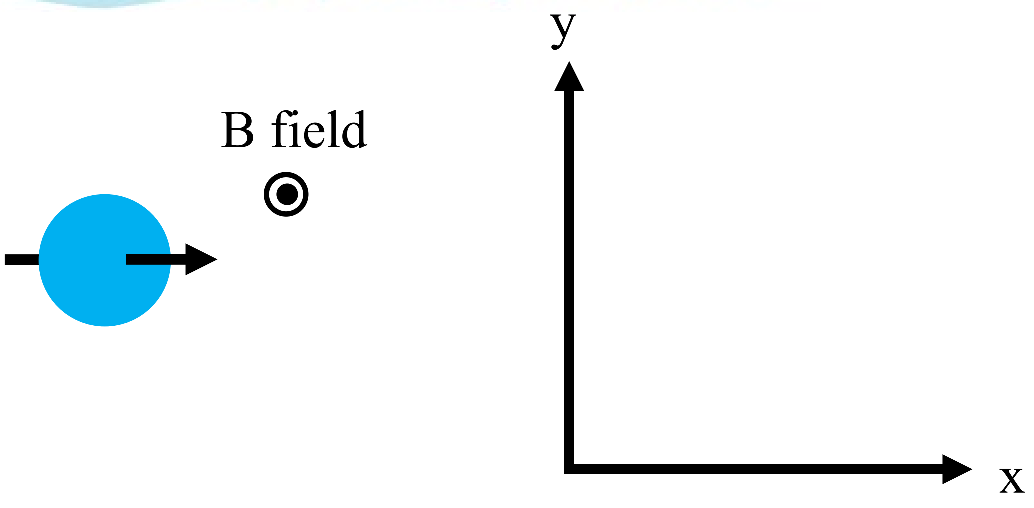
If we know the value of ω,
We also know the value of B

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

$$H = \frac{1}{2}\omega\hat{\sigma}_z$$

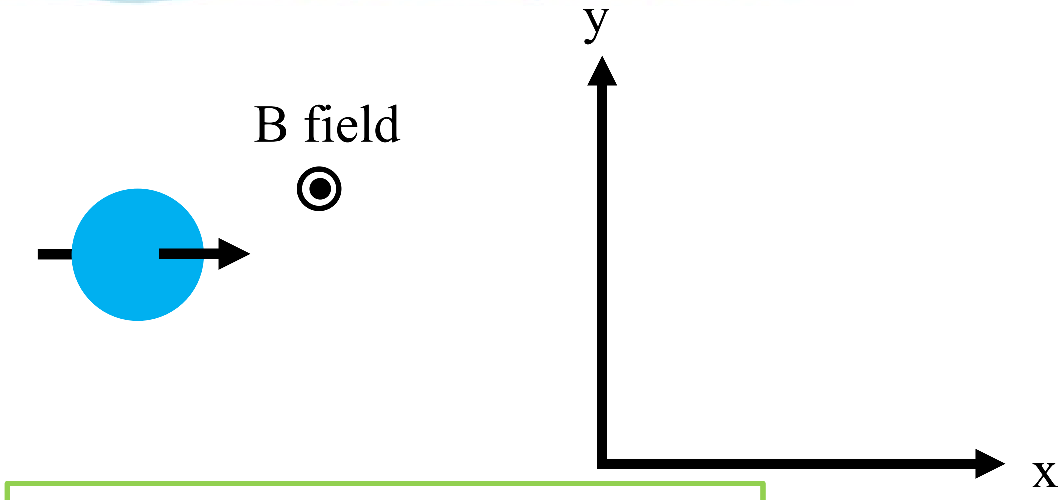


Prepare the state of $|+\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$



$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

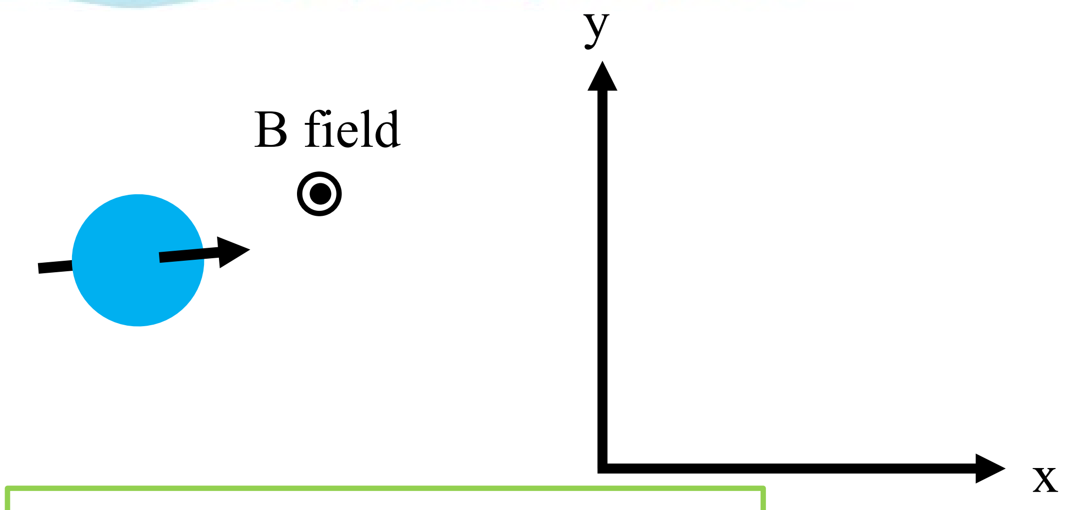
Rotation in the X-Y plane with an angle of ωt

$$H = \frac{1}{2}\omega\hat{\sigma}_z$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{\frac{i\omega t}{2}}|0\rangle + e^{-\frac{i\omega t}{2}}|1\rangle)$$

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$



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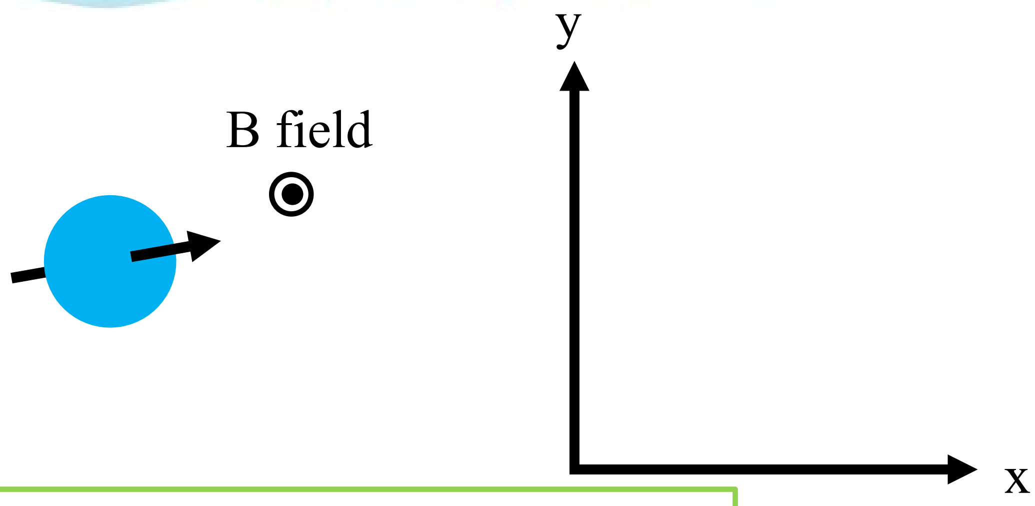

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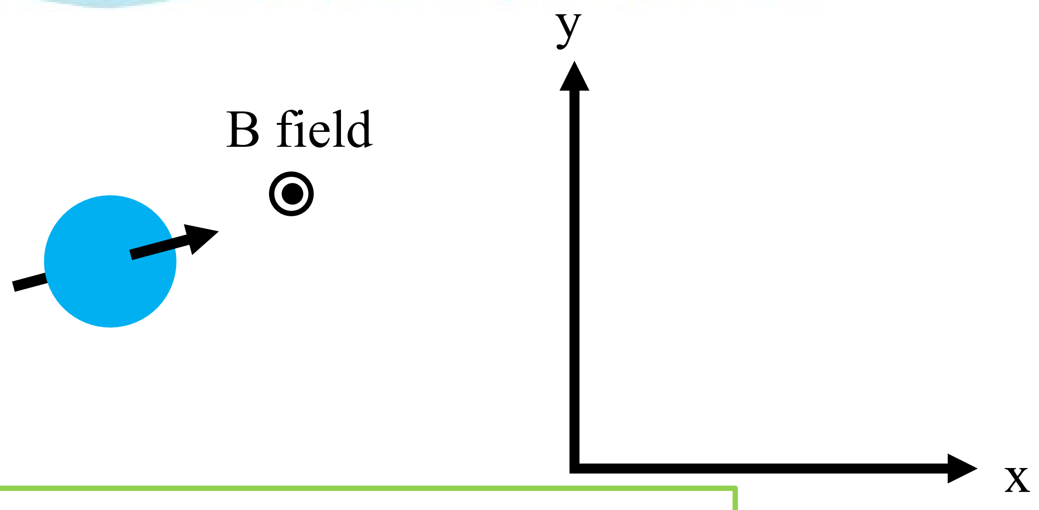


$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

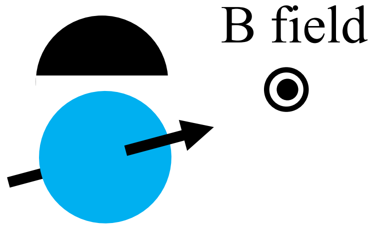
Rotation in the X-Y plane with an angle of ωt

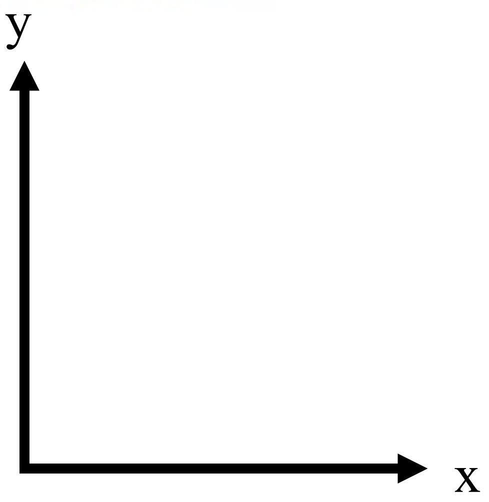
$$H = \frac{1}{2}\omega\hat{\sigma}_z$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{\frac{i\omega t}{2}}|0\rangle + e^{-\frac{i\omega t}{2}}|1\rangle)$$

Projective measurement along y direction

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{\frac{i\omega t}{2}}|0\rangle + e^{-\frac{i\omega t}{2}}|1\rangle)$$





Projection operator along y direction

$$\hat{\mathcal{P}}_{\pm} = \frac{1}{2}(\hat{\mathbb{1}} \pm \hat{\sigma}_y)$$

Probability to project the state into y direction

$$P_{\pm} = \langle\psi(t)|\mathcal{P}_{\pm}|\psi(t)\rangle = \frac{1}{2} \pm \frac{1}{2}\sin\omega t$$

Quantum sensing with a finite resource

If the number of repetition N is infinity large, the estimation of B is perfect.
However, the actual experiment has a finite resource.

Goal

To minimize the estimation uncertainty by using a finite resource

Typical resource for sensing

- L qubits
- T seconds

Assumption

- Necessary time for state preparation and readout is negligibly small

Quantum field sensing with a qubit

Goal **Estimating B or ω by using a qubit (electron spin)**

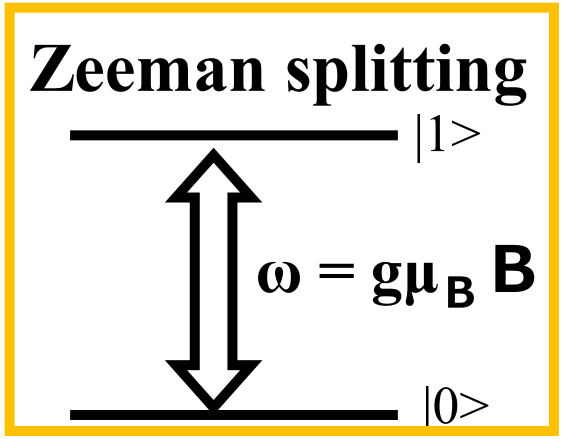
1. State preparation

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

2. Time evolution (interaction with B field)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{\frac{i\omega t}{2}}|0\rangle + e^{-\frac{i\omega t}{2}}|1\rangle)$$

3. Read out the spin (projection to the y direction)
Getting the measurement results either 0 or 1



S. Huelga, C. Macchiavello, T. Pellizzari, A. Ekert, M. Plenio, and J. Cirac, Phys. Rev. Lett. **79**, 3865 (1997).

Quantum field sensing with a qubit

Goal **Estimating B or ω by using a qubit (electron spin)**

Repeat N times

1. State preparation

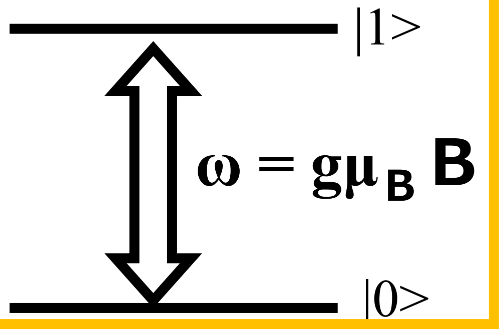
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

2. Time evolution (interaction with B field)

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{\frac{i\omega t}{2}}|0\rangle + e^{-\frac{i\omega t}{2}}|1\rangle\right)$$

3. Read out the spin (projection to the y direction)

Zeeman splitting



Projection probability

$$P_+ = \frac{1}{2} + \frac{1}{2}\sin\omega t \simeq \frac{1}{2} + \frac{1}{2}\omega t$$

➡ **Estimating the value of ω**

Procedure to estimate the value of ω

Theoretical calculation

$$P_+ = \frac{1}{2} + \frac{1}{2}\sin\omega t \simeq \frac{1}{2} + \frac{1}{2}\omega t$$

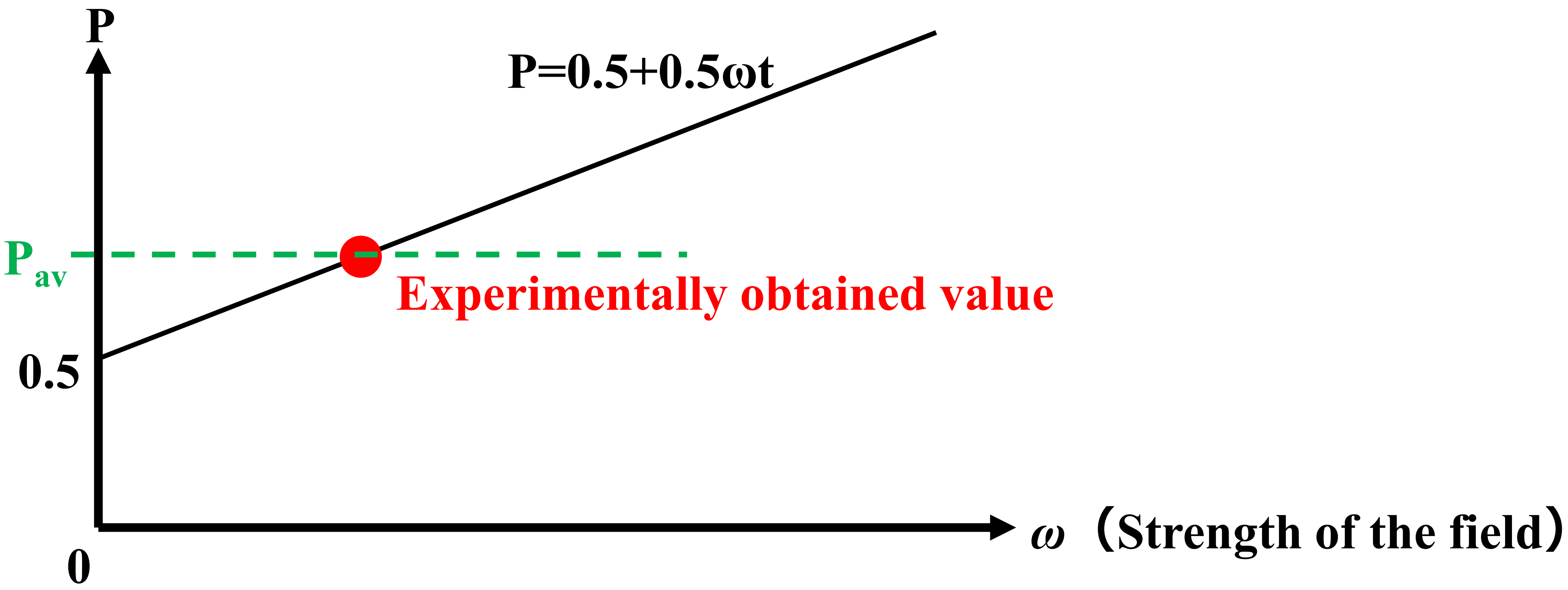
Experimentally obtained values

$$Y_1、Y_2、\cdot\cdot\cdot、Y_N \Rightarrow P_{av} = (Y_1 + Y_2 + \cdot\cdot\cdot + Y_N)/N$$

Estimation value

$$\omega_{est} = (2\,P_{av} - 1)/t$$

Uncertainty of the estimation



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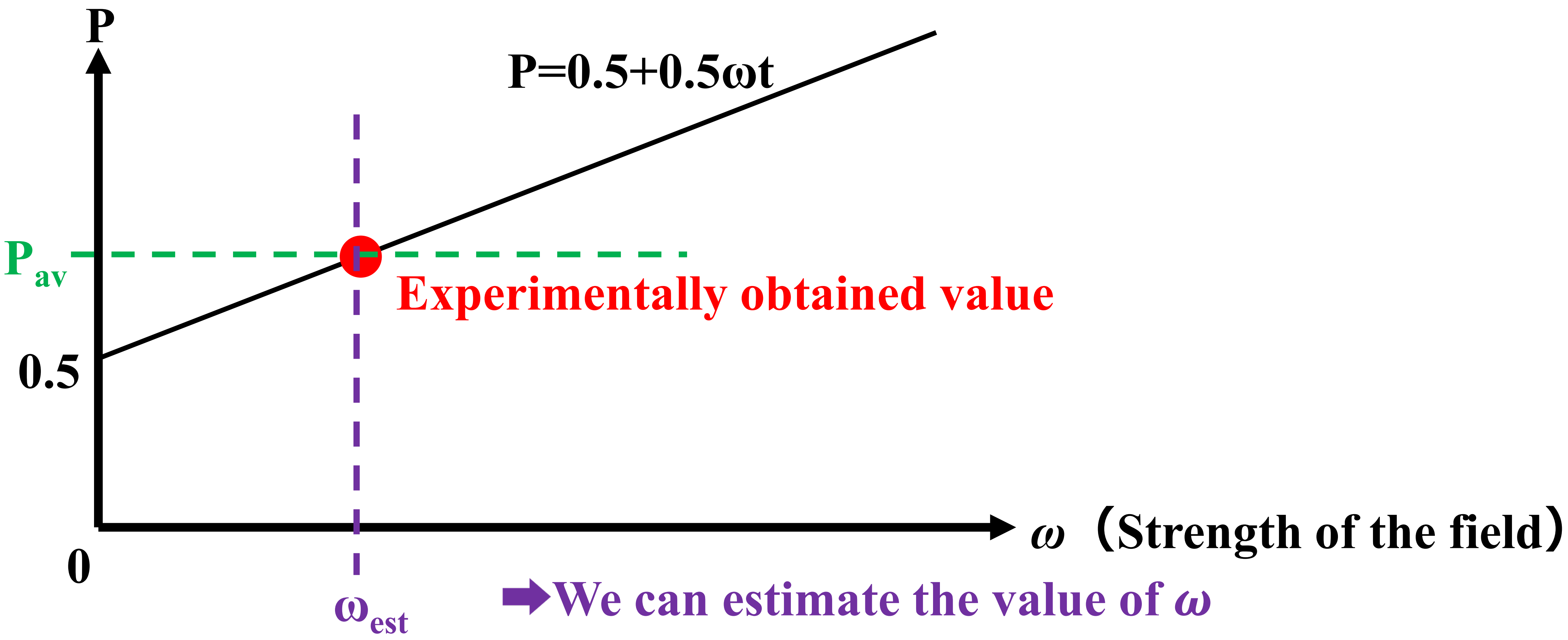
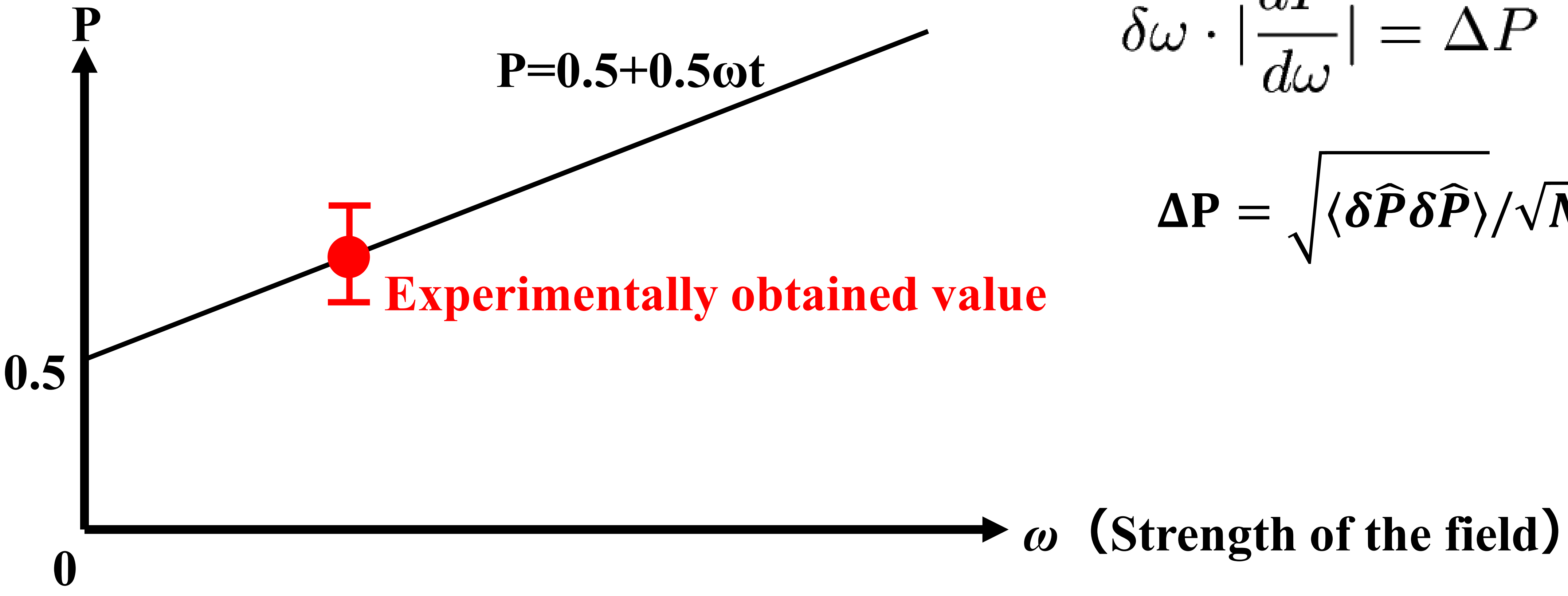


➡We can estimate the value of *ω*

However, in the actual setup, there must be uncertainty due to the finite repetition

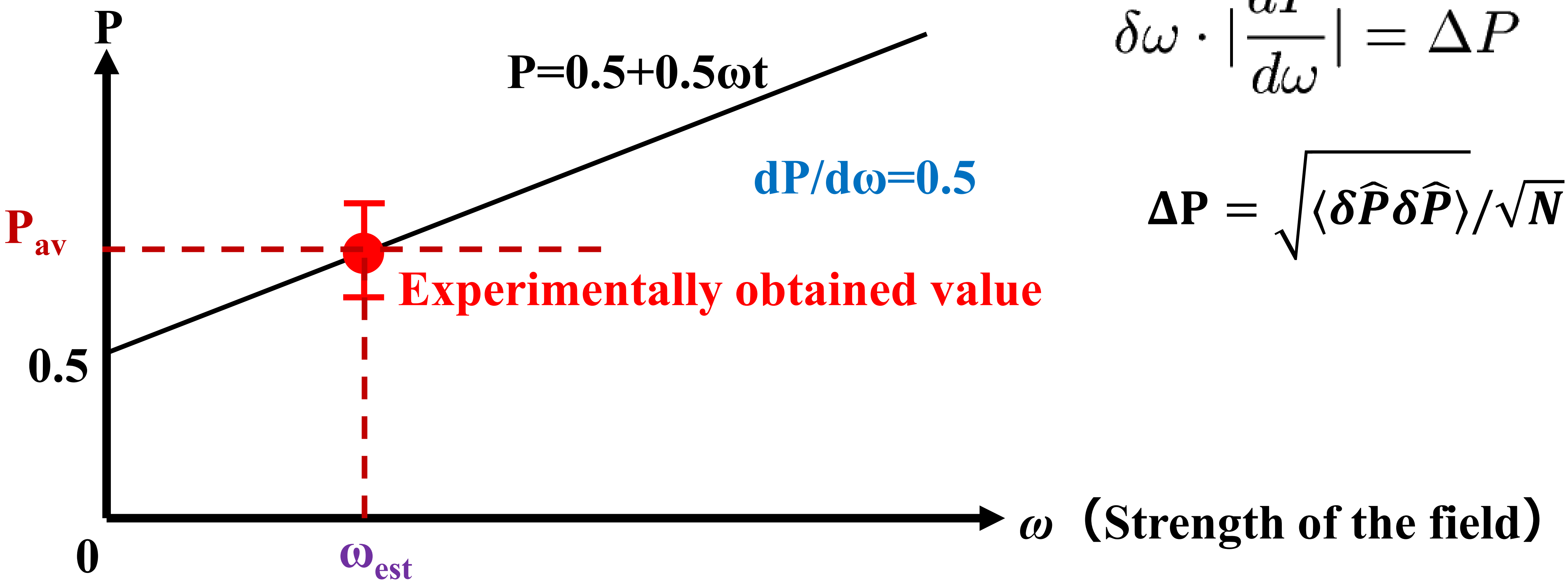
Uncertainty of the estimation



$$\delta\omega \cdot \left|\frac{dP}{d\omega}\right| = \Delta P$$

$$\Delta P = \sqrt{\langle \delta\hat{P}\delta\hat{P} \rangle}/\sqrt{N}$$

Uncertainty of the estimation

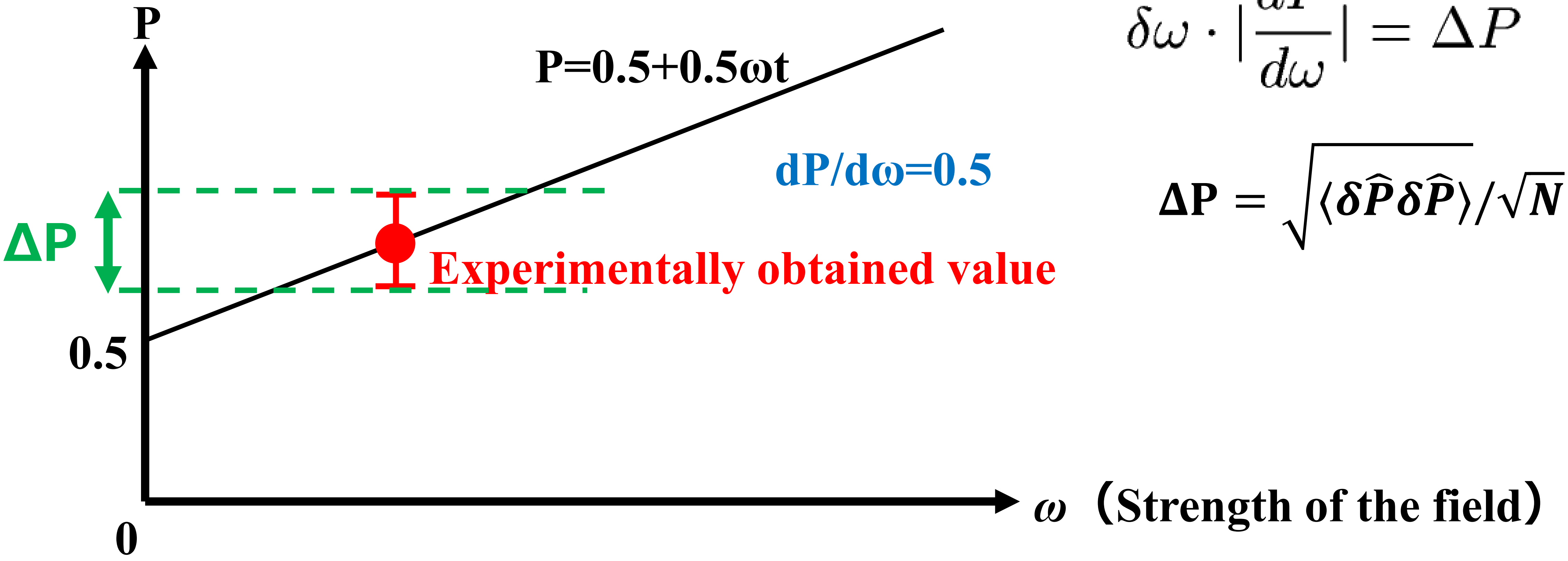


$$\delta\omega \cdot \left|\frac{dP}{d\omega}\right| = \Delta P$$

$$\Delta P = \sqrt{\langle \delta\hat{P}\delta\hat{P}\rangle / \sqrt{N}}$$

We obtain ω_{est}, but this contain an uncertainty

Uncertainty of the estimation



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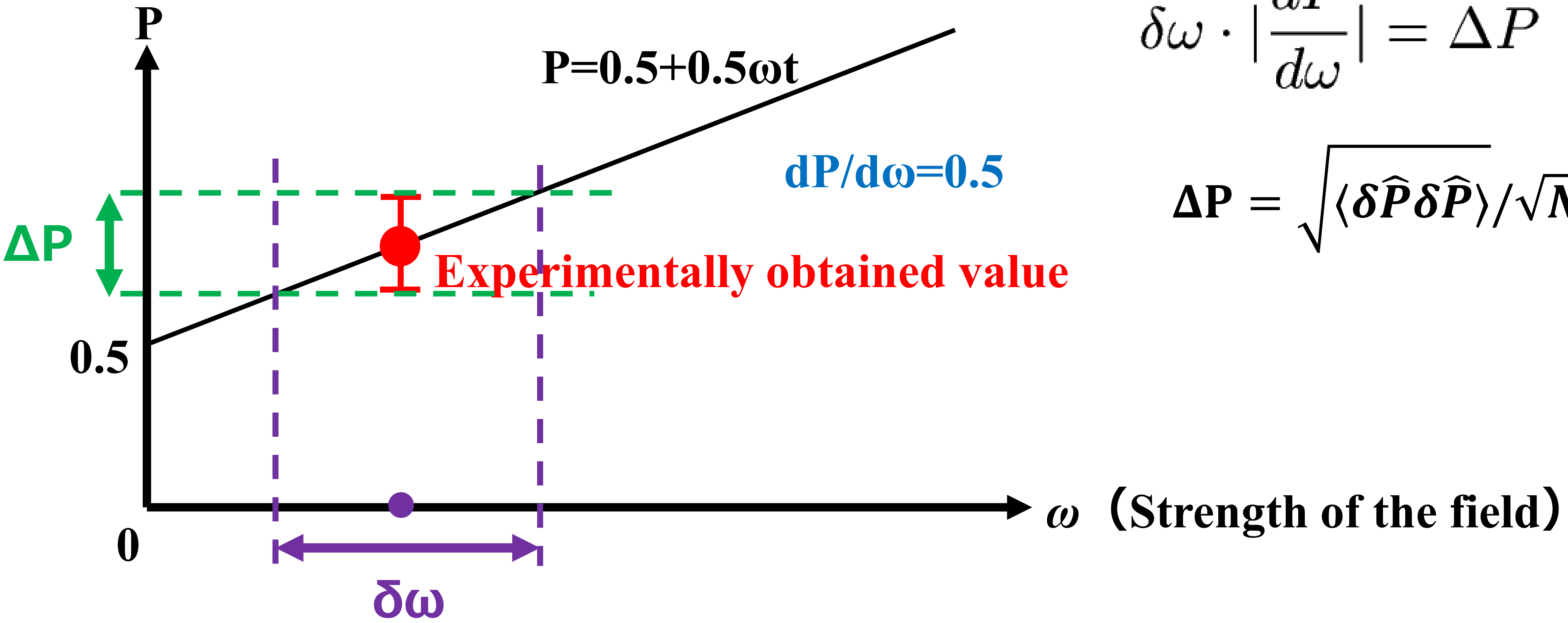

$$\delta\omega \cdot \left|\frac{dP}{d\omega}\right| = \Delta P$$

$$\Delta \mathbf{P} = \sqrt{\langle \delta\hat{P}\delta\hat{P}\rangle}/\sqrt{N}$$

How to calculate the uncertainty (L separable qubits)

$$\delta\omega = \frac{\sqrt{\langle \delta\hat{\mathcal{P}}\delta\hat{\mathcal{P}}\rangle}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}} = \frac{\sqrt{P(1-P)}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}}$$

How to calculate the uncertainty (L separable qubits)

$$\delta\omega = \frac{\sqrt{\langle \delta\hat{\mathcal{P}}\delta\hat{\mathcal{P}}\rangle}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}} = \frac{\sqrt{P(1-P)}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}}$$

$$P \simeq \frac{1}{2} + \frac{1}{2}\omega t$$

$$N = \frac{T}{t}L$$

Using L qubits, the uncertainty becomes L times smaller due to central limit theorem

How to calculate the uncertainty (L separable qubits)

$$\delta\omega = \frac{\sqrt{\langle \delta\hat{\mathcal{P}}\delta\hat{\mathcal{P}}\rangle}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}} = \frac{\sqrt{P(1-P)}}{|\frac{dP}{d\omega}|}\frac{1}{\sqrt{N}}$$

Substituting these into $\delta\omega$

$$P \simeq \frac{1}{2} + \frac{1}{2}\omega t$$

$$N = \frac{T}{t}L$$

$$\delta\omega = \frac{1}{t\sqrt{N}} = \frac{1}{\sqrt{tTL}}$$

Uncertainty (inverse of the sensitivity)

Decoherence

Quantum state loses coherence due to the environment

Pure state | **Mixed state**

$$|\psi_0\rangle \Rightarrow \quad \rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

Example (dephasing)

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \Rightarrow \quad \rho = 0.5|0\rangle\langle 0| + 0.5|1\rangle\langle 1|$$

Dephasing model (single qubit)

$$H_{total} = H + H_N(t)$$

Target field

$$H = \frac{\omega}{2}\sigma_z$$

Noise term

$$H_N(t) = \lambda f(t)\hat{\sigma}_z$$

$f(t)$ denotes a classical random variable

Non-bias

$$\overline{f(t)} = 0$$

White noise (small correlation time τ_c)

$$\overline{f(t')f(t'')} = \tau_c \delta(t' - t'')$$

Overline denotes an ensemble average

De Lange, G., et al. ‚*Science* 330.6000 (2010): 60-63.

Analytical solution of the master equation

Master equation to represent decoherence

$$\frac{d\overline{\rho(t)}}{dt} = -\lambda^2 \tau_c (\overline{\rho(t)} - \sigma_z \rho(t) \sigma_z)$$

Analytical solution of the master equation

$$\overline{\rho(t)} = \frac{1 + e^{-2\lambda^2 \tau_c t}}{2} \rho(0) + \frac{1 - e^{-2\lambda^2 \tau_c t}}{2} \hat{\sigma}_z \rho(0) \hat{\sigma}_z$$

Analytical solution of the master equation

Master equation to represent decoherence

$$\frac{d\overline{\rho(t)}}{dt} = -\lambda^2\tau_c(\overline{\rho(t)} - \sigma_z\rho(t)\sigma_z)$$

Analytical solution of the master equation

$$\overline{\rho(t)} = \frac{1+e^{-2\lambda^2\tau_c t}}{2}\rho(0) + \frac{1-e^{-2\lambda^2\tau_c t}}{2}\hat{\sigma}_z\rho(0)\hat{\sigma}_z$$

$P = (1 + e^{-2\lambda^2\tau_c t})/2 \rightarrow$ no effect from the noise

$P' = (1 - e^{-2\lambda^2\tau_c t})/2 \rightarrow$ Phase flip (σ_z) due to noise

Time evolution with magnetic field and noise

Time evolution with noise

$$\overline{\rho(t)} = \frac{1+e^{-t/T_2^*}}{2}\rho(0) + \frac{1-e^{-t/T_2^*}}{2}\hat{\sigma}_z\rho(0)\hat{\sigma}_z$$

$$T_2^* = 1/(2\lambda^2\tau_c)$$

(T_2^* is a coherence time)

Time evolution with magnetic field and noise

$$\overline{\rho(t)} = \frac{1+e^{-t/T_2^*}}{2}|\psi\rangle\langle\psi| + \frac{1-e^{-t/T_2^*}}{2}\hat{\sigma}_z|\psi\rangle\langle\psi|\hat{\sigma}_z$$

$$|\psi\rangle = (e^{-\frac{i\omega t}{2}}|0\rangle + e^{\frac{i\omega t}{2}}|1\rangle)/\sqrt{2}$$

Uncertainty of the estimation with noise

Time evolution with magnetic field and noise

$$\overline{\rho(t)} = \frac{1+e^{-t/T_2^*}}{2}|\psi\rangle\langle\psi| + \frac{1-e^{-t/T_2^*}}{2}\hat{\sigma}_z|\psi\rangle\langle\psi|\hat{\sigma}_z$$

$$|\psi\rangle = (e^{-\frac{i\omega t}{2}}|0\rangle + e^{\frac{i\omega t}{2}}|1\rangle)/\sqrt{2}$$

Projection probability along y direction ($\omega t \ll 1$)

$$P = \frac{1}{2} + \frac{e^{-t/T_2^*}}{2}\sin\omega t \simeq \frac{1}{2} + \frac{e^{-t/T_2^*}}{2}\omega t$$

Uncertainty

$$\delta\omega = \frac{\sqrt{P(1-P)}}{|dP/d\omega|\sqrt{N}} \simeq \frac{e^{t/T_2^*}}{\sqrt{LTt}}$$

$$N = LT/t$$

Uncertainty of the estimation with noise

Time evolution with magnetic field and noise

$$\overline{\rho(t)} = \frac{1+e^{-t/T_2^*}}{2}|\psi\rangle\langle\psi| + \frac{1-e^{-t/T_2^*}}{2}\hat{\sigma}_z|\psi\rangle\langle\psi|\hat{\sigma}_z$$

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Projection probability along y direction ($\omega t \ll 1$)

$$P = \frac{1}{2} + \frac{e^{-t/T_2^*}}{2}\sin\omega t \simeq \frac{1}{2} + \frac{e^{-t/T_2^*}}{2}\omega t$$

Uncertainty

$$\delta\omega = \frac{\sqrt{P(1-P)}}{|dP/d\omega|\sqrt{N}} \simeq \frac{e^{t/T_2^*}}{\sqrt{LTt}} = \frac{\sqrt{2}e^{1/2}}{\sqrt{LTT_2^*}}$$

$t = T_2^*/2$ is optimal

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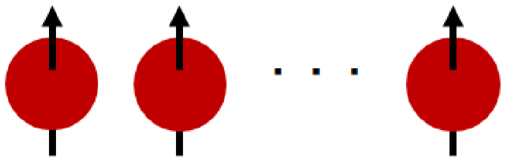
Quantum sensing with a cat (GHZ) state



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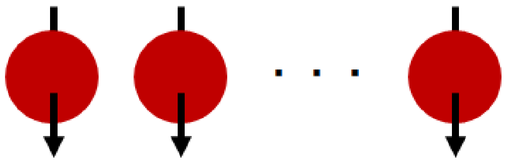

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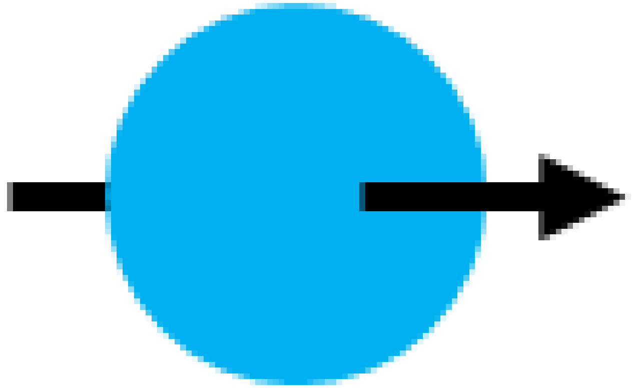


Cat state

$$(|00\cdots0\rangle + |11\cdots1\rangle)/\sqrt{2}$$

Superposition between
All spin up and all spin down
➡ a cat (GHZ) state

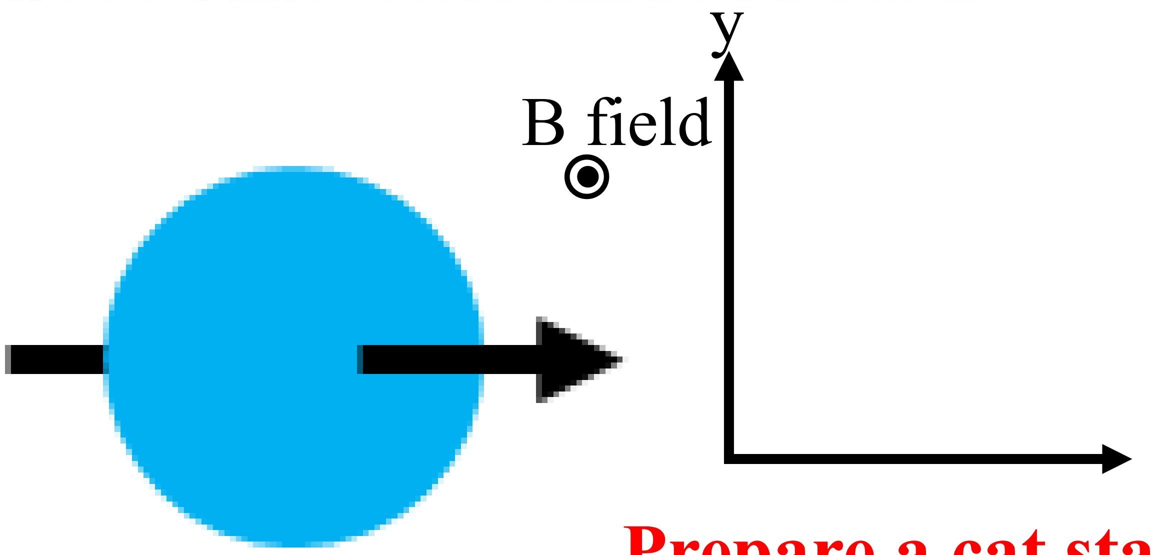
S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

$$H = \sum_{j=1}^{L} \frac{\omega}{2}\sigma_z^{(j)}$$



Prepare a cat state

$$|\psi_{cat}\rangle = (|00\cdots 0\rangle + |11\cdots 1\rangle)/\sqrt{2}$$

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

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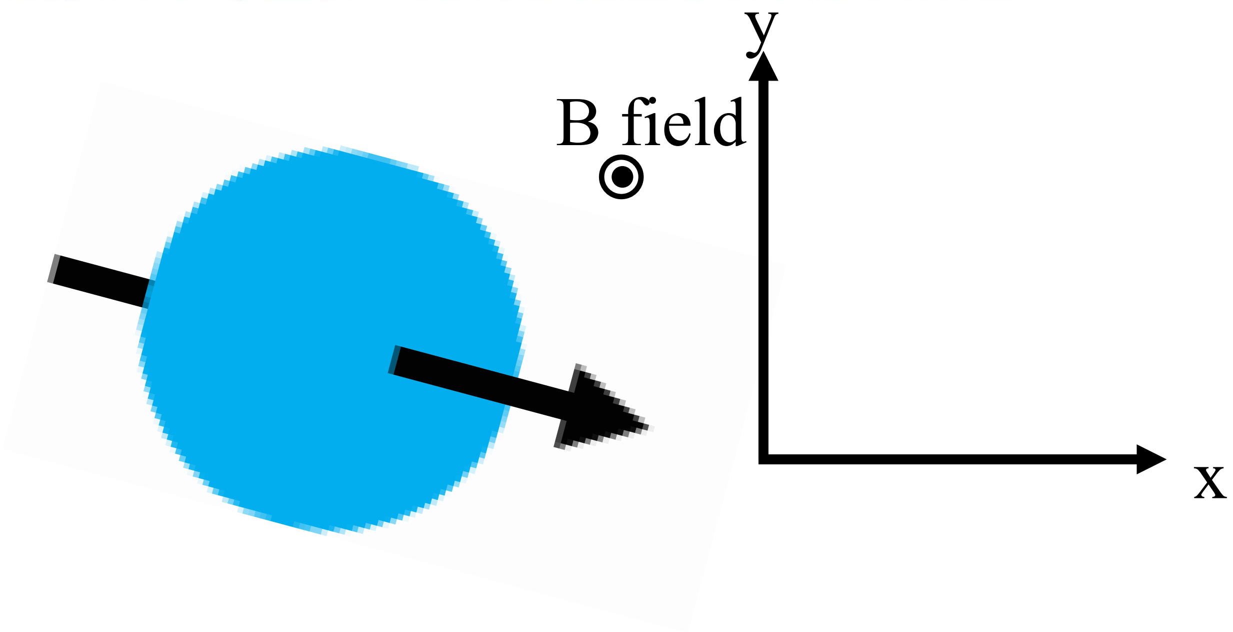


$$|\psi_{cat}\rangle = (|00\cdots0\rangle + |11\cdots1\rangle)/\sqrt{2}$$

Time evolution with magnetic field

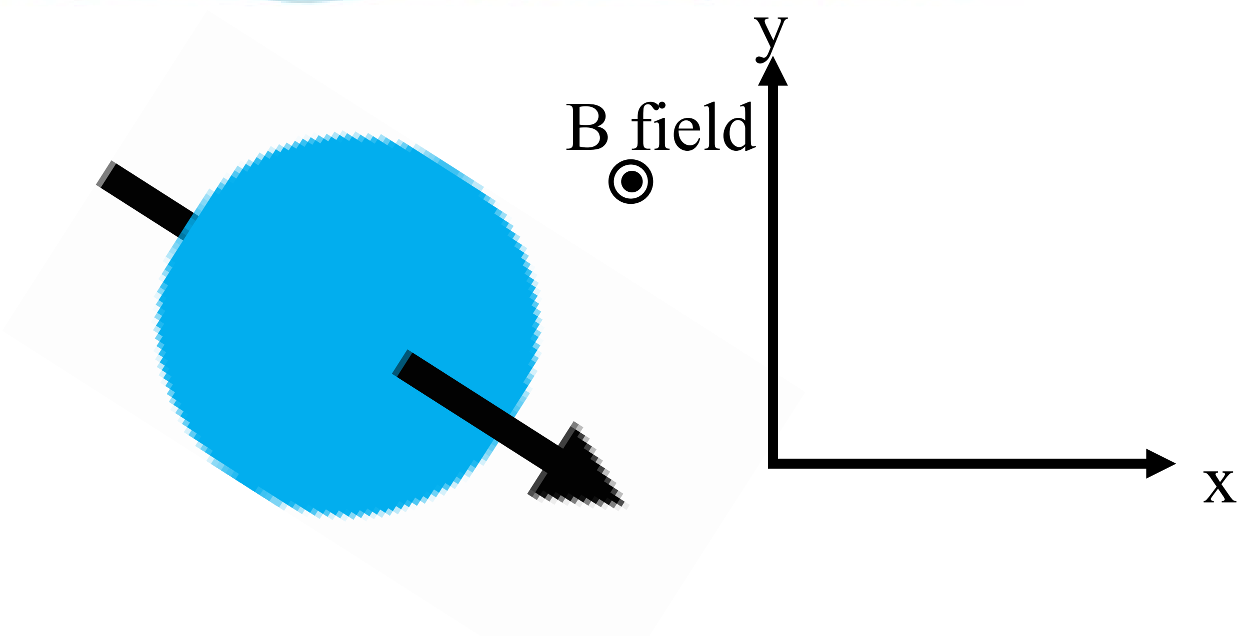
$$e^{-iHt}|\psi_{cat}\rangle = (e^{-\frac{iL\omega t}{2}}|00\cdots0\rangle + e^{\frac{iL\omega t}{2}}|11\cdots1\rangle)/\sqrt{2}$$

Time evolution

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

$$H = \sum_{j=1}^{L}\frac{\omega}{2}\sigma_z^{(j)}$$



$$|\psi_{cat}\rangle = (|00\cdots 0\rangle + |11\cdots 1\rangle)/\sqrt{2}$$

Time evolution with magnetic field

$$e^{-iHt}|\psi_{cat}\rangle = (e^{-\frac{iL\omega t}{2}}|00\cdots 0\rangle + e^{\frac{iL\omega t}{2}}|11\cdots 1\rangle)/\sqrt{2}$$

Time evolution

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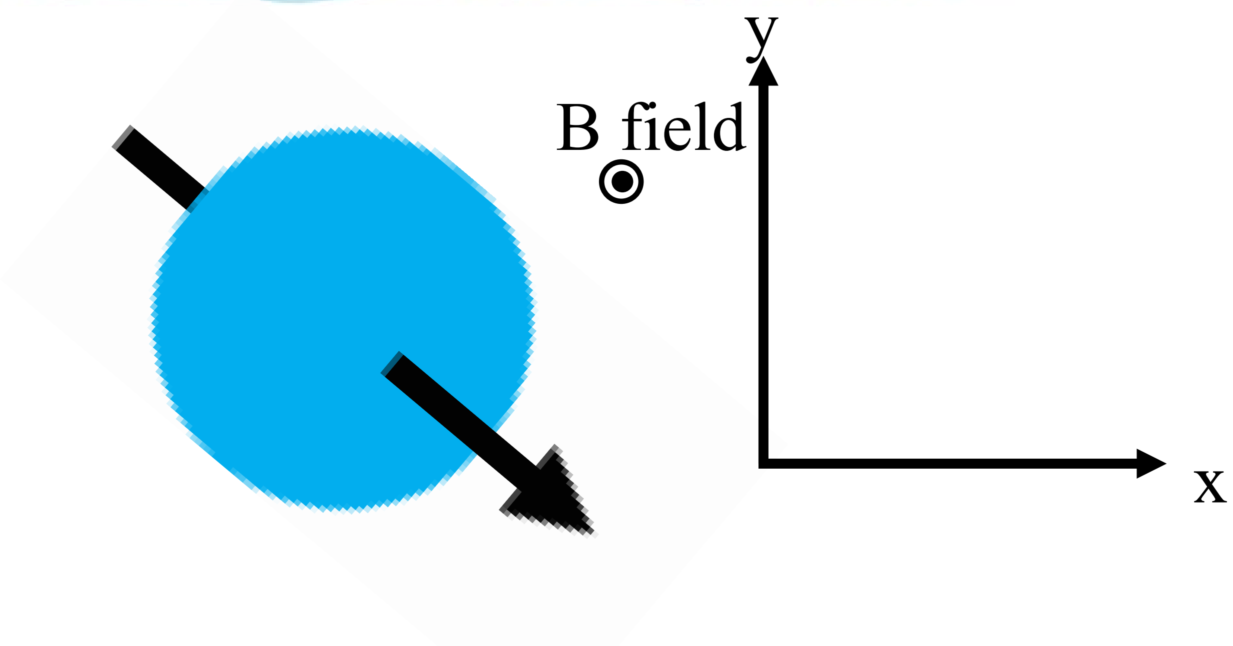

$$|\psi_{cat}\rangle = (|00\cdots0\rangle + |11\cdots1\rangle)/\sqrt{2}$$

Time evolution with magnetic field

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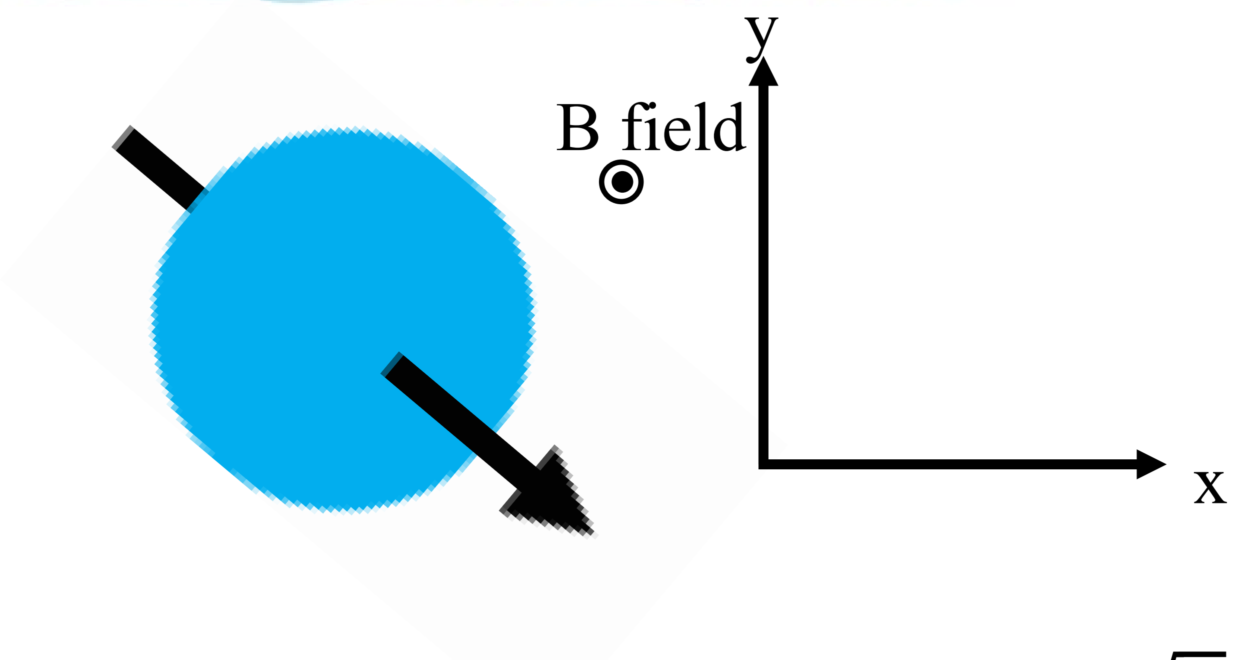


$$|\psi_{cat}\rangle = (|00\cdots 0\rangle + |11\cdots 1\rangle)/\sqrt{2}$$

Phase acquisition is *L* times larger than that of a qubit

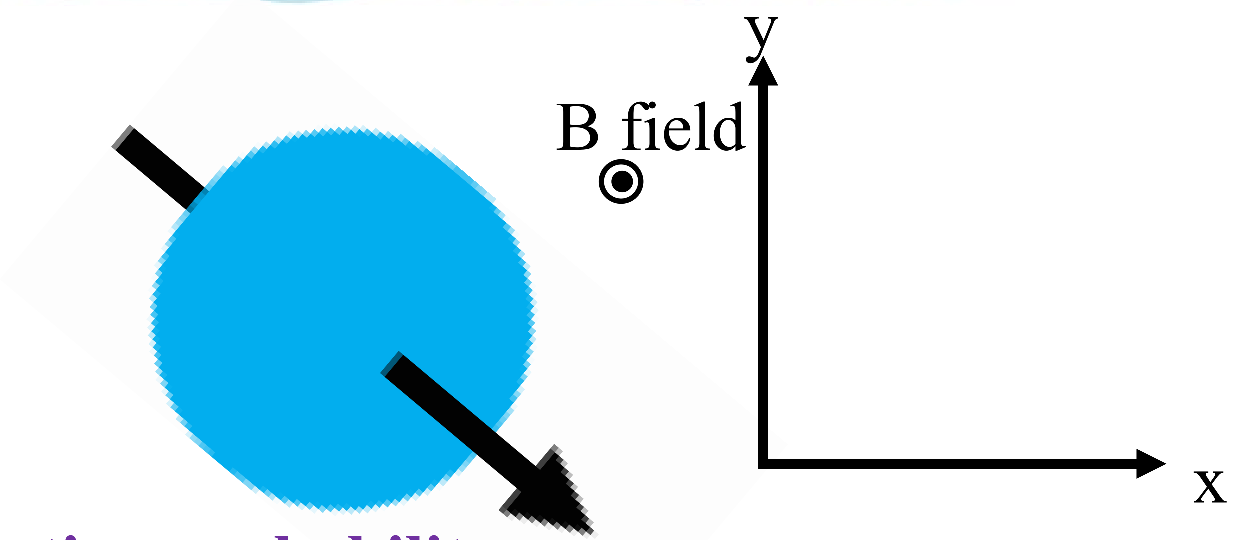
$$e^{-iHt}|\psi_{cat}\rangle = (e^{-\frac{iL\omega t}{2}}|00\cdots 0\rangle + e^{\frac{iL\omega t}{2}}|11\cdots 1\rangle)/\sqrt{2}$$

Projection probability for a cat state

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

$$H = \sum_{j=1}^{L}\frac{\omega}{2}\sigma_z^{(j)}$$



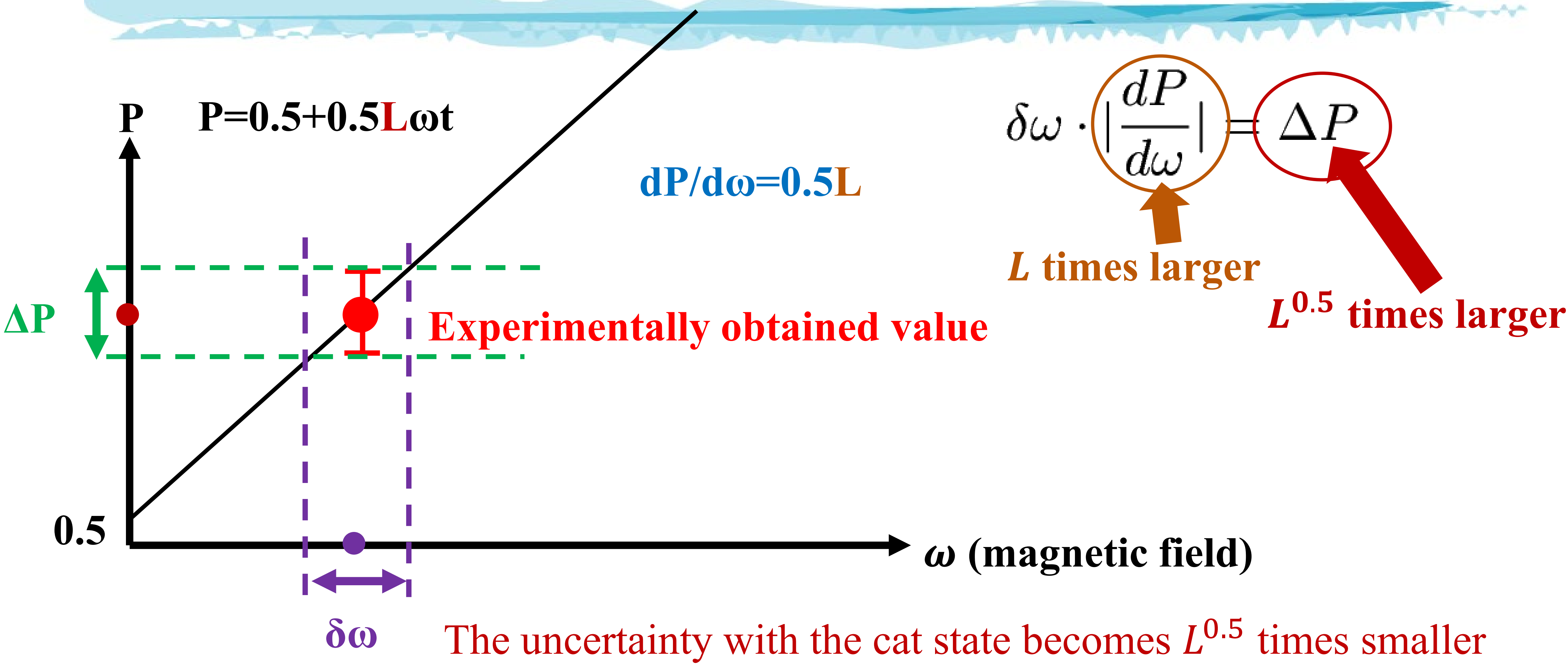
Projection probability

$$P_{cat} \simeq \frac{1}{2} + \frac{1}{2}\boldsymbol{L}\omega t$$

Phase acquisition is *L* times larger than that of a qubit

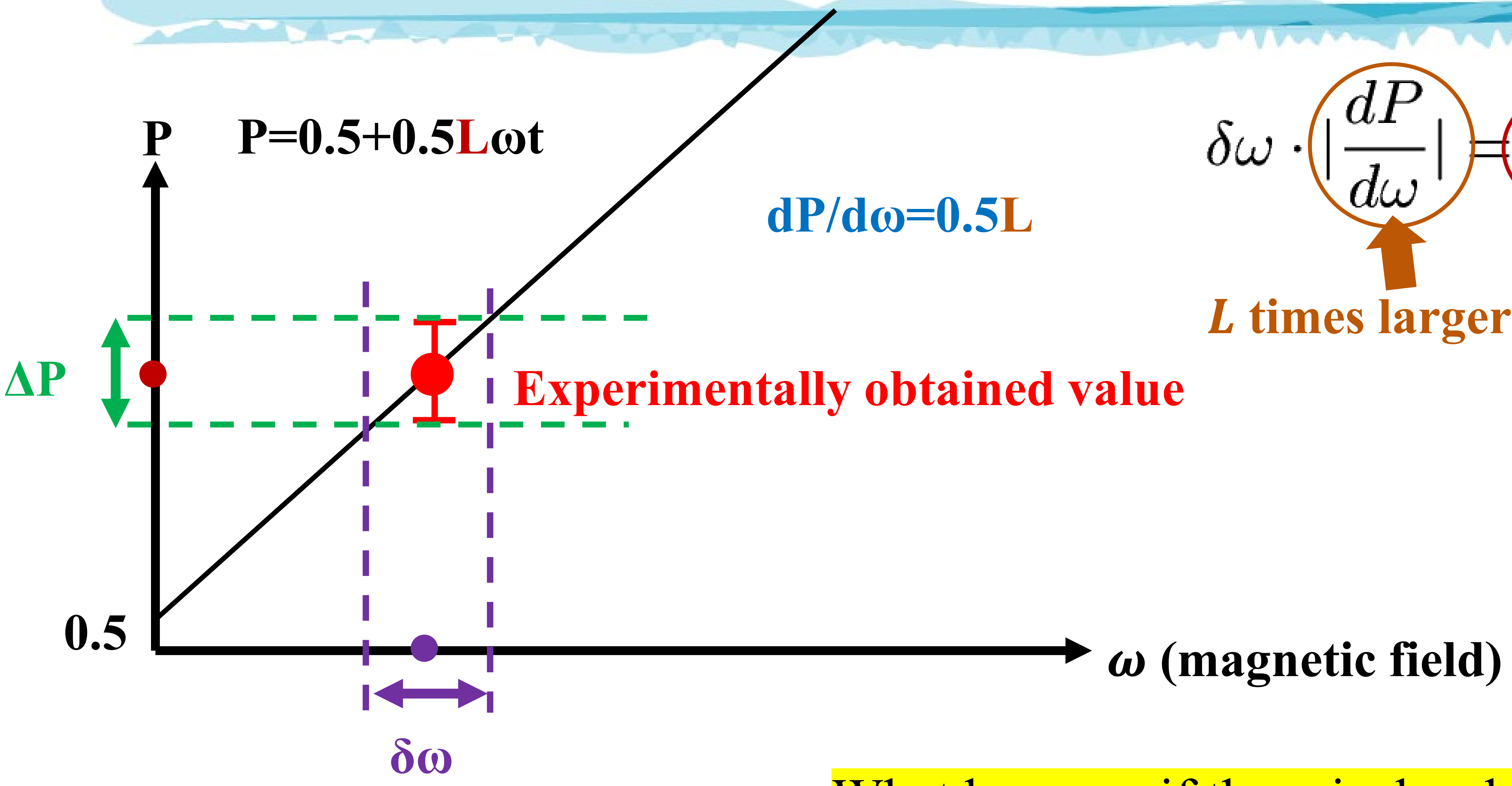
$$e^{-iHt}|\psi_{cat}\rangle = (e^{-\frac{iL\omega t}{2}}|00\cdots0\rangle + e^{\frac{iL\omega t}{2}}|11\cdots1\rangle)/\sqrt{2}$$

Uncertainty of the estimation



The uncertainty with the cat state becomes $L^{0.5}$ times smaller than that with L separable qubits!

Uncertainty of the estimation



$$\delta\omega \cdot \left|\frac{dP}{d\omega}\right| = \Delta P$$

L times larger

$L^{0.5}$ times larger

What happens if there is decoherence?

Decoherence for a single qubit

$$\overline{\rho(t)} = \frac{1+e^{-t/T_2^*}}{2}|\psi\rangle\langle\psi| + \frac{1-e^{-t/T_2^*}}{2}\hat{\sigma}_z|\psi\rangle\langle\psi|\hat{\sigma}_z$$

$$|\psi\rangle = (e^{-\frac{i\omega t}{2}}|0\rangle + e^{\frac{i\omega t}{2}}|1\rangle)/\sqrt{2}$$

For simplicity, we use a short-time approximation

$$\rho(t) \simeq \left(1 - \frac{t}{2T_2^*}\right)|\psi\rangle\langle\psi| + \frac{t}{2T_2^*}\sigma_z|\psi\rangle\langle\psi|\sigma_z$$

Error probability

Decoherence for a cat state

$$\rho(t) \simeq \left(1 - \frac{Lt}{2T_2^*}\right)|\psi_{cat}\rangle\langle\psi_{cat}| + \frac{Lt}{2T_2^*}\sigma_z|\psi_{cat}\rangle\langle\psi_{cat}|\sigma_z$$

Error probability

$$|\psi_{cat}\rangle = (e^{-\frac{iL\omega t}{2}}|00\cdots 0\rangle + e^{\frac{iL\omega t}{2}}|11\cdots 1\rangle)/\sqrt{2}$$

Probability to be affected by noise becomes *L* times larger

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

Decoherence for a cat state

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→If $Lt/2T_2^*$ approach to 1, a cat state is destroyed

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→If $Lt/2T_2^*$ approach to 1, a cat state is destroyed

→$Lt/2T_2^*$ should be smaller than 1,and so we should set $t \simeq T_2^*/L$

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Error probability

Acquired phase $L\omega t = L\omega(T_2^*/L) = \omega T_2^*$ → no dependence on L

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→If $Lt/2T_2^*$ approach to 1, a cat state is destroyed

→$Lt/2T_2^*$ should be smaller than 1,and so we should set $t \simeq T_2^*/L$

→ no signal enhancement by the cat state

→no improvement of the sensitivity

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

Usefulness of the cat (GHZ) state

To measure DC magnetic field, the use of the cat state does not improve the sensitivity under the effect of high-frequency parallel noise

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Usefulness of the cat (GHZ) state

To measure DC magnetic field, the use of the cat state does not improve the sensitivity under the effect of high-frequency parallel noise

↓

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

To measure AC magnetic field, the use of the cat state can improve the sensitivity under the effect of high-frequency parallel noise for some cases

T. Sichanugrist, H. Fukuda, T. Moroi, K. Nakayama, S. Chigusa, N. Mizuochi, M. Hazumi, and **Y. M.** , arXiv:2410.21699 (2024)

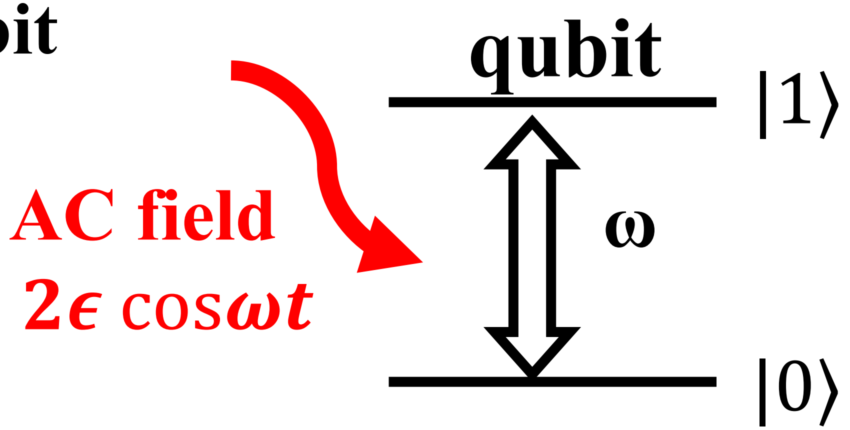
AC magnetic field sensing with a qubit (resonant)

Applying AC field along x direction to the qubit

$$H = H_0 + \Delta H$$

$$H_0 = -\frac{\omega}{2}\sigma_z$$

$$\Delta H = -2\epsilon \cos \omega t \, \sigma_x$$



AC field frequency is the same as the qubit energy (resonant)

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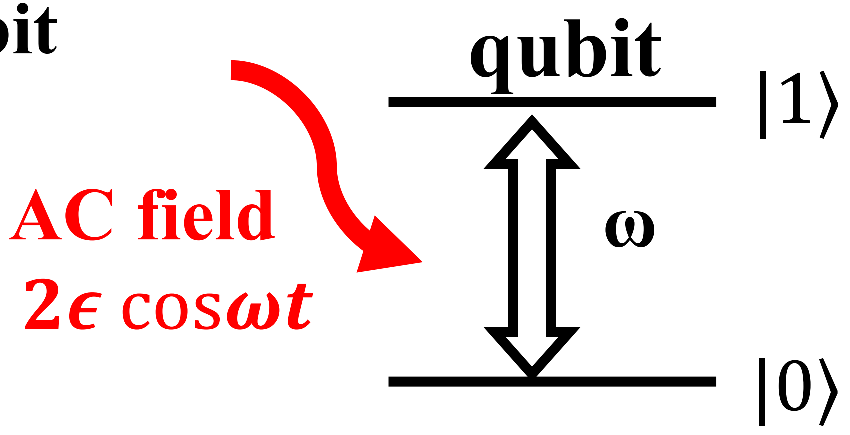

AC field frequency is the same as the qubit energy (resonant)

Going into a rotating frame (interaction picture)

$$H_I = e^{iH_0 t}\Delta H e^{-iH_0 t}, \ \rho_I = e^{-iH_0 t}\rho e^{iH_0 t}$$

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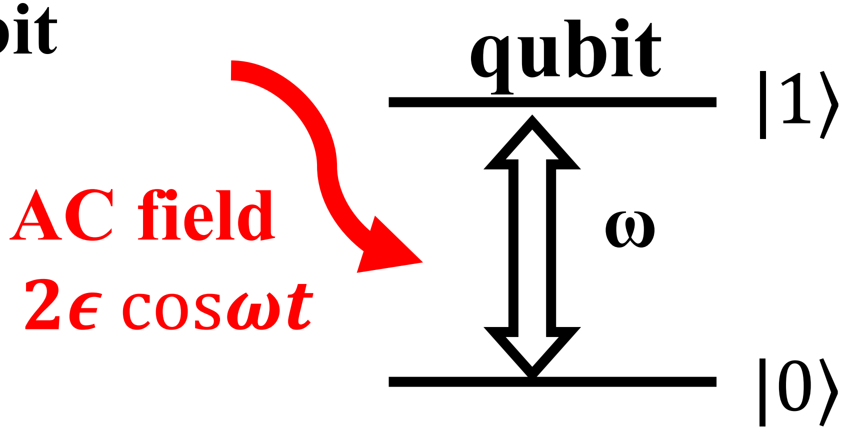

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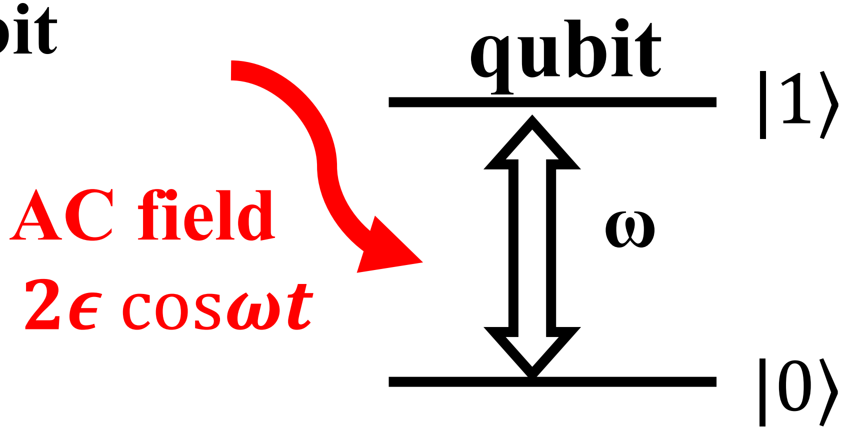

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$$H_I \simeq -\epsilon\,\sigma_x$$

The same as the DC field in the rotating frame

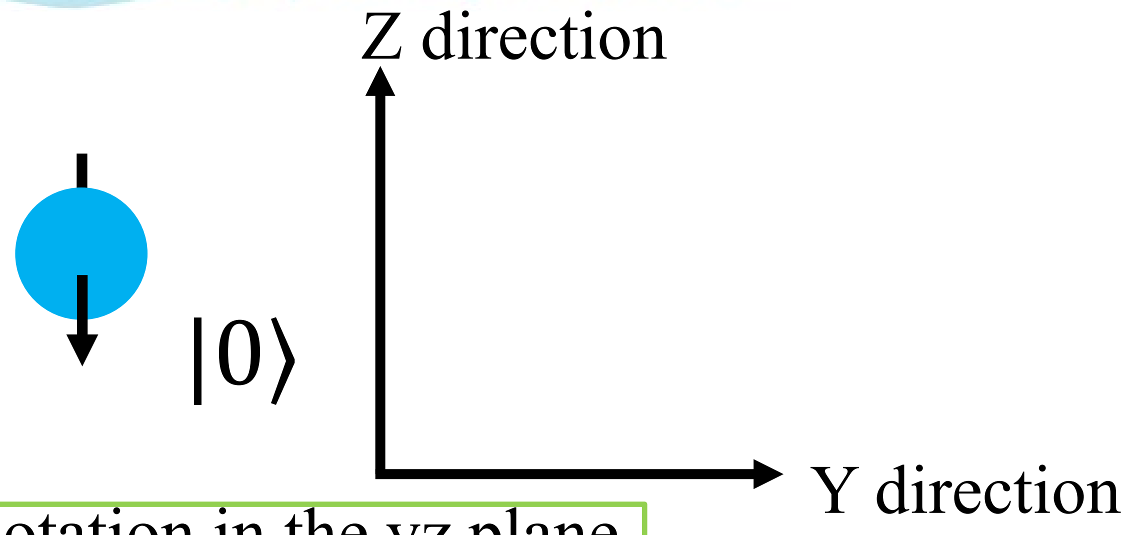
→The same sensitivity as the DC field

Time evolution in the rotating frame

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$$

$$H = -\epsilon\sigma_x$$



Rotation in the yz plane with an angle of 2εt

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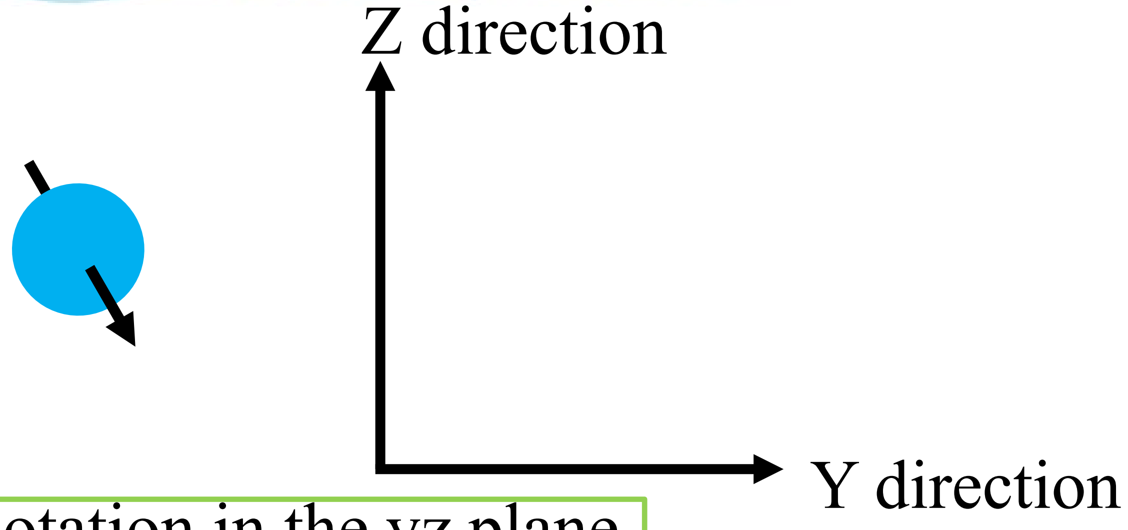

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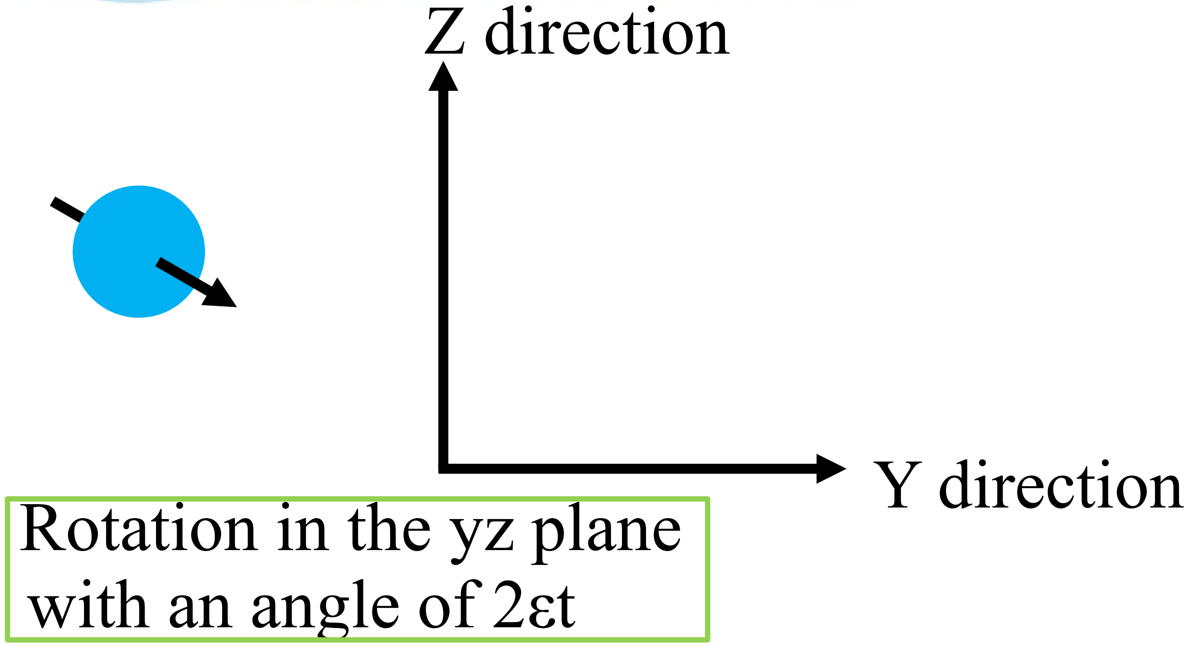
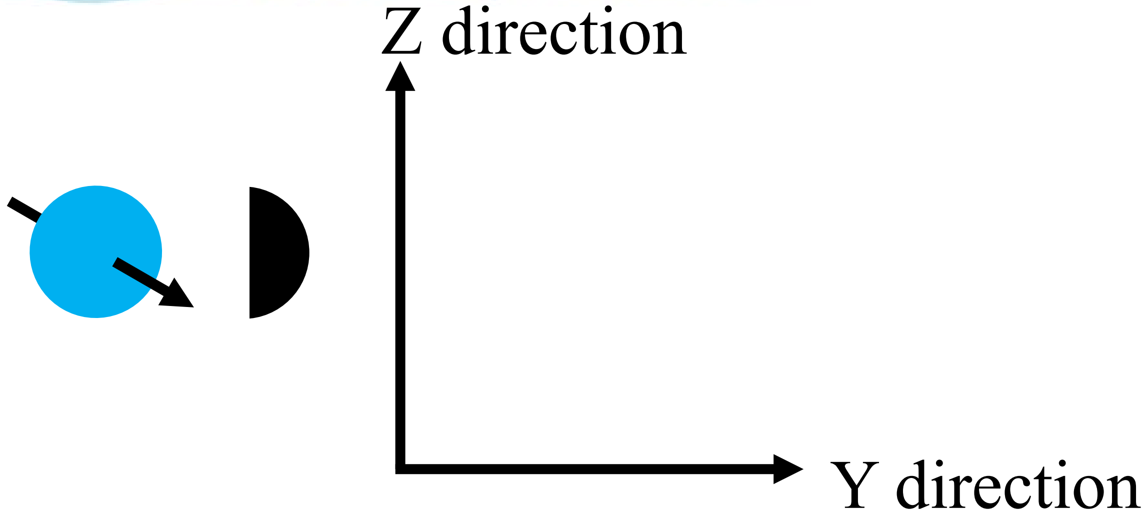


Projective measurement



Projection operator

$$\hat{\mathcal{P}}_{\pm} = \frac{1}{2}(\hat{1} \pm \hat{\sigma}_y)$$

Projection probability to y direction

$$P = \frac{1}{2} - \sin \epsilon t$$

AC magnetic field sensing with a qubit (non-resonant)

Applying AC field along x direction to the qubit

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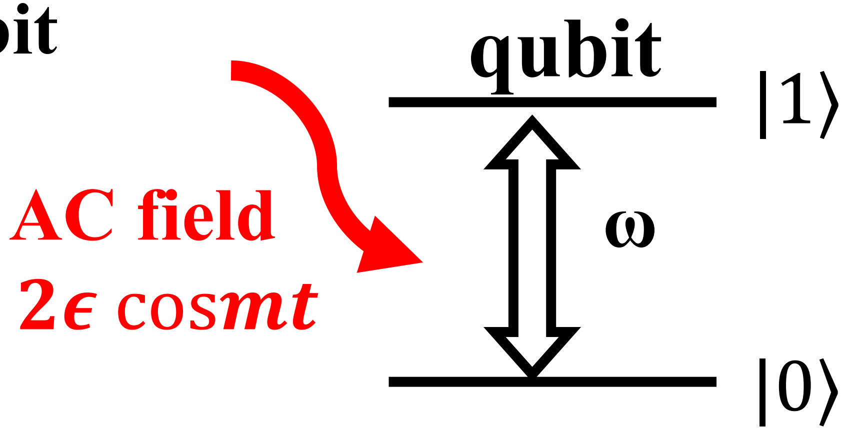
$$\Delta H = -2\epsilon \cos mt\, \sigma_x$$



AC field frequency is different as the qubit energy (non-resonant)

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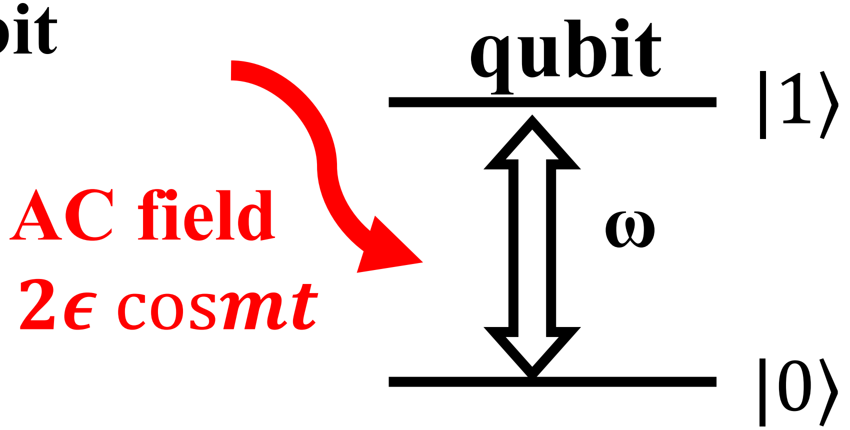
Going to the rotating frame

$$H_I \simeq -\frac{\omega - m}{2}\sigma_z - \epsilon \, \sigma_x$$



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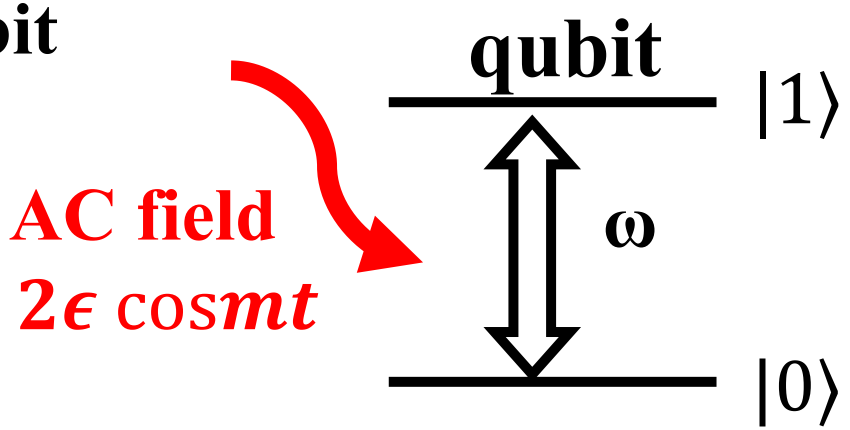

AC field frequency is different as the qubit energy (non-resonant)

For a large detuning $(\omega - m)$,
the signal becomes smaller, and estimating ϵ becomes difficult

Time evolution with noise for L qubits

$$H^i = H_0^i + \Delta H^i, \quad H_0^i = -\frac{1}{2}\omega\sigma_Z^i, \quad \Delta H^i = -2\epsilon\sigma_X^i \cos mt$$

Hamiltonian for *i*-th qubit

Time evolution with noise for L qubits

$$H^i = H_0^i + \Delta H^i, \quad H_0^i = -\frac{1}{2}\omega\sigma_Z^i, \quad \Delta H^i = -2\epsilon\sigma_X^i \cos mt$$

Total Hamiltonian

$$H = H_0 + \Delta H, \qquad H_0 = \sum_{j=1}^{L} H_0^j, \quad \Delta H = \sum_{j=1}^{L} \Delta H^j$$

$$H_I \equiv e^{iH_0 t}\Delta H e^{-iH_0 t} \qquad \rho \equiv e^{-iH_0 t}\rho_I e^{iH_0 t}.$$

$$\frac{d\rho_I(t)}{dt} = -i[H_I, \rho_I] + D_I[\rho_I], \qquad D_{I,X}[\rho] = \frac{\Gamma_X}{2}\sum_{j=1}^{L}\left(\sigma_X^j \rho \sigma_X^j - \frac{1}{2}\rho\right),$$

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Interaction picture

$$H_I \equiv e^{iH_0 t}\Delta H e^{-iH_0 t} \qquad \rho \equiv e^{-iH_0 t}\rho_I e^{iH_0 t}.$$

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Master equation

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$T_1 = 1/\Gamma_x$ is the energy relaxation time

Noise direction is the same as that with AC field

$$H_I \equiv e^{iH_0 t} \Delta H e^{-iH_0 t} \qquad \rho \equiv e^{-iH_0 t} \rho_I e^{iH_0 t}.$$

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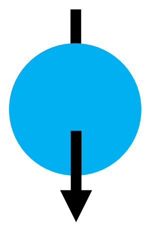
Initial state

Initial state for AC field sensing with separable states

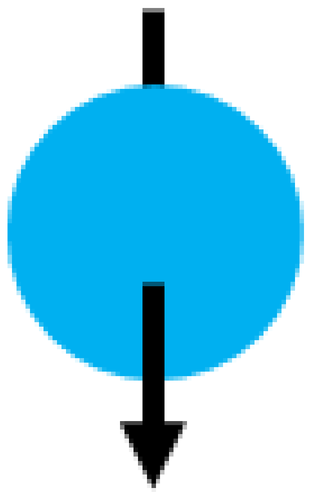
$$|\psi_{\mathrm{indv}}(0)\rangle \equiv |0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

$$|\pm\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$



Initial state for entanglement enhanced AC field sensing

$$|\psi_{\mathrm{GHZ}}(0)\rangle = \frac{|+\rangle^{\otimes L} + |-\rangle^{\otimes L}}{\sqrt{2}}$$



Projection probability after the interaction with AC field

Projection probability for separable states

$$p_Y = \frac{1}{2} - \epsilon t e^{-\Gamma_X t} W(t)$$

Projection probability for the cat (GHZ) state

$$p_{\mathrm{GHZ},Y} \simeq \frac{1}{2} - e^{-L\Gamma_X t} L\epsilon t W(t)$$

$$W(t) \simeq \begin{cases} 2 & \text{for} \quad t \lesssim |\omega + m|^{-1} \\ 1 & \text{for} \quad |\omega + m|^{-1} \lesssim t \lesssim |\omega - m|^{-1} \\ \dfrac{1}{|\omega - m| t} & \text{for} \quad |\omega - m|^{-1} \lesssim t \end{cases}$$

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When the detuning $(\omega - m)$ is much smaller than $1/t$, the signal is similar with that of DC field sensing

Projection probability after the interaction with AC field

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When the detuning $(\omega - m)$ is much larger than $1/t$, the signal becomes smaller

Projection probability after the interaction with AC field

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Interpretation

Uncertainty between time and energy

$t \cdot \delta\omega \simeq 1$

the detectable bandwidth becomes larger

Estimation uncertainty of the AC field strength

Uncertainty with L separable qubits

$$\delta\epsilon^{(\mathrm{indv})} \simeq \begin{cases} \sqrt{\dfrac{|\omega - m|}{T}}\dfrac{1}{\sqrt{L}} & \text{for } \Gamma_X \lesssim |\omega - m| \\ \sqrt{\dfrac{\Gamma_X}{T}}\dfrac{1}{\sqrt{L}} & \text{for } \Gamma_X \gtrsim |\omega - m| \end{cases}$$

Uncertainty with a cat (GHZ) state

$$\delta\epsilon^{(\mathrm{GHZ})} \simeq \begin{cases} \sqrt{\dfrac{|\omega - m|}{T}}\dfrac{1}{L} & \text{for } L\Gamma_X \lesssim |\omega - m| \\ \sqrt{\dfrac{\Gamma_X}{T}}\dfrac{1}{\sqrt{L}} & \text{for } L\Gamma_X \gtrsim |\omega - m| \end{cases}$$

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For a large detuning $(\omega - m)$, the uncertainty with the cat state is L times smaller than that of the separable states

Estimation uncertainty of the AC field strength

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An interaction time with AC field for a cat state is much shorter than that of the separable state

Estimation uncertainty of the AC field strength

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For a large detuning $(\omega - m)$, the uncertainty with the cat state is L times smaller than that of the separable states

⇩

An interaction time with AC field for a cat state is much shorter than that of the separable state

⇩

The detectable bandwidth becomes larger for the cat state!

Estimation uncertainty of the AC field strength

Uncertainty with L separable qubits

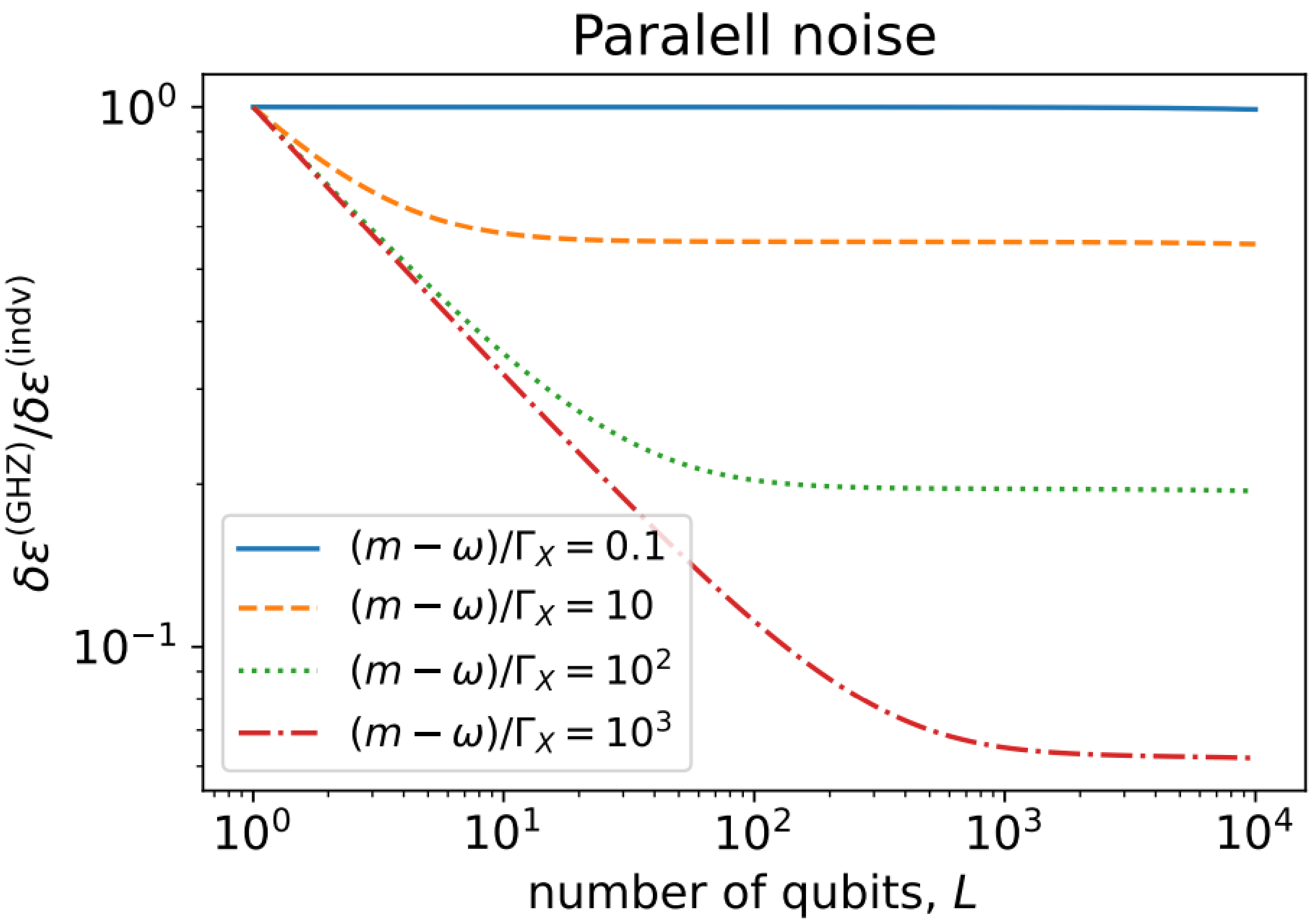
$$\delta\epsilon^{(\mathrm{indv})} \simeq \begin{cases} \sqrt{\frac{|\omega - m|}{T}}\frac{1}{\sqrt{L}} & \text{for } \Gamma_X \lesssim |\omega - m| \\ \sqrt{\frac{\Gamma_X}{T}}\frac{1}{\sqrt{L}} & \text{for } \Gamma_X \gtrsim |\omega - m| \end{cases}$$

Uncertainty with a cat (GHZ) state

$$\delta\epsilon^{(\mathrm{GHZ})} \simeq \begin{cases} \sqrt{\frac{|\omega - m|}{T}}\frac{1}{L} & \text{for } L\Gamma_X \lesssim |\omega - m| \\ \sqrt{\frac{\Gamma_X}{T}}\frac{1}{\sqrt{L}} & \text{for } L\Gamma_X \gtrsim |\omega - m| \end{cases}$$

For a small detuning $(\omega - m)$, there are no advantage to use the cat state!

Comparison of the sensitivity



For a large detuning $(\omega - m)$, we can decrease the uncertainty by using the cat state!

T. Sichanugrist, H. Fukuda, T. Moroi, K. Nakayama, S. Chigusa, N. Mizuochi, M. Hazumi, and **Y. M.** arXiv:2410.21699 (2024)

Conclusion

- To measure DC magnetic field with qubits, we cannot improve the sensitivity by using a cat (GHZ) state under the effect of high-frequency parallel noise

S. Huelga *et al*. Physical Review Letters 79.20 (1997): 3865.

- To measure AC magnetic field with qubit, we can improve

the sensitivity by using a cat state under the effect of high-frequency parallel noise for some cases.

T. Sichanugrist, H. Fukuda, T. Moroi, K. Nakayama, S. Chigusa, N. Mizuochi, M. Hazumi, and **Y. M.**
arXiv:2410.21699 (2024)