Extending Starobinsky's formalism to general relativity

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From Inflation to Structure Formation (Nov 6-8, 2024), November 7, 2024

Based on arXiv: 2405.01024, 2409.12003

with Nick Tsamis & R. P. Woodard

Starobinsky's Formalism ➔ The Proof (gr-qc/0505115)

- Exact field equation for scalar potential model on de Sitter
  - $\mathcal{L} = -\frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi g^{\mu\nu}\sqrt{-g} - V(\Phi)\sqrt{-g}$ ➔ $\partial_\mu\left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu\Phi\right] = V'(\Phi)\sqrt{-g}$
- Yang-Feldman equation
  - $\Phi(t,\vec{x}) = \Phi_0(t,\vec{x}) - \int d^4x' \sqrt{-g(t',\vec{x}')}\, i\theta(t-t')[\Phi_0(t,\vec{x}),\Phi_0(t',\vec{x}')]\, V'(\Phi(t,\vec{x}))$
  - $\Phi_0(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left\{u(t,k)e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + u^*(t,k)e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$
  - $u(t,k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right] = \frac{H}{\sqrt{2k^3}}\left[1 + \frac{1}{2}\left(\frac{k}{Ha}\right)^2 + \frac{i}{3}\left(\frac{k}{Ha}\right)^3 + \cdots\right]$

Starobinsky's Formalism ➔ The Proof (gr-qc/0505115)

- IR truncation changes everything but preserves leading logarithms
  - Every pair of Φ_0's must contribute an IR log for leading logarithm order ➔ can IR truncate Φ_0 to φ_0
    - $\varphi_0(t,\vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3}\theta(k-H)\theta(Ha-k)\left\{\frac{H}{\sqrt{2k^3}}e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + \frac{H}{\sqrt{2k^3}}e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$

    (1st imaginary for commutator)
  - $\varphi(t,\vec{x}) = \varphi_0(t,\vec{x}) - \int d^4x' \, \frac{\theta(t-t')}{3H} \, \delta^3(\vec{x}-\vec{x}') \, V'\big(\varphi(t',\vec{x}')\big) = \varphi_0(t,\vec{x}) - \frac{1}{3H}\int_0^t dt' \;\; V'(\varphi(t',\vec{x}))$
    - $\varphi_0(t,\vec{x}) \neq \Phi_0(t,\vec{x})$ ➔ no UV divergences, φ_0 & $\dot{\varphi}_0$ commute, but their correlators agree at leading log order
- Taking the time derivative gives Starobinsky's Langevin equation!
  - $\dot{\varphi} = \dot{\varphi}_0 - \frac{1}{3H}V'(\varphi)$ ➔ $3H(\dot{\varphi} - \dot{\varphi}_0) = -V'(\varphi)$
- Importance of every pair of free fields contributing an IR log to reach leading log order
  - This fails for fields other than MMC scalars and undifferentiated gravitons
  - Doesn't work for photons, fermions, MCC scalars
  - Even fails when some MMC scalars and gravitons are differentiated (cf. $\sqrt{16\pi G} \times h\partial h\partial h$)

Explicit Computations Show Discrepancies beyond Scalar Potential Models

- Order 1 contributions come from both UV & IR
  - Exact dim. reg. gives $\langle \partial_\mu \phi(x) \partial_\nu \phi(x) \rangle = -g_{\mu\nu} \times \frac{H^D}{2(4\pi)^{D/2}} \frac{\Gamma(D)}{\Gamma\left(\frac{D}{2}+1\right)}$
  - Any purely IR stochastic result must be positive for $\mu = \nu$
- Simple rule for ``GR + Fermions'' deviates from Starobinsky's (gr-qc/0802.2377)
- Renormalization matters
  - Primitive $\left(\frac{(2H)^{D-4}}{D-4}\right)$ – Counterterm $\left(\frac{(\mu a)^{D-4}}{D-4}\right) = -\ln\left(\frac{\mu a}{2H}\right) + O(D-4)$
  - No stochastic formalism will recover these logs, but RG was designed to do it
- Crucial to stay focused on large logarithms
  - Always check formalism against explicit computations

Distinguish between "Active" & "Passive" Fields

- Undifferentiated Actives can cause IR logs, Passives cannot
  - MMC scalars & gravitons [$h_{\mu\nu}$ in $g_{\mu\nu} \equiv a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$] are Active
  - MCC scalars, fermions & photons are Passive
- Integrate out Passives & differentiated Actives for constant Active
  - Induces scalar potential model for Actives ➔ use Starobinsky formalism
- Constant Actives induce effective potentials three ways (at least):
  - Through masses, Yukawa: $-f\phi\bar{\Psi}\Psi\sqrt{-g}$
  - Through field strengths, Nonlinear Sigma: $-f(A)^2\partial_\mu B\partial_\nu B g^{\mu\nu}\sqrt{-g}$
  - Through the Hubble constant, gravity: constant $h_{\mu\nu}$ corresponds to de Sitter different H ➔ just change this parameter in the propagators

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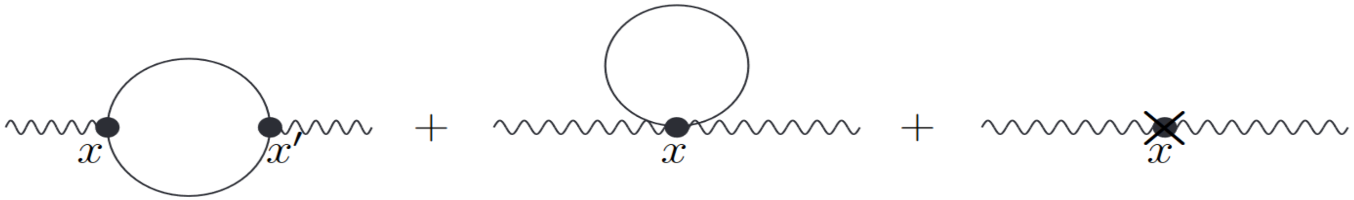
Double Field Model/ Single Field Model VS. Matter loops to GR/ pure GR

- Same derivative interactions as gravity
  - And same sorts of large logarithms but no indices and no gauge fixing issues
- Double Field Model
  - $\mathcal{L} = -\frac{1}{2}\partial_\mu A \partial_\nu A g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\left(1 + \frac{\lambda}{2}A\right)^2 \partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g}$
- MMCS Corrections to Gravity (arXiv:2405.00116)
- $-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')$



- Single Field Model
  - $\mathcal{L} = -\frac{1}{2}(1 + \frac{1}{2}\lambda\Phi)^2 \partial_\mu \Phi \partial_\nu \Phi g^{\mu\nu}\sqrt{-g}$ ➔ $\Phi[\Psi] = \frac{2}{\lambda}\left[\sqrt{1+\lambda\Psi} - 1\right]$ for Ψ free
  - Unit S-matrix but interactions affect background and particle kinematics
- Pure Gravity (no dimensionally regulated & fully renormalized result yet)

Explicit computation : MMCS Loops to GR

- Effective field equation for linearized gravity ($\kappa^2 = 16\pi G$)
  - $\mathcal{L}^{\mu\nu\rho\sigma}\kappa h_{\rho\sigma}(x) - \int d^4x' \, [^{\mu\nu}\Sigma^{\rho\sigma}](x;x')\kappa h_{\rho\sigma}(x') = 8\pi G T^{\mu\nu}(x)$
  - Used Schwinger-Keldysh formalism for real & causal field equations
- Gravitational radiation
  - $C_{0i0j}(t,\vec{x}) = C^{(0)}_{0i0j}(t,\vec{x}) \left\{1 - \frac{3\kappa^2 H^2}{160\pi^2} \times \ln[a(t)] + O(\kappa^4)\right\}$
- Response to $T^{\mu\nu} = -\delta^\mu_0 \delta^\nu_0 M a \delta^3(\vec{x})$
  - $ds^2 = -[1 - 2\Psi(t,r)]dt^2 + a^2(t)[1 - 2\Phi(t,r)]d\vec{x} \cdot d\vec{x}$
  - $\Psi(t,r) = \frac{GM}{ar}\left\{1 + \frac{\kappa^2}{320\pi^2 a^2 r^2} - \frac{3\kappa^2 H^2}{160\pi^2} \times \ln[aHr] + O(\kappa^4)\right\}$
  - $\Phi(t,r) = -\frac{GM}{ar}\left\{1 - \frac{\kappa^2}{960\pi^2 a^2 r^2} - \frac{3\kappa^2 H^2}{160\pi^2}(\ln[aHr] + 1) + O(\kappa^4)\right\}$
- NB the negative sign
  - Inflationary creation of scalars sucks energy from gravity

What We Did (approximation technique)in arXiv:2405.01024

- Integrated Scalars out of Einstein Equation at leading logarithm order
  - $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{1}{2} g_{\mu\nu} \Lambda = 8\pi G\{\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi\}$
  - Extended to a fully conserved form
- Explained 1-loop leading log results using variant of Renormlization Group
- Re-summed leading logarithm results to all orders
  - $C_{0i0j}(t, \vec{x}) \to C^{(0)}_{0i0j}(t, \vec{x}) \times [a(t)]^{-\frac{3GH^2}{10\pi}}$
  - $\Psi(t, r) \to \frac{GM}{a(t)r} \times [a(t)Hr]^{-\frac{3GH^2}{10\pi}}$
  - $\Phi(t, r) \to -\frac{GM}{a(t)r} \times [a(t)Hr]^{-\frac{3GH^2}{10\pi}}$

$$G_N(x_1, \ldots, x_N; \lambda; \mu) = G_N(x_1, \ldots, x_N; \lambda; \mu_0) \times \left[\frac{\mu_0}{\mu}\right]^{N\gamma(\lambda)}$$

Integrating out differentiated fields

- Logs come from undifferentiated $h_{\mu\nu}$ ➔ constant $\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$
  - But $g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu}$ with constant $\tilde{g}_{\mu\nu}$ is de Sitter with $H^2 \to -\tilde{g}^{00} H^2$!

  $\Gamma^{\rho}_{\mu\nu} = aH\left(\delta^{\rho}_{\mu}\delta^{0}_{\nu} + \delta^{\rho}_{\nu}\delta^{0}_{\mu} - \tilde{g}^{0\rho}\tilde{g}_{\mu\nu}\right)$ ➔ $R^{\rho}_{\sigma\mu\nu} = -\tilde{g}^{00} H^2\left(\delta^{\rho}_{\mu} g_{\sigma\nu} - \delta^{\rho}_{\nu} g_{\sigma\mu}\right)$

- Integrate out $\partial\phi\partial\phi$ with $\partial_\mu \partial'_\nu i\Delta(x;x')_{x'=x} = -\frac{3H^4}{32\pi^2} \times g_{\mu\nu}$
  - E.g. $T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \partial_\rho \varphi \partial_\sigma \varphi \;\; \rightarrow \;\; \frac{3}{32\pi^2}\left[-\tilde{g}^{00} H^2\right]^2 g_{\mu\nu}$
  - NB a negative contribution to the cosmological constant & arbitrarily large
  - Explains the finite renormalization of Λ but none of the large logarithms
- Induced stress tensor only conserved at leading log order ➔ extend
  - $\Delta\mathcal{L} = \frac{R^2 \ln(R)\sqrt{-g}}{2^8 \cdot 3 \cdot \pi^2}$ gives fully conserved $T_{\mu\nu}$
  - Agrees with induced $T_{\mu\nu}$ for constant $\tilde{g}_{\mu\nu}$
- Induced $T_{\mu\nu}$ for QG more complicated
  - But can reconstruct using solutions (explicit computations) for potentials (if not RG effects)

Curvature-Dependent Field Strength Renormalization

- 1-Loop C-terms: $\Delta\mathcal{L} = c_1 R^2\sqrt{-g} + c_2 C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}\sqrt{-g}$
  - $c_1 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{D/2}}\frac{(D-2)}{(D-1)^2(D-3)(D-4)}$ $\quad c_2 = \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{2^8\pi^{D/2}}\frac{2}{(D+1)(D-1)(D-3)^2(D-4)}$
  - R^2 induces a Λ-dependent renormalization of graviton field strength

$$R^2 = [R - D\Lambda]^2 + 2D\Lambda[R - (D-2)\Lambda] + D(D-4)\Lambda^2$$

  - C^2 does also from ∂_0^2 (which surprised us!)

$$\delta Z = D[2(D-1)c_1 - c_2]\kappa^2 H^2 \quad \rightarrow \quad \gamma \equiv \frac{\partial\ln(1+\delta Z)}{\partial\ln(\mu^2)} = \frac{\kappa^2 H^2}{320\pi^2} = \frac{3GH^2}{20\pi}$$

- Callan-Symanzik Equation explains all three leading logs
  - $\left[\frac{\partial}{\partial\ln(\mu)} + \beta_G\frac{\partial}{\partial G} + 2\gamma\right]G^{(2)}(x;x') = 0$ $\quad (-\frac{3\kappa^2H^2}{160\pi^2}\times\ln[a(t)])$
  - $\beta_G = 0$ (at this order) and factors of $\ln(\mu)$ are really $\ln\left(\frac{\mu a}{2H}\right)$

$$G_N(x_1,\ldots,x_N;\lambda;\mu) = G_N(x_1,\ldots,x_N;\lambda;\mu_0)\times\left[\frac{\mu_0}{\mu}\right]^{N\gamma(\lambda)}$$

A variant of RG group

- Callan-Symanzik Equation
  - $\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\lambda}+2\gamma_B\right]P_B(t,r)=0$ and $P_B(t,r)\to\frac{KH}{4\pi}\ln(Hr)+O(\lambda^2)$
  - $\mu\to\frac{1}{r}$ ➔ $P_B(t,r)\to\frac{KH}{4\pi}\ln(Hr)\left\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(Hr)+O(\lambda^4)\right\}$
- $\mu\frac{\partial}{\partial\mu}$ ➔ $\frac{1}{r}\frac{\partial}{\partial\frac{1}{r}}=\frac{1}{r}\frac{\partial r}{\partial\frac{1}{r}}\frac{\partial}{\partial r}=\frac{1}{r}(-r^2)\frac{\partial}{\partial r}=-r\frac{\partial}{\partial r}$
- $\beta\sim O(\lambda^3)$
- $-r\frac{\partial}{\partial r}P_B(t,r)=-r\frac{\partial}{\partial r}\left[\frac{KH}{4\pi}\ln(Hr)\left\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(Hr)+O(\lambda^4)\right\}\right]$

  ➔ $-\mathrm{r}\times\frac{1}{r}\frac{KH}{4\pi}\ln(Hr)\times\left(-\frac{\lambda^2H^2}{16\pi^2}\right)$
- $2\gamma_B\times P_B(t,r)=2\times\left(-\frac{\lambda^2H^2}{32\pi^2}\right)\times\left[\frac{KH}{4\pi}\ln(Hr)\left\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(Hr)+O(\lambda^4)\right\}\right]$

  $=\left(-\frac{\lambda^2H^2}{16\pi^2}\right)\times\frac{KH}{4\pi}\ln(Hr)$

Differences between Matter + GR and Pure GR

- The same $h_{\mu\nu}$ provide the Langevin kinetic term $\dot{h}_{\mu\nu} - \dot{\chi}_{\mu\nu}$ and the induced potential out of $\partial h_{\mu\nu}$
  - Strange but true: check against the explicit computation in single field model
  - $\left(1+\frac{\lambda}{2}\Phi\right)\partial_\mu\left[\left(1+\frac{\lambda}{2}\Phi\right)\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi\right] \rightarrow -3Ha^3\left(1+\frac{\lambda}{2}\varphi_0\right)^2(\dot{\varphi}-\dot{\varphi}_0)+\left(1+\frac{\lambda}{2}\varphi_0\right)\partial_\mu\left[\frac{\lambda}{4}\sqrt{-g}\,g^{\mu\nu}\partial_\nu\langle\Omega|\Phi^2|\Omega\rangle\right]$
- Background changes $\eta_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu}$ ➔ gauge fixing changes
  - Old: $\mathcal{L}_{GF} = -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu$, $F_\mu = \eta^{\rho\sigma}\left[h_{\mu\rho,\sigma} - \frac{1}{2}h_{\rho\sigma,\mu} + (D-2)aHh_{\mu\rho}\delta^0_\sigma\right]$
  - New: $\tilde{\mathcal{L}}_{GF} = -\frac{1}{2}a^{D-2}\tilde{g}^{\mu\nu}\tilde{F}_\mu\tilde{F}_\nu$, $\tilde{F}_\mu = \tilde{g}^{\rho\sigma}\left[h_{\mu\rho,\sigma} - \frac{1}{2}h_{\rho\sigma,\mu} + (D-2)aHh_{\mu\rho}\delta^0_\sigma\right]$
- Ghost contributions

Graviton propagator in $\tilde{g}_{\mu\nu}$ background

- $\tilde{g}_{\mu\nu} = \begin{pmatrix} -N^2 + \gamma_{k\ell}N^k N^\ell & -\gamma_{j\ell}N^\ell \\ -\gamma_{ik}N^k & \gamma_{ij} \end{pmatrix} = \begin{pmatrix} \gamma_{k\ell}N^k N^\ell & -\gamma_{j\ell}N^\ell \\ -\gamma_{ik}N^k & \gamma_{ij} \end{pmatrix} - \begin{pmatrix} -N \\ 0 \end{pmatrix}_\mu \begin{pmatrix} -N \\ 0 \end{pmatrix}_\nu \equiv \bar{\gamma}_{\mu\nu} - u_\mu u_\nu$
- $\tilde{g}^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & -\frac{N^j}{N^2} \\ -\frac{N^i}{N^2} & \gamma^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \gamma^{ij} \end{pmatrix} - \begin{pmatrix} \frac{1}{N} \\ \frac{N^i}{N} \end{pmatrix}^\mu \begin{pmatrix} \frac{1}{N} \\ \frac{N^j}{N} \end{pmatrix}^\nu \equiv \bar{\gamma}^{\mu\nu} - u^\mu \mu^\nu$
- $-N\delta^0_\mu = u^\mu$, $u^\mu = -N\tilde{g}^{0\mu}$, $\tilde{g}^{\mu\nu}u_\mu u_\nu = -1 = \tilde{g}_{\mu\nu}u^\mu u^\nu$, $\widetilde{H}^2 = \frac{H^2}{N^2} = -\tilde{g}^{00}H^2$
- New GR propagator takes the same expression but Replace $\bar{\eta}_{\alpha\beta}$ with $\bar{\gamma}_{\alpha\beta}$. $i\widetilde{\Delta}_A(x;x')$: replace H with $\widetilde{H}$

$$i\left[{}_{\alpha\beta}\widetilde{\Delta}_{\rho\sigma}\right](x;x') = \left[2\,\overline{\gamma}_{\alpha(\rho}\overline{\gamma}_{\sigma)\beta} - \frac{2}{D-3}\,\overline{\gamma}_{\alpha\beta}\overline{\gamma}_{\rho\sigma}\right] i\widetilde{\Delta}_A(x;x')$$
$$-4\,u_{(\alpha}\overline{\gamma}_{\beta)(\rho}u_{\sigma)}\; i\widetilde{\Delta}_B(x;x')$$
$$+\frac{2}{(D-3)(D-2)}\left[(D-3)u_\alpha u_\beta + \overline{\gamma}_{\alpha\beta}\right]\left[(D-3)u_\rho u_\sigma + \overline{\gamma}_{\rho\sigma}\right] i\widetilde{\Delta}_C(x;x')$$

Rules for the leading log eqn. & induced stress tensors

- $\frac{\delta S[h]}{\delta h_{\mu\nu}}_{LLOg} = a^4\sqrt{\tilde{g}}T[h]^{\mu\nu}|_{ind}$
- LHS (dropping unimportant terms):
- Temporal changes on the fields are much slower than the one on $a(t)$
  - E.g. $\frac{d}{dt}[a^3 h_{\mu\nu}]$ ➔ $3Ha^3 h_{\mu\nu}$ & drop $a^3\dot{h}_{\mu\nu}$
- Only keep the first time derivative on the field & subtract it with stochastic jitter
  - E.g. $\partial_\alpha\partial_\beta h_{\mu\nu} + \partial_\alpha h_{\mu\nu}$ ➔ $\delta^0_\alpha[\dot{h}_{\mu\nu} - \dot{\chi}_{\mu\nu}]$
- RHS (integrating out differentiated fields):

$$h_{\alpha\rho,\gamma} h_{\beta\sigma,\delta} \longrightarrow \partial_\gamma \partial'_\delta \, i\left[{}_{\alpha\rho}\widetilde{\Delta}_{\beta\sigma}\right](x;x')\Big|_{x=x'}$$

$$h_{\alpha\rho,\gamma} h_{\beta\sigma} \longrightarrow \partial_\gamma \, i\left[{}_{\alpha\rho}\widetilde{\Delta}_{\beta\sigma}\right](x;x')\Big|_{x=x'}$$

Example

$$\mathcal{L}_{inv} + \widetilde{\mathcal{L}}_{GF} \;\equiv\; \mathcal{L}_{1+2+3} + \mathcal{L}_{4+5} + \mathcal{L}_{6}$$

$$\begin{aligned}\mathcal{L}_{1+2+3} \;=\; & a^2\sqrt{-\widetilde{g}}\,\widetilde{g}^{\alpha\beta}\widetilde{g}^{\rho\sigma}\widetilde{g}^{\gamma\delta} \\ & \times\Big\{-\tfrac{1}{4}h_{\alpha\rho,\gamma}h_{\beta\sigma,\delta} + \tfrac{1}{8}h_{\alpha\beta,\gamma}h_{\rho\sigma,\delta} + a^2\widetilde{H}^2 h_{\gamma\rho}u_\sigma h_{\delta\alpha}u_\beta\Big\}\end{aligned}$$

$$\begin{aligned}& a^4\sqrt{-\widetilde{g}}\,\Big\{\tfrac{3}{2}\,\widetilde{g}^{00}H\Big[\widetilde{g}^{\rho\mu}\widetilde{g}^{\sigma\nu} - \tfrac{1}{2}\widetilde{g}^{\rho\sigma}\widetilde{g}^{\mu\nu}\Big]\big[\dot{h}_{\rho\sigma} - \dot{\chi}_{\rho\sigma}\big] + 2\widetilde{H}^2 u^{(\mu}\widetilde{g}^{\nu)(\alpha}u^{\beta)}h_{\alpha\beta} \\ & + \kappa\widetilde{H}^2\Big[\tfrac{1}{2}\,\widetilde{g}^{\mu\nu}u^{(\alpha}\widetilde{g}^{\beta)(\rho}u^{\sigma)} - 2u^{(\mu}\widetilde{g}^{\nu)(\alpha}\widetilde{g}^{\beta)(\rho}u^{\sigma)} - u^{(\alpha}\widetilde{g}^{\beta)(\mu}\widetilde{g}^{\nu)(\rho}u^{\sigma)}\Big]h_{\alpha\beta}h_{\rho\sigma}\Big\} \;=\; a^4\sqrt{-\widetilde{g}}\,\frac{\kappa\widetilde{H}^4}{8\pi^2}\Big[-\tfrac{1}{2}\,\widetilde{g}^{\mu\nu} + 6u^\mu u^\nu\Big]\end{aligned}$$

Some remarks

- Inflationary production of MMC scalars & gravitons induces large logs
  - Eventually overwhelm small couplings ➔ perturbation theory breaks down
  - Can change force laws & evolution of the backgrounds even at late times
- Leading Logarithm Resummation is accomplished by
  - A variant of Starobinsky formalism and a variant of RG

Pure GR

- R^2, C^2 counterterms are gauge dependent:
  - Need 1-loop computation in constant $\tilde{g}_{\mu\nu}$ gauge ➔ a variant of RG technique
- Check the induced-graviton stress tensor:
  - compute the 1-loop, 1-point function in $\tilde{g}_{\mu\nu}$ =const. for pure GR
- Check 1-loop stochastic RG predictions:
  - Compute 2-point function of graviton (dimensionally regulated and BPHZ renormalized)
  - Solve the linearized Einstein equation with graviton self-energy in a new gauge
- Reorganized the Langevin equation to distinguish h_{ij} from $h_{0\mu}$