EFT for inflation and structure formation

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Effective Field Theory (EFT)

EFT is a modern name for an old practice:

1. Identify the relevant Degrees Of Freedom (DOFs),
2. Identify the symmetries,
3. Write a local model, compatible with the symmetries.

Examples: Hydrodynamics, General Relativity, Fermi Theory, String Theory, ...

Power of EFT

The locality and symmetry make EFTs very predictive.

- The leading behavior at large distance and long time is universal
  1. Turbulent cascade (Hydro)
  2. Kerr black hole (GR),
  3. Parity violation (Weak interactions), ...
- Deviation are systematically organized via a derivative expansion.

Outline

1. EFT of Inflation
2. EFT for open systems and dissipation
3. EFT of LSS

Single-Field Inflation

Simplest model of inflation adds one scalar field to GR

$$S = \frac{1}{2}\int \sqrt{-g}[M_{\rm pl}^2 R - (\partial\phi)^2 - 2V(\phi)],$$

with a sufficiently flat potential:

$$\frac{M_{\rm pl}^2 V_{,\phi}^2}{V^2} \ll 1, \qquad \left|\frac{M_{\rm pl}^2 V_{,\phi\phi}}{V}\right| \ll 1.$$

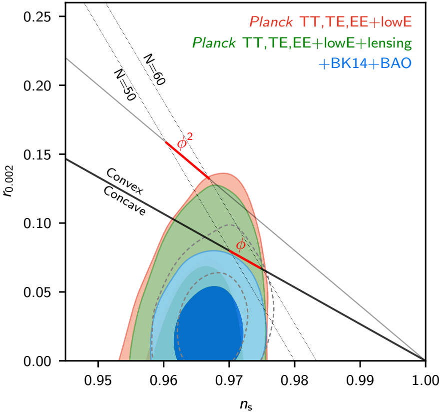
This is already an EFT with cutoff $\leq M_{\rm pl}$.

Cosmology as a BSM lab

- Many inflationary models solve flatness and horizon puzzles.
- They leave their signature in cosmological perturbations;
  1. Observed adiabatic scalar perturbations: $\delta\phi \to \zeta \to \delta\rho/\rho$
  2. Illusive tensor perturbations: γ_{ij}.
- It's tempting to use observations to learn about inflationary physics.
- $H_{\rm inf}$ could be as high as 10^{14}GeV, $\Delta\phi \gg M_{\rm pl}$.

Cosmology as a BSM lab, $V(\phi)$

Different $V(\phi)$ can be distinguished using the observed $P_\zeta(k)$ and constrained $P_\gamma(k)$:



Cosmology as a BSM lab, derivative interactions

- Flatness of the potential implies an approximate shift symmetry.
- There can be large derivative interactions like $(\partial\phi)^4$ or

$$(\partial\phi)^2 \to -\sqrt{1-(\partial\phi)^2}, \qquad \text{DBI.}$$

- These can be distinguished by their non-Gaussianity predictions

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} \right\rangle \sim B(k_1, k_2, k_3).$$

Systematic approach to NG

- Information in $B(k_1, k_2, k_3)$ is more than $P(k)$.
- In noisy data, we need templates to look for signal.
- EFT of Infl. separates the theory of background from perturbations to
  1. reduce many underlying models to what is essential for predicting ζ, γ_{ij} spectra,
  2. organize the set of templates compatible with symmetries.

EFT of Inflation

- The rolling inflaton field introduces preferred time slices, $\phi(t_u, \vec{x})$ =constant.
- Single-field inflation $\equiv$ Massive gravity

$$S = M_{\rm pl}^2 \int \sqrt{-g}[R - 3H^2(t) - \dot{H}(t)\delta g^{00} + c(t)(\delta g^{00})^2 + \cdots]$$

- Different underlying models differ in the coefficients of the expansion.
- This allows a systematic study of NG templates:

$$B(k_1, k_2, k_3) = f_{\rm NL}^{\rm eq} F^{\rm eq} + f_{\rm NL}^{\rm orth} F^{\rm orth} + \cdots$$

Pros and cons

Pros

- Ideal for precision modeling,
- Conceptual; identification of ζ with Stueckelberg field, $t \to t + \pi$.
  - Eg. revealing a universal coupling during particle production:

$$\mathcal{L} = \pi \partial_\mu T^\mu_0.$$

- Conceptual; identification of the symmetry breaking pattern with super-fluids.

Cons

- Underlying physics is important and we have theoretical priors.
- Most interesting variants of vanilla slow-roll inflation are not captured.

Beyond single-field inflation

Observations have ruled out simplest single-field models, which motivates multi-field ones:

- Extra scalar fields coupled to the inflaton.
- Models with particle production, gauge field production, axion inflation.
- Warm inflation.
- Different symmetry breaking patterns (solid, chromonatural, gaugid, ...).

EFT for open systems and dissipation

Warm inflation Fang 80', Moss 85', Yokoyama, Maede 86', Berera,Fang 95',...

- Inflation is a theory of initial condition.
- It erases the pre-existing structures by stretching them to unobservably long wavelength.
- ~~It makes the inflationary universe classically cold and empty.~~
  Repeated particle production keeps the universe warm and populated.
- ~~It stretches vacuum fluctuations in the UV into the observed cosmological perturbations.~~
  The origin of what we see are the subhorizon thermal fluctuations.

Energy budget

Warm inflation needs a continuous energy transfer

$$\phi \to X \quad \text{(another sector)}$$

such that

$$\frac{\rho_X}{\rho_{\rm tot}} \sim \epsilon \qquad \text{small but approximately fixed.}$$

Assuming thermalization, the temperature can be much greater than H:

$$T \gg H \qquad \text{is compatible with} \qquad T^4 \ll M_{\rm pl}^2 H^2.$$

Background evolution

Particle production back-reacts on the inflaton evolution

$$\ddot{\phi} + (3H + \gamma)\dot{\phi} + V'(\phi) = 0,$$

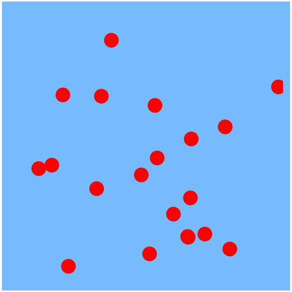
$$\dot{\rho}_X + 4H\rho_X = \gamma\dot{\phi}^2 + \cdots$$

This can have a warm slow-roll attractor.

Therefore, not only conceptually different but also the predictions of warm inflation for a given $V(\phi)$ are dramatically different from cold inflation. E.g. the number of e-folds.

Origin of perturbations

The transfer of energy $\phi \to X$ is not uniform. It is a random microscopic process.



- This induces large (effectively classical) fluctuations already inside the horizon

$$\delta\phi \gg \delta\phi_{\rm vac}.$$

- By the central limit theorem the observed spectrum is nearly Gaussian if $T \gg H$.
- But the non-Gaussian features can be distinct from other scenarios. (We are sensitive to $\mathcal{O}(10^{-4})$ deviation from Gaussianity.)

EFT for Warm inflation: fluid + inflaton

- At $\lambda \gg 1/T$ the thermal bath is described by a fluid

$$T_{\mu\nu} = \frac{4}{3}\rho u_\mu u_\nu + \frac{1}{3}\rho g_{\mu\nu},$$

plus $\mathcal{O}(H/T)$ dissipative corrections.

- However, there is one dissipative term that is essential

$$-\nabla^2\phi + V'(\phi) = \frac{1}{f}O_X = \underbrace{-\gamma(\rho)u^\mu\partial_\mu\phi}_{\langle\rangle\text{on long-}\lambda\text{ bgr}} + \underbrace{\xi}_{\text{noise}}.$$

This couples ϕ to the fluid:

$$\nabla^\nu T_{\mu\nu} = \partial_\mu\phi(\gamma u^\mu\partial_\mu\phi - \xi).$$

Bastero-Gil, Berera, Moss, Ramos '14

Predictivity of EFT

- We need $\gamma(\rho)$ and the statistics of ξ to calculate correlators of ζ.
- If $[O_X] = M^4$, then $\gamma \propto T^3$. By Fluctuation-Dissipation theorem

$$\langle \xi(x_{\rm ph})\xi(y_{\rm ph})\rangle \approx 2\gamma T\ \delta^4(x_{\rm ph} - y_{\rm ph}).$$

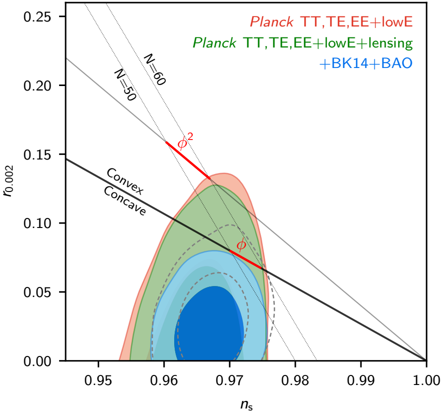
- This allows to compute the correlation functions as a function of γ/H, without the knowledge of the underlying mechanism.
- The only reliable underlying mechanism is sphaleron heating Berghaus,Graham,Kaplan '19.

Warm ϕ^4 inflation MM, Gruzinov '22

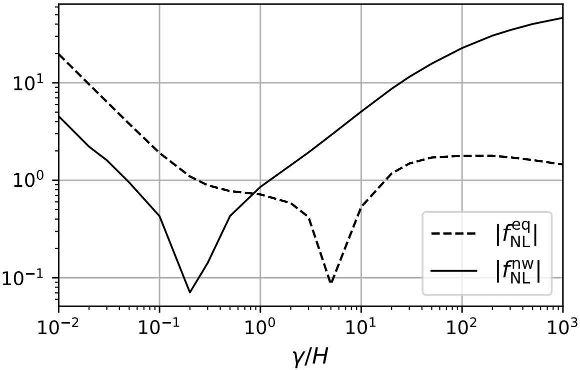
$$N_e = 55, \qquad \phi \approx 11.6 M_{\rm pl}, \qquad 1 - n_s \approx 0.0337, \qquad \gamma \approx 5.34 H.$$

For $SU(2)$ gauge group

$$\frac{T}{H} \approx 1200, \qquad r \approx 4.7 \times 10^{-7}.$$



Warm inflation at the verge of discovery!



Open EFTs and stochastic inflation

Break-down of perturbations theory

- Light fields have large excursions

$$\langle \phi^2 \rangle \sim \frac{H^4}{m^2}$$

- The ratio of $\lambda\phi^4$ interaction to the mass term is

$$\frac{\lambda\phi^4}{m^2\phi^2} \sim \frac{\lambda H^4}{m^4}$$

- This can be large, even for a technically natural mass $m^2 \sim \lambda H^2$

$$\frac{\lambda\phi^4}{m^2\phi^2} \sim \frac{1}{\lambda} \gg 1.$$

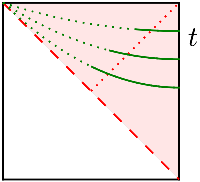
Stochastic method in cosmological slicing

Starobinsky showed that

$$\varphi(t) = \int d^3\vec{x}\ W_L(a(t)\vec{x})\ \phi(t,\vec{x}), \qquad W_L \text{ a window-function}$$

satisfies the Fokker-Planck eq. as in Brownian motion

$$\partial_t p(t,\varphi) = \frac{1}{8\pi^2}\partial_\varphi^2 p(t,\varphi) + \frac{1}{3}\partial_\varphi(V'(\varphi)p(t,\varphi)).$$



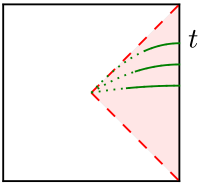
Stochastic method in the static patch

MM '20

- In the static patch there are thermal fluctuations.
- There is a close analogy with Browning motion:

$$\varphi(t) = \int d^3\vec{x} W_L(\vec{x})\phi(t,\vec{x}),$$

in the environment of all other DOFs, which have short-lived correlations.



Open EFT for φ

- The reduced density matrix

$$p(t,\varphi) = [\mathrm{Tr}_{\mathrm{env.}}\rho(t)]_{\varphi_L=\varphi_R=\varphi}$$

satisfies FP eq.

$$\partial_t p(t,\varphi) = \frac{1}{8\pi^2}\partial_\varphi^2 p(t,\varphi) + \frac{1}{3}\partial_\varphi(V'(\varphi)p(t,\varphi)) + \cdots$$

- One can show that FP is just the leading term in a systematic expansion in $1/(Ht_\lambda) \sim \sqrt{\lambda}$.

- The evolution remains Markovian to all orders, as in hydrodynamics.

EFT of LSS

Cosmological perturbations as seen today

- Cosmological perturbations grow during matter domination and collapse.
- To extract information from LSS surveys, this nonlinear evolution has to be accounted for.
- EFT of LSS is a tool to organize the nonlinear effects without making assumptions about galaxy formation details.
- This is important because surveys are reaching sub-percent precision in the weakly nonlinear regime.

Cosmic fluid?

- The idea is to treat matter at large scales as a pressure-less fluid

$$\partial_t(a^3\rho) + \partial_i(a^2\rho v^i) = 0,$$

$$\partial_t v^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\phi = [-\frac{1}{a^2\rho}\partial_j\tau^{ij}],$$

where $\rho = \bar{\rho}(1+\delta)$, v^i is peculiar velocity.

- This is not a normal fluid, because

1. It's atoms (halos) are getting bigger by mergers.
2. The collision time between the atoms is the age of the universe.

Cosmic fluid?

- Generically, an EFT with no separation of time-scales is useless.
- EFT of LSS is useful because we only want to make a limited use:
  - start from $\delta_{\rm in} \ll 1$ and evolve until $\delta < 1$.
- In contrast, classical EFTs like hydrodynamics and GR are fully nonlinear systems.
  - They break when gradients are big (e.g. $R \sim M_{\rm pl}^2$), not perturbations ($h_{\mu\nu} \sim 1$).

Perturbative expansion

- We can expand

$$\delta(t,\vec{k}) = \sum_{n=1}^{\infty} \int F_n(t,\vec{k};\{\vec{q}_i\})\delta_{\rm in}(\vec{q}_1)\cdots\delta_{\rm in}(\vec{q}_n).$$

  and keep a finite number of terms for a given precision.

- At a fixed order in $\delta_{\rm in}$, any non-local term on the RHS of

$$\partial_t v^i + Hv^i + \frac{1}{a}v^j\partial_j v^i + \frac{1}{a}\partial_i\phi = [-\frac{1}{a^2\rho}\partial_j\tau^{ij}],$$

  is equivalent to a finite number of local terms.

- This allows a systematic way of producing templates $\tilde{F}_n$ that capture the short scale unknowns.

EFT of LSS, pros and cons

Pros

- It is robust.
- It can reach very high precision at low k.

Cons

- We know some things about galaxy formation, and interested to learn about it.
- Introduces lots of parameters, and throws away short-distance data.

An application

Ivanov, Simonović, Zaldarriaga '19