7th Nov. 2024 @ KEK

STOchastic LAttice Simulation of cosmic inflation

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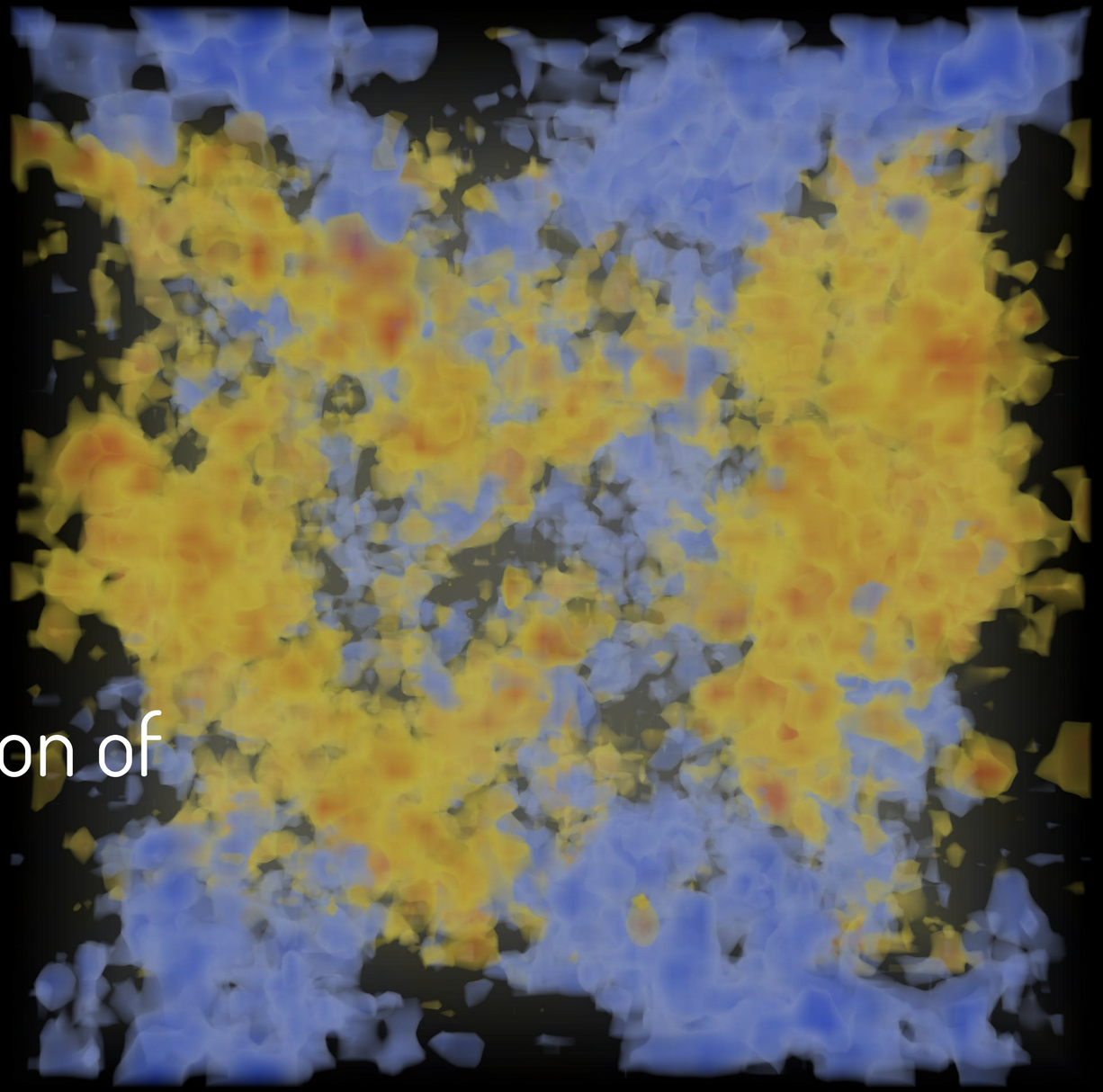
w/ Mizuguchi, Murata 2405.10692

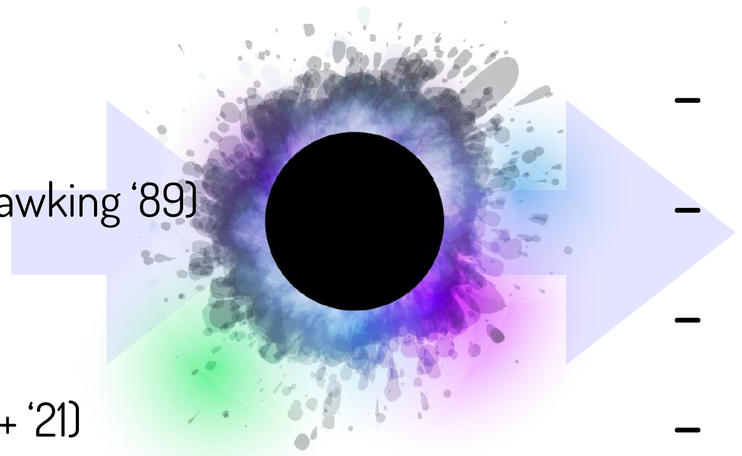


Primordial BH

Carr & Hawking '74

- $\sim \mathcal{O}(1)$ overdensity (Carr '75)
- Isocurvature (Dolgov & Silk '93)
- Quark Confinement (Dvali+ '21)
- Collapse of topological defect (Hawking '89)
- Bubble collision (Hawking+ '82)
- Particle trapping in bubble (Baker+ '21)
- Asynchronous 1st PT (Liu+ '21, Lewicki+ '24)
- Scalar 5th force (Flores & Kusenko '20)

⋮



before Star Formation

Primordial Black Hole

- **Dark Matter** (Chapline '75)
- LVK merger GW? (Sasaki+ '16)
- SMBH seeds? (Düchting '04)
- OGLE lensing obj.? (Niikura+ '19)
- Planet 9? (Scholtz & Unwin '19)
- Trigger of r-process? (Fuller+ '17)
- Baryogenesis? (Baumann+ '07)
- JWST luminous gals? (Hutsi+ '22)

⋮

Primordial BH

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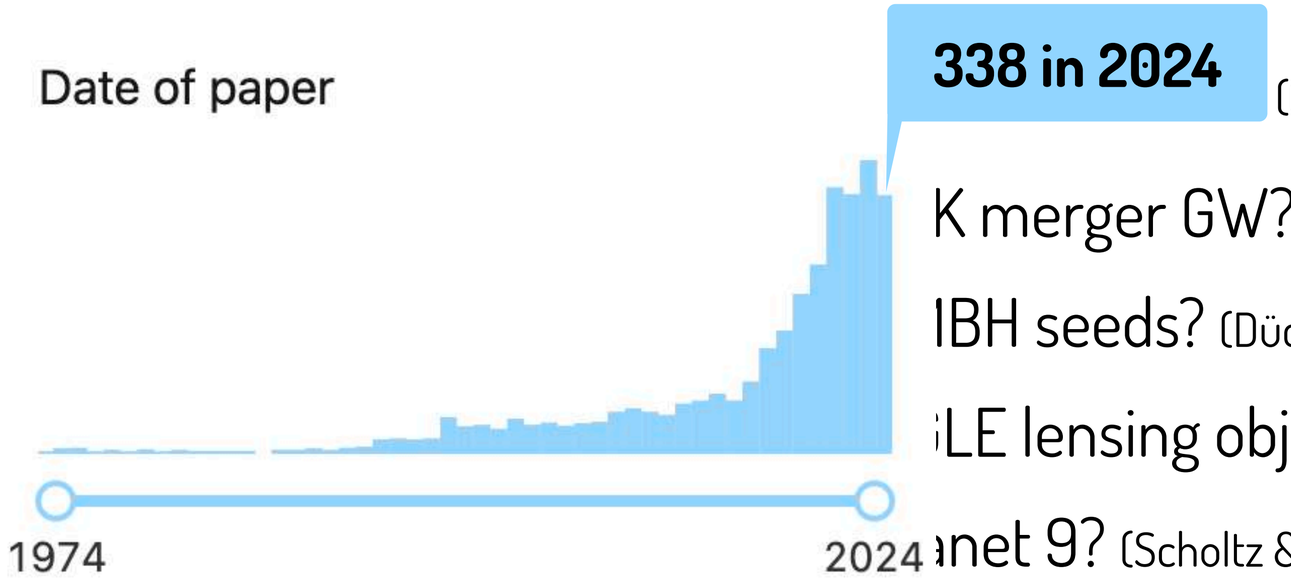

(Chapline '75)

- K merger GW? (Sasaki+ '16)
- IBH seeds? (Düchting '04)
- LE lensing obj.? (Niikura+ '19)
- net 9? (Scholtz & Unwin '19)
- Trigger of r-process? (Fuller+ '17)
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- JWST luminous gals? (Hutsi+ '22)

⋮

before Star Formation

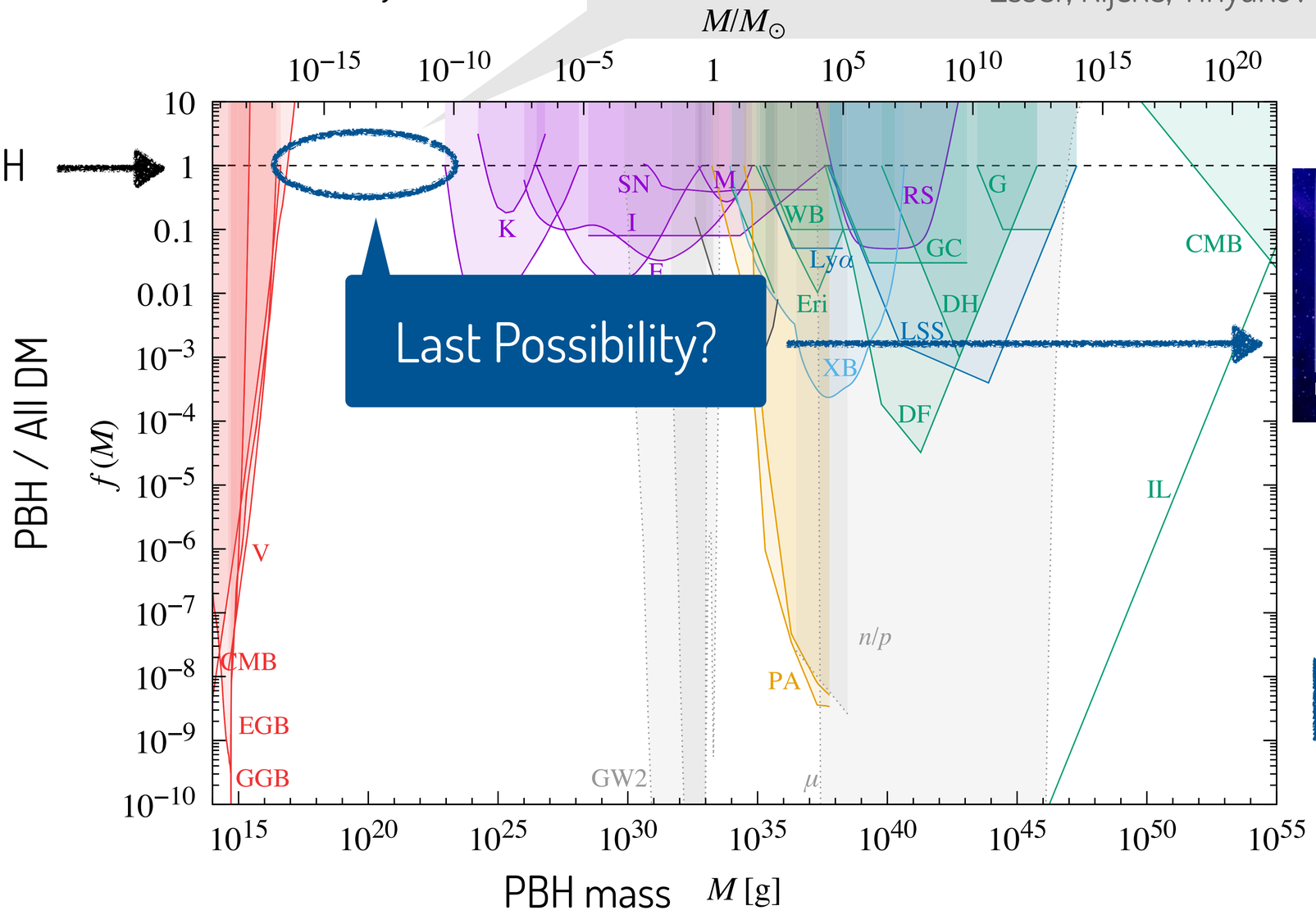
Primordial Black Hole

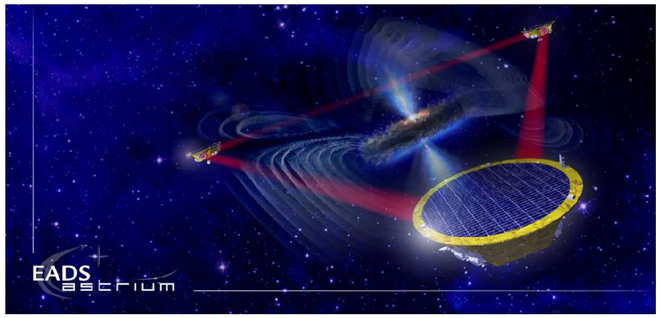
Obs. Consts.

Carr, Kohri, Sendoda, Yokoyama ‘20

Stellar Mass Function in Ultra-Faint Dwarfs...?
Esser, Rijcke, Tinyakov ‘23

All DM = PBH





Corresponding induced GWs of mHz ↔ LISA (2037-) !!

Prove or Falsify PBH-DM?

Triangle study

Universal Criterion

Atal+ '19
Escrivà, YT, Yokoyama, Yoo '22

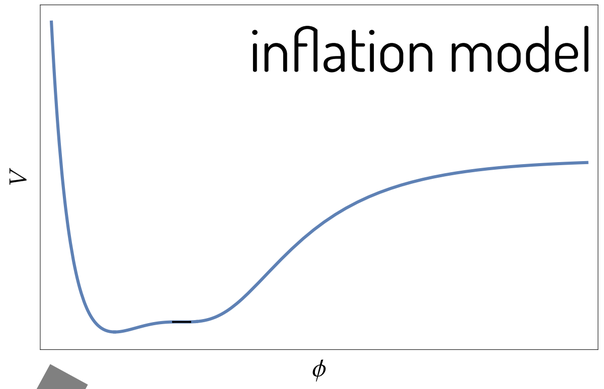
$$\mathscr{C}(r) = \frac{2}{3}\left[1 - (1 + r\zeta'(r))^2\right]$$

$$\bar{\mathscr{C}} = \frac{1}{V_{R_\mathrm{m}}}\int_0^{R_\mathrm{m}} 4\pi R^2 \mathscr{C}\,\mathrm{d}R > \bar{\mathscr{C}}_\mathrm{th} = \frac{2}{5}$$

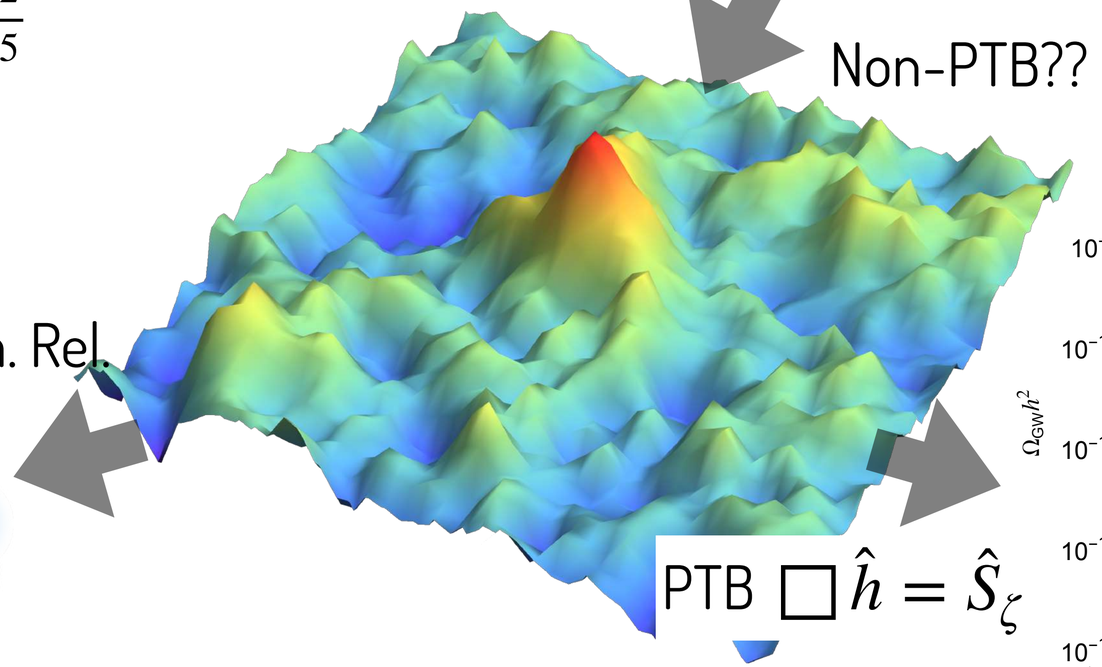
Mass Formula

Choptuik+ '93

$$M \sim M_{R_\mathrm{m}}(\bar{\mathscr{C}} - \bar{\mathscr{C}}_\mathrm{th})^{0.36}$$

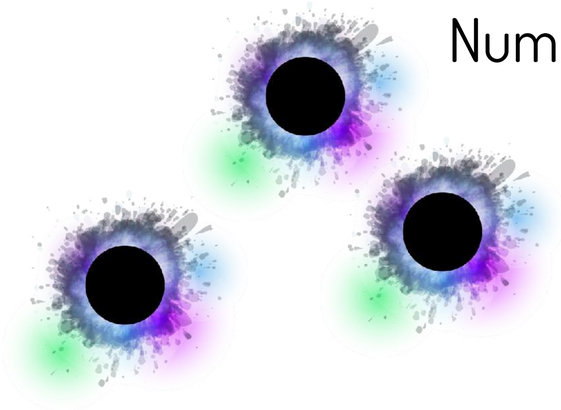


Non-PTB??



Num. Rel.

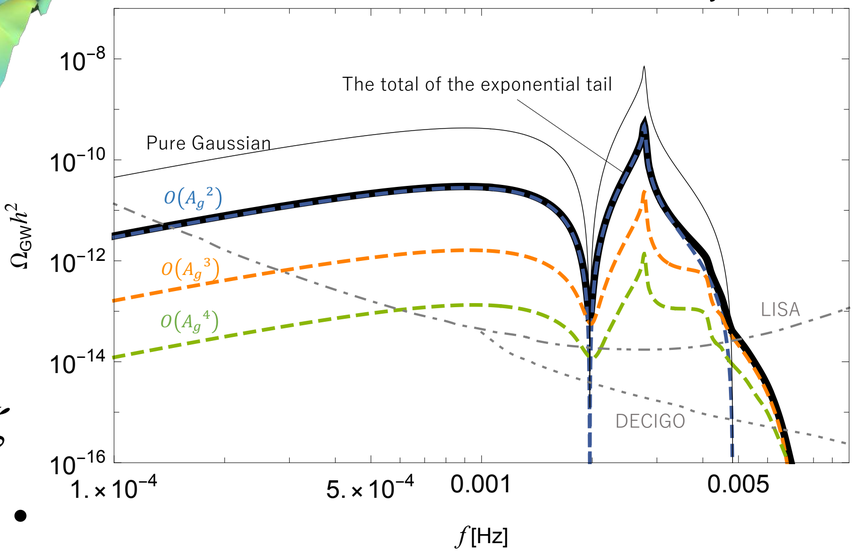
PTB $\square\hat{h} = \hat{S}_\zeta$



PBH abundance

indirect evidence

Abe, Inui, YT, Yokoyama '22

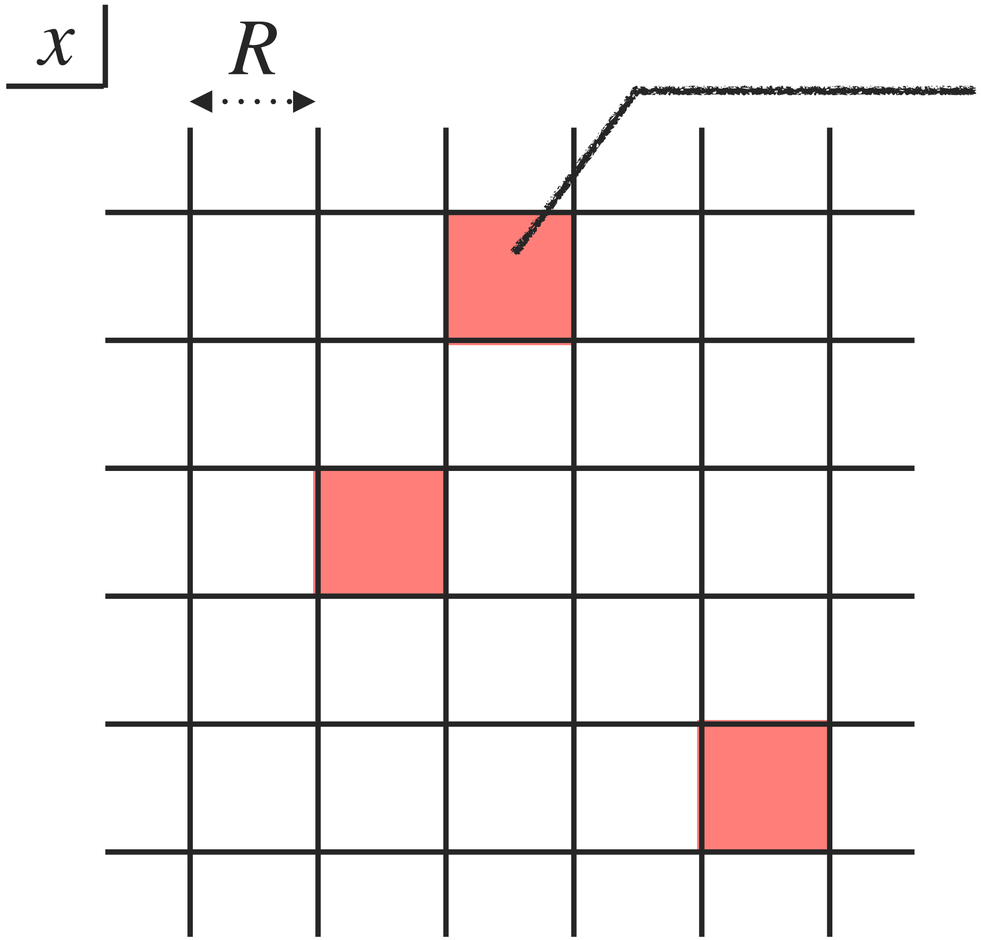


induced GW b.g.

Carr's simplest way

Press—Schechter

$$\delta \sim -\frac{4}{9}\frac{\Delta}{(aH)^2}\zeta, \quad \sigma_R^2 = \left(\frac{4}{9}\right)^2 \int \mathrm{d}\ln k\, (kR)^2 \mathscr{P}_\zeta(k) W^2(kR)$$



$$\delta_R(\mathbf{x}) = \int \mathrm{d}^3 y\, W_R(\mathbf{x}-\mathbf{y})\delta(\mathbf{y}) \gtrsim \frac{1}{3}\left(= \frac{p}{\rho}\right)$$

➡ PBH!!

* Abundance : $\dfrac{\rho_{\rm PBH}}{\rho_{\rm tot}} = \displaystyle\int_{1/3}^{\infty} \frac{1}{\sqrt{2\pi\sigma_R^2}} \mathrm{e}^{-\delta_R^2/2\sigma_R^2}$

* Mass : $M_{\rm PBH} \sim M_H\big|_{R=H^{-1}} = \dfrac{4\pi}{3}\rho R^3\Big|_{R=H^{-1}}$

Compaction & Peak th.

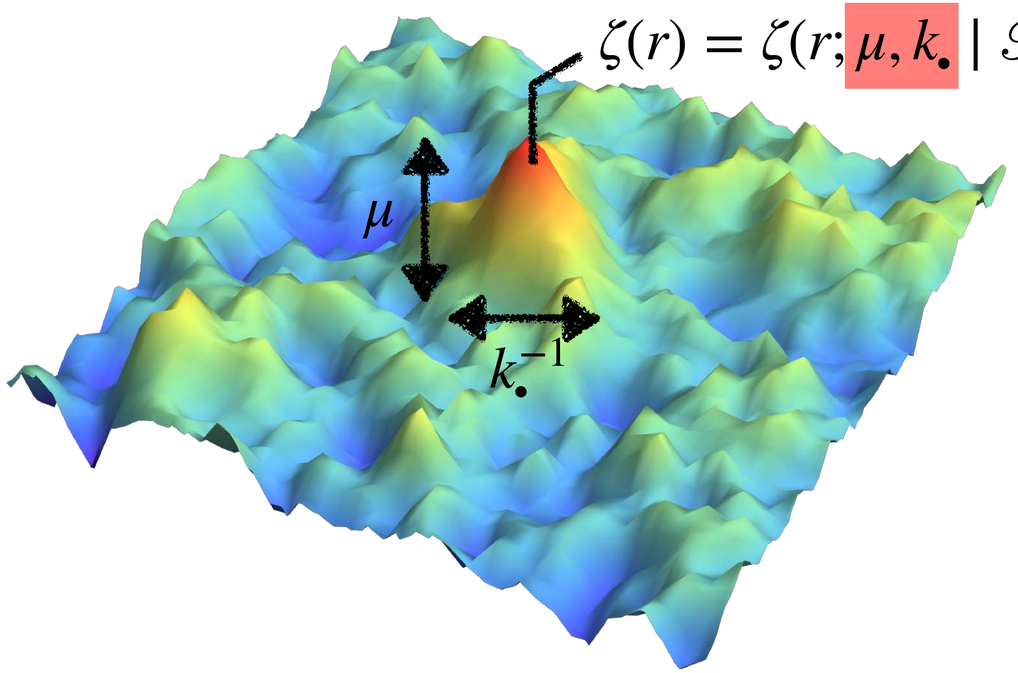
Yoo, Harada, Garriga, Kohri '18

$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}e^{-5\zeta/2}\Delta e^{\zeta/2}$$

if ζ is Gaussian (Bardeen, Bond, Kaiser, Szalay '86)

comb. Gaussian

$$\zeta(r) = \zeta(r; \mu, k_* \mid \mathcal{P}_\zeta)$$



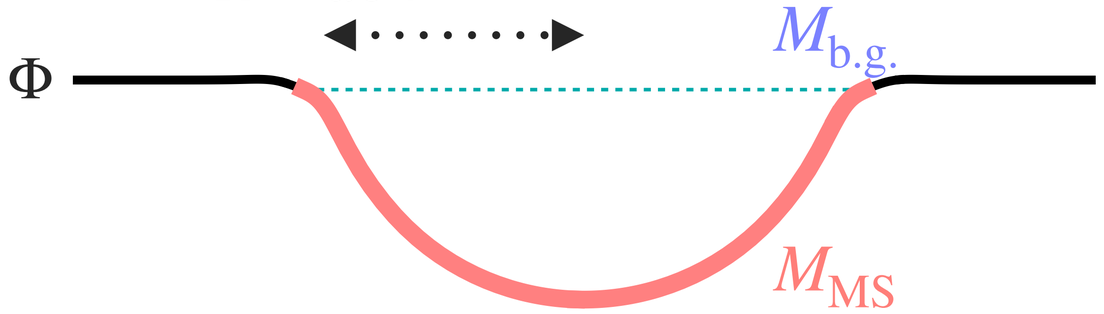
Compaction Function Shibata & Sasaki '99

$$\mathscr{C} = 2G\frac{M_{\rm MS} - M_{\rm b.g.}}{R}$$

$$= \frac{1}{V(R)}\int_0^R \delta \times 4\pi R^2 {\rm d}R\Big|_{R=H^{-1}}$$

$$= \frac{2}{3}\left[1 - (1 + r\zeta')^2\right]$$

$$R = ae^{\zeta}r$$



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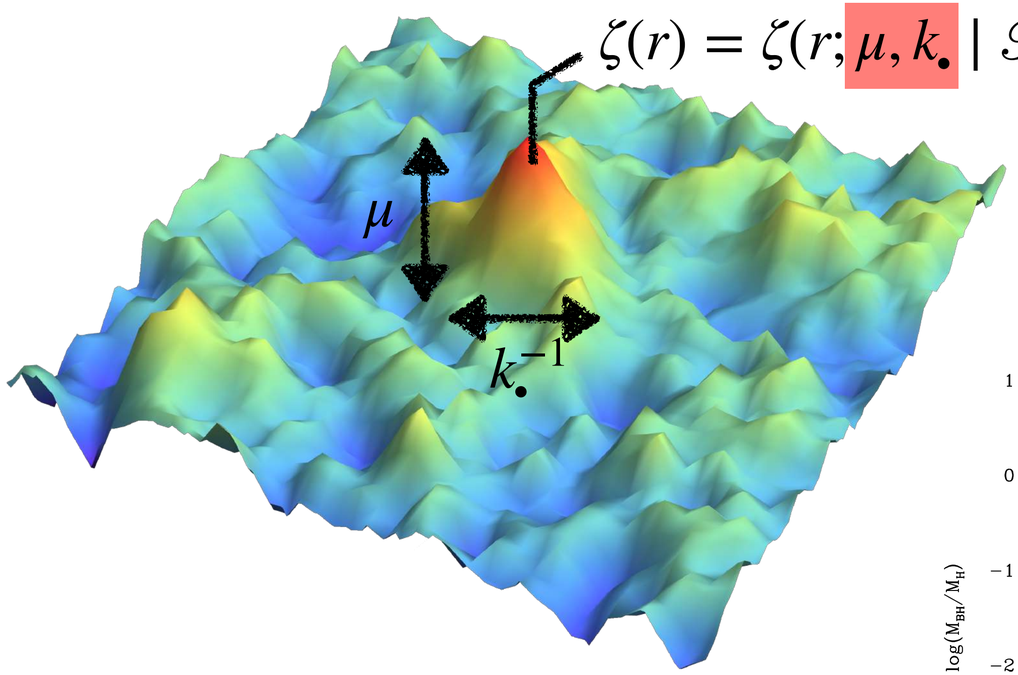


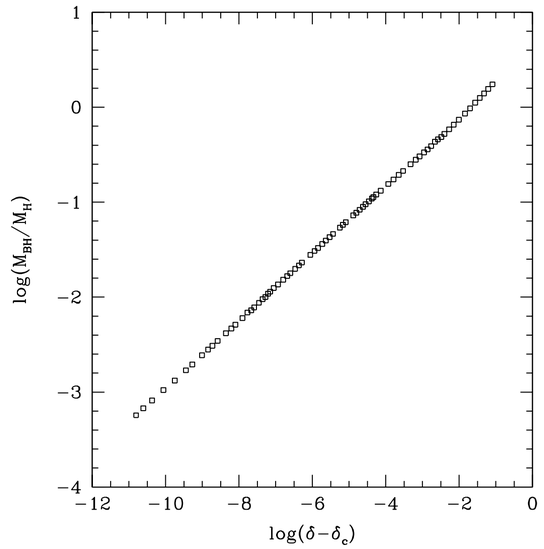
Compaction Function Shibata & Sasaki '99

$$\mathscr{C} = \frac{2}{3}\left[1 - (1 + r\zeta')^2\right]$$

Universal Criterion Atal, Cid, Escriva, Garriga, '19

$$\bar{\mathscr{C}} = \frac{1}{V(R)}\int_0^R \mathscr{C} \times 4\pi R^2 > \bar{\mathscr{C}}_{\rm th} \simeq \frac{2}{5}$$

cf. non-sphericity, Escrivà & Yoo '24

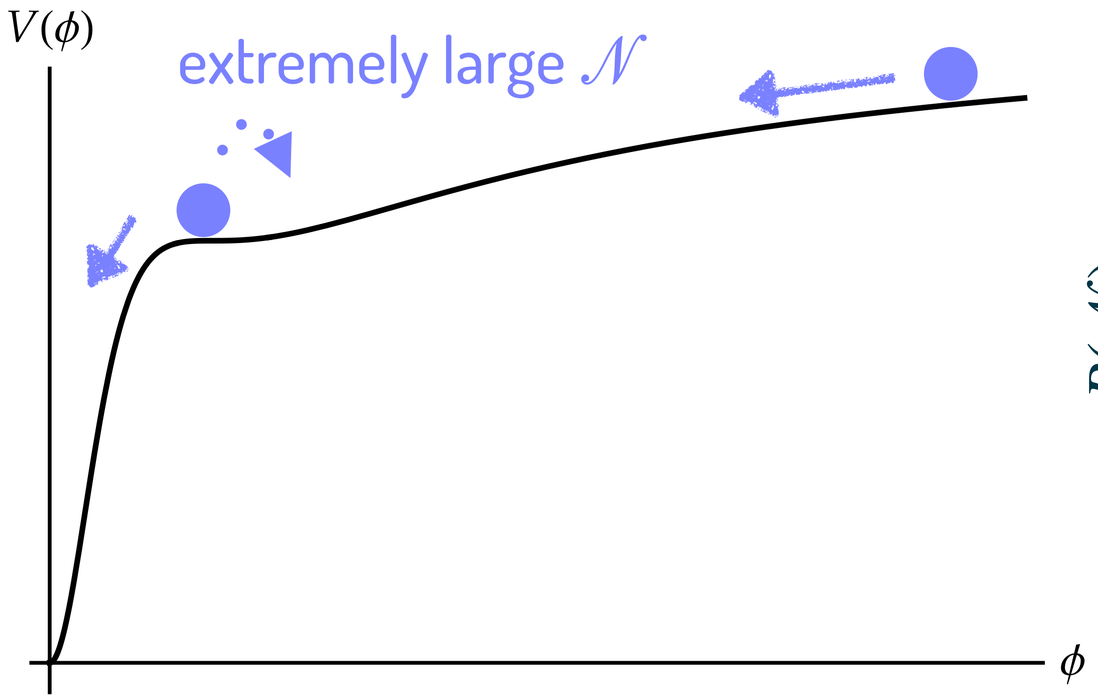


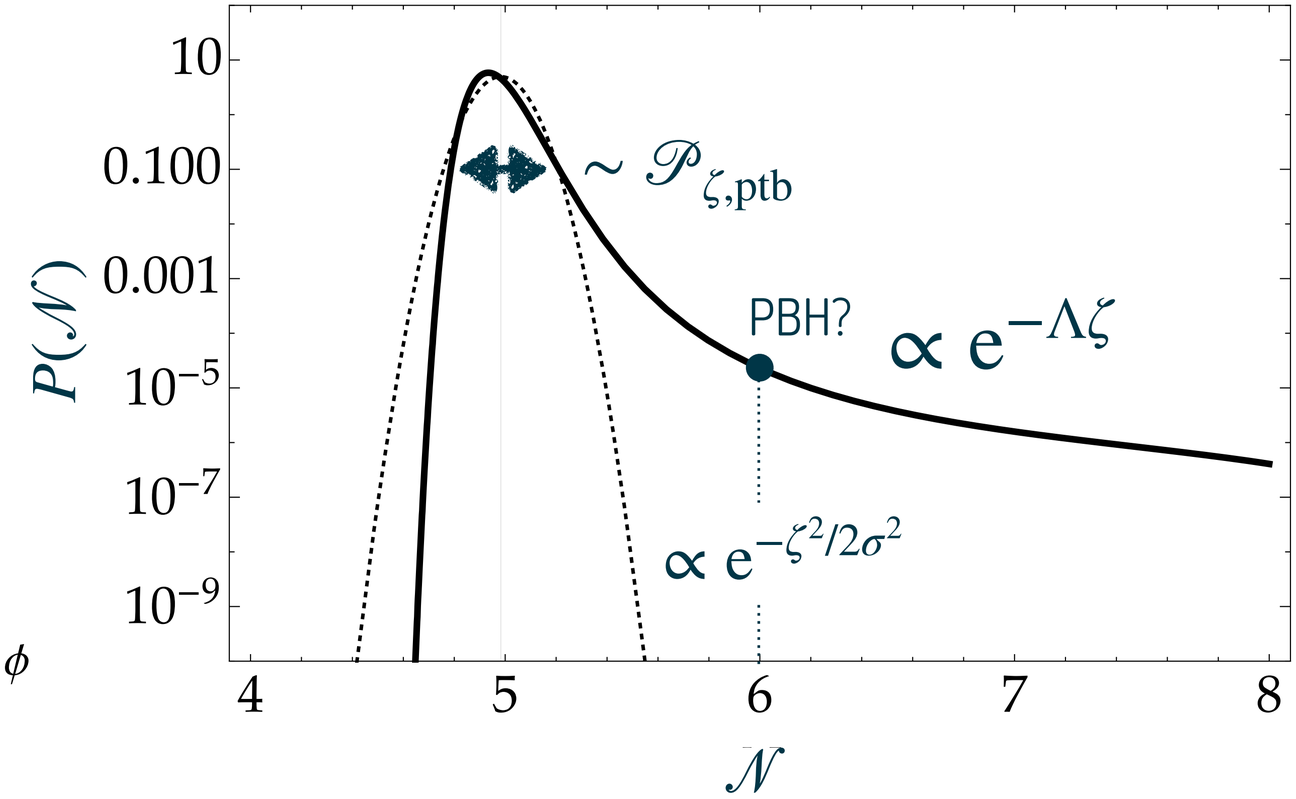
Mass Musco, Miller, Polnarev '08

$$M_{\rm PBH} \simeq \left(\mu - \mu_{\rm th}(k_*, \cdots)\right)^{0.36} M_H\Big|_{R=H^{-1}}$$

Exp-tail

Pattison+ '17, '21, Ezquiaga+ '20, Figueroa+'20





$$\zeta(\mathbf{x}) = \delta\mathcal{N}(\mathbf{x}) = -\frac{1}{\Lambda}\ln\left(1-\Lambda g(\mathbf{x})\right) \quad \left(= g(\mathbf{x}) + \frac{\Lambda}{2}g^2(\mathbf{x}) + \cdots\right)$$

Lattice of Inflation

Standard Lattice Simulation

Original: Caravano, Komatsu, Lozanov, Weller '21
EM in axion: Caravano, Komatsu, Lozanov, Weller '22
Butterfly effect: Caravano, Inomata, Renaux-Petel '24
USR: Caravano, Franciolini, Renaux-Petel '24

✓ Non-PTB on sub-Hubble ElectroMagnetic field

✗ No metric-ptb.
Not directly ζ

STOchastic LAttice Simulation

Mizuguchi, Murata, YT '24

= EFT of superH fields

= Local FLRW + Correlated Brownian motion

Starobinsky '86, +Yokoyama '94

✓ Non-PTB on superH
Directly ζ via $\delta\mathcal{N}$

✗ PTB on subH

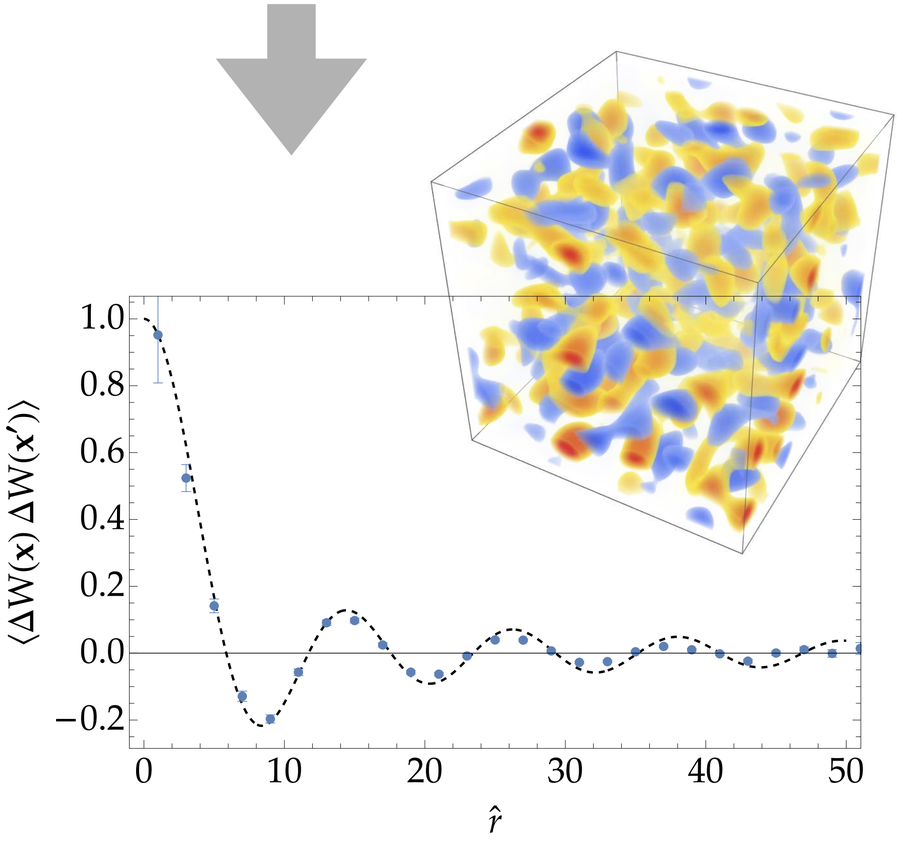
EoM & Noise

Mizuguchi, Murata, YT '24

$$\approx \frac{H(N,\mathbf{x})}{2\pi}$$

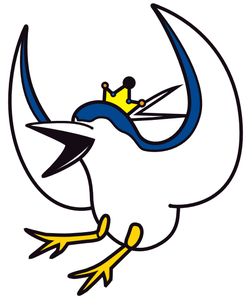
$$\begin{cases} \Delta\phi(N,\mathbf{x}) = \dfrac{\pi(N,\mathbf{x})}{H(N,\mathbf{x})}\Delta N + \sqrt{\mathscr{P}_\phi(N,\mathbf{x})}\,\Delta W(N,\mathbf{x}), \\ \Delta\pi(N,\mathbf{x}) = \left(-3\pi(N,\mathbf{x}) - \dfrac{V'(\phi(N,\mathbf{x}))}{H(N,\mathbf{x})}\right)\Delta N, \\ 3M_{\rm Pl}^2 H^2(N,\mathbf{x}) = \dfrac{1}{2}\pi^2(N,\mathbf{x}) + V(\phi(N,\mathbf{x})), \\ \langle \Delta W(N,\mathbf{x})\Delta W(N',\mathbf{y})\rangle = \dfrac{\sin k_\sigma(N)\,|\mathbf{x}-\mathbf{y}|}{k_\sigma(N)\,|\mathbf{x}-\mathbf{y}|}\delta_{NN'}\Delta N \end{cases}$$

$$\langle \Delta W_{\mathbf{k}} \Delta W^*_{\mathbf{k}'}\rangle \propto \delta_{\mathbf{k}\mathbf{k}'}\,\delta(k-k_\sigma)$$



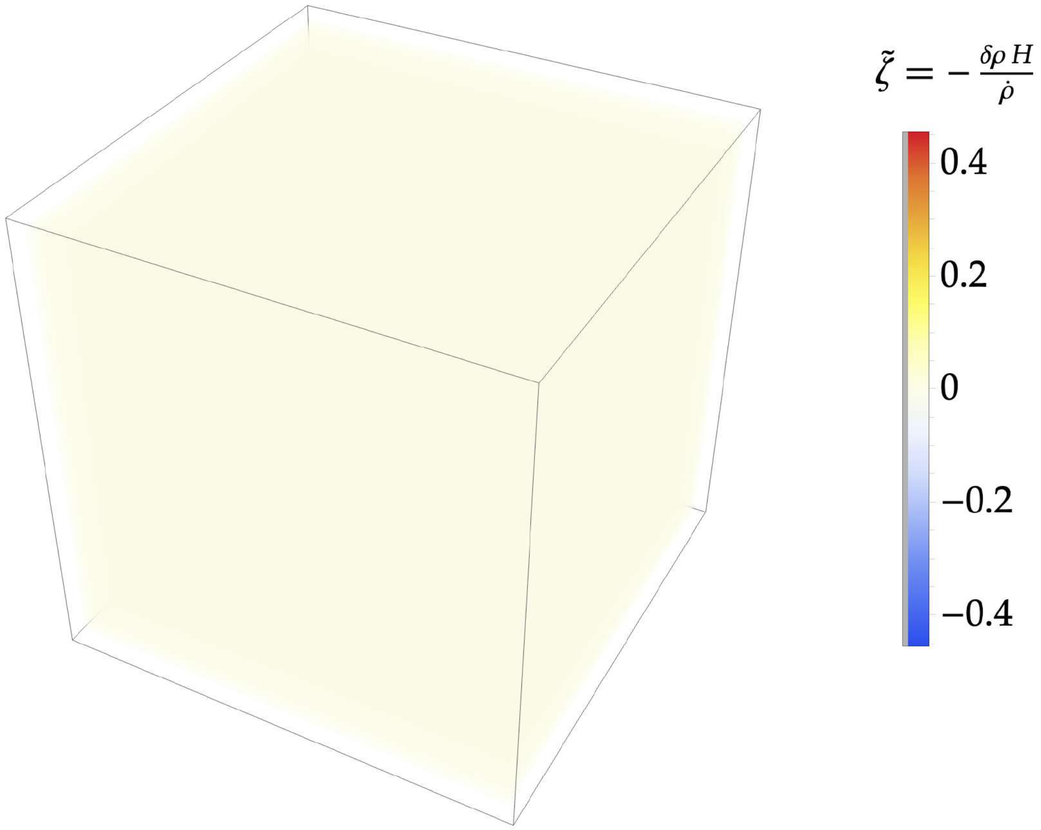
STOLAS

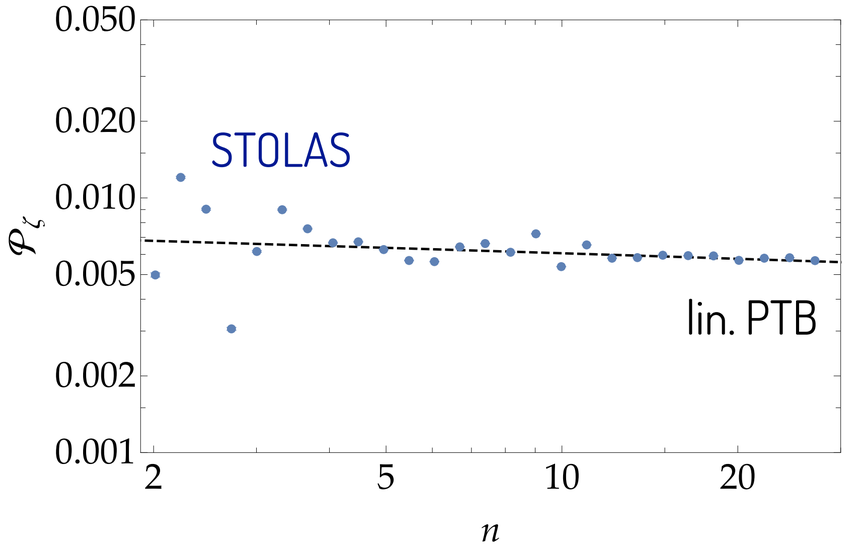
Mizuguchi, Murata, YT '24

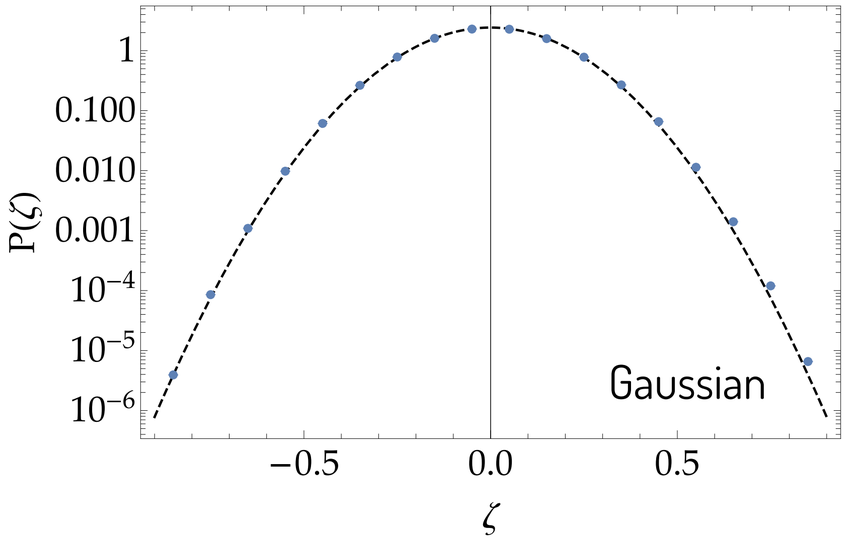


Ex. 1: Chaotic $V = \frac{1}{2}m^2\phi^2$

$N = 0.1$

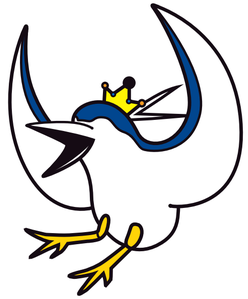






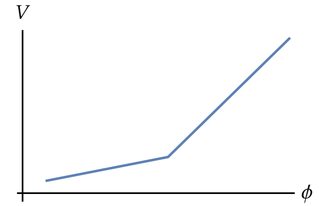
STOLAS

Mizuguchi, Murata, YT '24

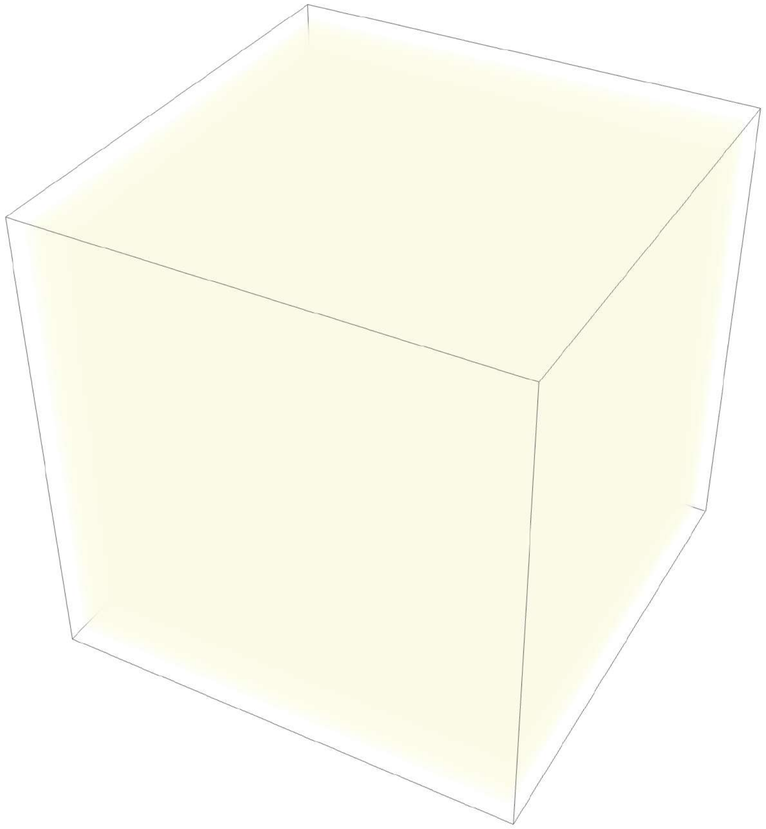


Ex. 2: Starobinsky's linear

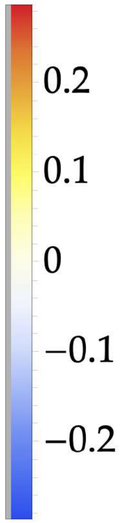
Starobinsky '92

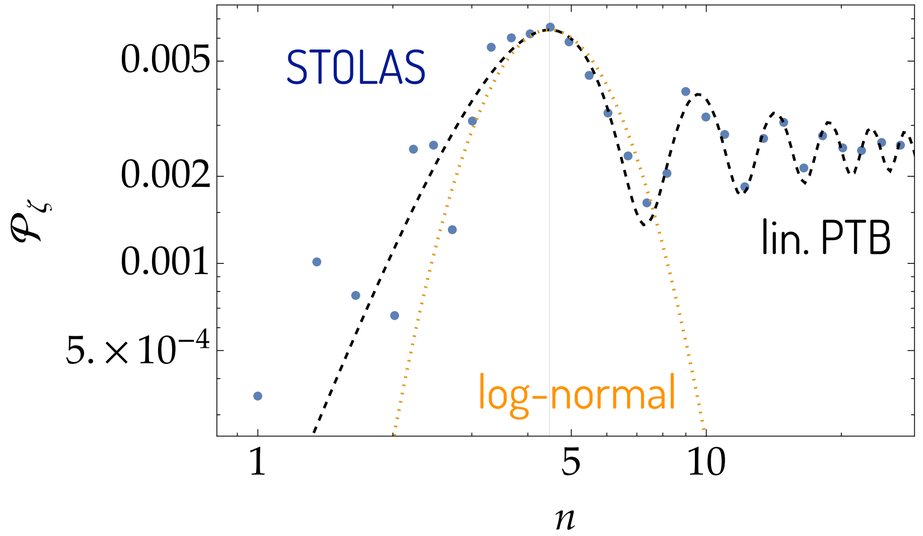


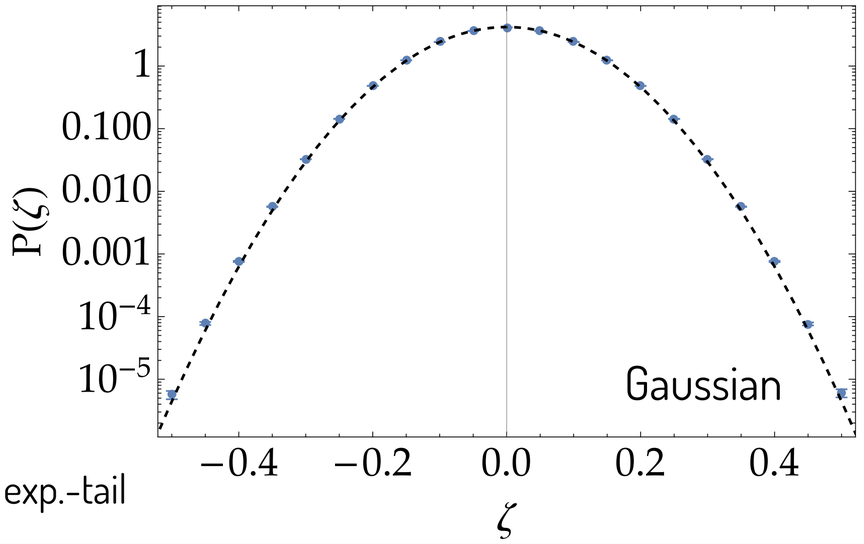
$N = 0.1$



$\tilde{\zeta} = -\frac{\delta\rho\, H}{\dot{\rho}}$

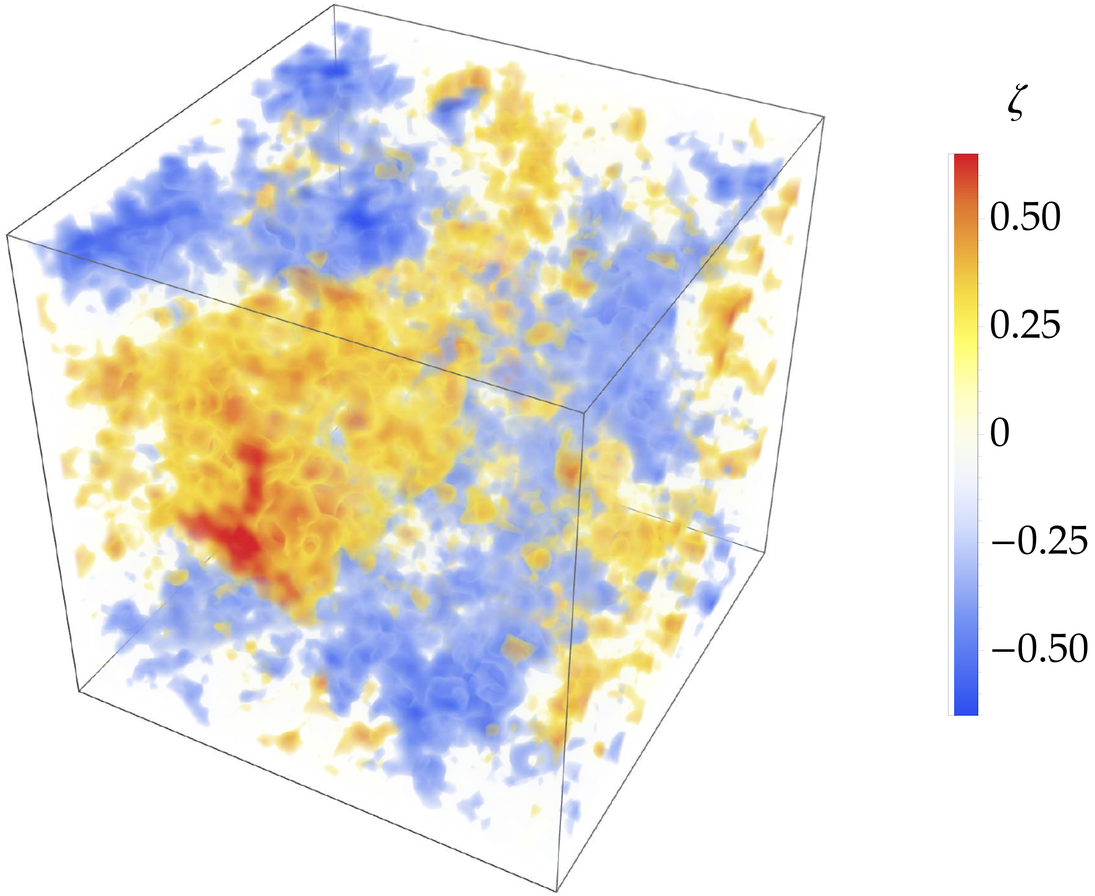




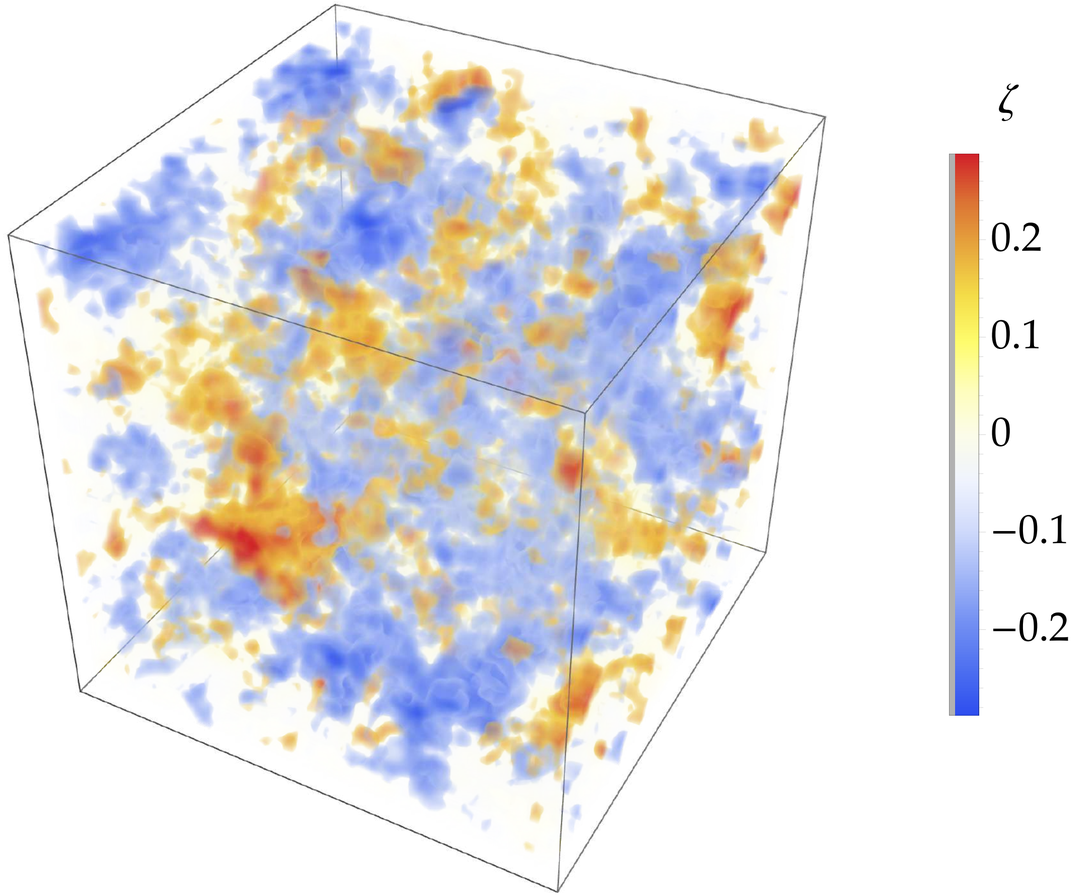


✱ sharp end of USR needed for exp.-tail

Ex. 1 : Chaotic



Ex. 2 : Starobinsky's linear

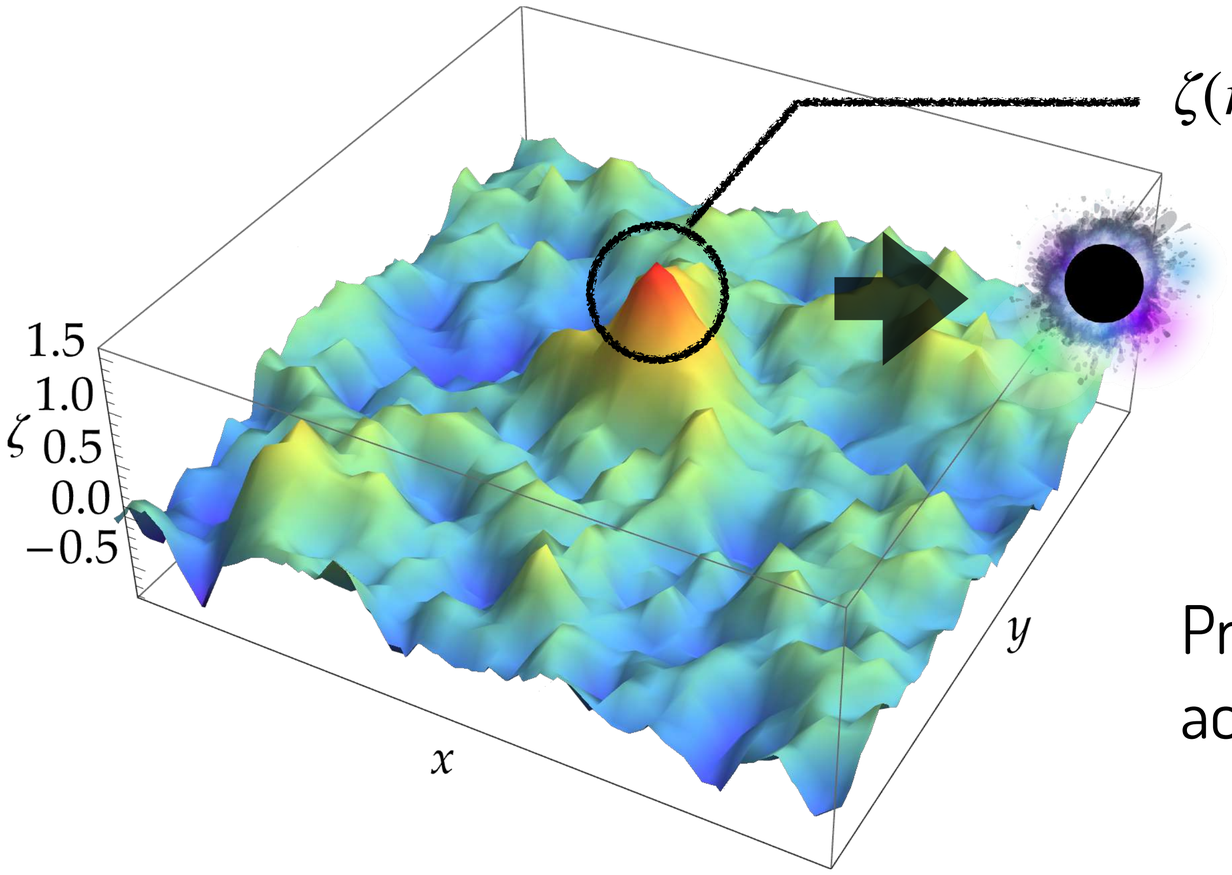


Importance Sampling

see, e.g., Jackson+ '22

Intentionally large noise $\propto \mathrm{sinc}\left(k_\sigma(N_{\rm PBH})r\right)$

Ex. 2 : Starobinsky's linear

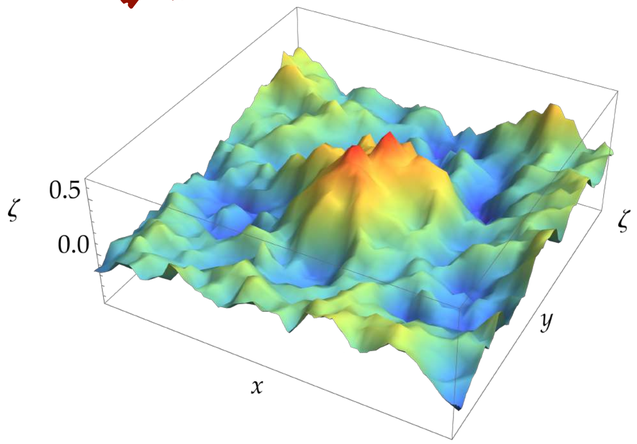


$$\zeta(r) \to \mathscr{C}(r) \to \bar{\mathscr{C}}_{\rm m} = 0.56 > \bar{\mathscr{C}}_{\rm th} = \frac{2}{5}$$

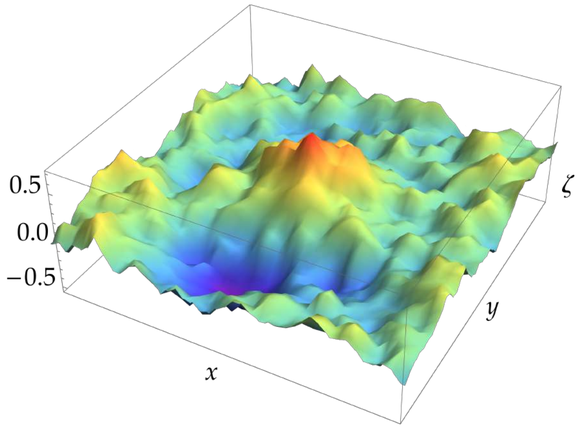
Probability is re-weighted
according to the probability of large noise!!

$M = 1.58 \times 10^{20}\,\mathrm{g}$

✗ $\bar{\mathscr{C}}_\mathrm{m} = 0.387$

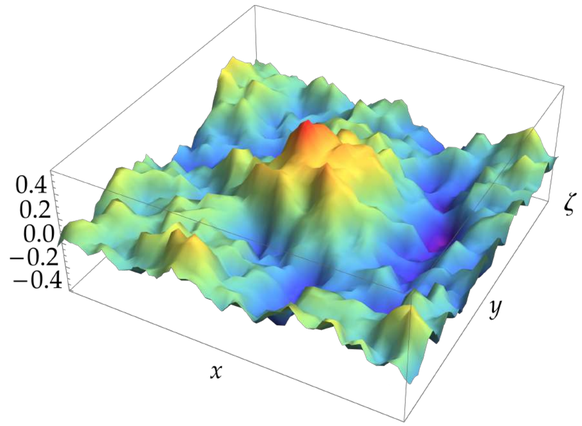


✓ $\bar{\mathscr{C}}_\mathrm{m} = 0.442$

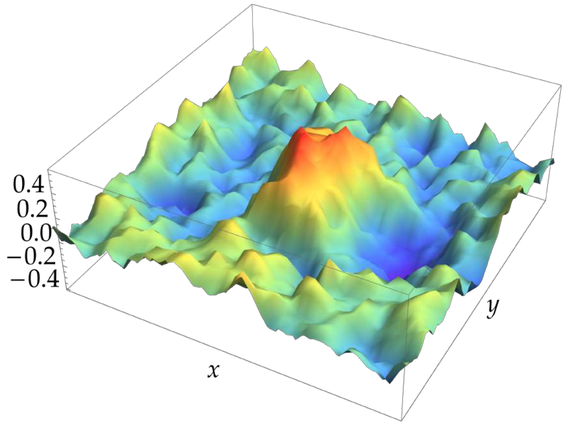


$M = 0.846 \times 10^{20}\,\mathrm{g}$

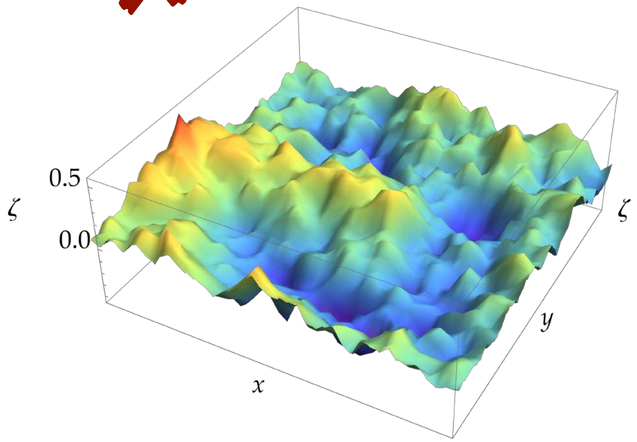
✗ $\bar{\mathscr{C}}_\mathrm{m} = 0.372$



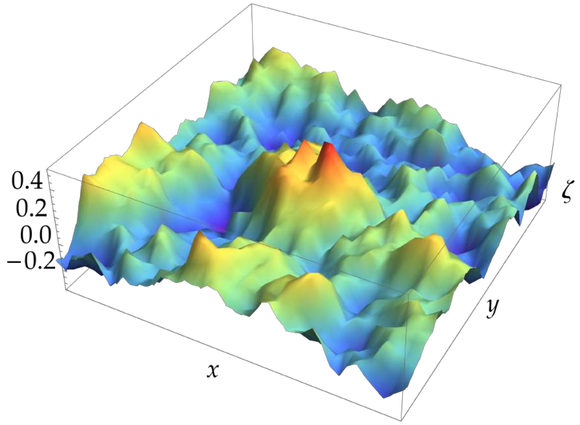
✓ $\bar{\mathscr{C}}_\mathrm{m} = 0.409$



✗ $\bar{\mathscr{C}}_\mathrm{m} = 0.268$

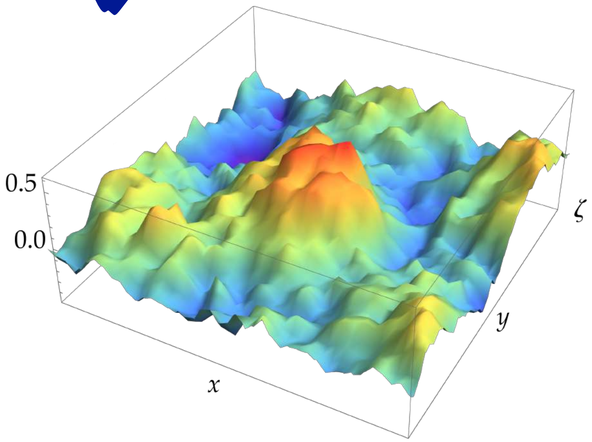


✗ $\bar{\mathscr{C}}_\mathrm{m} = 0.310$

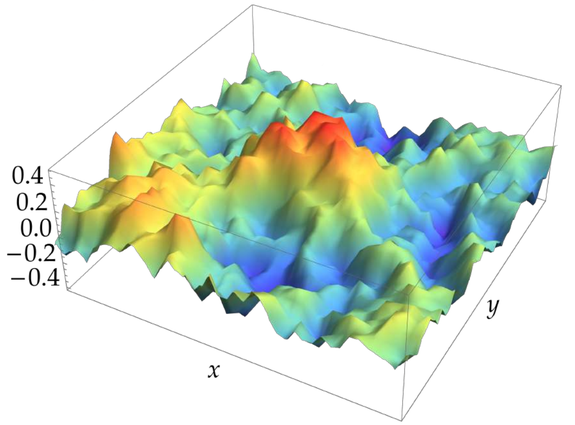


$M = 1.21 \times 10^{20}\,\mathrm{g}$

✓ $\bar{\mathscr{C}}_\mathrm{m} = 0.416$



✗ $\bar{\mathscr{C}}_\mathrm{m} = 0.383$



Importance Sampling

Mizuguchi, Murata, YT '24

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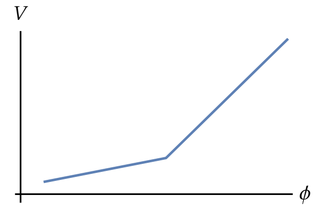
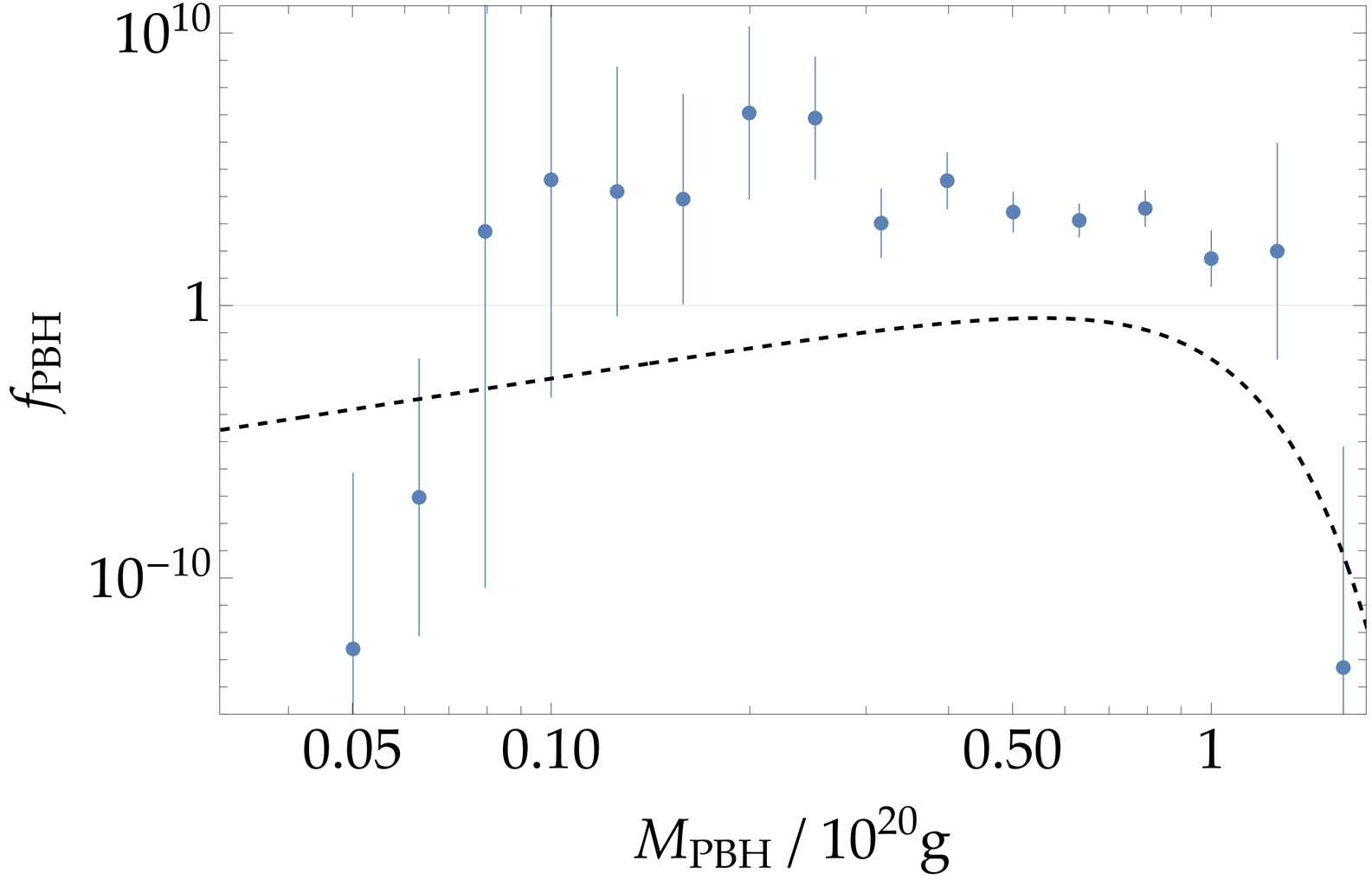
Starobinsky '92





Peak th w/
Gaussian & $\mathcal{P}_\zeta \propto \delta(\ln k - \ln k_*)$

Summary

- STOLAS can directly sample the PBH abundance
- High resolution in FUGAKU superC
- EM? GW? ...
- Initial Condition for Late-U. cosmological simulation?