Robust quantum sensing with nitrogen-vacancy centres for light dark matter detection

QUPosium 2024, Tsukuba, December 9th

Ernst David Herbschleb[1], So Chigusa[2], Riku Kawase[1], Hiroyuki Kawashima[1], Masashi Hazumi[3], Kazunori Nakayama[4], Norikazu Mizuochi[1]

[1]Kyoto university

[2]University of California, Lawrence Berkeley National Laboratory

[3]QUP, KEK, IPNS, JAXA, ISAS, Kavli IPMU, WPI, SOKENDAI

[4]Tohoku University, QUP, KEK

Outline

Introduction

Method

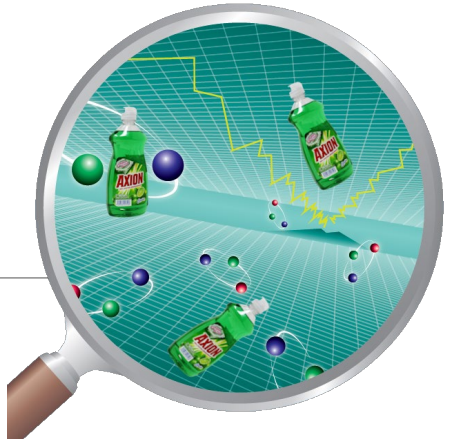
Sensitivity

Measurement

Detection results

Summary

Introduction



Kazunori Nakayama

- Nuclear spin metrology for axion dark matter detection

$$H_{int} = \sum_{\chi} \frac{g_{a\chi\chi}}{m_\chi} \vec{\nabla} a \cdot \vec{S}_\chi$$

Electron spin metrology

- Coupling to electron spin
- Effective magnetic field

$$H_{int} = \sum_{\chi} \gamma_\chi \vec{B}_\chi(t) \cdot \vec{S}_\chi$$

$$\vec{B}_{eff}(t) \approx \frac{g_{aee}}{e} \sqrt{2\rho_a} \vec{v}_a \cos(m_a t + \phi)$$

$$\vec{B}_\chi(t) \approx \frac{g_{a\chi\chi}}{m_\chi \gamma_\chi} \sqrt{2\rho_a} \vec{v}_a \cos(m_a t + \phi)$$

For $\rho_a = 0.4 \frac{GeV}{cm^3}$

$v_a = 0.001$

$\sqrt{2\rho_a} v_a \approx 13\ nT$

$B_{eff} \approx 4\ aT \times \frac{g_{aee}}{10^{-10}}$

Sensitive magnetometers

Introduction

Challenges

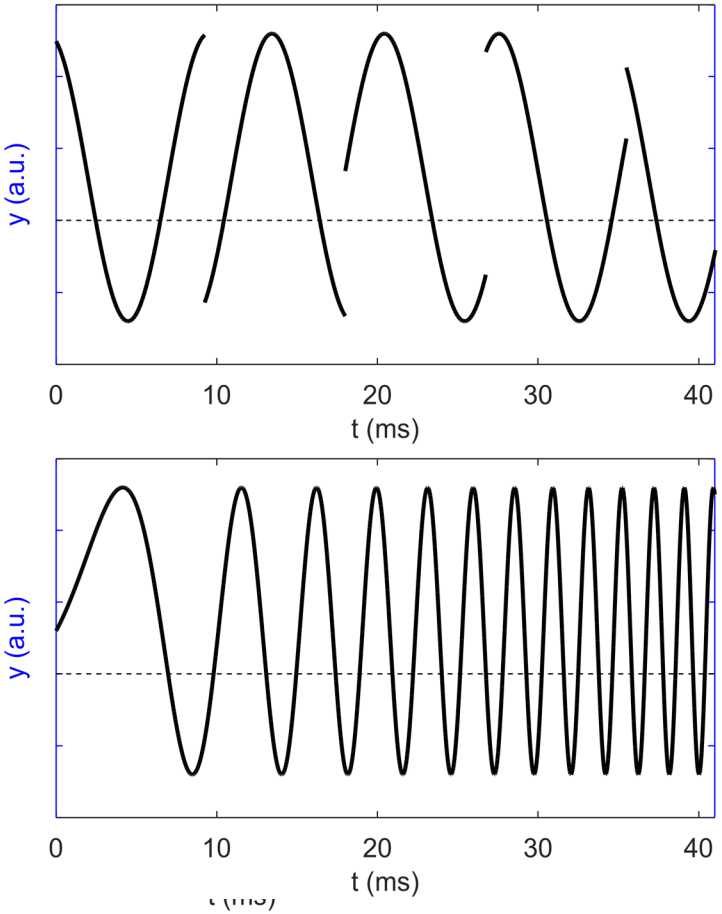
- Limited coherence time
- Unknown direction
- Unknown amplitude
- Unknown frequency (mass)
- Unknown phase

$$\tau_a = \frac{1}{mv_a^2}$$

Randomness in the signal to measure

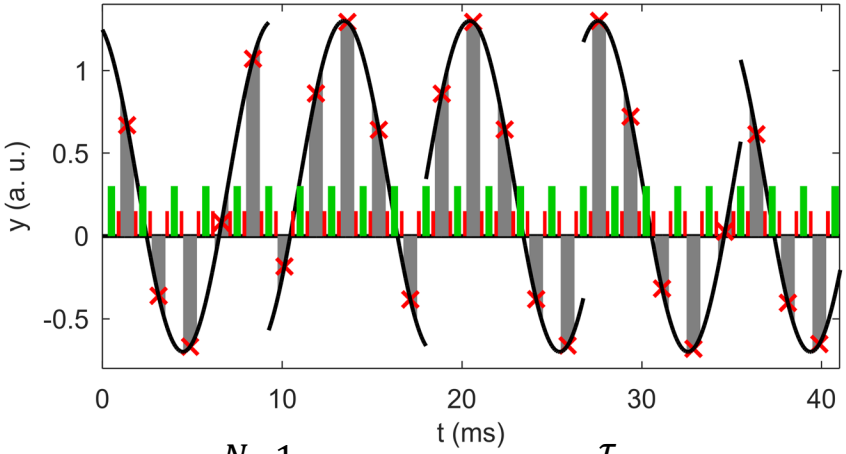
Regular sensing methods

- Hahn-echo
- Q-scope
- Dynamical decoupling



Method – “std sensing”

Measure standard deviation (std)



$$= \frac{A}{\sqrt{2}}$$

$$VAR = \sigma_y^2 = \frac{1}{N-1}\sum_{k=0}^{N-1}(y_i - \mu)^2 \approx \frac{1}{\tau}\int_0^{\tau}(A\sin(2\pi f t + \theta) + C - C)^2\,dt =$$

$$A^2 f \int_0^{1/f} \sin^2(2\pi f t + \theta)\,dt = \frac{A^2 f}{2}\int_0^{1/f} 1 - \cos(4\pi f t + \theta)\,dt = \frac{A^2}{2}$$

Sensitivity – definition

Sensitivity $\eta = \sigma_{variable}\sqrt{T_{measurement}}$

Variable: standard deviation of the signal σ_S

If signal and noise independent, variances add

$$VAR(data) = \sigma_D^2 = VAR(noise) + VAR(signal) = \sigma_n^2 + \sigma_S^2$$

Thus, the (measured) data standard deviation is

$$\sigma_D = \sqrt{\sigma_n^2 + \sigma_S^2}$$

Sensitivity $\eta = \sigma_{\sigma_S}\sqrt{T_{measurement}}$

Uncertainty in standard deviation

Noise on top of result

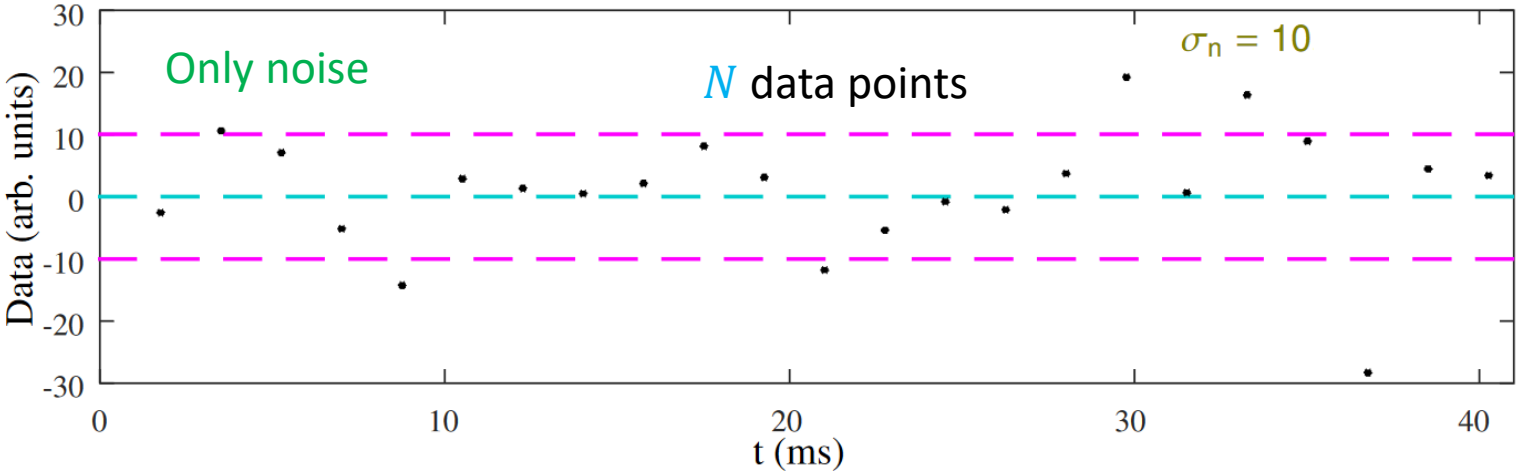
Noise on signal, e.g. magnetic noise

Noise due to measurement, e.g. shot noise

Noise distribution: normal with average 0 and std σ_n

How to find σ_n?

$$VAR = \sigma_n^2 = \frac{1}{N-1}\sum_{k=0}^{N-1}(y_i - \mu)^2$$



However, noise is random, thus σ_n will be slightly different each time measured

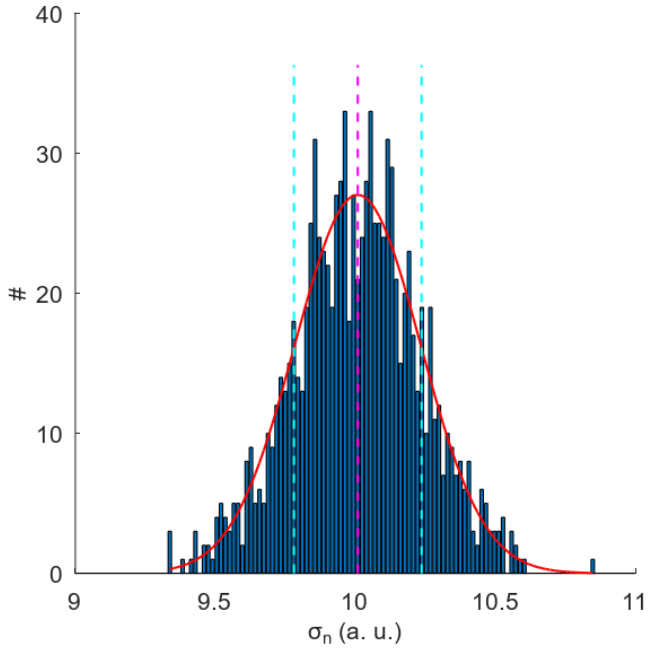
Uncertainty in standard deviation

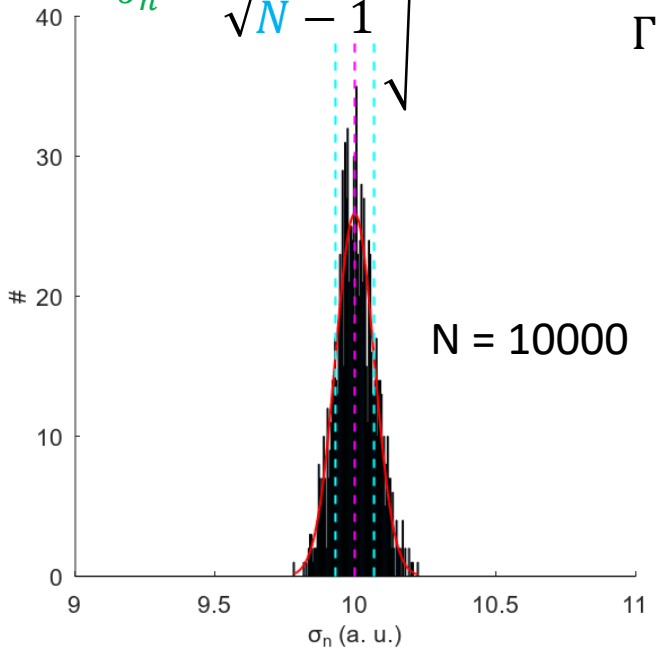
Repeat the measurement to find σ_n with N = 1000: distribution

Sampled variance: Chi-squared distribution

Sampled std: Chi distribution

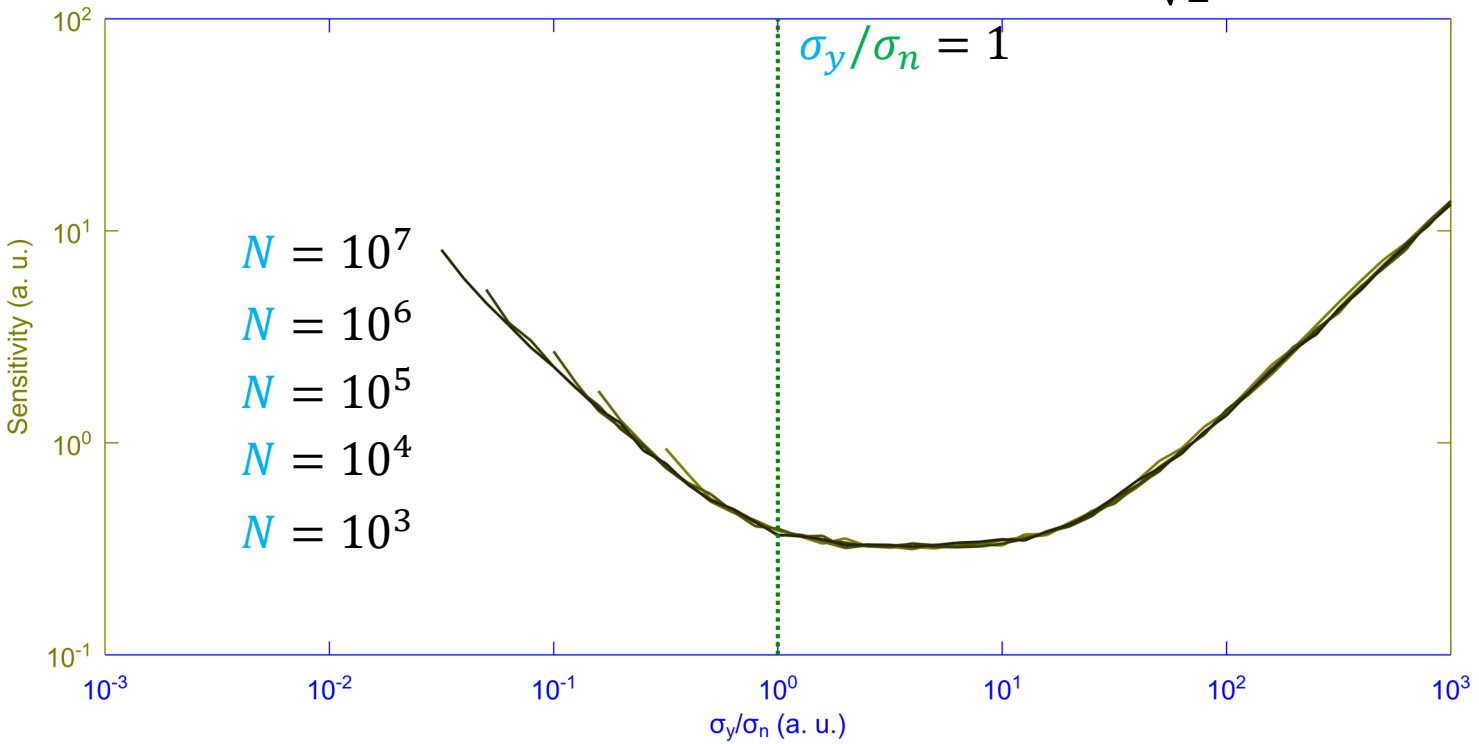
$$\sigma_{\sigma_n} = \frac{\sigma_n}{\sqrt{N-1}} \sqrt{N - 1 - 2\frac{\Gamma^2\left(\frac{N}{2}\right)}{\Gamma^2\left(\frac{N-1}{2}\right)}} \approx \frac{\sigma_n}{\sqrt{2N}} \quad (N \gg 1)$$





Sensitivity

Varying signal amplitude (numerical simulation) $\sigma_y = \frac{A}{\sqrt{2}}$



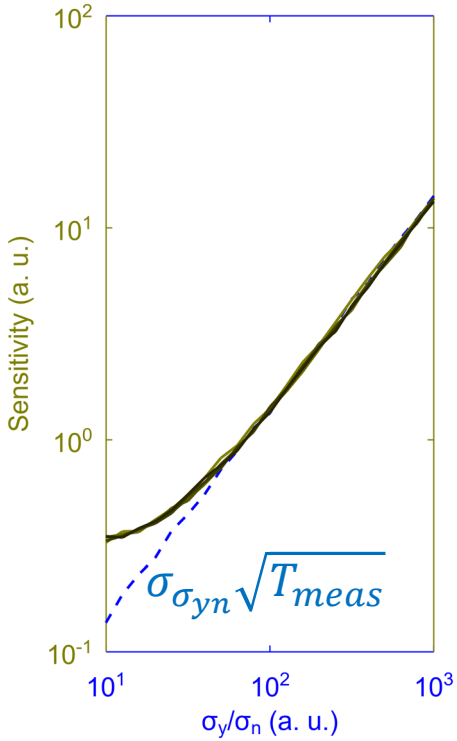
Sensitivity – large signal

For large signal

$$\sigma_S \gg \sigma_n \Rightarrow \sigma_D = \sqrt{\sigma_n^2 + \sigma_S^2} = \sigma_S \sqrt{\frac{\sigma_n^2}{\sigma_S^2} + 1} \approx \sigma_S$$

Changes in phase at random time adds noise

$$\sigma_S \equiv \sigma_y + \sigma_{yn} \qquad \boxed{\begin{aligned} \mu_{\sigma_{yn}} &= 0 \\ \sigma_{\sigma_{yn}} &\propto A \end{aligned}}$$



Axion signal not that large

(For detection of larger signal, less sensitivity required)

Sensitivity – decent signal

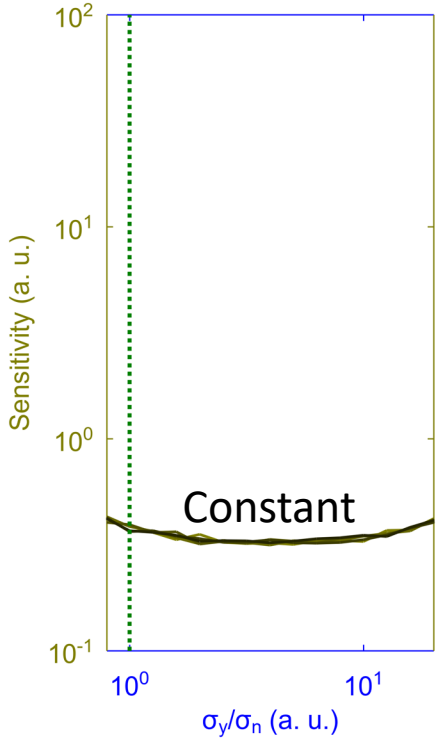
$\sigma_S \approx \sigma_y$ For sufficiently large signal, still

$$\sigma_S > \sigma_n \Rightarrow \sigma_D = \sqrt{\sigma_n^2 + \sigma_S^2} = \sigma_S \sqrt{\frac{\sigma_n^2}{\sigma_S^2} + 1} \approx \sigma_S$$

With $\sigma_{\sigma_{yn}}$ negligible, the uncertainty becomes

$$\sigma_{\sigma_D} \approx \sigma_{\sigma_S} \approx \frac{\sigma_n}{\sqrt{N}}$$

$$\left.\begin{array}{l} \eta = \sigma_{\sigma_S}\sqrt{T_{measurement}} \\ T_{measurement} \propto N \end{array}\right\} \Rightarrow \eta \propto 1$$



Sensitivity – small signal

For small signal $\sigma_S \ll \sigma_n \Rightarrow \Delta\sigma = \sigma_D - \sigma_n = \sqrt{\sigma_n^2 + \sigma_S^2} - \sigma_n =$

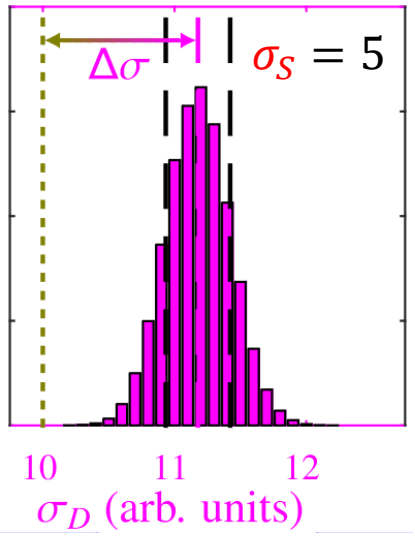
$$\sigma_n \sqrt{1 + \frac{\sigma_S^2}{\sigma_n^2}} - \sigma_n \approx \sigma_n \left(1 + \frac{\sigma_S^2}{2\sigma_n^2} - \cdots\right) - \sigma_n \approx \frac{\sigma_S^2}{2\sigma_n}$$

Offset, thus same distribution

$$\sigma_{\sigma_n} \approx \frac{\sigma_n}{\sqrt{2N}} \approx \sigma_{\sigma_D} = \sigma_{\Delta\sigma}$$

For σ_{σ_S}, need square root of this distribution

- Smaller signal, negative values



Sensitivity – small signal

No analytical solutions

Get an idea at the detection limit

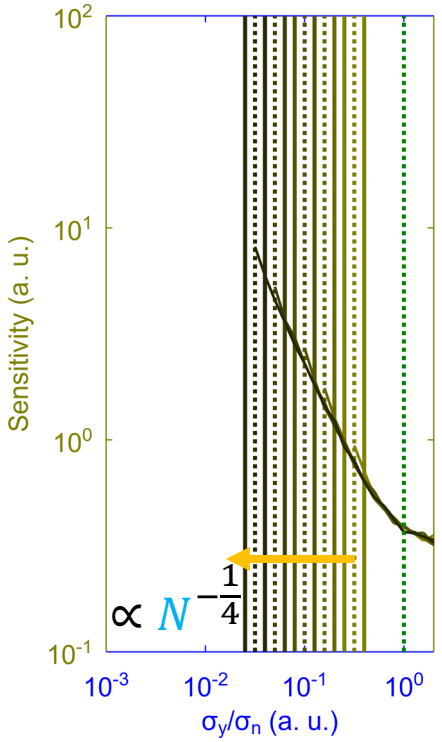
$H_0: \sigma_y = 0$
$H_1: \sigma_y > 0$

Confidence level of 95%

$$\Delta\sigma \geq 1.645\sigma_{\sigma_n}$$

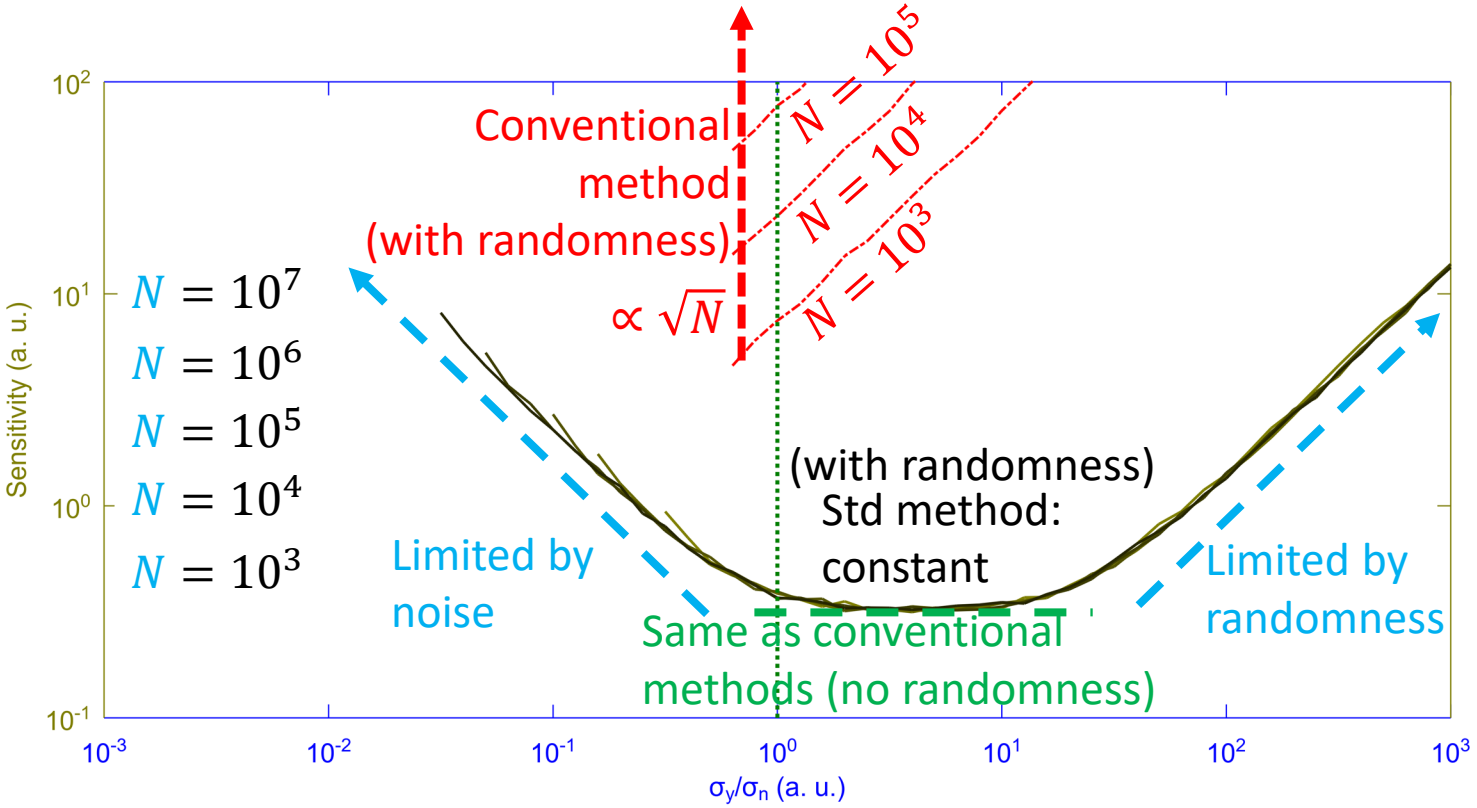
$$\frac{\sigma_S^2}{2\sigma_n} \geq 1.645\frac{\sigma_n}{\sqrt{2N}} \Rightarrow \sigma_S \geq \frac{1.5\sigma_n}{N^{\frac{1}{4}}}$$

$\sigma_s \approx \sigma_y$



Sensitivity – comparison

Compare with conventional method



Advantages std method

Standard deviation does not depend on order data

- Robust against phase changes
- Robust against frequency changes
- Essentially, each data point can be indexed at random

$$\sigma_y = \frac{A}{\sqrt{2}}$$

Std method requires limited data

- For single NV: about 4 numbers (0 photons, 1, 2, 3+)
- For ensemble: choose bin size
  - Bin size not equal to accuracy: if two bins for 0 and 1 with 5000 measurements, the maximum accuracy of the std is 4×10^{-8}
- Alternative: single-pass methods (standard: two-pass method: mean, then std)
  - Always only 2 numbers
- Alternative: average zero (often the case)
  - Only 1 number

Std method computational complexity: $\mathcal{O}(N)$

Outline

Introduction

Method

Sensitivity

Measurement

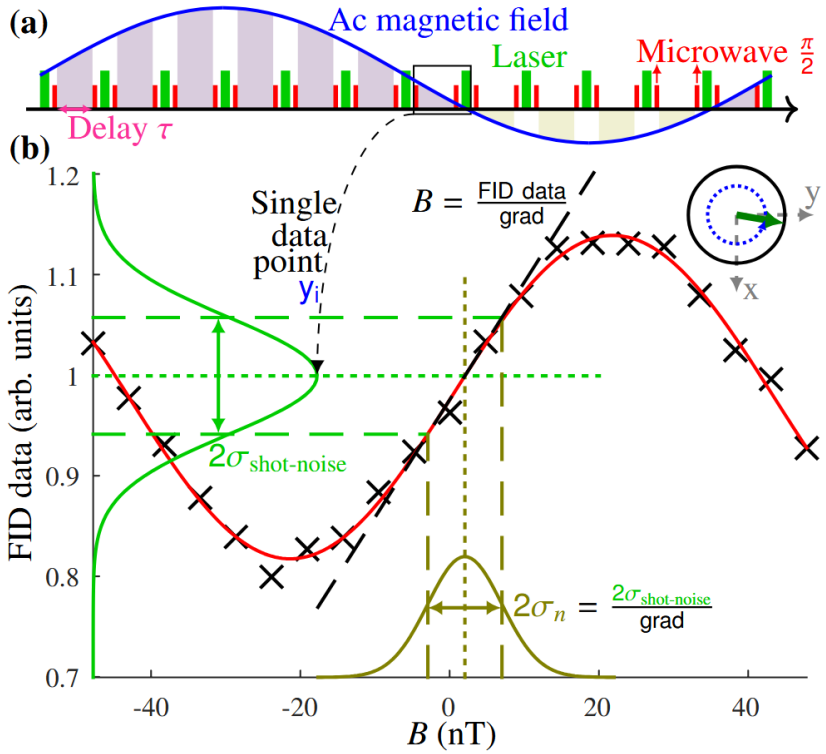
Detection results

Summary

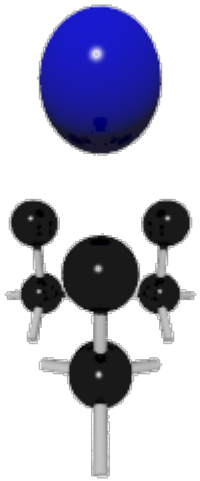
Measurement

Norikazu Mizuochi

- Nitrogen vacancy centres



Single nitrogen-vacancy centre



Measurement settings

Single NV centre

5000x accumulation
1 s time window
T = 5000 s

$$\tau = 0.4\ ms$$

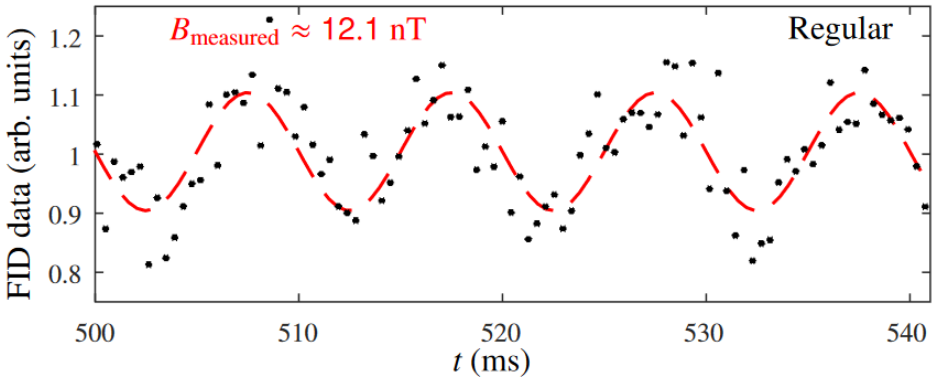
Effectively filter

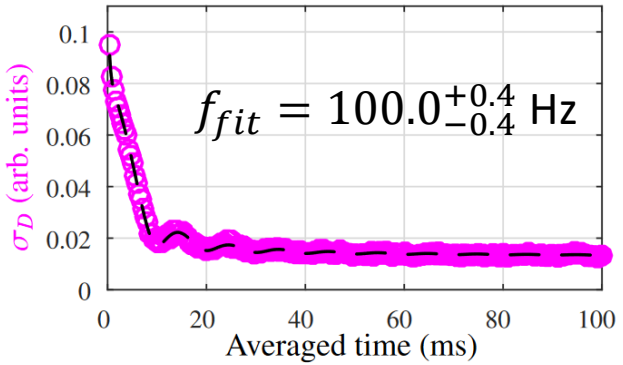
- Most sensitive to 1 Hz and harmonics
- Normally broadband

Measurement results

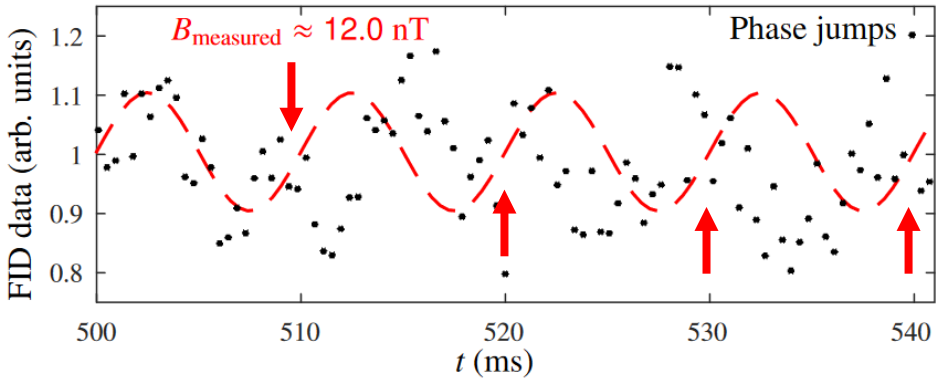
5000x accumulation
1 s time window
T = 5000 s
$\tau = 0.4\ ms$

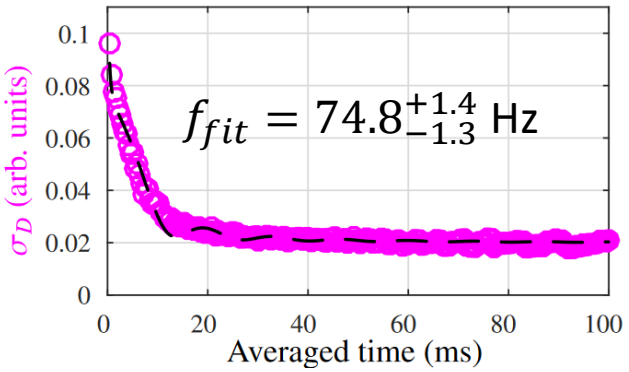
Regular





Limited coherence time by phase jumps (here one per period)





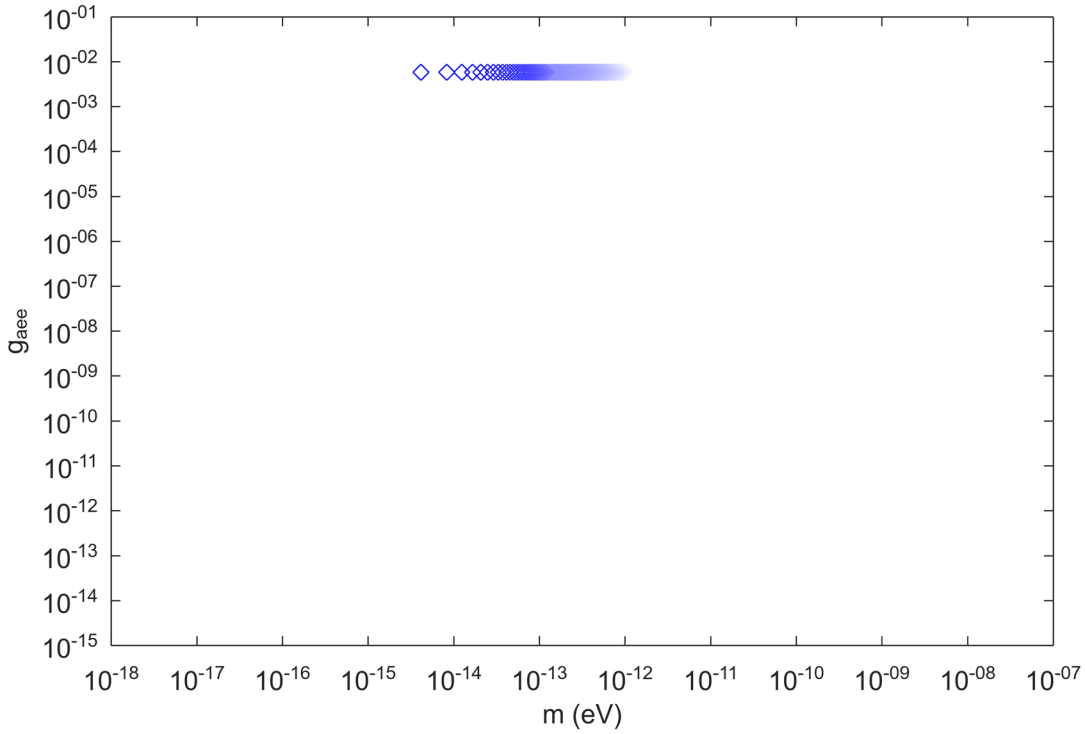
Detection results

5000x accumulation
1 s time window
T = 5000 s
$\tau = 0.4\ ms$

Dark matter detected?

1 Hz and harmonics

$\tau_a \gg T$



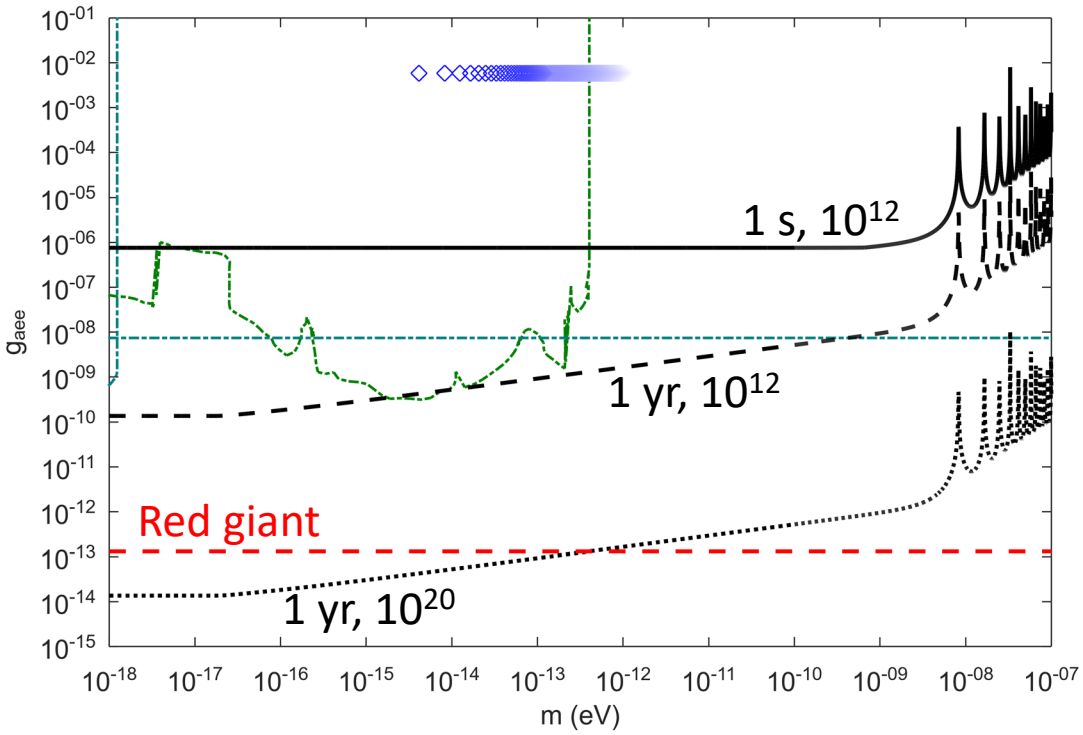
Detection results

Previous results

Improvement (theory)

- Ensembles
  - 1 s, 10^{12} NVs
  - 1 yr, 10^{12} NVs
  - 1 yr, 10^{20} NVs
- Quantum projection noise

  S. Chigusa, M. Hazumi, E. D. Herbschleb, N. Mizuochi, K. Nakayama, arXiv:2302.12756 (2023).



Summary

Sensitivity depends on signal size compared to noise

- Large, same as conventional methods, which is best possible
- Smaller, worse sensitivity (“demerit”)

Several merits of the std method

- Robustness, memory usage, computational complexity

E. D. Herbschleb, S. Chigusa,
R. Kawase, H. Kawashima,
M. Hazumi, K. Nakayama, N. Mizuochi,
APL Quantum 1, 046106 (2024).

Search for light dark matter

S. Chigusa, M. Hazumi,
E. D. Herbschleb,
N. Mizuochi, K. Nakayama,
arXiv:2302.12756 (2023).

S. Chigusa, M. Hazumi,
E. D. Herbschleb, Y. Matsuzaki,
N. Mizuochi, K. Nakayama,
arXiv:2407.07141 (2024).

Thank you for your kind attention.

The research is supported by KAKENHI (No. 22K14560), MEXT Q-LEAP (No. JPMXS0118067395), CREST (No. JPMJCR23I5), and the Collaborative Research Program of ICR, Kyoto University (2021-114).

Questions?